

Analysis of Urban Population Growth Pattern for Chennai – Zone IX, TEYNAMPET

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ABSTRACT— Graph-cellular automata (GCA) is implemented for zone IX of Chennai city. The population, population density, educational institution, industries, etc were taken as attribute. It is shown that GCA with particular structural properties defined in terms of the relationships between subsets of cells is a useful generalization of traditional cellular automaton (CA). The population data from Chennai census 1981, 1991, and 2001 were used to develop the GCA and the results are enumerated. The obtained results are compared with census data 2011.

Keywords- Graph cellular automata, urban growth.

I. INTRODUCTION

The work focuses to develop a computational modeling which (i) reproduce efficiently the underlying pattern of the urban growth dynamics, (ii) express in linguistic terms the ideas and results acquired and (iii) sustain a generic model independent from data limitations for Chennai Metropolitan Area.

Formally, a graph-CA model consists of a directed graph G , and a set of allowed cell (or vertex) states $S = \{s_j\}$. G consists of spatial elements represented by the vertex set $V(G)$ and directed relations between those elements represented by the arc set $E(G)$. In any graph, the neighbourhood of a vertex v_i consists of all the other vertices (or locations) in $V(G)$ to which it is adjacent. In the context of building a dynamic proximal model we may regard the in-neighbourhood of v_i , $N_{in}(v_i)$ as the set of influencing locations. We use a '(t)' superscript to indicate the particular state attaining at a vertex, or the collective state in a neighbourhood, at time t , as opposed to the set of allowed states S and its elements $\{s_j\}$ (with no superscript). Thus, $s_j^{(t)}$ is the state of location v_j at time t , and the cellular automaton aspect of the model is incorporated according to

$$s_i^{(t+1)} = f[S_i^{(t)}],$$

where $S_i^{(t)}$ is the set of states of the in-neighbours of v_i at time t , that is,

$$S_i^{(t)} = \{s_j^{(t)} \mid v_j \in N_{in}(v_i)\}.$$

Note that $S_i^{(t)}$ will always be some combination of the allowed cell states in S . Also, in most cases, $v_i v_j \in E(G)$, so

that $v_i \in N_{in}(v_j)$, and the state of a location at time $(t + 1)$ is partly dependent on its state at time t , as we would expect.

II. ZONE IX

Teynampet zone with seventeen wards is one amongst the busiest commercial areas within the town of Chennai metropolis. Teynampet gets its name from Thennam' and pettai which implies place of coconut trees as a result of the coconut groves that once existed there. Located in Latitude 13.05495° , Longitude 80.23922° it covers an area of 25.19 Sq.km. Its neighbouring areas include Alwarpet, T. Nagar, Nandanam, and Nungambakkam. Some the important residential areas in this zone are Gopalapuram, Poes road, Poes Garden, Seethamal Colony, Vellala Teynampet, and Ellaiyamman colony. Teynampet zone is also well known for housing the most celebrated residences of political leaders and film stars. Its notable that the house of the late Tamil Nadu Chief Minister and AIADMK Chief J. Jayalaithea is in this zone. Mylapore which has been from time immemorial famous for Hindu Religious lore also belongs to this zone. The zone is noted for the famous temple of Sri Kapaleeswarar, one amongst the traditional temples of South India. The famous Diocesan Church and the Gemini Flyover are also in this region.

Teynampet zone houses two of Chennai's famous arts and science women's college, the Justice Basheer Ahmed Sayeed College (SIET) and the Stella Maris College in Cathedral Road. The Schools in and around Teynampet are SIET Boys and Girls School, DAV Girls and D.A.V Boys School in Gopalapuram, Sri Sarada Matriculation School in Gopalapuram, National Public School (N.P.S) in Gopalapuram, Church Park Convent. Teynampet also houses Anna Arivalaya, which is the head office of the DMK party,

and upscale hotels such as Hyatt Regency and Marriott Courtyard.

In this zone, there are 1785 roads, out of which 1700 roads are Interior Roads, 84 Roads are Bus Route Roads. Also there are six canals in this zone namely the Virugambakkam Canal, the Buckingham canal, the Trustpura Canal, the Mambalam Canal, the Nadanam Canal and the Nugambakkam Canal.

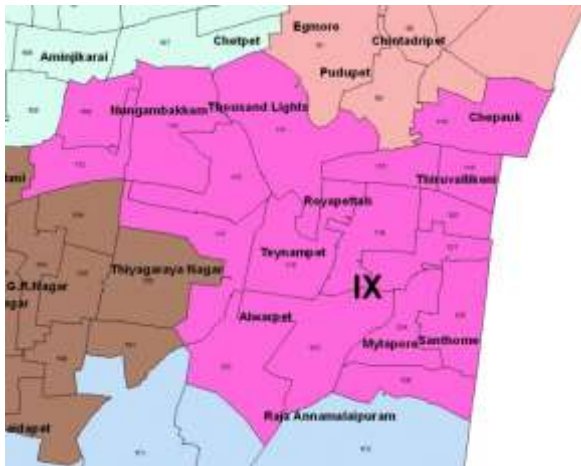


Fig. 1: Ward map of Teynampet (Zone IX)

III. GCA FOR TEYNAMPET

Graph-cellular automata (GCA) is implemented for Teynampet for the population taken as attribute. It is shown that GCA with particular structural properties defined in terms of the relationships between subsets of cells is a useful generalization of traditional cellular automaton (CA).

Each ward is considered as a vertex. Two vertices are connected if the two wards are adjacent on the map. Thus obtaining a ward graph of Teynampet zone (see Fig 2.)



Fig. 2: The graph obtained by the Teynampet zone (highlighted), where each vertices represents the wards and the edges represents their adjacency.

IV. ANALYSIS OF POPULATION GROWTH BASED ON GCA:

Consider, s_i^t is the state value of the ward i at time t , and the cellular automaton aspect of the model is incorporated according to which,

$$s_i^{t+1} = f(s_k^t), k \in [N(i)]$$

where s_k^t is the state values of the in-neighbours of s_i at time t . The state value could be any attributes that signifies certain factors such as populations, workers, transportation, educational institution, migrations, geological factor, etc. Initially we consider the population, which is a major factor in influencing the urbanization. To analyze the growth pattern of population through graph cellular automata model, vary many transition rules had been experimented and among them six certain transition rules have been identified which mimics the growth behavior of wards in Chennai city. The identified transition rules are as follows:

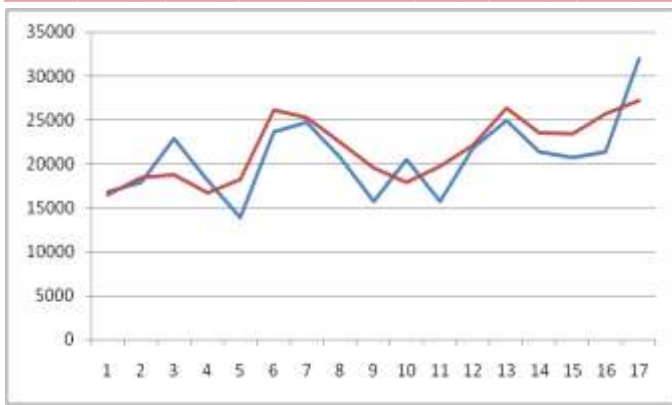
RULE 1: The s_i^{t+1} the population for the ward i at the time $t + 1$, is given by

$$s_i^{t+1} = \frac{\sum_{k \in [N(i)]} (s_k^t)}{|[N(i)]|}$$

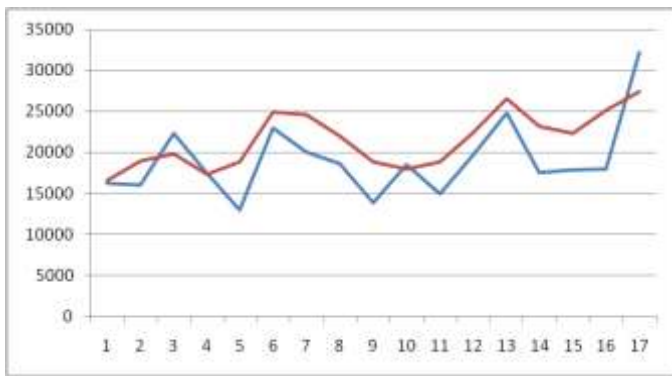
where, s_k^t is the population for the ward k at the time t , $[N(i)]$ represents the closed neighborhood of ward i and $|[N(i)]|$ represents its cardinality.

Ward No.	Ward Name	1991	2001	2011	1991	2001	2011	Ward No.
0086	Chepauk	16057	16763	16185	16442	16615	15505	0086
0089	Triplicane	18494	17896	16014	18395	18990	18301	0089
0091	Krishnampet	22148	22859	22285	18718	19848	19694	0091
0093	Azad Nagar (North)	17689	18171	17368	16710	17380	16766	0093
0096	Vivekanandapuram	13504	13871	12954	18146	18888	18346	0096
0107	Ko Su Mani Nagar	23198	23692	22902	26116	24936	22473	0107
0108	Nakkeerar Nagar	28294	24810	20042	25251	24630	22724	0108
0109	Thousand Lights	21521	20676	18546	22455	22033	20526	0109
0110	Azhagiri Nagar	16190	15671	13867	19438	18912	17329	0110
0111	Amir Mahal	21268	20516	18480	17863	17992	17103	0111
0112	Royapettah	16792	15698	14912	19679	18873	17101	0112
0113	Teynampet	22623	21804	19701	22144	22455	21748	0113
0115	Alwarpet (North)	23925	24992	24775	26364	26578	25508	0115
0146	Santhome	23928	21376	17539	23502	23141	21495	0146
0147	Mylapore	22285	20725	17881	23425	22353	19997	0147
0148	Avvai Nagar (North)	23564	21381	17914	25711	25172	23348	0148
0149	Raia Annamalaiapuram	30678	32050	32138	27218	27398	26294	0149

Table 1. Census and Rule 1 GCA estimated populations for the zone 9(Teynampet)



Graph 1.Census data 2001 (blue) and Rule 1 GCA estimated data from 1991 (red)



Graph 2.Census data 2011 (blue) and Rule 1 GCA estimated data from 2001 (red)

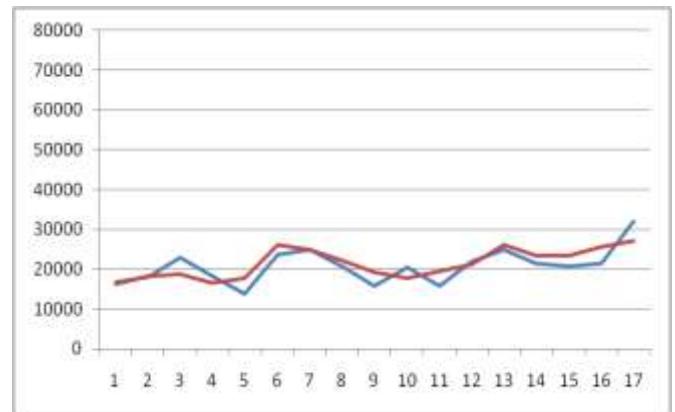
RULE 2: The s_i^{t+1} the population for the ward i at the time $t + 1$, is given by

$$s_i^{t+1} = \left(\prod_{k \in [N(i)]} (s_k^t) \right)^{1/|[N(i)]|}$$

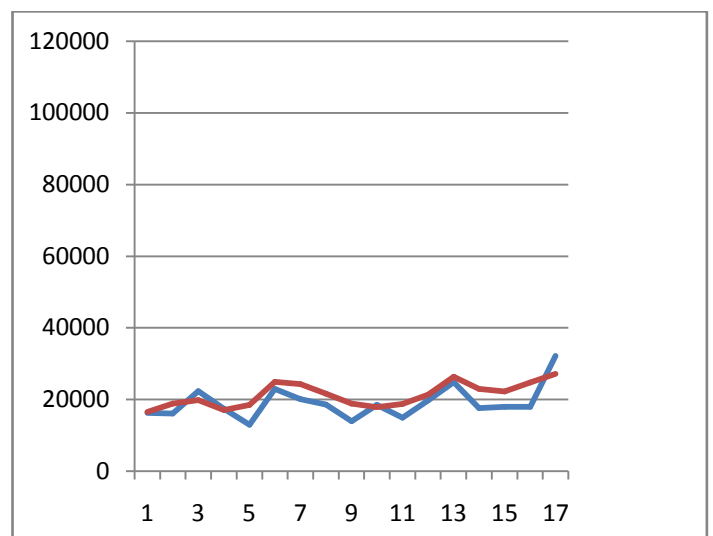
where, s_k^t is the population for the ward k at the time t , $[N(i)]$ represents the closed neighborhood of ward i and $|[N(i)]|$ represents its cardinality.

Ward No.	Ward Name	1991	2001	2011	1991	2001	2011
0086	Chepauk	16057	16763	16185	16290	16478	15337
0089	Triplicane	18494	17896	16014	18305	18847	17998
0091	Krishnampet	22148	22859	22285	18658	19756	19445
0093	Azad Nagar (North)	17689	18171	17368	16422	17062	16324
0096	Vivekanandapuram	13504	13871	12954	17747	18464	17820
0107	Ko Su Mani Nagar	23198	23692	22902	26041	24922	22427
0108	Nakkeerar Nagar	28294	24810	20042	24943	24307	22293
0109	Thousand Lights	21521	20676	18546	22044	21559	19891
0110	Azhagiri Nagar	16190	15671	13867	19319	18772	17192
0111	Amir Mahal	21268	20516	18480	17794	17851	16807
0112	Royapettah	16792	15698	14912	19495	18682	16944
0113	Teynampet	22623	21804	19701	21169	21348	20496
0115	Alwarpet (North)	23925	24992	24775	26184	26343	25153
0146	Santhome	23928	21376	17539	23422	22955	21064
0147	Mylapore	22285	20725	17881	23379	22250	19743
0148	Avvai Nagar (North)	23564	21381	17914	25525	24708	22406
0149	Raja Annamalaipuram	30678	32050	32138	27113	27140	25733

Table 2. Census and Rule 2 GCA estimated populations for the zone 9(Teynampet)



Graph 3.Census data 2001 (blue) and Rule 2 GCA estimated data from 1991 (red)

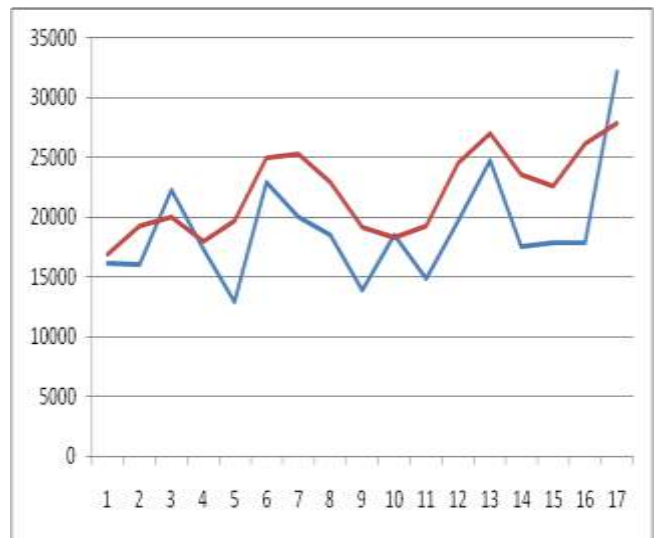


Graph 4.Census data 2011 (blue) and Rule 2 GCA estimated data from 2001 (red)

RULE 3:The s_i^{t+1} the population for the ward i at the time $t + 1$, is given by

$$s_i^{t+1} = \frac{\sum_{k \in [N(i)]} (\mu_k^t \times s_k^t)}{\sum_{k \in [N(i)]} (\mu_k^t)}$$

where, s_k^t is the population for the ward k at the time t , $[N(i)]$ represents the closed neighborhood of ward i and μ_k^t represents membership value between s_k^t and s_i^t . The μ_k^t signifies how much the s_k^t has its influence on s_i^t by considering various parameters such as geological factors, educational institution, industries, transportation etc.



Graph 6. Census data 2011 (blue) and Rule 3 GCA estimated data from 2001 (red)

Ward No.	Ward Name	1991	2001	2011	1991	2001
0086	Chepauk	16057	16763	16185	16715	16859
0089	Triplicane	18494	17896	16014	18586	19289
0091	Krishnampet	22148	22859	22285	18851	20037
0093	Azad Nagar (North)	17689	18171	17368	17263	17984
0096	Vivekanandapuram	13504	13871	12954	18895	19673
0107	Ko Su Mani Nagar	23198	23692	22902	26262	24966
0108	Nakkeerar Nagar	28294	24810	20042	25860	25309
0109	Thousand Lights	21521	20676	18546	23245	22947
0110	Azhagiri Nagar	16190	15671	13867	19670	19181
0111	Amir Mahal	21268	20516	18480	18013	18285
0112	Royapettah	16792	15698	14912	20035	19242
0113	Teynampet	22623	21804	19701	23904	24520
0115	Alwarpet (North)	23925	24992	24775	26738	27066
0146	Santhome	23928	21376	17539	23676	23562
0147	Mylapore	22285	20725	17881	23521	22580
0148	Avvai Nagar (North)	23564	21381	17914	26100	26145
0149	Raja Annamalaipuram	30678	32050	32138	27427	27893

Table 3. Census and Rule 3 GCA estimated populations for the zone 9(Teynampet)

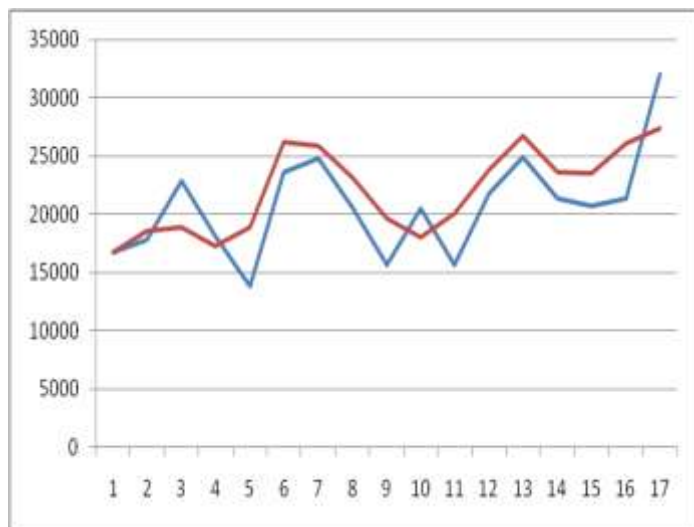
RULE 4:The s_i^{t+1} the population for the ward i at the time $t + 1$, is given by

$$s_i^{t+1} = s_i^t + \frac{\sum_{k \in (N(i))} (\mu_k^t \times s_k^t)}{\sum_{k \in (N(i))} (\mu_k^t)}$$

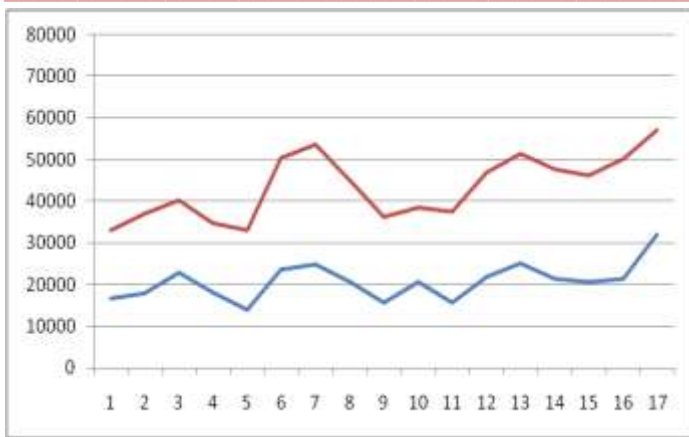
where, s_k^t is the population for the ward k at the time t , $(N(i))$ represents the open neighborhood of ward i and μ_k^t represents membership value between s_k^t and s_i^t . The μ_k^t signifies how much the s_k^t has its influence on s_i^t by considering various parameters such as geological factors, educational institution, industries, transportation etc.

Ward No.	Ward Name	1991	2001	2011	1991	2001	2011
0086	Chepauk	16057	16763	16185	32899	33642	31916
0089	Triplicane	18494	17896	16014	37099	37444	35454
0091	Krishnampet	22148	22859	22285	40189	42225	41989
0093	Azad Nagar (North)	17689	18171	17368	34861	36115	35100
0096	Vivekanandapuram	13504	13871	12954	32952	34130	32898
0107	Ko Su Mani Nagar	23198	23692	22902	50335	49055	45349
0108	Nakkeerar Nagar	28294	24810	20042	53691	50202	44265
0109	Thousand Lights	21521	20676	18546	45000	43924	40911
0110	Azhagiri Nagar	16190	15671	13867	36330	35323	31943
0111	Amir Mahal	21268	20516	18480	38475	38278	36079
0112	Royapettah	16792	15698	14912	37495	35647	32844
0113	Teynampet	22623	21804	19701	46790	46848	44605
0115	Alwarpet (North)	23925	24992	24775	51286	52539	51371
0146	Santhome	23928	21376	17539	47561	45270	40671
0147	Mylapore	22285	20725	17881	46096	43728	39039
0148	Avvai Nagar (North)	23564	21381	17914	50233	48501	44607
0149	Raja Annamalaipuram	30678	32050	32138	57158	58673	57914

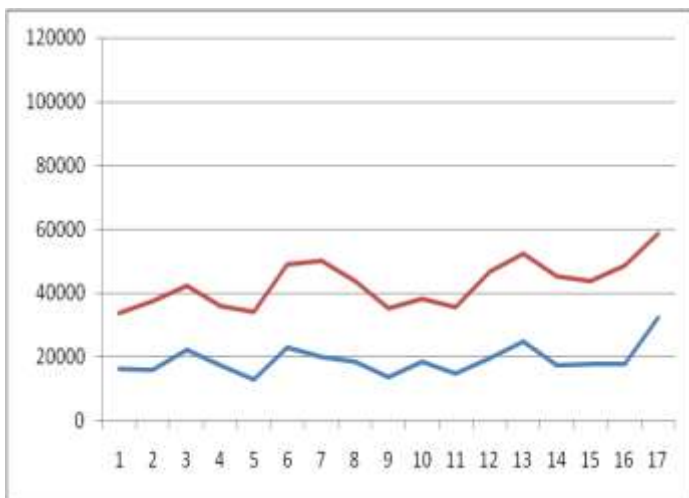
Table 4. Census and Rule 4 GCA estimated populations for the zone 9(Teynampet)



Graph 5. Census data 2001 (blue) and Rule 3 GCA estimated data from 1991 (red)



Graph 7. :Census data 2001 (blue) and Rule 4 GCA estimated data from 1991 (red)



Graph 8.Census data 2011 (blue) and Rule 4 GCA estimated data from 2001 (red)

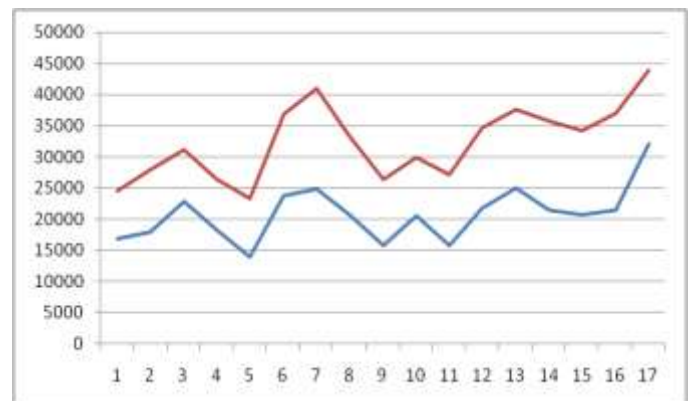
RULE 5:The s_i^{t+1} the population for the ward i at the time $t + 1$, is given by

$$s_i^{t+1} = s_i^t + \frac{\sum_{k \in (N(i))} (\mu_k^t \times s_k^t)}{2 \times \sum_{k \in (N(i))} (\mu_k^t)}$$

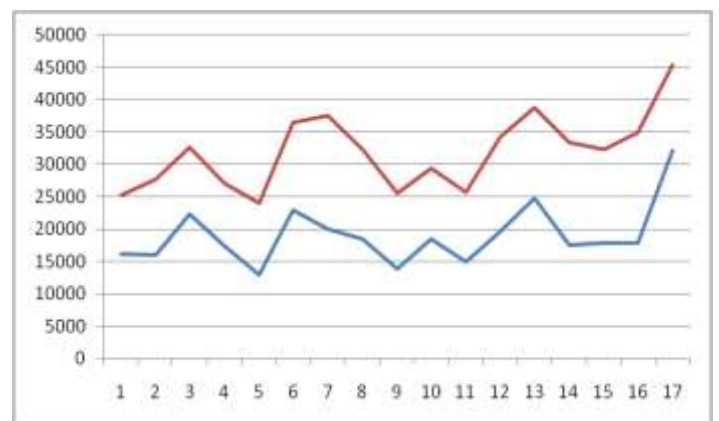
where, s_k^t is the population for the ward k at the time t , $(N(i))$ represents the open neighborhood of ward i and μ_k^t represents membership value between s_k^t and s_i^t . The μ_k^t signifies how much the s_k^t has its influence on s_i^t by considering various parameters such as geological factors, educational institution, industries, transportation etc.

Ward No.	Ward Name	1991	2001	2011	1991	2001	2011
0086	Chepauk	16057	16763	16185	24478	25203	24051
0089	Triplicane	18494	17896	16014	27796	27670	25734
0091	Krishnampet	22148	22859	22285	31168	32542	32137
0093	Azad Nagar (North)	17689	18171	17368	26275	27143	26234
0096	Vivekanandapuram	13504	13871	12954	23228	24001	22926
0107	Ko Su Mani Nagar	23198	23692	22902	36767	36373	34126
0108	Nakkeerar Nagar	28294	24810	20042	40992	37506	32154
0109	Thousand Lights	21521	20676	18546	33261	32300	29729
0110	Azhagiri Nagar	16190	15671	13867	26260	25497	22905
0111	Amir Mahal	21268	20516	18480	29871	29397	27279
0112	Royapettah	16792	15698	14912	27143	25672	23878
0113	Teynampet	22623	21804	19701	34706	34326	32153
0115	Alwarpet (North)	23925	24992	24775	37606	38765	38073
0146	Santhome	23928	21376	17539	35745	33323	29105
0147	Mylapore	22285	20725	17881	34191	32226	28460
0148	Avvai Nagar (North)	23564	21381	17914	36898	34941	31260
0149	Raja Annamalaipuram	30678	32050	32138	43918	45361	45026

Table 5. Census and Rule 5 GCA estimated populations for the zone 9(Teynampet)



Graph 9.Census data 2001 (blue) and Rule 5 GCA estimated data from 1991 (red)

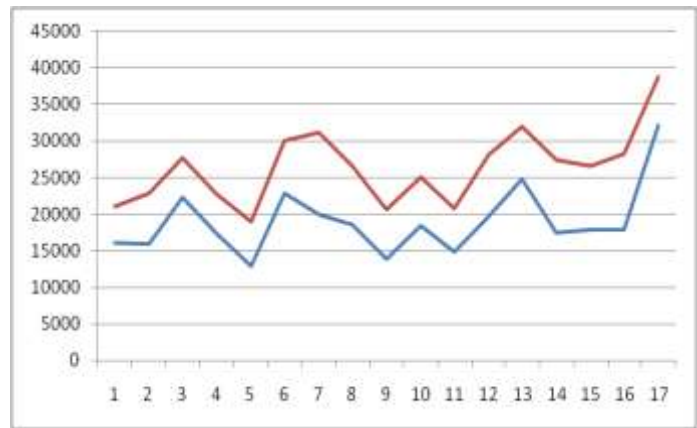


Graph 10.Census data 2011 (blue) and Rule 5 GCA estimated data from 2001 (red)

RULE 6: The s_i^{t+1} the population for the ward i at the time $t + 1$, is given by

$$s_i^{t+1} = s_i^t + \frac{\sum_{k \in (N(i))} (\mu_k^t \times s_k^t)}{4 \times \sum_{k \in (N(i))} (\mu_k^t)}$$

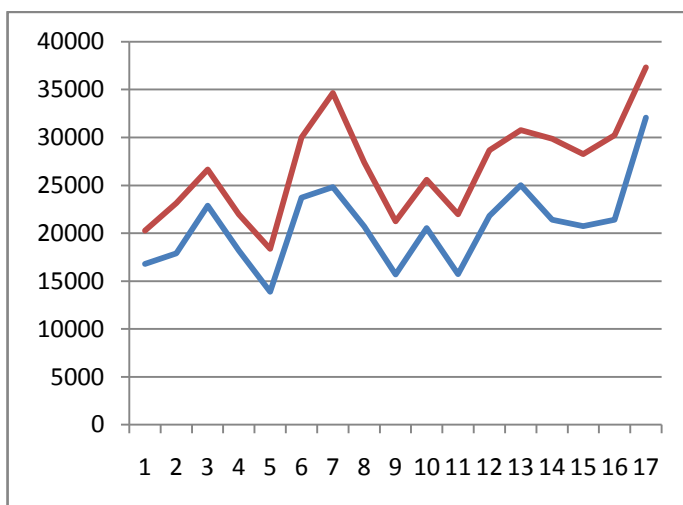
where, s_k^t is the population for the ward k at the time t , $(N(i))$ represents the open neighborhood of ward i and μ_k^t represents membership value between s_k^t and s_i^t . The μ_k^t signifies how much the s_k^t has its influence on s_i^t by considering various parameters such as geological factors, educational institution, industries, transportation etc.



Graph 12. Census data 2011 (blue) and Rule 6 GCA estimated data from 2001 (red)

Ward No.	Ward Name	1991	2001	2011	1991	2001	2011
0086	Chepauk	16057	16763	16185	20268	20983	20118
0089	Triplicane	18494	17896	16014	23145	22783	20874
0091	Krishnampet	22148	22859	22285	26658	27700	27211
0093	Azad Nagar (North)	17689	18171	17368	21982	22657	21801
0096	Vivekanandapuram	13504	13871	12954	18366	18936	17940
0107	Ko Su Mani Nagar	23198	23692	22902	29982	30033	28514
0108	Nakkeerar Nagar	28294	24810	20042	34643	31158	26098
0109	Thousand Lights	21521	20676	18546	27391	26488	24137
0110	Azhagiri Nagar	16190	15671	13867	21225	20584	18386
0111	Amir Mahal	21268	20516	18480	25570	24956	22880
0112	Royapettah	16792	15698	14912	21968	20685	19395
0113	Teynampet	22623	21804	19701	28665	28065	25927
0115	Alwarpet (North)	23925	24992	24775	30765	31879	31424
0146	Santhome	23928	21376	17539	29836	27349	23322
0147	Mylapore	22285	20725	17881	28238	26476	23170
0148	Avvai Nagar (North)	23564	21381	17914	30231	28161	24587
0149	Raja Annamalaiapuram	30678	32050	32138	37298	38706	38582

Table 6. Census and Rule 6 GCA estimated populations for the zone 9(Teynampet)



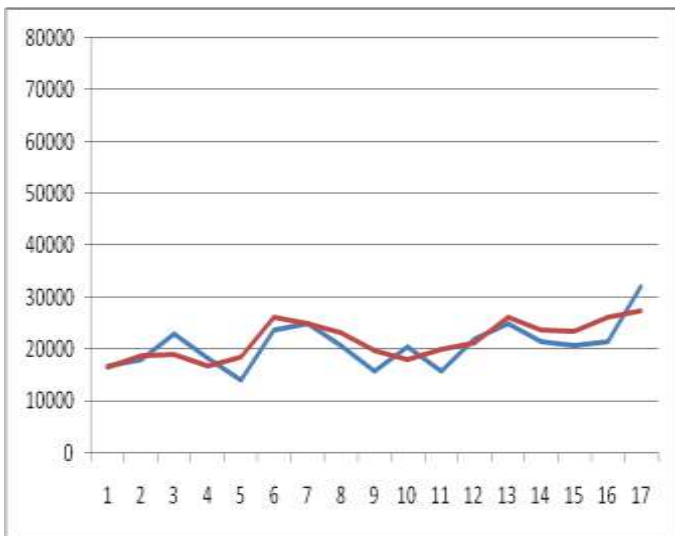
Graph 11. Census data 2001 (blue) and Rule 6 GCA estimated data from 1991 (red)

TRANSITION RULES:

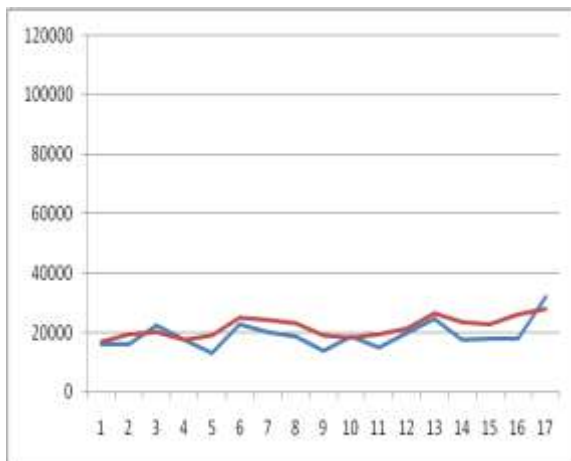
From the above estimation it is observed that certain GCA rule attain more accuracy for certain wards. Considering all possible decadal comparison with all possible GCA rule, we the best GCA rule is assigned to the particular wards. The following table 7 illustrates the rule assignments. The Graph 13, 14 that the estimated data almost coincides with the actual census data.

Ward No.	Ward Name	Transition Rule No	Actual census			Best Estimation		
			1991	2001	2011	1991	2001	2011
0086	Chepauk	1	16057	16763	16185	16442	16615	15505
0089	Triplicane	2	18494	17896	16014	18586	19289	18940
0091	Krishnampet	2	22148	22859	22285	18851	20037	20191
0093	Azad Nagar (North)	1	17689	18171	17368	16710	17380	16766
0096	Vivekanandapuram	6	13504	13871	12954	18366	18936	17940
0107	Ko Su Mani Nagar	2	23198	23692	22902	26262	24966	22563
0108	Nakkeerar Nagar	2	28294	24810	20042	24943	24307	22293
0109	Thousand Lights	2	21521	20676	18546	23245	22947	21934
0110	Azhagiri Nagar	2	16190	15671	13867	19670	19181	17594
0111	Amir Mahal	2	21268	20516	18480	18013	18285	17757
0112	Royapettah	2	16792	15698	14912	20035	19242	17405
0113	Teynampet	2	22623	21804	19701	21169	21348	20496
0115	Alwarpet (North)	2	23925	24992	24775	26184	26343	25153
0146	Santhome	2	23928	21376	17539	23676	23562	22480
0147	Mylapore	2	22285	20725	17881	23521	22580	20572
0148	Avvai Nagar (North)	2	23564	21381	17914	26100	26145	25346
0149	Raja Annamalaiapuram	2	30678	32050	32138	27427	27893	27331

Table 7. Census and Rule 6 GCA estimated populations for the zone 9(Teynampet)



Graph 13. :Census data 2001 (blue) and assigned transition GCA rules estimated data from 1991 (red)



Graph 14. Census data 2011 (blue) and assigned transition GCA rules estimated data from 2001 (red)

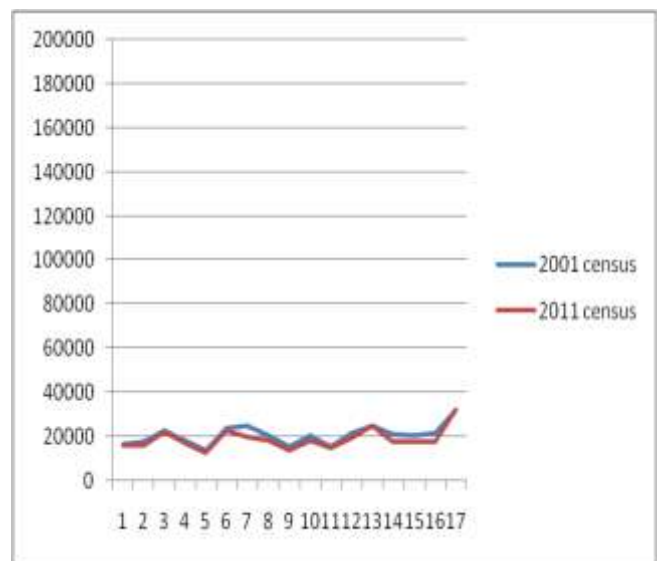
With respect to the assigned transition rule, the estimated data for few decades are obtained. With which the polynomials which describes the growth pattern is computed using the following function. The years were considered as x_i and the estimated population were considered its corresponding f_i .

$$f(x) = \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)} f_1 + \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)} f_2 + \frac{(x - x_1)(x - x_2)}{(x_3 - x_2)(x_3 - x_1)} f_3$$

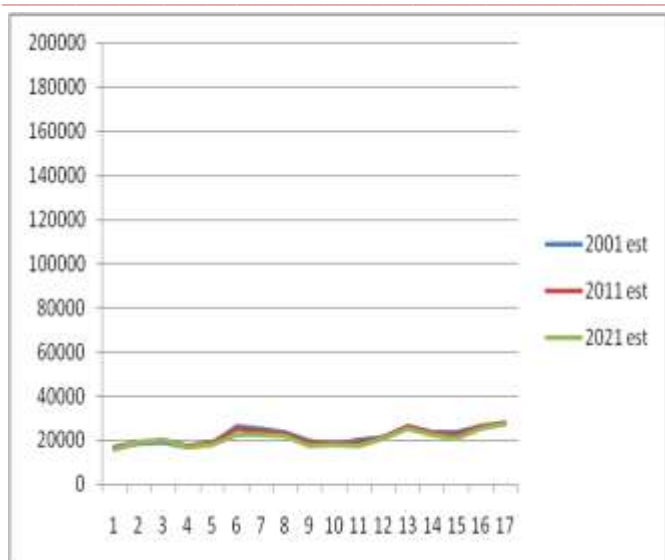
The obtained growth polynomial for each of the wards from their respective transition GCA rules is given below:

No	Ward name	Polynomial
0086	Chepauk	$-5.97x^2 + 23826.68x - 23776461.72$
0089	Triplicane	$-5.26x^2 + 21068.22x - 21077173.96$
0091	Krishnampet	$-5.16x^2 + 20717.32x - 20774675.16$
0093	Azad Nagar (North)	$-5.18x^2 + 20750.66x - 20763361.48$
0096	Vivekanandapuram	$-7.83x^2 + 31314.36x - 31289770.53$
0107	Ko Su Mani Nagar	$-5.54x^2 + 21966.12x - 21767094.58$
0108	Nakkeerar Nagar	$-5.30x^2 + 21122.46x - 20999508.16$
0109	Thousand Lights	$-3.58x^2 + 14241.60x - 14160191.03$
0110	Azhagiri Nagar	$-5.49x^2 + 21867.18x - 21755080.69$
0111	Amir Mahal	$-4.00x^2 + 15995.20x - 15972106.20$
0112	Royapettah	$-5.22x^2 + 20758.94x - 20618511.72$
0113	Teynampet	$-5.09x^2 + 20380.88x - 20377255.79$
0114	Sathyamurthy Nagar	$-5.22x^2 + 20785.44x - 20665303.22$
0115	Alwarpet (North)	$-5.76x^2 + 23026.72x - 22986354.96$
0146	Santhome	$-4.84x^2 + 19309.88x - 19236143.04$
0147	Mylapore	$-5.34x^2 + 21203.22x - 21043717.89$
0148	Avvai Nagar (North)	$-4.22x^2 + 16850.74x - 16795301.52$
0149	Raja Annamalaipuram	$-5.14x^2 + 20565.48x - 20543067.34$

Table 8. Population growth polynomial ward-wise for the zone 9(Teynampet)



Graph 15. Census 2001 and 2011 for zone 9(Teynampet)



Graph 16. Estimated data from GCA transition rules for zone 9(Teynampet)

Further, the increase in error for few wards which was observed during analysis were due to influence of various other factors such as, the wards lying on the peripheral of Chennai city, or influence of other factors such as industries, educational institution, transportation, etc.

V. CONCLUSION

It is found that the results obtained for Chennai populations using GCA and that of the actual census data coincides almost similar. The GCA modeling is more appropriate for investigation. Further Fuzzy GCA would be studied as future work. Also GCA for other causal attribute would be further analyzed.

VI. ACKNOWLEDGEMENT

This work is supported by Project No. 5LCTOI14MAT0021, Loyola College - Times of India, Chennai, India.

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