An Integer Optimization Model for the Time Windows Periodic Vehicle Routing Problem with Delivery, Fleet, and Driver Scheduling

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Abstract—The vehicle routing problem (VRP) is a well-known combinatorial optimization problem which describes a homogeneous set of vehicles and routes, in which each vehicle starts from a depot and traverses along a route in order to serve a set of customers with known geographical locations. If the delivery routes are constructed for period of time, the VRP is generalized as periodic VRP (PVRP). This paper develops a model for the optimal management of periodic deliveries of meals of a catering company. The PVRP incorporates time windows, deliveries, fleet and driver scheduling in the periodic planning.. The objective is to minimize the sum of the costs of travelling and elapsed time over the planning horizon. We model the problem as a linear mixed integer program and we propose a feasible neighbourhood direct search approach to solve the problem.

Keywords-Catering problem, modeling, delivery scheduling, elapsed time, direct search

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I. INTRODUCTION

Companies which serve their customers through delivery goods or services needs a system to plan their services. The system called logistic system is expected could give benefit not only to the companies but also to their customers. Vehicle Routing Problem (VRP) is one of the important issues related to the logistic system. This well known combinatorial optimization problem aims at minimizing the total travel cost (proportional to the travel times or distances) and operational cost (proportional to the number of vehicles used) such that customers' demand is fulfilled in time. [13] who were the first to introduce a version of VRP in the Truck Dispatching Problem. They modelled a problem for a fleet of homogeneous trucks that serve the demand for oil of a number of gas stations from a central hub and with a minimum travelled time. [32] give a comprehensive overview of the VRP which discusses problem formulations, solution techniques, important variants and applications. Due to its challenging nature many researchers have been working in this area to discover new approaches in selecting the best routes in order to find the better solutions. Some interesting survey of VRP can be found in [33], [18], [24], [19], [23], [22], and [21].

By assuming that the customers for deliveries of goods and services are known in advance the classical vehicle routing problem (*VRP*) can be defined as follows: vehicles with a fixed capacity Q must deliver order quantities q_i ($i = 1, \ldots, n$) of goods to n customers from a single depot (*i* = 0). Given the distance d_{ij} between customers *i* and *j* ($i, j = 1, \ldots, n$), the objective of the problem is to minimize the total distance traveled by the vehicles. There are requirements are necessarily to be satisfied:

a) only one vehicle handles the deliveries for a given customer
b) the total quantity of goods that a single vehicle delivers is not larger than Q
c) each customer is assigned to one and only one vehicle
d) every vehicle starts and end at the depot (i = 0)

Taking into account scheduled time of planning for routing problems is another variant of the VRP, known as the priodic VRP (PVRP). In PVRP, within a given time horizon, there is a set of customers needs to be delivered once or several times. There would be a delivering schedules associated with each customer. A fleet of vehicles is available and each vehicle leaves the depot, serves a set of customers, when its work shift or capacity is over, returns to the depot. The problem is to minimize the total length of the routes travelled by the vehicles on the time horizon. This problem has several important real world applications, such as, distribution for bakery companies [27], blood product distribution [20], or pick-up of raw materials for a manufacture of automobile parts [1]. [5] describe a case for periodic maintenance of elevators at different customer locations. Further case studies concerning waste collection and road sweeping can be found in [4] and [15]. [10] describe a milk collection problem where it is important that the goods are collected when fresh. [7] implement periodic capacitated PVRP for retail distribution of fuel oils.

A survey on PVRP and its extensions can be found in [17]. Due to the combinatorial nature of the problem most of the works present heuristic approaches, nevertheless [3] proposed an exact method based on a set partitioning integer linear programming formulation of the problem. The first work on the PVRP is proposed by [4]. Then there are other works by [9], [31], and, [28. [7] are the first to provide a two-phase heuristic that allows escaping from local optima. They use an integer linear program to assign visit day combinations to customers in order to initialize the system. Moreover, the capacity limit of the vehicles is temporarily. [6] addressed a combined of heuristic and exact method for solving PVRP.

Early formulations of the PVRP were developed by [4] and by [28]. They proposed heuristics technique applied to waste collection problem. [21] use the idea of the generalized

assignment method proposed by [14] and assign a delivery schedule to each vertex. Eventually a heuristic for the VRP is applied to each day. [29] developed a heuristic organized in four phases.[11] present another algorithm: The solution algorithm is a tabu search heuristic which allow infeasible solutions during the search process. They considered two types neighborhood operators. Similarly, good results were obtained in the work of [14], [2], and [5] who provide specific practical applications of the PVRP. [25] used particle swarm optimization to tackle the problem.

[17] introduce the Period Vehicle Routing Problem with Service Choice (PVRP-SC) which allows service levels to be determined endogenously. They develop an integer programming formulation of the PVRP-SC with exact and heuristic solution methods. Due to the computational complexity of the problem, solutions to the discrete PVRP-SC are limited by instance size. Continuous approximation models are better suited for large problem instances, yet the use of continuous approximation models for periodic routing problems has been limited.

[13] presents modeling techniques for distribution problems with varying service requirements. [30] develop continuous approximation models for distribution network design with multiple service levels. These references show that continuous approximations can be powerful tools for strategic and tactical decisions when service choice exists. In continuous approximation models, aggregated data are used instead of more detailed inputs. [26] present PVRP with time windows (PVRPTW). The problem requires the generation of a limited number of routes for each day of a given planning horizon. The objective is to minimize the total travel cost while satisfying several constraints.

This paper is about to deliver daily meals from a catering company to customers spread across the city of Medan. The requests from customers are varied in time. Therefore the company needs to schedule the time of delivery, vehicle to be used and the drivers. Coverage area of the operation of this catering company is large, in such a way, the company divides the whole area into several sub-area, with a consequence that it is necessarily to include the scheduling of the sub-area in the PVRP. [17] address this type of PVRP as PVRP with service choice (PVRPSC). However, the schedule of service is within a set of days. In our case the schedule of service is a set of time in a day. We call our PVRP as PVRP with delivery scheduling (PVRPDS).

The catering company has a limited number of fleet of vehicles and drivers. Therefore it needs to plan a schedule that can organize these vehicles and drivers in order to satisfy their customers. This paper concerns with a comprehensive modeling for the PVRP incorporated with time windows, fleet and driver, and delivery scheduling (PVRPDFDS). Due to the fact that heterogeneous vehicle with different capacities are available, the basic framework of the vehicle routing can be viewed as a Heterogeneous Vehicle Routing Problem with Time Windows (HVRPTW). We address a mixed integer programming formulation to model the problem. A feasible neighborhood heuristic search is proposed to get the integer feasible solution after solving the continuous model of the problem.

Section 2 reviews the mixed integer programming formulation of the PVRP with time windows from [26]

Section 3 describes the mathematics formulation of the (PVRPDFDS). The basic approach is given in Section 4. The algorithm is described in Section 5. Finally Section 6 describes the conclusions.

II. THE BASIC FRAMEWORK MODEL OF THE PVRPTW

Firstly, we consider the basic frame work model for the catering problem as a PVRPTW By using graph the formulation of the model can be written as follows (based on [26]). Let $G = (V, A)$ be a graph, where $V = \{0, 1, \dots, n\}$ is the vertex set and $A = \{(i, j) : i, j \in V, i \neq j\}$ is the set of route. For each route $(i, j) \in A$ a distance (or travel) cost c_{ij} is defined. The center of catering, called depot vertex $i = 0$. Define V_c □ is the set of customers' vertex. Each customer $i \in V_c$ has a known daily demand $q_i \geq 0$ within the planning horizon of time □ in a day. As the problem has time windows, then the service time $s_i \geq 0$ requires a_i as the earliest time of service may start and b_i is the latest time. Another requirement is the frequency number of visits f_i need to be performed according to one of the allowable visit-time patterns taken from the list of schedule L_i. At the center of catering ($i = 0$), the interval of time for vehicles to leave and to return to the depot is given by $[a_0, b_0]$. There are m vehicles, each with capacity Q_k is based at the central. Elapsed time, u_{ik}^t with $i \in V_c, k \in K, t \in T$, may occur. Vehicles are grouped into set □. Vehicle routes are restricted to a maximum duration of D_k, $k = 1, \dots, m$.

Assume that the vehicle fleet is homogen with $Q_k = Q$ and a common duration restriction $D_k = D$, $\forall k = 1, \dots, m$. The PVRPTW can then be regarded as the problem of generating (at most) m vehicle routes for each time in a day , such that to minimize the total cost over the entire planning horizon. There are some constraints that must be satisfied, such as, 1) each vertex i is visited the required number of times, f_i, corresponding to a single time schedule of visit chosen from L, and is serviced within its time window, i.e., a vehicle may arrive before a_i and wait to begin service; 2) each route starts from the central, visits the customers vertex selected for that time, with a total demand not exceeding Q, and returns to the central after a duration (travel time) not exceeding D.

Let α_{lt} be 1 if time $t \in$ □ belongs to list of schedule l, and 0 otherwise. Binary variables referred to Route-selection, time list-selection, and continuous timing decision variables are defined as follows

- $x_{ijk}^t = \begin{cases} 1 & \text{if vehicle } k \in K \text{ traverses route } (i,j) \in A \\ & \text{on time } t \in T; \\ 0 & \text{otherwise;} \end{cases}$

- $y_{ir} = \begin{cases} 1 & \text{if schedule } l \in L \text{ is assigned} \\ & \text{to customer } i \in V_c; \\ 0 & \text{otherwise;} \end{cases}$

- w_{ik}^t indicate the service time begins for vehicle $k \in K$ at customer $i \in V_c$ at time $t \in T$

- $z_{ik}^t = \begin{cases} 1 & \text{if vehicle } k \in K \text{ visit customer } i \in V_c \\ & \text{on time } t \in T; \\ 0 & \text{otherwise;} \end{cases}$

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The PVRPTW for the catering problem can be written mathematically as follows. The objective function is to minimize travel time for each vehicle and elapsed time.

$$\text{minimize} \sum_{t\in T}\sum_{(i,j)\in A}\sum_{k\in K} c_{ij}x_{ijk}^{t} + \sum_{i\in V_c}\sum_{t\in T}\sum_{k\in K} u_{ik}^{t}z_{ik}^{t} \quad (1)$$

There are constraints that must be fulfilled formulated as follows

$$\sum_{l\in L} y_{il} = 1, \qquad \forall i \in V_c, \quad (2)$$

$$\sum_{j\in V} x_{ijk}^{t} = \sum_{j\in V} x_{jik}^{t}, \quad \forall i \in V, k \in K, t \in T, \quad (3)$$

$$\sum_{k\in K}\sum_{j\in V} x_{ijk}^{t} = \sum_{l\in L} y_{il}\alpha_{lt} \quad \forall i \in V_c, t \in T, \quad (4)$$

$$\sum_{k\in K}\sum_{j\in V} x_{0jk}^{t} \leq m \quad \forall t \in T, \quad (5)$$

$$\sum_{j\in V_c} x_{0jk}^{t} \leq 1, \quad \forall k \in K, t \in T, \quad (6)$$

$$\sum_{i\in V_c} q_i \sum_{j\in V} x_{ijk}^{t} \leq Q, \quad \forall k \in K, t \in T, \quad (7)$$

$$w_{ik}^{t} + s_i + c_{ij} - M(1-x_{ijk}^{t}) \leq w_{jk}^{t}, \quad \forall (i,j)\in A, k\in K, t\in T, \quad (8)$$

$$e_i \sum_{i,j\in V} x_{ijk}^{t} \leq w_{ik}^{t} \leq l_i \sum_{j\in V} x_{ijk}^{t}, \quad \forall i \in V_c, k\in K, t\in T, \quad (9)$$

$$w_{ik}^{t} + s_i + c_{i0} - M(1-x_{i0k}^{t}) \leq D, \quad \forall i \in V_c, k\in K, t\in T, \quad (10)$$

$$x_{ijk}^{t} \in \{0,1\}, \quad \forall (i,j)\in A, k\in K, t\in T, \quad (11)$$

$$y_{il} \in \{0,1\}, \quad \forall i \in V_c, l\in L, \quad (12)$$

$$z_{ik}^{t} \in \{0,1\}, \quad \forall i \in V_c, k\in K, t\in T \quad (13)$$

$$w_{ik}^{t} \geq 0, \quad \forall i \in V_c, k\in K, t\in T, \quad (14)$$

Expression (2) ensures that a feasible pattern is assigned to each customer. Eq. (3) shows the enforcment of flow conservation to ensure that a vehicle arriving at a customer on a given time, leaves that customer on the same time. Eq. (4) guarantee that each customer is visited on the times corresponding to the assigned schedule, while expression (5) makes sure that the number of vehicles used on each time scheduled does not exceed *m*. To ensure that each vehicle is used at most once in time schedule is presented in Eq. (6), while (7) guarantee that the load charged on a vehicle does not exceed its capacity. Constraints (8) enforce time feasibility, i.e., vehicle *k* cannot start servicing *j* before completing service at the previous customer *i* and traveling from *i* to *j*, i.e., not before $w_{ik}^{t} + s_i + c_{ij}$. Constraints (9) ensure that customer time window restrictions are respected, while (10) constrains the route length. Expressions (11) - (14) define the sets of decision variables.

III. MODELING THE PVRDFDS

Now we formulate the catering problem as a Periodic Vehicle Routing with Time Windows considering Delivery, Fleet, and Driver Scheduling (PVRPDFDS) Define the planning horizon by *T* and the set of drivers by *D*. The set of

scheduled time from the list L for driver $l \in D$ is denoted by $T_l \subseteq T$. The start working time and latest ending time for driver $l \in D$ on time $t \in T$ are given by g_l^t and h_l^t, respectively. For each driver $l \in D$, let *H* denote the maximum daily working duration.

Let *K* denote the set of vehicles. For each vehicle $k \in K$, let Q_k and P_k denote the capacity in weight and in volume, respectively. We assume the number of vehicles equals to the number of drivers. Denote the set of *n* customers (nodes) by $N = \{1, 2, \ldots, n\}$ and the central depot by $\{0\}$. Each vehicle starts from central of catering $\{0\}$ and terminates at central $\{0\}$. Each customer $i \in N$ specifies a set of times to be visited, denoted by $T_i \subseteq T$. At each scheduled time $t \in T_i$, customer $i \in N$ requests service with demand of q_i^t in weight and p_i^t in volume, service duration d_i^t and time window $[a_i, b_i]$. Note that, for the central depot $i \in \{0\}$ at time *t*, we set $q_i^t = p_i^t = d_i^t = 0$. The travel time between customer *i* and *j* is given by c_{ij}. Denote the cost coefficients of the travel time of the drivers by μ and the elapsed time by ρ.

We define binary variable x_{ijk}^t to be 1 if vehicle *k* travels the route (i,j)∈ *V* on time *t*. Other variables can be found at the following notations.

This notations used are given as follows :

Set:

T	The set of scheduled time in the planning horizon,
D	The set of drivers *D* ,
T_l	The set of working time for driver $l \in D$,
K	The set of vehicles,
N	The set of customers,
T_i	The set of time on which customer $i \in N$ orders,

Parameter:

Q_k	The weight capacity of vehicle $k \in K$,
P_k	The volume capacity of vehicle $k \in K$,
c_{ij}	The travel time of route $(i, j) \in V$,
$[a_i, b_i]$	The earliest and the latest visit time at node $i \in V$,
d_i^t	The service time of node $i \in V$ on time $t \in T_i$,
q_i^t	The weight demand of node $i \in V$ on time $t \in T_i$,
p_i^t	The volume demand of node $i \in V$ on time $t \in T_i$,

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$[\,g_l^t,\ h_l^t\,]$ The start time and the latest ending time of driver $l \in D$ on time $t \in T$,

β_i^t Delivery quantity for customer i on time $t \in T_i$,

F The maximum elapsed driving time ,

μ The cost factor on the total travel time of drivers,

ρ The cost factor on the total working hour of drivers

λ The cost factor for the total elapsed time

Variables:

x_{ilk}^t Binary variable indicating whether vehicle $k \in K$ travels from node $i \in V$ to $j \in V$ on time $t \in T$,

v_{ik}^t The time at which vehicle $k \in K$ starts service at node $i \in V$ on time $t \in T$,

z_{ik}^t Binary variable indicating whether vehicle $k \in K$ takes break after serving node $i \in N$ on time $t \in T$,

u_{ik}^t The elapsed driving time of vehicle $k \in K$ at node $i \in V$ on time $t \in T$,

y_{lk}^t Binary variable indicating whether vehicle $k \in K$ is assigned to driver $l \in D$ on time $t \in T$,

r_l^t The total working duration of driver $l \in D$ on time $t \in T$,

s_l^t The total travel distance of driver $l \in D$ on time $t \in T$,

σ_{jk}^t Number of delivery demands of customer j served by vehicle $k \in K$ on time $t \in T$

The mathematical formulation for this problem is presented as follows.

The objective of the catering company is to minimize the cost of working hours, travel time, and elapsed time of drivers.

$$\min\ \mu \cdot \sum_{l \in D}\sum_{t \in T_i} s_l^t + \rho \cdot \sum_{l \in D}\sum_{t \in T_i} r_l^t + \lambda \sum_{i \in V}\sum_{k \in K}\sum_{t \in T_i} u_{ik}^t \qquad (15)$$

Constraints

$$\sum_{k \in K}\sum_{j \in V} x_{ijk}^t = 1 \qquad\qquad \forall\ i\ \in\ N,\ t\ \in\ T \tag{16}$$

$$\sum_{i \in N}\sum_{j \in V} q_i^t x_{ijk}^t \le Q_k \qquad\qquad \forall\ k \in K, t \in T \tag{17}$$

$$\sum_{i \in N}\sum_{j \in V} p_i^t x_{ijk}^t \le P_k \qquad\qquad \forall\ k \in K, t \in T \tag{18}$$

$$u_{jk}^t \ge u_{ik}^t + c_{ij} - M(1 - x_{ijk}^t) - M z_{ik}^t$$
$$\forall i, j \in V,\ k \in K, t \in T \tag{19}$$

$$u_{jk}^t \ge c_{ij} - M(1 - x_{ijk}^t) \qquad \forall i,\ j\ \in\ N,\ k\ \in\ K,\ t\ \in\ T \tag{20}$$

$$u_{ik}^t + \sum_{j \in N_0} c_{ij} x_{ijk}^t - F \le M z_{ik}^t \quad \forall i \in V,\ k \in K, t \in T \tag{21}$$

$$v_{jk}^t \ge v_{ik}^t + d_i^t + c_{ij} - M(1 - x_{ijk}^t)$$
$$\forall i, j \in V,\ k \in K, t \in T \tag{22}$$

$$b_i \ge v_{ik}^t \ge a_i \qquad\qquad \forall i \in N,\ k \in K, t \in T_i \tag{23}$$

$$v_{0k}^t \ge \sum_{l \in D}(g_l^t \cdot y_{lk}^t) \qquad\qquad \forall k \in K, t \in T \tag{24}$$

$$v_{n+1,k}^t \le \sum_{l \in D}(h_l^t \cdot y_{ik}^t) \qquad\qquad \forall k \in K, t \in T \tag{25}$$

$$s_l^t \ge \sum_{i \in V}\sum_{j \in V} c_{ij} x_{ijk}^t - M(1 - y_{ik}^t) \quad \forall\ l \in D_l, k \in K, t \in T_i \tag{26}$$

$$r_l^t \ge v_{n+1,k}^t - g_l^t - M(1 - y_{lk}^t) \quad \forall\ l \in D, k \in K, t \in T_i \tag{27}$$

$$\sum_{k \in K} \sigma_{jk}^t = \beta_j^t \qquad\qquad \forall j \in N, t \in T \tag{28}$$

$$x_{ijk}^t, w_i^t, z_{ik}^t, y_{lk}^t \in \{0,1\} \quad \forall i, j \in N_0, l \in D,\ k \in K, t \in T \tag{29}$$

$$v_{ik}^t, u_{ik}^t, r_l^t, s_l^t \ge 0 \quad \forall i, j \in N_0, l \in D,\ k \in K, t \in T \tag{30}$$

$$\theta_{jk}^t, \sigma_{jk}^t \in \{0,1,2,...\} \quad \forall j \in N, k \in K, t \in T \tag{31}$$

Constraints (16) state that each customer must be delivered by one vehicle on each of its delivery time.) The vehicle capacities which are restricted to their weight and volume, expressed in Constraints (17-18). In Constraints (19-24) we define the elapsed driving time and the time spent at each delivery. Algebraically, for the vehicle (k) travelling from customer i to j on day t, the elapsed driving time at j equals the elapsed driving time at i plus the driving time for route i to j (i.e., $u_{jk}^t \ge u_{ik}^t + c_{ij}$) if the driver does not take a break when it delivers to customer i (i.e., $z_{ik}^t = 0$); Otherwise, if the driver takes a break (i.e., $z_{ik}^t = 1$), the elapsed driving time at j will be constrained by (20) which make sure it is greater than or equal to the travel time between i and j (i.e., $u_{jk}^t \ge c_{ij}$). Constraints (21) state that the elapsed driving time never

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exceeds an upper limit F by imposing a break at customer i (i.e., $z_{ik}^t = 1$.

Constraints (23) determine the time to start the delivery at each customer. If j is delivered immediately after i, the time v_{jk}^t to start the delivery at j should be greater than or equal to the delivery starting time v_{ik}^t at i plus its delivery duration d_i^t, the travel time between the two customers c_{ij}, and the break time G if the driver takes a break after serving I (i.e., $z_{ik}^t = 1$). Constraints (23) make sure the services start within the customers' time window.

Constraints (24-25) ensure that the starting time and ending time of each route must lie between the start time and latest delivery job ending time of the assigned driver. Constraints (26) calculate the total travel time for each driver. Constraints (27) define the working duration for each driver on every workday, which equals the time the driver returns to the central minus the time the driver starts the delivery. Constraints (28 – 29) define the delivery for each customer. Constraints (30-31) define the variables used in this formulation.

IV. THE BASIC APPROACH

Consider a Mixed Integer Linear Programming (MILP) problem with the following form

$$\text{Minimize} \quad P = c^T x \quad (32)$$
$$\text{Subject to} \quad Ax \leq b \quad (33)$$
$$x \geq 0 \quad (34)$$
$$x_j \text{ integer for some } j \in J \quad (35)$$

A component of the optimal basic feasible vector $(x_B)_k$, to MILP solved as continuous can be written as

$$(x_B)_k = \beta_k - \alpha_{k1}(x_N)_1 - \cdots - \alpha_{kj}(x_N)_j - \cdots \alpha_{kn} - m(x_N)_N n \quad (36)$$

Note that, this expression can be found in the final tableau of Simplex procedure. If $(x_B)_k$ is an integer variable and we assume that β_k is not an integer, the partitioning of β_k into the integer and fractional components is that given

$$[\beta_k] + f_k, \quad 0 \leq f_k \leq 1 \quad (37)$$

suppose we wish to increase $(x_B)_k$ to its nearest integer, $([\beta] + 1)$. Based on the idea of suboptimal solutions we may elevate a particular nonbasic variable, say $(x_N)_{j*}$, above its bound of zero, provided α_{kj*}, as one of the element of the vector α_{j*}, is negative. Let Δ_{j*} be amount of movement of the non variable $(x_N)_{j*}$, such that the numerical value of scalar $(x_B)_k$ is integer. Referring to Eqn. (17), Δ_{j*} can then be expressed as

$$\Delta_{f*} = \frac{1 - f_k}{-\alpha_{kj*}} \quad (38)$$

while the remaining nonbasic stay at zero. It can be seen that after substituting (37) into (38) for $(x_N)_{j*}$ and taking into account the partitioning of β_k given in (18), we obtain

$$(x_B)_k = [\beta] + 1$$

Thus, $(x_B)_k$ is now an integer.

It is now clear that a nonbasic variable plays an important role to integerized the corresponding basic variable. Therefore, the following result is necessary in order to confirm that must be a non-integer variable to work with in integerizing process.

Theorem 1. *Suppose the MILP problem* (32)‑(35) *has an optimal solution, then some of the nonbasic variables.* $(x_N)_j, j = 1, \ldots, n$, *must be non-integer variables.*

Proof:
Solving problem as a continuous of slack variables (which are non-integer, except in the case of equality constraint). If we assume that the vector of basic variables consists of all the slack variables then all integer variables would be in the nonbasic vector x_N and therefore integer valued.

V. THE ALGORITHM

Stage 1. For the integerizing process.

Step 1. Get row $i*$ the smallest integer infeasibility, such that
$$\delta_{i*} = \min\{f_i, 1 - f_i\}$$
(This choice is motivated by the desire for minimal deterioration in the objective function, and clearly corresponds to the integer basic with smallest integer infeasibility).

Step 2. Do a pricing operation
$$v_{i*}^T = e_{i*}^T B^{-1}$$

Step 3. Calculate $\sigma_{ij} = v_{i*}^T \alpha_j$
With j corresponds to
$$\min_j \left\{ \left\| \frac{d_j}{\alpha_{ij}} \right\| \right\}$$
Calculate the maximum movement of nonbasic j at lower bound and upper bound.
Otherwise go to next non-integer nonbasic or superbasic j (if available). Eventually the column $j*$ is to be increased form LB or decreased from UB. If none go to next $i*$.

Step 4.
Solve $B\alpha_{j*} = \alpha_{i*}$ for α_{j*}

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic $j*$ from its bounds.

Step 6. Exchange basis

Step 7. If row $i* = \{\varnothing\}$ go to Stage 2, otherwise Repeat from step 1.

Stage 2. Pass1 : adjust integer infeasible superbasics by fractional steps to reach complete integer feasibility.
Pass2 : adjust integer feasible superbasics. The objective of this phase is to conduct a highly localized neighbourhood search to verify local optimality.

VI. CONCLUSIONS

This paper was intended to develop a model of Periodic vehicle Routing with Time Windows, Delivery, Fleet, and Driver Scheduling Problem This model is used for solving a

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catering problem located in Medan city, Indonesia. The result model is in the form of mixed integer linear programming and then solved by employing nearest neighbor heuristic algorithm. This algorithm offers appropriate solutions in a very small amount of time.

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