Performance Assessment of Polyphase Sequences Using Cyclic Algorithm

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Abstract—Polyphase Sequences (known as P1, P2, Px, Frank) exist for a square integer length with good auto correlation properties are helpful in the several applications. Unlike the Barker and Binary Sequences which exist for certain length and exhibits a maximum of two digit merit factor. The Integrated Sidelobe level (ISL) is often used to define excellence of the autocorrelation properties of given Polyphase sequence. In this paper, we present the application of Cyclic Algorithm named CA which minimizes the ISL (Integrated Sidelobe Level) related metric which in turn improve the Merit factor to a greater extent is main thing in applications like RADAR, SONAR and communications. To illustrate the performance of the P1, P2, Px, Frank sequences when cyclic Algorithm is applied. we presented a number of examples for integer lengths. CA(Px) sequence exhibits the good Merit Factor among all the Polyphase sequences that are considered.

Keywords-*Polyphase sequence, Cyclic Algorithm, Correlation level, Integrated sidelobe level, Merit Factor,*

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I. INTRODUCTION

Radar waveform designs have examined Polyphase sequences for a long time as a productive contracting option to the diverse classes of Frequency-modulated signals [1][2]. In radar & communication, sequences with good autocorrelation properties are of main interest. The “goodness” depends on the application we use. Frank sequence merit factors are analyzed in [3]. P2 &Px sequences derived from linear-frequency-modulated [4]. Frank & P1 are designed from step approximation-to-linear frequency-modulation waveform [5]. Frank sequence merit factors are better than Chu. The two of them significantly outperform binary sequences [6][7].

The sequences can be processed digitally even though they are deriving from phase history of chirp or step-chirp Analog signals. By and by, radar waveforms might be upgraded in a first plan step by utilizing relationship measures and thus connects with the uncertainty capacity to assess the effect of phase shift changes on the execution. Prevalent execution as far as the incorporated Sidelobe levels contrasted with the Frank and P1 sequence is given by Px sequences that have been presented by Rapajic and Kennedy [8].

The ideas driving Frank sequences have later been summed up to encourage plans of Polyphase sequence of any length and related work was combined in the plain Zad-off-Chu(FZC) sequences or Chu Sequence[9][10]. A few execution parts of the previously mentioned classes of Polyphase sequences have been accounted for in writing fined years [11][12]. These sequences were initially presented inside the specific circumstance of utilizations for code division various to (CDMA) frame works, while these conducts inside radar situations have not been considered so for to the best of our insight [13].

This paper is organized as follows. Section II characterizes the measures used to encourage a quantitative execution assessment of good correlation sequence. Section III presents the essentials on classes of Polyphase sequences that are utilized with radar applications. Section IV presents the basic cyclic algorithm. On the basis of numerical results for different are given in Section V. Section VI concludes the paper.

II. PERFORMANCE MEASURES

Let N denotes the length of each Polyphase sequences C_k=[$C_k(0)$,$C_k(1)$,….,$C_k(N-1)$…..] of a size P where $1 \le k \le N-1$

A. Correlation Function

The correlation function at a discrete shift k between a Polyphase sequence is given by

$$C_k = \sum_{n=k+1}^{N} x_n x_{n-k}^{*} = C_{-k}^{*}, k = 0,1,\ldots\ldots N-1 \qquad (1)$$

Where $(\cdot)^*$ denotes the complex conjugate for scalar & the conjugate transpose for vector & matrices.

B. Integrated Sidelobe Level (ISL)

TThe ISL for the Polyphase sequence C_k=[$C_k(0)$,$C_k(1)$,….,$C_k(N-1)$ can be defined as follows

$$ISL = \sum_{k=1}^{N-1} \left| C_k \right|^2 \text{ is the ISL metric} \qquad (2)$$

The primary concentration of this paper is on calculation for limiting the ISL metric or ISL related measurements over the arrangement of Polyphase sequences. Note that minimization of ISL metric is proportional to the improvement of the merit factor defined as a performance metric in the below.

C. Merit Factor(MF)

The MF for the Polyphase sequence C_k=[$C_k(0)$,$C_k(1)$,….,$C_k(N-1)$ is defined as follows

$$MF = \frac{\left| C_0 \right|^2}{\sum_{k=-(N-1), k \ne 0}^{N-1} \left| C_k \right|^2} = \frac{N^2}{2ISL} \qquad (3)$$

Polyphase sequences with good merit factor are desired in many applications including range compression radar and sonar and wireless communication.

III. POLYPHASE SEQUENCES CLASSES

This section, we will describe the definitions of the considered Polyphase sequence classes in terms of phases & autocorrelation function. We adopted the sequence as it is used in many radar-related publication work & communication systems. In particular the P1, P2,Px, Frank sequences will be described for radar applications. on behalf of that these sequences beneficial properties and remarks also said.

A. *Frank Sequence*

Let the Polyphase sequence $X = (\chi_1, \chi_2, \dots\dots\dots, \chi_N)$ of a square integer length N=M^2(where M is a prime number). Due to work on phase shift pulse codes in [14].The history of complex-valued back as far as the 1950's.

The sequence elements are arranged as a M x M Matrix and are given by Mth roots of unity $w = \exp\left(j2\Pi \big/ M\right)$ (5)

From the above, the actual Length 'N' of Polyphase sequence can be produced by matrix of roots of units row-by-row.

Polyphase sequence of perfect square length N=M^2 are shown in [15]. the related sequence are referred to as Frank sequence.

Frank Sequence: The elements $x_k(m,n)$ of k^{th} Frank sequence is given as a matrix

$$X_K = [x_k(m,n)]_{M\times M} = \left[\exp\left(j\frac{2\Pi k}{M}mn\right)\right]_{M\,X\,M} \quad (6)$$

The Phase components are $\theta_k = \frac{2\Pi}{M}kmn$

Where $1 \le k \le M-1$, $0 \le m,n \le M-1$ and gcd(k,m)=1is required. (m,n)th element s of (8) can be point to the ith element of a sequence length N in terms of the phase sequence as follows

$$i = mM + n : \theta_k(m,n) \rightarrow \theta_k(i) = \theta_k(mM+n) \quad (7)$$

B. *P1, P2, Px Sequence*

This sequence can be considered for perfect square length N=M^2 only. In P1, P2, Px the phase components are rearranged version of Frank phase components [16] by cluster of zeros placed in the central part of the sequence.

$$p(i) = p(mM+n) = p(m,n) = \exp[\,j\theta(m,n)] \quad (8)$$

here $0 \le m,n \le M-1$ and the phase components are

P1sequence: $\theta(m,n) = \frac{-2\Pi}{M}\left(\frac{M-1}{2} - m\right)(mM+n)$ (9)

P2 sequence: $\theta(m,n) = \frac{+2\Pi}{M}\left(\frac{M-1}{2} - m\right)\left(\frac{M-1}{2} - n\right)$ (10)

Similarly for Px sequence Px:

$$\theta(m,n) = \frac{2\Pi}{M}\left(\frac{M-1}{2} - m\right)\left(\frac{M-1}{2} - n\right) \qquad M\ even \quad (11)$$

$$\theta(m,n) = \frac{2\Pi}{M}\left(\frac{M-1}{2} - m\right)\left(\frac{M-2}{2} - n\right) \qquad M\ odd \quad (12)$$

Here $0 \le m,n \le M-1$. Note that the phase elements of Px are similar to that of P2 for M even.

IV. CYCLIC ALGORITHM

The approach in the following is much simpler & computationally efficient than applying the optimization technique for the Polyphase sequence [16] and [17]. This makes feasible to work with quite large values of N (in some radar and imaging applications we can choose $N \sim 1000$). It means we can choose Q first from practical consideration and select $N \ge Q$ on computational as well as practical operation accounts.

Let $\tilde{C}$ be the following block-Toeplitz matrix

$$\tilde{C} = \begin{bmatrix} \overbrace{C_1(1) & \cdots & C_1(L) & 0}^{L+K-1} \\ \vdots & \ddots & & \ddots \\ 0 & C_1(1) & \cdots & C_1(L) \\ & \vdots & & \\ C_N(1) & \cdots & C_N(L) & 0 \\ \vdots & \ddots & & \ddots \\ 0 & C_N(1) & \cdots & C_N(L) \end{bmatrix} \quad (13)$$

Note that $\tilde{C}$ is $NL \times C(L+K-1)$. The auto & cross correlation appeared in below are the elements of the positive –semi finite matrix $\tilde{C}\tilde{C}^*$.

$$\sum_{n=1}^{N}\sum_{L=L+1,L\neq 0}^{L-1}\left|r_{nn}(p)\right|^2 + \sum_{n=1}^{N}\sum_{\tilde{n}=1,\tilde{n}\neq n}^{L-1}\sum_{p=-p+1}^{p-1}\left|r_{n\tilde{n}}(p)\right|^2 \quad (14)$$

Where

$$r_{n\tilde{n}}(p) = \sum_{l=p+1}^{L} x_n(l)\,x^*_{\tilde{n}}(l-p) = x^*_{\tilde{n}n}(-p), \quad p = 0,1,2,\dots \quad (15)$$

denote the (cross)- correlation of $\chi_n(l)$ and $\chi_{\tilde{n}}(l)$ at lag .Consequently, a criterion related to above equation (15) which has more compact form of the following

$$\frac{1}{NLK^2}\left\|\tilde{C}\tilde{C}^* - KI\right\|^2 \quad (16)$$

The above equation is the generalized correlation coefficient; and we use this equation to evaluate the correlation quality of a waveform. In such a case where $\tilde{C}\tilde{C}^*$ is singular, it follows that the maximum magnitude of its off-diagonal elements must be of the order $O(1)$ or larger; consequently, the ratio between $\max_{p,n,\tilde{n}}\left|r_{n\tilde{n}}(p)\right|$ and $r_{nn}(0) = K$ is of the order $O(1/L)$ but not smaller.

In the following we assume that

$$NL < L + K - 1 \Leftrightarrow L < \frac{K-1}{N-1} \left(posibly\ L << \frac{K-1}{N-1}\right) \quad (17)$$

Under the above equation, if we relax any requirement on the elements and the structure of $\tilde{C}$, then the class matrix $\tilde{C}$ that satisfies the equality $\tilde{C}\tilde{C}^* - KI$ is given by

$$\tilde{C} = \sqrt{K}U \quad (18)$$

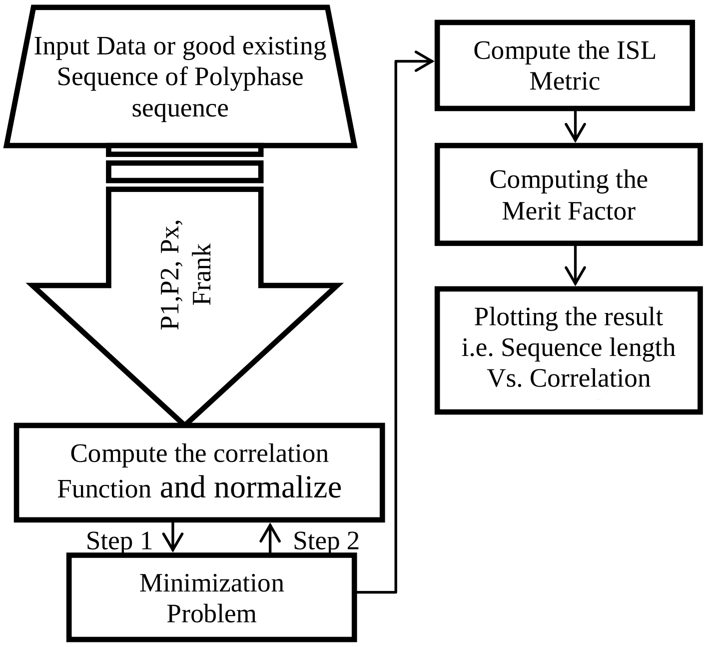
Where U is an arbitrary semi-unitary matrix [18] i.e. $UU^* = I$ usually the observation, we can reformulate (18) or (20) in the following related(but bot equivalent) way:

$$\min_{\{\theta_n(l)\},U}\left\|\tilde{C} - \sqrt{K}U\right\|^2 \quad (19)$$

This is a non-convex problem, the following cyclic minimization algorithm [19] [20], that is conceptually

&computationally simple and also have good local convergence properties.

A. BLOCK DIAGRAM OF THE CA



Cyclic Algorithm

*Step 0:*Initialize *U*or possibly $\tilde{C}$ (in which case the sequence of the next steps should be inverted), at some value suggested by "prior knowledge". i.e. initializing C with possibly good existing sequence of Polyphase sequence (P1, P2, Px, Frank)

Step 1: Compute the semi-unitary matrix *U*& minimize the (19) with respect to $\{\theta_n(l)\}$.

Step 2: with $\{\theta_n(l)\}$ set to the most-recent values, minimize the (19) w.r.to U.

Iteration: repeat step 1 and 2 until a practical convergence criterion is satisfied.

The iteration can be terminated, when the relative difference of the cost in (19) (i.e. the cost difference normalized by the cost of the previous iteration) is less than or equal to the 10^{-3} value in the numerical example illustrated.

The minimization problem in step 1 has the following generic form

$$\min_{\theta} \sum_{p=1}^{L} \left| e^{j\theta} - Zp \right|^2 = \min_{\theta} \left\{ const - 2\,\mathrm{Re}\left[\left(\sum_{p=1}^{L} Zp \right) e^{-j\theta} \right] \right\} \quad (20)$$

$$= \max_{\theta} \cos\left[\arg\left(\sum_{p=1}^{L} Zp \right) \right] - \theta \quad (21)$$

Where $\{Zp\}$are numbers given in (20) (21). The solution to the above equation is given by

$$\theta = \arg\left(\sum_{p=1}^{L} Zp \right) \quad (22)$$

In step 2 of the CA, the minimization problem solution can be easily computed as. Let

$$\sqrt{K}\tilde{C} = \bar{U}\,\Sigma\,\tilde{U}^{*} \quad (23)$$

Above equation denotes the singular value decomposition (SVD) of $\sqrt{K}\tilde{C}$, when $\bar{U}$ is $NL \times NL$, and $\tilde{U}$ is $(L+K-1) \times NL$ then the said solution is given by [21][22].

$$\bar{U}\,\Sigma\,\tilde{U}^{*} \quad (24)$$

Consider the simple illustration that with N=4 and L=22. Fig 1 shows the generalized correlation coefficient of the waveform given by the CA, as a function of Sampler Number L.(Note that for L<94, the GCC is too large to be of any practical interest). As L increases, we can achieve the goal of obtaining sequence with small value of auto & cross correlation effectively which is sign of improving the Merit Factor [23]. Additional simulation examples are shown in the next section for different values of N.

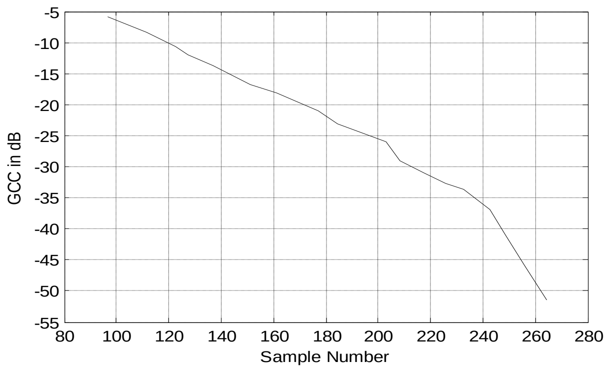


Fig 1 Sample number Vs. GCC via Cyclic Algorithm

V RESULTS

We compared the merit factors of the Polyphase sequence (P1, P2, Px, Frank), and that of CA Algorithm initialized by sequence said above(denoted as CA(P1), CA(P2), CA(Px), CA(Frank).

Note that the above sequences can be calculated for any value of N of possible practical interest, with the only restriction that N must be perfect square for Frank, P1, P2, Px sequences. We computed the Merit factors of above eight type sequences (P1, P2,Px, Frank) for the following length shown in Table I. the results are shown in Fig (1 & 2). The correlation level is defined as

$$Correlation\ Level = 20\log_{10}\left|\frac{c_k}{c_0}\right| \ , \quad k = 1,\ldots,N-1 \quad (25)$$

We calculated the Merit Factor of Polyphase sequences for the lengths N=100 and N=256 and note that the correlation levels of the CA(Px) and CA(Frank)sequence are comparatively small from k close to zero and *N*-1.

The Merit Factor of Polyphase sequences using conventional and cyclic algorithm for N=100 and N=256 are shown in TABLE I. In conventional method the P2, Frank exhibits the nearly same merit factor for length N=100 and P1, P2 have the same merit factor for length N=256. The Px exhibits the good merit factor among all the sequences when cyclic algorithm is applied for both the lengths N=100 & 256.

Merit Factor of Polyphase sequences (p1, p2, Px, frank) for integer values M=2 to 16 (i.e. N=4 to 256 where $N=M^2$), the merit factor are exist lengths N=16 to N=256 are shown in TABLE II. We notice that for integer values M=11 & 13 all the sequences such as CA(P1), CA(P2), CA(Px),

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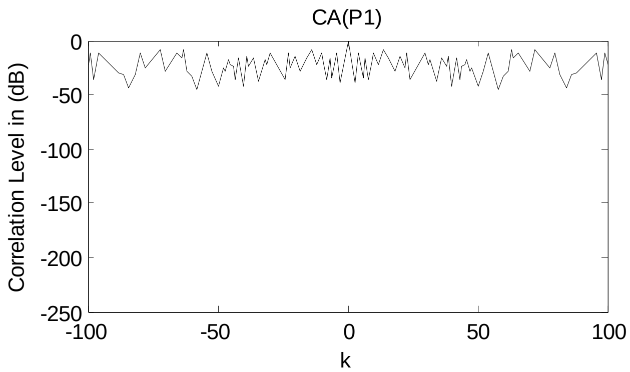
CA(Frank) exhibits the approximate value of the merit factor for the co-integer values. Where the 11 and 13 are the prime numbers. So, the P1, P2, Px, Frank can exhibit the good merit factor for the sequences length N which are obtained from the prime integer. Merit factor vs. sequence length are shown in Fig. 3 and Fig. 4 for the conventional and when cyclic algorithm applied.

TABLE I MERIT FACTOR OF POLYPHASE SEQUENCES USING CONVENTIONAL AND CYCLIC ALGORITHM FOR N=100 AND N=256.

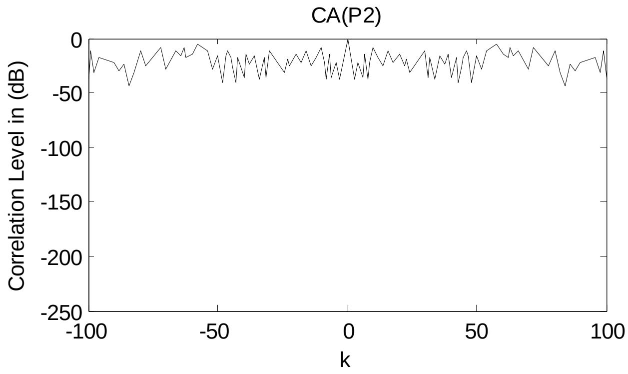
Polyphase Sequences	Conventional Method Merit Factor		Polyphase Sequences with Cyclic Algorithm	CA Merit Factor	
	N=100	N=256		N=100	N=256
P1	22.452	36.103	CA(P1)	60.014	92.342
P2	23.121	36.014	CA(P2)	60.213	93.789
Px	25.012	40.012	CA(Px)	67.344	107.732
Frank	23.592	38.214	CA (Frank)	61.414	94.355

TABLE II MERIT FACTOR OF POLYPHSE SEQUENCES (P1, P2, Px, Frank) FOR LENGTHS N=4 TO N=256.

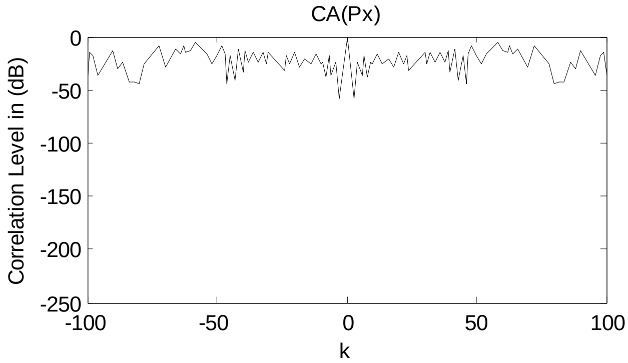
Sequence Length $N=M^2$					
M	N	MF for CA(P1)	MF for CA(P2)	MF for CA(Px)	MF for CA(Frank)
2	4	---	----	----	----
3	9	---	----	----	----
4	16	8.614	8.081	12.808	10.671
5	25	13.459	14.0193	21.325	19.215
6	36	19.380	19.942	34.172	31.587
7	49	29.408	30.718	46.536	45.345
8	64	38.411	39.415	56.020	51.421
9	81	48.615	50.615	59.610	56.192
10	100	60.014	60.213	67.344	61.414
11	121	75.612	76.354	87.486	73.405
12	144	72.850	73.015	81.438	68.147
13	169	88.412	91.031	105.417	93.012
14	196	79.451	81.247	91.325	85.410
15	225	81.159	84.564	97.142	87.621
16	256	92.342	93.789	107.732	94.355



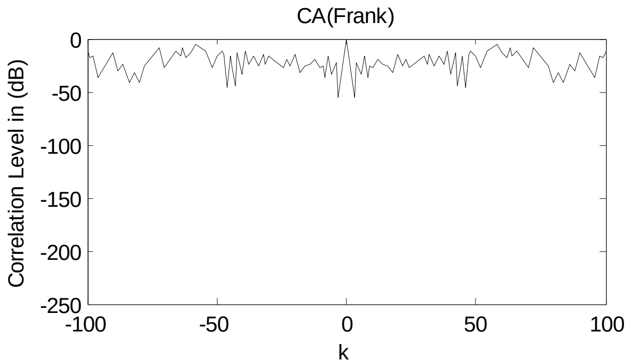
(a)



(b)

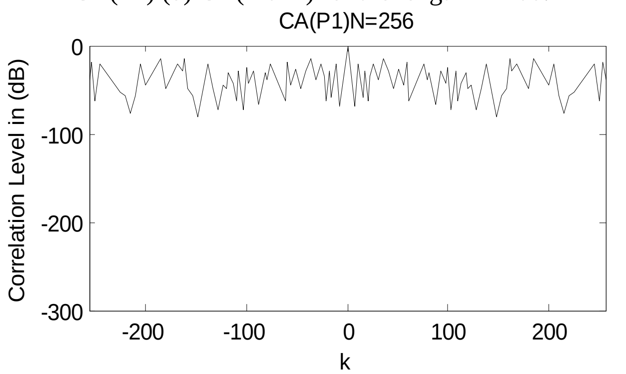


(c)

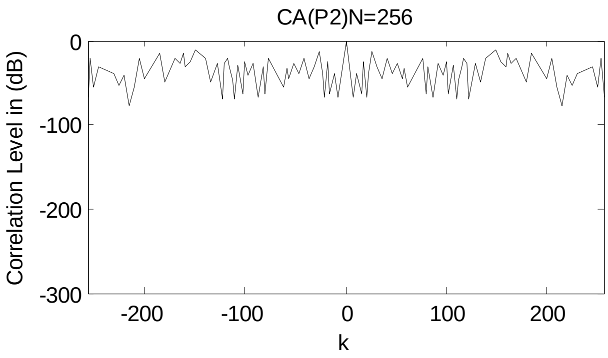


(d)

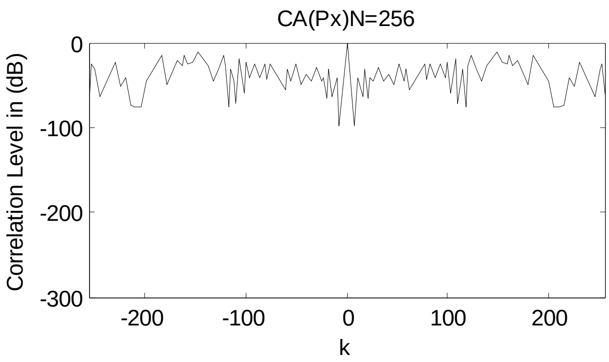
Fig 1. Correlation levels of the (a) CA(P1), (b) CA(P2), (c) CA(Px) (d) CA(Frank) for the length N=100.



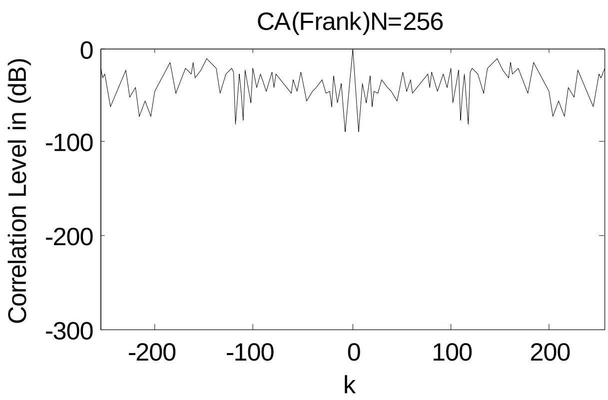
(a)



(b)



(c)



(d)

Fig 2. Correlation levels of the (a) CA(P1), (b) CA(P2), (c) CA(Px) (d) CA(Frank) for the length N=256.

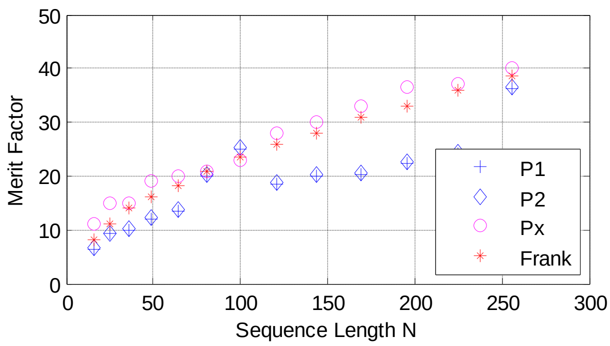


Fig 3. Merit Factor of the P1, P2, Px, Frank for the length N=0 to 256.

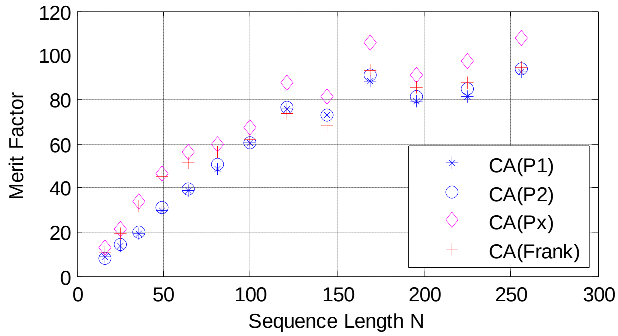


Fig 4 Merit Factor of CA(P1), CA(P2), CA(Px), CA(Frank) for the lengths N=0 to 256.

VI CONCLUSION

This paper presents the cyclic algorithm namely CA, which can be applied to the Polyphase sequences such as P1, P2, Px, Frank that have good correlation properties. The CA algorithm makes use of SVD of matrix $NL \times NL$ can be computationally efficient upto the length of N=256. In conventional method the best Merit Factor is obtained for Px sequence only of 25.012& 40.012 for length N=100& 256 respectively. But when Cyclic Algorithm is appliedthe CA(Px) and CA(Frank) sequences exhibits the better merit Factor 61.344 & 61.414 for N=100, 107.732 & 94.355 for N=256 respectively. The P1, P2, Px, Frank express good merit factor for the M=11, 13 which are prime numbers (i.e. $N=M^2$, N=121, 169). The merit factor comparison between P1, P2, Px, Frank is Px>Frank>P2>P1. The minimum integer value in $N=M^2$ we can apply the CA is M=4 and maximum is 16.

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