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On Cubic Graceful Labeling

Mathew Varkey T. K [1] ,
[1]Asst.Prof, Department of Mathematics,
T.K.M College of Engineering,
Kollam, Kerala, India
Email: *mathewvarkeytk@gmail.com*

Mini. S. Thomas [2]
[2] Research Scholar, Department of Mathematics
St. Albert's College,
Eranakulam, Kerala, India
minirenjan1994@gmail.com

Abstract- A graph with *n* vertex and *m* edges is said to be cubic graceful labeling if its vertices are labeled with distinct integers {0,1,2,3,……..,m^3} such that for each edge f^**(uv)* induces edge mappings are {1^3,2^3,3^3,……,m^3}. A graph admits a cubic graceful labeling is called a cubic graceful graph. In this paper, we proved that < $K_{1,a}$, $K_{1,b}$, K_{1c} , $K_{1,d}$>, associate u_{rp} with $u_{(r+1)1}$ of $K_{1,P}$, fixed vertices v_r and u_r of two copies of P_n are cubic graceful.

Keywords*: Cubic graceful graph, cubic graceful labeling, Path graph, Star graph,*

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I. Introduction

Graphs labeling, where the vertices are assigned values subject to certain conditions, have often been motivated by practical problems. Labeled graphs serves as useful mathematical models for a broad range of applications such as coding theory, including the design of good radar type codes, missile guidance codes and convolution codes with optimal autocorrelation properties. They facilitate the optimal nonstandard encoding of integers. For all terminology and notation in this paper we follow Harary[1].

A function *f* is called a *graceful labeling* of a (*p*,*q*) graph G= (V,E) if *f* is an injection from the vertices of G to the set {0,1,2,3,….*q*} such that, each edge *uv* is assigned the label |*f(u)-f(v)*\, the resulting edge labels are distinct. T. Tharmaraj and P.B.Sarasija [5] introduced *square graceful labeling* concept in 2014. The concept of *cubic graceful labeling* of graphs was introduced in [4]. A systematic presentation of survey of graph labeling is presented in [2]. We use the following definitions in the subsequent sections.

Definition 1.1

The ***Path graph*** P_n is the *n*- vertex graph with *n-1* edges, all on a single path.

Definition1.2

A complete bipartite graph $K_{1,n}$ is called a ***star*** and it has (n+1) vertices and n edges

Definition 1.3

The ***Trivial graph*** K_1 or P_1 is the graph with one vertex and no edges.

Definition 1.4

A graph G = (V,E) with *n* vertices and *m* edges is said be ***Cubic Graceful Graph (CGG)*** if there exists an injective function $f: V(G)\rightarrow\{0,1,2,\ldots\ldots,m^3\}$ such that the induced mapping $f^* : E(G)\rightarrow \{1^3,2^3,\ldots\ldots,m^3\}$ defined by $f^*(uv): \rightarrow|f(u)- f(v)|$ is a bijection . The function *f* is called a ***Cubic Graceful Labeling (CGL)*** of G

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II. MAIN RESULTS

Theorem 1

Let G be the graph < $K_{1,a}$, $K_{1,b}$, K_{1c} , $K_{1,d}$> obtained by joining the middle vertices of $K_{1,a}$, $K_{1,b}$, $K_{1,c}$, $K_{1,d}$ to another vertex 'u' is cubic graceful for all a ,b ,c, d ≥ 1

Proof:

Let G be a combination of the star graph < $K_{1,a}$, $K_{1,b}$, $K_{1,c}$, $K_{1,d}$>

Let $V(G) = \{ u, u_n, u_{1p}, u_{2q}, u_{3r}, u_{4s} \}$; $1 \le n \le 4$; $1 \le p \le a$

$1 \le q \le b$, $1 \le r \le c$, $1 \le s \le d$

And

$$E(G) = \begin{cases} uu_n \ ; & 1 \le n \le 4 \\ u_1u_{1p} & 1 \le p \le a \\ u_2u_{2q} \ ; & 1 \le q \le b \\ u_3u_{3r} \ ; & 1 \le r \le c \\ u_4u_{4s} \ ; & 1 \le s \le d \end{cases}$$

So

$n(G) = a + b + c + d + 5$

$m(G) = a + b + c + d + 4$

Define $f: V(G) \rightarrow \{0, 1, 2, \dots\dots\dots, (a + b + c + d + 4)^3\}$ by

$f(u) = 0$

$f(u_n) = (n(G) - n)^3$; $1 \le n \le 4$

$f(u_{1p}) = (n(G) - 1)^3 - (a + b + c + d - (p - 1))^3$; $1 \le p \le a$

$f(u_{2q}) = (n(G) - 2)^3 - (b + c + d - (q - 1))^3$; $1 \le q \le b$

$f(u_{3r}) = (n(G) - 3)^3 - (c + d - (r - 1))^3$; $1 \le r \le c$

$f(u_{4s}) = (n(G) - 4)^3 - (d - (s - 1))^3$; $1 \le s \le d$

The induced edge mapping are

$f*(uu_n) = (n(G) - n)^3$; $1 \le n \le 4$

$f*(u_1u_{1p}) = (a + b + c + d + 1 - p)^3$; $1 \le p \le a$

$f*(u_2u_{2q}) = (b + c + d + 1 - q)^3$; $1 \le q \le b$

$f*(u_3u_{3r}) = (b + c + d + 1 - q)^3$; $1 \le r \le c$

$f*(u_4u_{4s}) = (d + 1 - s)^3$; $1 \le s \le d$

The vertex labels are in the set $\{0,1,2, \ldots.., [(a+b+c+d+4)]^3\}$ Then the edge label arranged in the set $\{1^3, 2^3, 3^3, \ldots \ldots \ldots, (a+b+c+d+4)^3\}$. So the vertex labels are distinct and edge labels are also distinct and cubic. So the graph G is cubic graceful for all $a, b, c, d \geq 1$.

Theorem 2

If $G_r = K_{1,p}$ be the given graph for $1 \leq r \leq q$ with vertex *set* $V(G_r) = \{u_r, u_{rs}; 1 \leq r \leq q, 1 \leq s \leq p\}$ Let G be the given graph formulated by associate v_{rp} with $v_{(r+1)1}$ for $1 \leq r \leq q-1$. Then G is a cubic graceful for all p and q.

Proof :

The graph G consisting of the vertex set $V(G) = \{U_r, U_{rs};\ 1 \leq r \leq q,\ 1 \leq s \leq p\}$ and the edge set E(G) = $\{u_r u_{rs};\ 1 \leq r \leq q,\ 1 \leq s \leq p$

Then $n(G) = pq$ and m(G) = pq

Let $f: V(G) \rightarrow \{0, 1, 2, 3, \ldots \ldots, (pq)^3\}$ be defined as follows.

$f(u_i) = 0$
$f(u_{1,s}) = (pq - s + 1)^3$; $1 \leq s \leq p$

$f(u_r) = f(u_{(r-1)p}) - [p(q - r + 1)]^3$; $2 \leq r \leq q$

$f(u_{rs}) = f(u_r) + [q - (r-1)p - (s-1)]^3$; $2 \leq r \leq q,\ 2 \leq s \leq n$

Then f induces a bijection

$f^*(u_r u_{rs}) = (pq - s + 1)^3$; $r = 1,\ 1 \leq s \leq n$

$f^*(u_r u_{(r-1)p}) = p(q - r + 1)^3$, $2 \leq r \leq p,\ s = p$

$f^*(u_r u_{rs}) = [p(q - r + 1 - i)]^3$; $2 \leq r \leq q, 2 \leq s \leq p\ 1 \leq i \leq p-1$,

The vertex labels are in the set $\{0, 1, 2 \ldots \ldots (pq)^3\}$. Then the edge label arranged in the set $\{1^3, 2^3, \ldots \ldots .. (pq)^3\}$. So the vertex labels are distinct and edge labels are also distinct and cubic. So the graph G is cubic graceful for all $p, q \geq 1$.

Theorem 3

Let G be the graph obtained from two copies of P_n with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, u_3 \ldots, u_n$ by joining the fixed vertices v_r and u_r by means of an edge, for $1 \leq r \leq n$. Then the graph $(P_n: G)$ is cubic graceful.

Proof: Let G denote the graph $(P_n: G)$

Let V(G) = $\begin{cases} u_i & for \quad i = 1,2,3, \ldots \ldots \ldots, (n-1) \\ v_i & for \quad i = 1,2,3, \ldots \ldots \ldots, (n-1) \end{cases}$

and E(G)= $\begin{cases} u_i u_i & \text{for} \quad i = 1,2,3 \ldots \ldots \ldots (n-1) \\ v_i v_{i++1} & \text{for} \quad i = 1,2,3 \ldots \ldots \ldots, (n-1) \\ u_r v_r & \text{for } r = 1,2, \ldots \ldots \ldots, n \quad \text{where `r` is a fixed vertex} \end{cases}$

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So, $p = 2n$ and $q = 2n-1$

Define $f: V(G) \rightarrow \{0,1,2,\ldots\ldots,(2n+1)^3\}$ by

$f(u_i) = 0$

$f(u_{i+1}) = \sum_{k=1}^{i}(-1)^{k+1}(2n-k)^3$ for $i = 1,2,\ldots\ldots,(n+1)$

$f(v_r) = f(u_r) - n^3$; for $r = 1,2,\ldots\ldots,n$ and it is a fixed vertex.

$f(v_r - j) = f(v_r) + \sum_{k=1}^{j}(-1)^K (n-r+j)^3$; for $j = 1,2,\ldots\ldots,(r-1)$

$f(v_r + j) = f(v_r) + \sum_{k=1}^{j}(-1)^K (n-r-j+1)^3$; for $j = 1,2,\ldots\ldots,(n-r)$

The induced edge mapping are

$f^*(u_i u_{i+1} + 1) = (2n-i)^3$ for $i = 1,2,\ldots\ldots,(n-1)$

$f^*(u_r v_r) = n^3$ for $r = 1,2,\ldots\ldots,n$; where 'r' is a fixed vertex

$f^*(v_i v_{i+1}) = (n-1)^3$ for $i = 1,2,\ldots\ldots,(n-1)$

The vertex labels are in the set$\{0,1,2\ldots.(2n-1)^3\}$. Then the edge labels are arranged in the set $\{1^3,2^3,3^3\ldots.(2n-1)^3\}$ So the vertex labels are distinct and the edge labels are also cubic and distinct . So the graph (P_n:G)) is a cubic graceful.

REFERENCES

[1] Frank Harary, *Graph Theory*, Narosa Publishing House, New Delhi, 2001[6]
[2] J.A . Gallian, *A dynamic survey of graph labeling, The Electronic journal of Combinatrorics,(2016)*
[3] V.J.Kaneria and H.M.Makadia, *Some Graceful Graphs J. of Math. Research*, 4 (1),(2012) 54-57
[4] Mini.S.Thomas and Mathew Varkey T.K, *Cubic Graceful Labeling* . Global Journal of Pure And Applied Mathematics, June (2017)
[5] T. Tharmaraj and P.B.Sarasija, *International Journal of Mathematics and Soft Computing Vol.4 No.1(2014).129-137.*