An Integrated Single Vendor-Buyer Stochastic Inventory Model with Partial Backordering under Imperfect Production and Carbon Emissions

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Abstract: This paper develops an integrated single vendor single buyer inventory model with imperfect quality and environmental impact. The demand during lead time is assumed to be stochastic and follows the normal distribution. An integrated system with controllable lead time and logarithmic investment to reduce the defective percentage is discussed in this model.100% error-free screening process is adopted by the buyer to separate defective and non-defective items. We assume that shortages are allowed and are partially backordered at the buyer's end. Logistics management is the component of supply chain management that focusses on how and when to get raw materials, intermediate products and finished goods from their respective origins to their destinations.Thus, transportation play a major role in supply chain. As transportation increases, it affects the weather by the matter of carbon emission.The fixed and variable carbon emission cost for both vendor and buyer is considered. The prime motive is to determine the optimal policies regarding optimal order quantity, reorder point, lead time and the number of lots delivered in a production run by minimizing the expected total cost of the system. Finally, a numerical example is provided to demonstrate the model.

Keywords: *Integrated model, Defective items, Partially backordered, Controllable lead time, Investment, Environmental impact, Carbon emission.*

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I. INTRODUCTION

In recent years, most inventory problems have their focus on the integration between the vendor and the buyer. For supply chain management, establishing long term strategic partnerships between the buyer and the vendor is advantageous for the two parties regarding costs, and therefore profits since both parties, to achieve improved benefits, cooperate and share information with each other. In classic inventory model, it is assumed that the items produced are of perfect quality. However, in the real life production environment, it can often be observed that there are defective items being produced due to imperfect production process. The defective items must be rejected, repaired, reworked, or if they have reached the customer, refunded.

In many real-life conditions , stock out is unavoidable because of various uncertainties in the related system. Therefore occurrence of shortages in inventory could be considered as a natural phenomenon. In a retail store, when an item is out of stock for a particular item, we obtain a rain check and wait to get the order filled. In market situations where the demand is not quite static backorders are often incurred due to demand uncertainty and variation. In treating the excessive demands, companies usually employ one of the following methods: place an emergency order to fill all the backorders; purchase the items from competitors; or satisfy a certain number of backorders and the rest is filled by purchasing from competitors. These situations are described in inventory texts as complete backordering, complete lost sales and partial backordering, respectively. Due to presence of imperfect items, shortages cannot be ignored. It seems to be more reasonable to assume that only a fraction of customers are ready to wait during stock out period for their delivery while the rest of go to other place.Therefore taking this in account, it is considered that shortage during stockout period is partially backordered.

One of the ways of measuring the amount of CO_2 emission is to consider factors such as distance travelled, vehicle type and gross vehicle weight. In order to determine CO_2 emission in transporting inventory, we consider distance between the vendor and buyer; and fuel efficiency and CO_2 emissions per gallon that depend on vehicle type, vehicle age and average speed. We categorize the CO_2 emission cost into fixed and variable. The fixed cost depends on several factors: distance between the vendor and buyer (eg., in miles), fuel efficiency (eg., in miles per gallon) and CO_2 emission per gallon(eg., in pounds per gallon). From this information one would be able to determine the emission for the forward as well as for the reverse supply chain. Then multiplying them with the emission cost per pound of CO_2, the fixed emission costs can be determined. The variable emission cost depends on actual weight of the shipment, which would be proportional to the shipment size.

The rest of the paper is structured as follows. Section 2 describes the relevant literature. Section 3 presents the fundamental notations and assumptions. Section 4 provides the mathematical model. A numerical example is illustrated in Section 5. The paper concludes in Section 6.A list of referencs is also provided.

II. LITERATURE REVIEW

Nowadays, the companies understand that the inventories across the supply chain can be managed efficiently by improving the cooperation and collaboration between parties in this system. Therefore, the cooperative inventory model involving vandor and buyer has been received a great deal of attention. An integrated inventory model for a single supplier-single buyer problem was first developed by Goyal (1988). Banerjee (1986) enhanced Goyal (1988) model under the assumption of lot-for-lot basis. Then Goyal (1988) extended Banerjee (1986) model and suggested that the vendor's economic production quantity per cycle should be a positive integer multiple of the buyer's purchase quantity. Several authors (e.g., Amasaka 2002; Ben-Daya and Raouf 1994; Bylka 2003, etc.) have performed on integrated inventory problems with different parameters and assumptions. The integrated inventory model by taking the annual customer demand to be stochastic and allowing shortages was studied by Ben-Daya and Hariga (2004). Priyan and Uthayakumar (2015) established a multi item multi-constraint product returns integrated inventory model with permissible delay in payments and variable lead time.Tersine (1982) considered the controllable lead time and it became a prominent role in the related literatures on inventory models. Lead time usually consists of the following components such as setup time, order preparation, production time, order transit, supplier lead time and delivery time, etc as in Tersine (1982). Ouyang et al. (2007) developed lead time reduction inventory models under various crashing cost function and practical situations. Pan and Yang (2002) extended the model of Goyal (1988) by considering lead time as a controllable factor and obtained a lower joint total expected cost and shorter lead time.

Traditional inventory models assume that all the items produced through a production process are of good quality/perfect. But in many manufacturing systems this assumption is not valid as some defective items often exist due to imperfect production process. During the production process, some items may be deteriorating, due to human mistakes, machine faults, etc. Huang (2002, 2004) flourished an integrated vendor-buyer cooperative inventory model with imperfect quality under equal lot size delivery policy and assumed that the number of defective items follows a given probability density function. Dey and Giri (2014) implemented the logarithmic and power investment as in Porteus (1986) to reduce the defectiveness by assuming the imperfect production process. The defectiveness is assumed as a random variable and demand follows unknown distribution in Lin (2012).

When we receive the lot size consisting of defective and non-defective products, it is necessary to implement screening process to separate them. Ouyang et al. (2004) investigate an integrated inventory model with fixed finite defective rate, and assumed that the buyer performs 100% screening process immediately on receiving a lot. Priyan and Uthayakumar (2015) and Khan et al. (2014) investigate the imperfect screening process. That is the buyer may misclassified the defective items as non-defective and vice versa. Holding cost is considered both for defective and non-defective items. The consumer give much importance to the usable products, not for the deteriorated products after the separation. Lin (2012) and Dey and Giri (2014) keeps the inventories at the different holding costs. Porteus (1985, 1986) developed a framework for investigating in reducing EOQ model setup cost. The logarithmic and power investment functions were implemented by Lin (2009) into the cost function to reduce the ordering cost. Further, he compared the optimal solutions from both investment functions and they conclude that which is the best investment for the given parameter. Ouyang et al. (2004) utilized the logarithmic investment function to reduce the production process can go out of control. Priyan and Uthayakumar (2014) created the trade credit financing in the vendor-buyer inventory system with ordering cost reduction. Annadurai (2016) developed the inventory model with variable lead time and a service level constraint by reducing order cost by using the logarithmic investment. The main purpose for using these investment functions is to reduce the parameter value so that the total cost gets minimized. Ivan Darma Wangsa and Hui Ming Wee (2017) proposed an integrated vendor-buyer inventory model with partial backordering and stochastic demand. Lin (2012) applies the minimax distribution free approach to determine all the decision variables (Q, k, l, n) and he assumed that the shortages to be partially backlogged. As there is increasing trend to develop environmentally sustainable supply chains, companies expect to implement green supply chain by concerning environmental impact as in Wahab et al (2011). Ganesh kumar and Uthaya kumar (2017) developed an integrated single vendor single buyer inventory model with imperfect quality and considered logarithmic and power investment functions to reduce the defectiveness.

This paper is an extension of " **An integrated single vendor-buyer inventory model for imperfect production process with stochastic demand in**

controllable lead time" by M. Ganesh Kumar and R. Uthayakumar (2017). In this paper an integrated single vendor-buyer inventory model is considered for an imperfect production with controllable lead time and the demand follows normal distribution. Shortages are assumed to be partially backordered; Fixed and variable emission costs are considered.

III. NOTATIONS AND ASSUMPTIONS

To develop the proposed model , the following notations and assumptions are used.

Notations

D	Expected Demand Rate
P	Production Rate $\left(P = \frac{1}{p}\right)$
Q	Order Quantity (A Decision Variable)
n	Number Of Shipments (A Decision Variable)
A	Buyer's Ordering Cost Per Order
B	Vendor's Setup Cost Per Setup
F	Transportation Cost Per Shipment
L	Length Of The Lead Time (A Decision Variable)
h_v	Vendor's Holding Cost Per Item
h_{b1}	Buyer's Holding Cost For Defective Items Per Item
h_{b2}	Buyer's Holding Cost For Non-Defective Items Per Item
r	Reorder Level (A Decision Variable)
s	Buyer's Unit Screening Cost
S	Buyer's Screening Rate
w	Vendor's Unit Warranty Cost For Defective Items
n	Number Of Shipments (A Decision Variable)
y_0	Original Percentage Of Defective Items Produced (Before Any Investment Is Made)
y	Percentage Of Defective Items (A Decision Variable)
K_b	The Fixed Emission Costs In The Forward Supply Chain.
K_v	The Fixed Emission Costs In The Reverse Supply Chain
L_b	The Variable Emission Costs Per Item In The Forward Supply Chain
L_v	The Variable Emission Costs Per Item In The Reverse Supply Chain
π_1	Buyer's Shortage Cost Per Unit Short
π_2	Buyer's Marginal Profit (I.E., Cost Of Lost Demand) Per Unit
β	Fraction Of The Demand During The Stock-Out Period That Will Be Backordered, $\beta \in [0,1]$
η	Fractional Opportunity Cost
$I(y)$	Capital Investment Required To Achieve Y
X	Lead Time Demand Which Has A Cumulative Distribution Function Φ With Finite Mean DL And Standard Deviation $\sigma\sqrt{L}$, Where σ Denotes The Standard Deviation Of The Demand Per Unit Time.
$E(\cdot)$	Mathematical Expectation
x^+	Maximum Value Of x And 0
$*$	The Superscripts Representing Optimal Values.

Assumptions

The following assumptions are used in this model:

1. The system involving single-vendor and single-buyer belongs to different corporate entities and both interested to minimize the total expected cost of the system.
2. The classical (Q, r) continuous review inventory policy is applied and the buyer faces a stochastic demand.
3. The vendor produces nQ non-defective items and transfers them to the buyer in n equal shipments, where n is a positive integer.
4. The lead time consists of m mutually independent components. The ith component has a normal duration b_i, minimum duration a_i , and crashing cost per unit time c_i, such that $c_1 \le c_2 \le \cdots \le c_m$.The components of lead time are crashed one at a time starting from the first component because it has the minimum unit crashing cost, and then the second component, and so on. Let $L_0 = \sum_{i=1}^{m} b_i$, and L_i be the length of the lead time with components $1,2,\ldots,i$ crashed to their minimum duration, then L_i can be expressed as $L_i = \sum_{i=1}^{m} b_i - \sum_{j=1}^{i}(b_j - a_j)$; $i = 1,2,\ldots,m$; and lead time crashing cost $C(L)$ per cycle is given by $C(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j\left(b_j - a_j\right), L \in [L_i, L_{i-1}]$.
5. The lead time demand X is normally distributed with finite mean DL, standard deviation $\sigma\sqrt{L}$. The reorder point $r =$ expected lead time demand $+$ safety stock $= DL + k\sigma\sqrt{L}$ where $k\sigma\sqrt{L}$ is the safety stock and k is the safety factor satisfies

$P(X > r) = q$, q represents the allowable stock out probability during L.

6. Shortages are allowed and are partially backlogged. The expected shortage quantity is $E(X - r)^+$.
7. The percentage of defective items produced in each batch size Q is $y\ (0 < y < 1)$.
8. The non-defective production rate is greater than the demand rate for the vendor. i.e., $(1 - y) > D$.
9. The screening rate S is greater than the demand rate. i.e., $S > D$.
10. Warranty cost is paid by the vendor for each defective item.

IV. MATHEMATICAL MODEL

The buyer places an order of size nQ for non-defective item to the vendor. The vendor produces nQ items and transfers n batches of Q items each at regular intervals of $Q(1 - y)/D$ units of time on an average in order to reduce the production cost. The length of the complete production cycle is $nQ(1 - y)/D$. The defective items nQy are shipped back to the vendor at the next shipment.

Buyer's perspective

As soon as the inventory of non-defective items reaches the level called the re-order point r, the buyer places an order of size Q for non-defective items to the vendor. The order quantity Q have y percentage of defectiveness. The buyer inspects the items at a fixed screening rate S.

The buyer's average inventory level for non-defective items (including those defective items which have not been detected before the end of the screening time Q/S) is given by

$$\frac{nQ(1-y)}{D}\left[r - DL + \frac{Q(1-y)}{2} + \frac{DQy}{2S(1-y)} + (1-\beta)E(X-r)^+\right]$$

. The average inventory level for defective items is given by $Q^2 y\left[\frac{1-y}{D} - \frac{1}{2S}\right]$.

The expected shortage quantity at the end of the cycle is given by $E(X - r)^+$.Thus, the expected number of backorders per ordering cycle is $\beta E(X - r)^+$ and the expected loss in sales per ordering cycle is $(1 - \beta EX - r+$. Thus the stock out cost per ordering cycle is $[\pi_1\beta + \pi_2(1 - \beta)]E(X - r)^+$.

The annual cost for the buyer including ordering cost, holding cost, screening cost, shortage cost, emission cost, lead time crashing cost for given $L \in [L_i, L_{i-1}]$ is given by

$$\begin{aligned} TCB(Q,n,y,L) = {} & \frac{D(A+nF)}{nQ(1-y)} + \frac{D(K_b + L_b Q)}{Q(1-y)} \\ & + h_{b1}\left[Qy - \frac{DQy}{2S(1-y)}\right] \\ & + h_{b2}\left[r - DL + \frac{Q(1-y)}{2} + \frac{DQy}{2S(1-y)} \right. \\ & \left. + (1-\beta)E(X-r)^+\right] \\ & + \frac{[\pi_1\beta + \pi_2(1-\beta)]D}{Q(1-y)}E(X-r)^+ \\ & + \frac{sD}{(1-y)} + \frac{D}{Q(1-y)}C(L) \end{aligned}$$

(1)

Vendor's perspective

During the production period, when the first Q units have been produced, the vendor will deliver them to the buyer, after that vendor will make a delivery on average every time interval $T = \frac{Q(1-y)}{D}$.

The vendor's average holding cost is $h_v \frac{Q}{2}\Big[n\Big(1 -$ $Dp1-y-1+2Dp1-y$. The vendor reduce the defective percentage by invest some amount to buy new equipment, improving machine maintenance and repair, worker training etc.The logarithmic investment $I(y)$ to reduce the defective percentage is given by $I(y) = \frac{1}{\delta}ln\left(\frac{y_0}{y}\right)$ for $0 < y \le y_0 < 1$, where δ is the percentage decrease in y per dollar increase in the investment $I(y)$.

The total annual cost incurred by the vendor is obtained as the sum of setup cost, holding cost, warranty cost for the defective items and investment is given by

$$TCV(Q,n,y) = \frac{D(B+nK_v)}{nQ(1-y)} + h_v\frac{Q}{2}\left[n\left(1 - \frac{Dp}{1-y}\right) - 1 + \frac{2Dp}{1-y}\right] + \eta I(y) + \frac{Dy(L_v+w)}{1-y} \qquad (2)$$

Integrated approach

The annual total cost of the integrated system is the sum of the vendor and the buyer's total costs and is given by

$$\begin{aligned} TC(Q,n,L,y) &= TCB(Q,n,L,y) + TCV(Q,n,y) \\ &= \frac{D(A+B+nF+nK_b+nK_v)}{nQ(1-y)} \\ &+ h_{b1}\left[Qy - \frac{DQy}{2S(1-y)}\right] \\ &+ h_{b2}\left[r - DL + \frac{Q(1-y)}{2} + \frac{DQy}{2S(1-y)} + (1-\beta)E(X-r)^+\right] \\ &+ h_v\frac{Q}{2}\left[n\left(1-\frac{Dp}{1-y}\right) - 1 + \frac{2Dp}{1-y}\right] \\ &+ \frac{[\pi_1\beta + \pi_2(1-\beta)]D}{Q(1-y)}E(X-r)^+ \\ &+ \frac{(s+L_b+wy+L_v y)}{(1-y)}D \\ &+ \frac{D}{Q(1-y)}C(L) + \frac{\eta}{\delta}ln\left(\frac{y_0}{y}\right). \end{aligned}$$

(3)

Where $E(X-r)^+ = \int_r^\infty (x-r)d\Phi(x)$

$=\int_r^\infty (x-r)\,f(x)dx$ where $f(x)$ is the probability density function of X

$=\sigma\sqrt{L}\psi(k)$

(4)

Where $\psi(k) = \int_k^\infty (z-k)\,dz = \phi(k) - k\left(1-\Phi(k)\right)$ and ϕ,Φ denote the standard normal probability function and cumulative distribution function respectively.

Then the annual joint expected total cost is

$$\begin{aligned} ETC(Q,n,L,y) &= \frac{D(A+B+nF+nK_b+nK_v)}{nQ(1-y)} \\ &+ h_{b1}\left[Qy - \frac{DQy}{2S(1-y)}\right] \\ &+ h_{b2}\left[k\sigma\sqrt{L} + \frac{Q(1-y)}{2} + \frac{DQy}{2S(1-y)} + (1-\beta)\sigma\sqrt{L}\psi(k)\right] \\ &+ h_v\frac{Q}{2}\left[n\left(1-\frac{Dp}{(1-y)}\right) - 1 + \frac{2Dp}{(1-y)}\right] \\ &+ \frac{[\pi_1\beta + \pi_2(1-\beta)]D}{Q(1-y)}\sigma\sqrt{L}\psi(k) \\ &+ \frac{(s+L_b+wy+L_v y)}{(1-y)}D \\ &+ \frac{D}{Q(1-y)}C(L) + \frac{\eta}{\delta}ln\left(\frac{y_0}{y}\right), \end{aligned}$$

subject to $0 < y \le y_0 < 1$

(5)

For a fixed integer n, let us take the partial derivatives of $ETC(Q,n,L,y)$ with respect to Q and y for a given $L \in [L_i, L_{i-1}]$ and equating to zero,we obtain

$$Q = \sqrt{\frac{DG(n) + [\pi_1\beta + \pi_2(1-\beta)]D\sigma\sqrt{L}\psi(k) + DC(L)}{H(n,y)}}$$

(6)

Where

$$G(n) = \frac{(A+B+nF+nK_b+nK_v)}{n}$$

And

$$H(n,y) = h_{b1}\left[y(1-y) - \frac{Dy}{2S}\right] + h_{b2}\left[\frac{(1-y)^2}{2} + \frac{Dy}{2S}\right] + \frac{h_v}{2}[(n-1)(1-y) - (n-2)Dp]$$

Next equating to zero the first derivative of $ETC(Q,n,L,y)$ with respect to y,

$$\begin{aligned} \frac{\partial}{\partial y}ETC(Q,n,L,y) &= \frac{D(L_v+w)}{1-y} + \frac{D(L_b+s+L_v y+wy)}{(1-y)^2} \\ &- \frac{\eta}{y\delta} + \frac{DG(n)}{Q(1-y)^2} \\ &+ Qh_{b1}\left[1 - \frac{D}{2S(1-y)}\right] - \frac{DQyh_{b1}}{2S(1-y)^2} \\ &+ h_{b2}\left[-\frac{Q}{2} + \frac{DQ}{2S(1-y)} + \frac{DQy}{2S(1-y)^2}\right] \\ &+ Q\frac{h_v}{2}\left[\frac{2Dp}{(1-y)^2} - \frac{Dnp}{(1-y)^2}\right] \\ &+ \frac{[\pi_1\beta + \pi_2(1-\beta)]D\sigma\sqrt{L}\psi(k)}{Q(1-y)^2} \\ &+ \frac{DC(L)}{(1-y)^2} = 0 \end{aligned}$$

(7)

The optimal value of n (denoted by n^*) can be obtained when

$$ETC(Q,n^*-1,L,y) \ge ETC(Q,n^*,L,y) \le ETC(Q,n^*+1,L,y).$$

The optimal solution of Q and y for given $L \in [L_i, L_{i-1}]$ and n can be obtained using (6) and solving (7) iterating until convergence.The following algorithm is used to find the optimal values of the order quantity,reorder level, process quality, lead time and number of shipments.

Algorithm:

Step 1. Set n=1.

Step 2. For each L_i perform (Step 2.1.) to (Step 2.3.), $i = 0,1,2,...,m$

Step 2.1. Start with $y_{i1} = y_0$.

Step 2.2. Substituting y_{i1} in (6) to evaluate Q_{i1}.

Step 2.3. Utilizing Q_{i1} to determine y_{i2} from (7).

Step 2.4. Repeat (Step 2.1.) to (Step 2.3.) until no change occus in the values of Q_i and y_i.

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Step 2.5. Compare y_i and y_0. If $y_i < y_0$, then the current solution is optimal for given L_i. Denote the solutions by $(Q_i{}^*, y_i{}^*)$. If $y_i \geq y_0$, then take $y_i = y_0$. And utilize (6) to determine the new $Q_i{}^*$ by procedure similar to (Step 2.1.) to (Step 2.3.) in Step 2. Result is denoted by (Q_i^*, y_i^*).

Step 2.6. Utilize (5) to calculate the corresponding $ETC(Q^*, n, L, y^*)$.

Step 3. Find $\min_{i=0,1,2,\dots,m} ETC(Q_i{}^*, n, L, y_i{}^*)$. Let $ETC\left(Q_{(n)}^*, n, L_{(n)}, y_{(n)}^*\right) = \min_{i=0,1,2,\dots,m} ETC(Q_i^*, n, L, y_i^*)$ then $\left(Q_{(n)}^*, L_{(n)}, y_{(n)}^*\right)$ is the optimal solution for fixed n.

Step 4. Set n by $n+1$ repeat steps 2 to 3 to get $ETC\left(Q_{(n)}^*, n, L_{(n)}; y_{(n)}^*\right)$.

Step 5. If $ETC\left(Q_{(n)}^*, n, L_{(n)}; y_{(n)}^*\right) \leq ETC\left(Q_{(n-1)}^*, n-1, L_{n-1}; y_{n-1}*\right.$ then go to Step 4, otherwise go to step 6..

Step 6. Set $(Q^*, n^*, L^*; y^*) = \left(Q_{(n-1)}^*, n-1, L_{n-1}; y_{n-1}*\right.$ and $Q*, n*, L*; y*$ is the optimal solution.

V. NUMERICAL EXAMPLE

The numerical example is given to illustrate the above procedure in this section.Let us consider the following data for finding the result.

$D = 600$ units per year, $P = 2000$ units per year, $A = \$200$ per order, $B = 1500$ per set-up, $\eta = \$0.1$ per dollar per year, $h_{b1} = \$15$ per unit, $h_{b2} = \$25$ per unit, $h_v = \$20$ per unit, $w = \$25$ per unit, $S = 175200$ units per unit time, $F = \$35$ per shipment, $s = \$0.5$ per unit, $y = 0.22$, $\pi_1 = 30$, $\pi_2 = 50$, $\beta = 0.5$, $K_b = 4$, $K_v = 100$, $L_b = 0.5$, $L_v = 10$, $\sigma = 7$ units per week, where 1 year = 52 weeks and the lead time has three components with the data as shown in Table 1.

Table 1 Lead time components with data

Lead time component i	Normal duration b_i(days)	Minimum duration a_i (days)	Unit crashing cost c_i (\$/ $days$
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

The lead time demand follows a normal distribution and the capital investment $I(y)$ is described by logarithmic function.We solve the cass for $q = 0.2$ (in this situation, the value of safety factor k can be found directly from the standard normal table, and is 0.845), and $\delta = 0.0002$.The initial defective percentage $y_0 = 0.22$.

Thus $\boldsymbol{n^* = 3}$, $\boldsymbol{L^* = 4}$, $\boldsymbol{Q^* = 128}$, $\boldsymbol{y^* = 0.0191}$,$\boldsymbol{I(y)}$=1222, $\boldsymbol{ETC^* = 10105}$ as shown in Table 2. The optimal reorder point when $L^* = 4$ is $\boldsymbol{r^* = 58}$ by using the relation $r^* = DL^* + k\sigma\sqrt{L^*}$.

Table 2 The optimal solutions are given as follows (L^* in weeks)

n	L^*	Q^*	y^*	$I(y)$	ETC^*
1	4	274	0.0177	1260	11125
2	4	171	0.0187	1233	10251
3	**4**	**128**	**0.0191**	**1222**	**10105**
4	4	104	0.0193	1217	10157

VI. CONCLUSION

An integrated inventory model with imperfect production process and stochastic demand is discussed. Shortages are allowed and partially backordered. The vendor makes logarithmic investment in improving the production process quality. Environmental impact is incorporated by taking into account the fixed and variable carbon emission costs. A numerical example is provided to support the model.

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