Kinematic Hardening Parameters Identification with Finite Element Simulation of Low Cycle Fatigue using Genetic Algorithm Approach

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Abstract: This paper deals with finite element (FE) simulation to characterize the low cycle fatigue (LCF) behavior using genetic algorithm (GA) approach. Non linear version of Chaboche's kinematic hardening material model is used to address the stable hysteresis cycles of the material. Cyclic hardening phenomenon is addressed by introducing exponential isotropic hardening rule in the material model. The elastic plastic FE code ABAQUS is used for finite element simulation of LCF behavior. The plastic modulus formulation is coupled with the isotropic/kinematic hardening rule together with the yield surface consistency condition Incremental plasticity theories is used to study the cyclic plastic stress-strain responses. The GA approach is used to optimize the isotropic/ kinematic hardening parameters of SS 316 steel. The validity of GA method is verified by comparing its simulation results with those of manual parameter determination approach available in the literature. The simulation results confirm the potentiality and efficacy of the Genetic algorithm.

Keywords: *Finite element analysis, Incremental plasticity, Low cycle fatigue, Genetic algorithm.*

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Nomenclature:

$\Delta\varepsilon$	Local strain range	σ_{ij}	Stress tensor
$\sigma\|_0$	Yield stress	σ_c	Current yield stress
γ	Knematic Hardening parameter	ε_{ij}	Total strain tensor
$\bar{\varepsilon}^p$	Equivalent plastic strain	ε_{ij}^e	Elastic strain tensor
C	kinematic Hardening parameter	$\dot{\varepsilon}_{ij}^e$	Incremental elastic strain tensor
E	Young's modulus	ε_{ij}^p	Plastic strain tensor
s_{ij}	Deviatoric stress tensor	$\dot{\varepsilon}_{ij}^p$	Incremental plastic strain tensor
Λ	Plastic multiplier	μ	Poisson's ratio
α_{ij}	Back stress tensor	ϕ	Yield function

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1. Introduction

Modeling the cyclic plastic behavior of a material is most important to estimate the fatigue life of the components. The experimental observations show a number of cyclic plastic behaviors such as bauschinger effect, cyclic hardening/softening, mean stress relaxation and ratcheting of a material. Some materials also show strength differential (S-D) effect [1]. Above all, there is additional hardening in case of non-proportional loading path. Low cycle fatigue [2-3] must be considered during design of nuclear pressure vessels, steam turbines and other type of power machineries where life is nominally characterized as a function of the strain range and the component fails after a small number of cycles at a high stress, and the deformation is largely plastic.

For simulating cyclic plastic behavior of the material various models are proposed by proper evolution laws of back stress tensor. A simplest choice like linear kinematic hardening law was proposed by Prager [4]. Later, a nonlinear kinematic hardening law with recall term was introduced by Armstrong and Frederick [5]. Thereafter, Armstrong–Frederick law was modified to have segment wise better matching with experimental results [6-7].

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With the advance growth of computer science and technology, various meta-heuristics intelligent techniques have been adopted in various field of optimization in last two decades [8-9]. The main advantage of these techniques is higher degree of avoiding local optimum and free from derivative structure. Amongst the various meta-heuristics techniques, genetic algorithm (GA) [10-11] has been adopted by many researchers for determining the optimal parameters of cyclic plasticity model.

2. Experimental procedure

2.1 Low cycle fatigue tests

SS 316 steel is the selected material for investigation of uniaxial cyclic plastic behaviour. Uniaxial cyclic experiments are performed at room temperature on 8mm diameter fatigue specimens, gauge length 18mm (Fig 1) under strain controlled (Fig 2) mode. A 100 KN servo-hydraulic universal testing machine (Instron UTM) (Fig 3) is used. The strain-controlled tests are performed on the specimens for symmetric tension-compression strain cycles with the strain amplitudes ±0.30%, ±0.50%,±0.60%, ±0.75%, and±1.0% for low cycle fatigue. A strain-controlled test was carried out up to 100 load cycles in strain-controlled mode with a constant strain rate of 10^{-3}/s. The stabilized hysteresis loops of σ-ϵ^{p} for various strain amplitudes are obtained from the test (Fig 4). It is observed that the material exhibits non Massing character. The kinematic hardening coefficients are obtained from ±1.0% strain amplitude. The kinematic hardening coefficients obtained from the experiments are used in FE simulation. Cyclic hardening is observed in the experiment as shown in Fig 5 .The material gets saturated after 100 cycles and the stabilized loop is obtained for all the cases. The variation of cyclic yield stress with strain amplitudes is obtained in Fig 4.

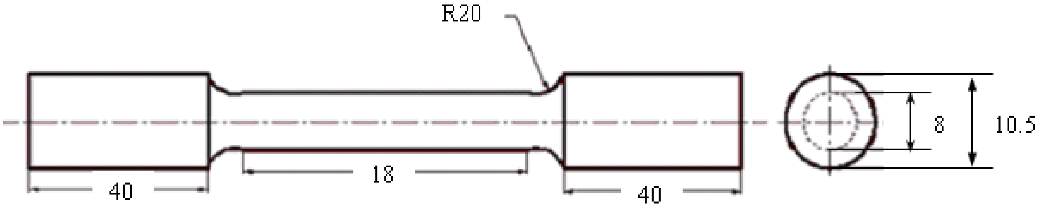

Fig 1 Uniaxial Fatigue Specimen

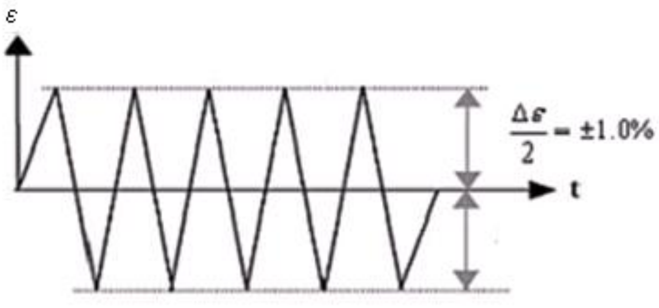

Fig 2 Loading history during strain-controlled test

Fig 3 Experimental setup

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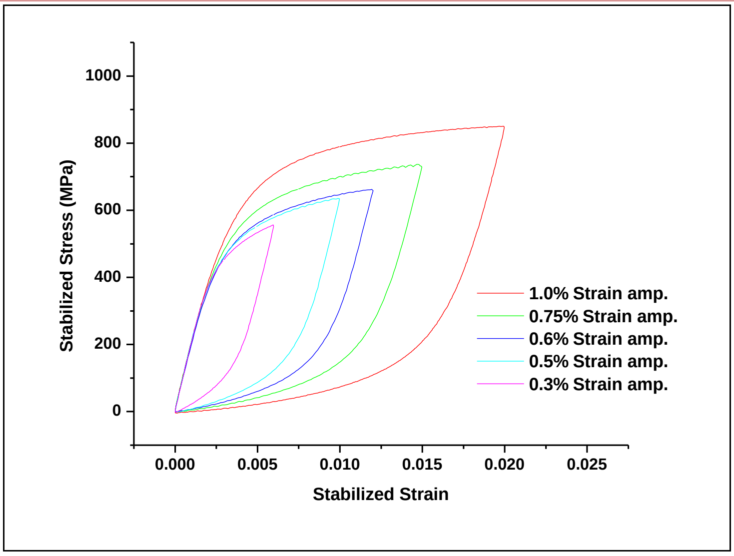

Fig 4 Stabilized hysteresis plots for different strain amplitudes

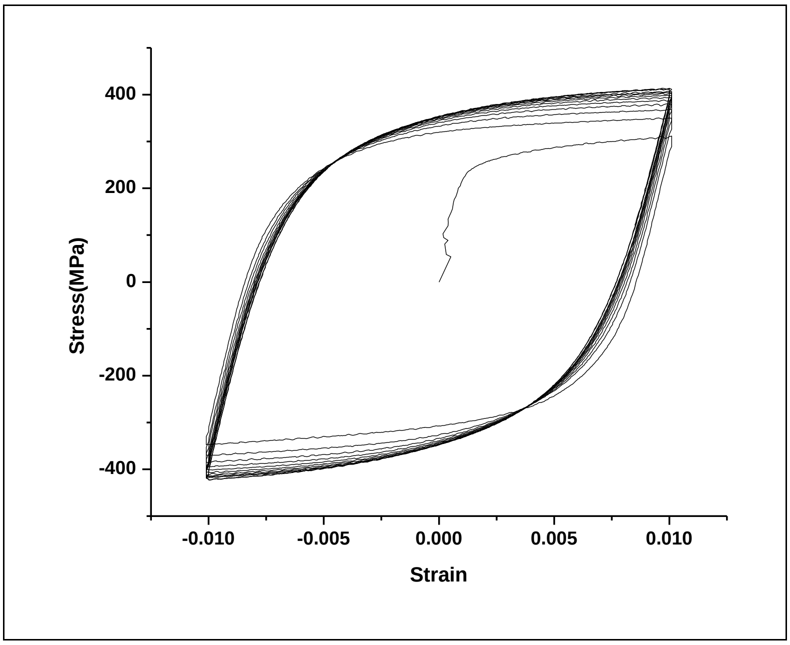

Fig 5 Experimental stress strain response up to 30th cycles for 1% strain amplitude curve.

3 Modelling of cyclic plasticity

Cyclic plasticity models are based on incremental plasticity theories. These plasticity relationships are given as

Strain rate decomposition: $d\bar{\varepsilon} = d\bar{\varepsilon}^{e} + d\bar{\varepsilon}^{p}$ (1)

Hook's law: $d\bar{\varepsilon}^{e} = \frac{1+\nu}{E} d\bar{\sigma} - \frac{\nu}{E} tr(d\bar{\sigma})\bar{I}$ (2)

Flow-rule: $d\bar{\varepsilon}^{p} = \frac{1}{H}\left\langle \frac{\partial f}{\partial \bar{\sigma}} : d\bar{\sigma} \right\rangle \frac{\partial f}{\partial \bar{\sigma}}$ (3)

Von-Mises yield criterion: $f(\bar{\sigma} - \bar{\alpha}) = \left[\frac{3}{2}(\bar{s} - \bar{\alpha})(\bar{s} - \bar{\alpha})\right]^{1/2} = \sigma_c$ (4)

Kkinematic hardening rule (e.g. for three segmented Chaboche model)

$$d\bar{\alpha} = \sum_{i=1}^{3} d\bar{\alpha}_i \tag{5}$$

$$d\bar{\alpha}_i = \frac{2}{3} C_i d\bar{\varepsilon}^p - \gamma_i \bar{\alpha}_i d\varepsilon_{eq}^p \tag{6}$$

where $\bar{\sigma}$ is the stress tensor, $\bar{\varepsilon}^p$ is the plastic strain tensor, $\bar{s}$ is the deviatoric stress tensor, $\bar{\alpha}$ is the current center of the yield surface known as back stress tensor and considered to be deviatoric in nature, σ_0 is the size of the yield surface, H is the plastic modulus.

$$d\varepsilon_{eq}^p = \left| d\bar{\varepsilon}^p \right| = \left[\frac{2}{3} d\bar{\varepsilon}^p : d\bar{\varepsilon}^p \right]^{1/2} \tag{7}$$

C's, γ's are model parameters of the Chaboche model [36]. Plastic modulus H is calculated using the consistency condition [37], and is given by the relationship

$$H = \sum_{i=1}^{3} H_i \tag{8}$$

Where $$H_i = C_i - \gamma_i \left(\bar{\alpha}_i : \frac{\partial f}{\partial \bar{\sigma}} \right) \tag{9}$$

For uniaxial loading it reduces and may be represented as follows:

$$H_i = C_i - \gamma_i \alpha_i \tag{10}$$

For uniaxial loading the back stress and plastic strain relationships are given by

$$\sum_{i=1}^{3} \alpha_i = \frac{2}{3} (\sigma_x - \sigma_0) \tag{11}$$

$$\alpha_i = \frac{C_i}{\gamma_i} \left[1 - \exp\left(-\gamma_i \varepsilon_x^p \right) \right] \tag{12}$$

4.1 Isotropic Hardening

The isotropic hardening behavior of the model defines the evolution of the yield surface size, σ_c as a function of the equivalent plastic strain, $\dot{\bar{\varepsilon}}^p$. This evolution can be introduced by specifying σ_c directly as a function of $\dot{\bar{\varepsilon}}^p$.

For the isotropic hardening rule, Chaboche proposed the following equation:

$$\dot{R}\left(\bar{\varepsilon}^p \right) = b \left(Q_\infty - e^{-b\bar{\varepsilon}^p} \right)$$

where Q_∞ and b are the isotropic hardening material parameters and are computed from experimental stress–strain loop results of LCF test of plain fatigue specimens. Using the initial condition $R\left(\bar{\varepsilon}^p \right) = 0$, on integration of the above differential equation, we get

$$R = Q_\infty \left(1 - e^{-b\bar{\varepsilon}^p} \right) \tag{13}$$

Now, the simple exponential law is

$$\sigma^c = \sigma|_0 + Q_\infty \left(1 - e^{-b\bar{\varepsilon}^p} \right) \tag{14}$$

where, $\sigma|_0$ is the yield stress at zero plastic strain and Q_∞ and b are material parameters. Q_∞ is the maximum change in the size of the yield surface, and 'b' defines the rate at which the size of the yield surface changes as plastic straining develops. When the equivalent stress defining the size of the yield surface remains constant ($\sigma_c = \sigma|_0$), the model reduces to a nonlinear kinematic hardening model.

4.3 Manual procedure for Parameter determination approaches

Chaboche's kinematic hardening coefficients are determined from saturated hystersis loop. For the material SS 316 steel saturation is obtained after 100 cycles. The experimental saturated loop of 100th cycle for strain amplitude of ±1.0% is used to obtain Chaboche's kinematic hardening coefficients [12].

To match the elastic to plastic transition part the value of C_1 and γ_1 are found by trial method keeping the value of C_1/γ_1 is constant. C_2 and γ_2 are evaluated by trials to produce a good representation of the experimental stable hysteresis curve which also satisfy the relationship at or close to plastic strain ε_L^p

$$\frac{C_1}{\gamma_1}+\frac{C_2}{\gamma_2}+\frac{C_3}{2}\left\{\varepsilon_x^p-\left(-\varepsilon_L^p\right)\right\}=\frac{2}{3}\left(\sigma_x-\sigma_0\right) \tag{15}$$

where ε_L^p is the strain limit of the stable hysteresis loop. The tuning is done with the values of C_2 and γ_2 keeping the ratio C_2/γ_2 same. C_1, γ_1 and C_3 are kept constant and γ_3 is set to zero. The values of Chaboche's kinematic hardening coefficients are listed in Table 1 along with other mechanical properties. Initial yield stress $\sigma_{0,}$ and Young modulus E are obtained from the monotonic uniaxial test data, while Poisson's ratio is presumed.

Table 1

Kinematic hardening parameters with isotropic hardening variables

Parameter	Value	Parameter	Value
Young's modulus, *E* (GPa)	200	γ_1	1500
Poisson's ratio, μ	0.3	γ_2	348
Yield strength, σ_{c0} (MPa)	240	γ_3	0
$C_{1,}$(MPa)	75000	Q_∞	50
C_2 (MPa)	35000	*b*	2.5
C_3 (MPa)	4000		

4.4 Parameters determination genetic algorithm

Determination of model parameters through manual operation is difficult as manual parameter determination for an advanced plasticity model requires vast knowledge of the model. Therefore, for structural analysis and design, parameter determination of advanced cyclic plasticity models has hardly been available in the literature. In this context, the GA technique is adopted in this research work to tune the parameters of advanced cyclic plasticity model.

The fitness function used to minimize the differences between predicted values and the experimental data of the hysteresis loop may be expressed as follows:

$$F = Min\frac{1}{K}\sum_{i=1}^{k}\left[\frac{\left(\sigma_i^{exp}-\sigma_i^{model}\right)}{\sigma_i^{exp}}\right]^2 \tag{16}$$

where K is the number of data points; σ_i^{exp} and σ_i^{model} are the stress from the experiments and the predicted stress using the kinematic hardening model.

The optimal values of Chaboche's kinematic hardening coefficients using GA are illustrated in Table 2. The comparative results of the proposed GA and the manual approaches are presented in Table 3. It is clearly noted form Table 3 that the fitness value obtained using GA is better compared to manual approach. Hence, it can be concluded from the aforesaid discussion that GA approach outperform the manual approach.

Table 2

Kinematic hardening parameters with isotropic hardening variables

Parameter	Value	Parameter	Value
Young's modulus, E (GPa)	200	γ_1	1409
Poisson's ratio, μ	0.3	γ_2	331
Yield strength, σ_{c0} (MPa)	240	γ_3	0
$C_{1,}$(MPa)	70411	Q_∞	50
C_2 (MPa)	37011	b	2.5
C_3 (MPa)	4978		

Table 3

Constitutive model parameters determination using manual calculation and GA for real response

Parameter	Manual calculation	GA	Parameter	Manual calculation	GA
$C_{1,(}$MPa)	75000	70411	γ_1	1500	1409
C_2 (MPa)	35000	37011	γ_2	348	331
C_3 (MPa)	4000	4978	γ_3	0	0
Fitness Value		Manual calculation			GA
Stable loop fitness, *fstb*		0.0034			0.0011

4 Simulation of Stable hysteresis loops and cyclic hardening

For simulation of stable hysteresis loops and cyclic hardening fully reversed tensile – compressive cyclic tests is conducted on a round bar specimen. In order to compare the simulated results with the experimental results the Chaboche kinematic hardening model has been used, plugged in elasto-plastic finite element FE code ABAQUS. The axial component of stress strain values, calculated at the center node of the specimen. GA optimization procedure has been applied to minimize the objective function and corresponding optimal values of the parameters are obtained. The GA results compared with the results that obtained using normal standard procedure. Fig 6 show the simulation results for stable hysteresis loop of strain amplitude ±1.0% using Chaboche's kinematic hardening model. Experimental result is compared with the simulated results obtained using normal procedure and GA approach. The results obtained using GA approach shows better matching with the experimental results than the result obtained using normal approach. Those results are compared with experimental stable hysteresis loops (at 100th cycle). Results of cyclic hardening for ±1.0% strain amplitude are also compared with the experimental result as show in Fig 7. It is observed that matching is quite satisfactory in engineering sense.

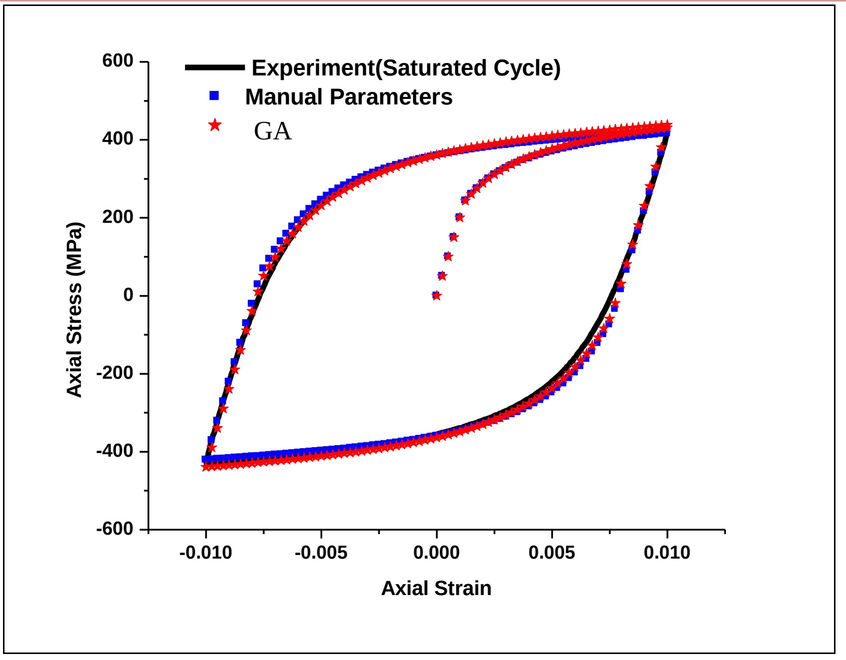

Fig 6 (a)

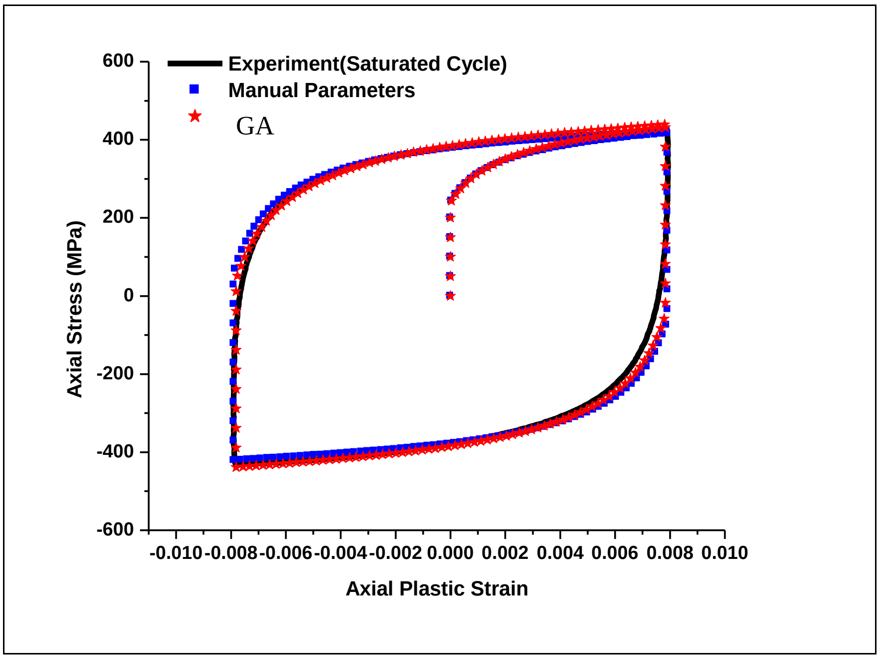

Fig 6 (b)

Fig 6 Stable stress strain hysteresis loop for ±1.0% strain amplitude using Chaboche rule (ABAQUS results).

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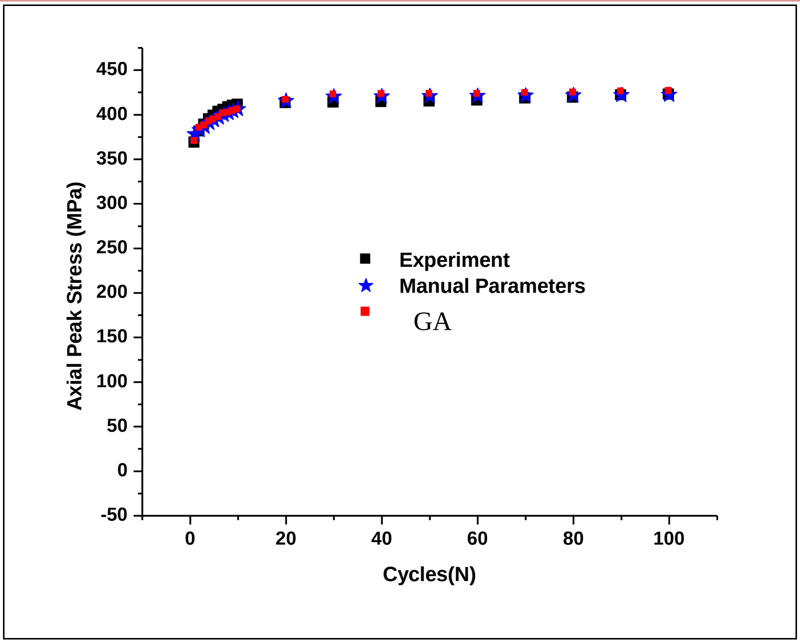

Fig 7 Variation of peak stress with no of cycles for ± 1.0% strain amplitude using Chaboche KH rule with isotropic hardening (ABAQUS result)

6 Conclusion

This paper presents, a newly developed meta-heuristics optimization method named genetic algorithm (GA) for performing material parameter identification of an SS 316 stainless steel. Moreover, tensile and strain controlled low cycle fatigue tests of various strain amplitude are performed to obtain an experimental database on this material. The upper branch of the stable hysteresis loop of 1% strain amplitude is taken into consideration for input data. The material model used to describe the material behavior is based on the isotropic hardening law and the non-linear kinematic hardening of Chaboche type with incremental plasticity theories. The simulation results clearly show that GA algorithm is able to fit the experimental behavior and Chaboche isotropic and kinematic hardening model. It is also observed that the GA provides better results for the case of uniaxial loading in comparison with manual approach. Therefore, it can finally be con- cluded that above mentioned GA approach is promising and encouraging for further research in this direction.

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