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The FM / FM / 1queue with Single Working Vacation

G. Kannadasan[1]
Department of Mathematics
Annamalai University
Annamalainager-608002,India
klsk.g.21@gmail.com[1]

N. Sathiyamoorthi[2]
Department of Mathematics
Annamalai University
Annamalainager-608002, India
n.satyamurthi@gmail.com[2]

Abstract—In this paper, we study a**FM / FM / 1** queue with single working vacation for this fuzzy queueing model. We obtain some system characteristic such as the number of customer in the system in steady state, the virtual time of a customer in the system, the server is in idle period, the server is in regular busy period. Finally, numerical results for all the performance measure are presented to show the effects of system parameters.

***Keywords-FM / FM / 1** queue, Single working vacation, Server is in idle period, Server is in regular busy period, The virtual time of a customer in the system*

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I. INTRODUCTION

During last two decades, vacation queue havebeen investigated extensively because of their applications in computer systems, communication networks, production managing and so forth. In various vacation queue models, the server completely stops serving customers during a vacation, but may perform other supplementary jobs. Proposing of various vacation policies provides more flexibility for optional design and operation control of the system. The details can be seen in the monographs of Takagi [13] and Tian and Zhang [15], the surveys of Doshi [2] and Teghem [14].

Recently, Servi and Finn [13] introduced a class of semi-vacation polices: the server works at a lower rate rather than completely stopping service during a vacation. Such a vacation is called a working vacation (wv). Part of servers keeps the system operating in a lower rate and the other parts of break or accomplish else assistant work during a vacation. If service rate degenerates into zero in a working vacation, then the working vacation queue becomes a classical vacation queue model. Therefore, the working vacation queue is an extension of classical vacation queue. Tian and Zhang [16] studied multi server queues with partial server's vacation where vacation servers completely stop service.

There is quite difference between working vacation queue and classical queue. During a vacation, customers in the forever may finish service and depart the system, however, customers in the latter can impossibly depart the system. On the other hand, during a vacation, the number of customers in the latter can only increases. Therefore, the working vacation models have more complicated modalities and the analysis of this kind of models is more difficult then classical vacation queue.

Liu, Xu and Tian [7] gave simple explicit expressions of distribution for the stationary queue length and waiting time which have intuituioustic probability sense. Furthermore, the authors got stochastic decomposition structures of stationary indices, derived the distributions of additional queue length and additional delay and obtained expected regular busy period and expected busy cycle. Kin, Choi and Chae [5], Wu and Takagi [18] generalized the work of servi and Finn [13] to an M/G/1 queue with multiple working vacations. Bala [1] investigated a GI/M/1 queue with multiple working vacations.

K.Julia Rose Mery and T.Gokilavani [4] investigate
the performance measure of an M^X/M/1 Multiple Working Vacation (MWV) queuing model in a fuzzy environment. Mary George and Jayalekshmi [9] studied on the analysis of G/M(n)/1/k queuing system with multiple exponential vacations and vacations of fuzzy length. J.Pavithra and K.Julia Rose Mary [11] gave the analysis of FM/M (a,b)/1/MWV queuing model. The analysis of the general bulk service queuing model to find the mean queue length probability that the system is in vacation and the probability that the system is in busy state are expressed in terms of crisp value for FM/M(a,b)/1 under multiple working vacation with fuzzy numbers. R.Ramesh and S.Kumara G Ghuru [12] constructs the membership function of the system characteristics of a batch-arrival queuing system with multiple servers, in which the batch-arrival rate and customer service rate are all fuzzy numbers.

II. THE CRISP MODEL

Introducing a single working vacation policy into a classical $M/M/1$ queue with arrival rate λ and service rate μ_b in a regular busy period. The server beings a working vacation of random length V at the instant when the queue becomes empty, and vacation duration V follows an exponential distribution with Parameter θ. During a working vacation an arriving customer is served at a rate of μ_v. When a vacation ends, if there are customers in the queue, the server changes service rate from μ_v to μ_b, and a regular busy period starts. Otherwise, the server enters idle period, and a new regular busy period starts when a customer arrival occurs. Working vacation V is a operation period in a lower rate. When the number of customers in the system is relatively few,

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we set a lower rate operation period in order to economize operation cost together with serving customers. Therefore, this single working vacation policy has practical significance in optimal design of the system.We assume that inter arrival times, service times, and working vacation times are mutually independent. In addition, the service order is first in first out (FIFO).

Let $Q(t)$ be the number of customers in the system at time t, and let

$$J(t)=\begin{cases}0, & \text{the system is in a working vacation period at time "}t\text{"}\\ 1, & \text{the system is not in a working vacation period at time "}t\text{"}\end{cases}$$

Then, $Q(t), J(t)$ is a Markov process with the state space $\Omega=\{(0,0)\}\cup\{(k,j)/k\geq 0, j=0,1\}$.

Where the state $(0,1)$ denotes that the system is in idle period; state $(k,1), k\geq 1$ indicates that the system is in regular busy period; state $(k,0), k\geq 0$ indicates that the system is in working vacation period and there are k customers in the queue.

III. THE MODEL IN FUZZY ENVIRONMENT

In this section the arrival rate λ, service rate μ_b in a regular busy period, working vacation an arriving customer is served at a rate of μ_v, working vacation θ are assumed to be fuzzy numbers respectively. Now:

$\bar{\lambda}=\{w,\mu_{\bar{\lambda}}(w); w\in S(\bar{\lambda})\}$, $\bar{\beta}=\{x,\mu_{\bar{\beta}}(x); x\in S(\bar{\beta})\}$,
$\bar{v}=\{y,\mu_{\bar{v}}(y); y\in S(\bar{v})\}$ and $\bar{\theta}=\{z,\mu_{\bar{\theta}}(z); z\in S(\bar{\theta})\}$

Where, $S(\bar{\lambda})$, $S(\bar{\beta})$, $S(\bar{v})$ and $S(\bar{\theta})$ are the universal sets of the arrival rate, service rate in a regular busy period, working vacation an arriving customer is served at a rate, and working vacation respectively. Define $f(w,x,y,z)$ as the system performance measure related to the above defined fuzzy queuing model which depends on the fuzzy membership function $f(\bar{\lambda},\bar{\beta},\bar{v},\bar{\theta})$.

Applying Zadeh's extension principle (1978) [15] the membership function of the performance measure $f(\bar{\lambda},\bar{\beta},\bar{v},\bar{\theta})$ can be defined as:

$$\mu_{\bar{f}(\bar{\lambda},\bar{\beta},\bar{v},\bar{\theta})}(H)=\sup_{\substack{w\in S(\bar{\lambda})\\ x\in S(\bar{\beta})\\ y\in S(\bar{v})\\ z\in S(\bar{\theta})}}\left\{\mu_{\bar{\lambda}}(w),\mu_{\bar{\beta}}(x),\mu_{\bar{v}}(y),\mu_{\bar{\theta}}(z)/H=f(w,x,y,z)\right\} \quad (1)$$

If the α - cuts of $f(\bar{\lambda},\bar{\beta},\bar{v},\bar{\theta})$ degenerate to some fixed value, then the system performance is a crisp number, otherwise it is a fuzzy number.

The Number of Customer in the System in Steady State

$$E(L)=\left(\frac{\lambda}{\mu_b-\lambda}\right)+E(Q_d)$$

$$=\left[\left[\frac{\lambda}{\mu_b-\lambda}\right]+\left(\frac{(\lambda+\theta)\left(2\mu_v-(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})\right)}{2\lambda\mu_v}\right)\right.$$
$$\left.+\left(\frac{(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})}{2\mu_v}\right)\left(\frac{\mu_b-\mu_v}{\mu_b}\right)\right]^{-1}$$

$$\times\left[\left(\frac{\mu_b-\mu_v}{\mu_b}\right)\times\left(\frac{(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})}{2\mu_v-(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})}\right)\right]$$

The Virtual Time of a Customer in the System

$$E(W)=\left[\left(\frac{1}{(\mu_b^3-\mu_b^2\lambda)}\right)+\left(\frac{(\theta+\lambda)(2\mu_v-(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v}))}{2\lambda\mu_v}\right)\right.$$
$$\left.\left(\frac{\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v}}{2\mu_v}\right)\left(\frac{\mu_b-\mu_v}{\mu_b}\right)\right]^{-1}$$
$$\times\left[\left(\frac{\mu_b-\mu_v}{\mu_b}\right)\left(\frac{\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v}}{\lambda(2\mu_v-(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v}))}\right)\right]$$

The server is in idle period

$$P_0=\frac{\theta}{\lambda}\left[\frac{\mu_b-\lambda}{\mu_b}\right]\times\left[\frac{2\mu_v-(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})}{2\mu_v}\right]$$
$$\times\left[\left(\frac{\mu_b-\lambda}{\theta}\right)+\left(\frac{2\mu_v\theta-\theta\lambda(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})}{2\lambda\mu_v}\right)\right.$$
$$\left(\frac{\mu_b-\lambda}{\mu_b}\right)+\left(\frac{\theta(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})}{2\mu_v[\mu_b 2\mu_v-\mu_b(\lambda+\theta+\mu_v)^2-4\lambda\mu_v]}\right)$$
$$\left.+\left(\frac{\theta 2\mu_v-\theta(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})}{2\mu_v\mu_b}\right)\right]^{-1}$$

The Server is in Regular Busy Period

$$P_1=\left[\frac{\theta(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})}{2\mu_b\mu_v-(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})}\right]$$

$$+\left[\frac{\theta(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2})(2\mu_v-(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v}))}{2\mu_b\mu_v}\right]$$

$$\left[\left(\frac{\mu_b-\lambda}{\mu_b}\right)+\left(\frac{\theta 2\mu_v-\theta(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})}{2\lambda\mu_v}\right)\right.$$

$$\times\left(\frac{\mu_b-\lambda}{\mu_b}\right)$$

$$+\left(\frac{\theta(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})}{2\mu_b\mu_v-\mu_b(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})}\right)$$

$$\left.+\left(\frac{\theta 2\mu_v-\theta(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})}{2\mu_b\mu_v}\right)\right]^{-1}$$

Under the study state condition $\rho=\frac{\lambda}{\mu_v}$ .We obtain the membership function of some performance measures namely the number of customer in the system in steady state, the virtual time of a customer in the system,the server is in idle period, the server is in regular busy period.

For the system in terms of membership function are:

$$\mu_{\overline{E(L)}}(A)=\sup_{\substack{w\in S(\bar{\lambda})\\ x\in S(\bar{\beta})\\ y\in S(\bar{\nu})\\ z\in S(\bar{\theta})}}\left\{\mu_{\bar{\lambda}}(w),\mu_{\bar{\beta}}(x),\mu_{\bar{\nu}}(y),\mu_{\bar{\theta}}(z)/A\right\}$$

(2)Where,

$$A=\left[\frac{w}{x-w}\right]$$

$$+\left[\left(\frac{(z+w)(2y-(w+z+y-\sqrt{(w+z+y)^2-4wy}))}{2wy}\right)\right.$$

$$\left.+\left(\frac{(w+z+y-\sqrt{(w+z+y)^2-4wy})}{2y}\right)\left(\frac{x-y}{x}\right)\right]^{-1}$$

$$\times\left[\left(\frac{x-y}{x}\right)\left(\frac{w+z+y-\sqrt{(w+z+y)^2-4wy}}{2y-(w+z+y-\sqrt{(w+z+y)^2-4wy})}\right)\right]$$

$$\mu_{\overline{E(W)}}(B)=\sup_{\substack{w\in S(\bar{\lambda})\\ x\in S(\bar{\beta})\\ y\in S(\bar{\nu})\\ z\in S(\bar{\theta})}}\left\{\mu_{\bar{\lambda}}(w),\mu_{\bar{\beta}}(x),\mu_{\bar{\nu}}(y),\mu_{\bar{\theta}}(z)/B\right\}(3)$$

Where,

$$B=\left(\frac{1}{x^3-x^2w}\right)+\left(\frac{(z+w)(2y-(w+z+y+\sqrt{(w+z+y)^2-4wy}))}{2wy}\right)$$

$$+\left(\frac{w+z+y-\sqrt{(w+z+y)^2-4wy}}{2y}\right)\left(\frac{x-y}{y}\right)$$

$$\times\left(\frac{x-y}{x}\right)\left(\frac{w+z+y-\sqrt{(w+z+y)^2-4wy}}{w(2y-(w+z+y-\sqrt{(w+z+y)^2}))}\right)$$

$$\mu_{\overline{P_0}}(C)=\sup_{\substack{w\in S(\bar{\lambda})\\ x\in S(\bar{\beta})\\ y\in S(\bar{\nu})\\ z\in S(\bar{\theta})}}\left\{\mu_{\bar{\lambda}}(w),\mu_{\bar{\beta}}(x),\mu_{\bar{\nu}}(y),\mu_{\bar{\theta}}(z)/C\right\}(4)$$

Where,

$$C=\left[\frac{x-w}{x}\right]$$

$$\times\left[\left(\frac{x-z}{z}\right)+\left(\frac{2yz+zw(w+z+y-\sqrt{(w+z+y)^2-4wy})}{2wy}\right)\right.$$

$$\left.+\left(\frac{2zy-z(w+z+y-\sqrt{(w+z+y)^2-4wy})}{2yx}\right)\right]^{-1}$$

$$\mu_{\overline{P_1}}(D)=\sup_{\substack{w\in S(\bar{\lambda})\\ x\in S(\bar{\beta})\\ y\in S(\bar{\nu})\\ z\in S(\bar{\theta})}}\left\{\mu_{\bar{\lambda}}(w),\mu_{\bar{\beta}}(x),\mu_{\bar{\nu}}(y),\mu_{\bar{\theta}}(z)/D\right\}(5)$$

Where,

$$D=\left[\frac{z(w+z+y-\sqrt{(w+z+y)^2-4wy})}{2xy-(w+z+y-\sqrt{(w+z+y)^2-4wy})}\right]$$

$$+\left[\frac{z(w+z+y-\sqrt{(w+z+y)^2-4wy}).(2y-(w+z+y-\sqrt{(w+z+y)^2-4wy}))}{2xy}\right]$$

$$+\left[\left(\frac{x-w}{x}\right)+\left(\frac{2zy-z(w+z+y-\sqrt{(w+z+y)^2-4wy})}{2wy}\right)\left(\frac{x-w}{x}\right)\right.$$

$$\text{x}\left(\frac{z(w+z+y-\sqrt{(w+z+y)^2-4wy})}{2zy-z(w+z+y-\sqrt{(w+z+y)^2-4wy})}\right)$$

$$\left.+\left(\frac{2zy-z(w+z+y-\sqrt{(w+z+y)^2-4wy})}{2xy}\right)\right]^{-1}$$

Using the fuzzy analysis technique explained we can find the membership of $\overline{E(L)},\ \overline{E(W)},\ \overline{P_0}\ and\ \overline{P_1}$ as a function of the parameter α, thus the α-cut approach can be used to develop the membership function of $\overline{E(L)},\ \overline{E(W)},\ \overline{P_0}$ and $\overline{P_1}$.

Performance of measure

The following Performance measure are studied for this model in fuzzy environment.

The Number of Customers in the System in Steady State

Based on Zadeh's extension Principle $\mu_{E(L)}(A)$ is the supremum of minimum over $\mu_{\bar{\lambda}}(w), \mu_{\bar{\beta}}(x), \mu_{\bar{v}}(y), \mu_{\bar{\theta}}(z)\}$ to satisfying $\mu_{E(L)}(A)=\alpha,\ 0<\alpha\le 1$.

The following four cases arise:

Case (i) $\mu_{\bar{\lambda}}(w)=\alpha, \mu_{\bar{\beta}}(x)\ge\alpha, \mu_{\bar{v}}(y)\ge\alpha, \mu_{\bar{\theta}}(z)\ge\alpha$,

Case (ii) $\mu_{\bar{\lambda}}(w)\ge\alpha, \mu_{\bar{\beta}}(x)=\alpha, \mu_{\bar{v}}(y)\ge\alpha, \mu_{\bar{\theta}}(z)\ge\alpha$,

Case (iii) $\mu_{\bar{\lambda}}(w)\ge\alpha, \mu_{\bar{\beta}}(x)\ge\alpha, \mu_{\bar{v}}(y)=\alpha, \mu_{\bar{\theta}}(z)\ge\alpha$,

Case (iv) $\mu_{\bar{\lambda}}(w)\ge\alpha, \mu_{\bar{\beta}}(x)\ge\alpha, \mu_{\bar{v}}(y)\ge\alpha, \mu_{\bar{\theta}}(z)=\alpha$.

For case (i) the lower and upper bound of α - cuts of $\overline{E(L)}$ can be obtained through the corresponding parametric non-linear programs:

$[\overline{E(L)}]_{\alpha}^{L_1}=\min_{\Omega}\{[A]\}$ & $[\overline{E(L)}]_{\alpha}^{U_1}=\max_{\Omega}\{[A]\}$.

Similarly, we can calculate the lower and upper bounds of the α -cuts of $\overline{E(L)}$ for the case $(ii),(iii)$& (iv) .By considering the cases, simulatuosly the lower and upper bounds of the α -cuts of $\overline{E(L)}$ can be written as:

$[\overline{E(L)}]_{\alpha}^{L}=\min_{\Omega}\{[A]\}$ & $[\overline{E(L)}]_{\alpha}^{U}=\max_{\Omega}\{[A]\}$

such that, $w_{\alpha}^{L}\le w\le w_{\alpha}^{U},\ x_{a}^{L}\le x\le x_{a}^{U},\ y_{\alpha}^{L}\le y\le y_{\alpha}^{U},\ z_{\alpha}^{L}\le z\le z_{\alpha}^{U}$.

If both $(\overline{E(L)})_{\alpha}^{L}$ and $(\overline{E(L)})_{\alpha}^{U}$ are invertible with respect to α, the left and right shape function, $L(A)=[(E(L))_{\alpha}^{L}]^{-1}$ and $R(A)=[(E(L))_{\alpha}^{U}]^{-1}$ can be derived from which the membership function $\mu_{\overline{E(L)}}(A)$ can be constructed as:

$$\mu_{\overline{E(L)}}(A)=\begin{cases} L(A), & (E(L))_{\alpha=0}^{L}\le A\le (E(L))_{\alpha=0}^{U} \\ 1, & (E(L))_{\alpha=1}^{L}\le A\le (E(L))_{\alpha=1}^{U} \\ R(A), & (E(L))_{\alpha=0}^{L}\le A\le (E(L))_{\alpha=0}^{U} \end{cases} \quad (6)$$

In the same way as we said before we get the following results.

The Virtual time of a Customer in the System

$$\mu_{\overline{E(W)}}(B)=\begin{cases} L(B), & (E(W))_{\alpha=0}^{L}\le B\le (E(W))_{\alpha=0}^{U} \\ 1, & (E(W))_{\alpha=1}^{L}\le B\le (E(W))_{\alpha=1}^{U} \\ R(B), & (E(W))_{\alpha=0}^{L}\le B\le (E(W))_{\alpha=0}^{U} \end{cases} \quad (7)$$

The Server is in idle period

$$\mu_{\overline{P_0}}(C)=\begin{cases} L(C), & (P_0)_{\alpha=0}^{L}\le C\le (P_0)_{\alpha=0}^{U} \\ 1, & (P_0)_{\alpha=1}^{L}\le C\le (P_0)_{\alpha=1}^{U} \\ R(C), & (P_0)_{\alpha=0}^{L}\le C\le (P_0)_{\alpha=0}^{U} \end{cases} \quad (8)$$

The Server is in Regular Busy Period

$$\mu_{\overline{P_1}}(D)=\begin{cases} L(D), & (P_1)_{\alpha=0}^{L}\le D\le (P_1)_{\alpha=0}^{U} \\ 1, & (P_1)_{\alpha=1}^{L}\le D\le (P_1)_{\alpha=1}^{U} \\ R(D), & (P_1)_{\alpha=0}^{L}\le D\le (P_1)_{\alpha=0}^{U} \end{cases} \quad (9)$$

Numerical study

The Number of Customers in the System in Steady State

Suppose the arrival rate $\bar{\lambda}$, the service rate $\bar{\beta}$, the vacation rate $\bar{v}$ & busy period $\bar{\theta}$ are assumed to be trapezoidal fuzzy numbers described by:

$\bar{\lambda}=[11,12,13,14]$, $\bar{\beta}=[91,92,93,94]$, $\bar{v}=[161,162,163,164]$ & $\bar{\theta}=[71,72,73,74]$ per mins respectively. Then:

$$\lambda(\alpha)=\min_{w\in s(\bar{\lambda})}\left\{w\in s(\bar{\lambda}), \begin{cases} w-11, & 11\le w\le 12 \\ 11\ \ , & 12\le w\le 13 \\ 14-w, & 13\le w\le 14 \end{cases} \ge\alpha\right\},$$

$$\max_{w\in s(\bar{\lambda})}\left\{w\in s(\bar{\lambda}), \begin{cases} w-11, & 11\le w\le 12 \\ 11\ \ , & 12\le w\le 13 \\ 14-w, & 13\le w\le 14 \end{cases} \ge\alpha\right\}.$$

(i.e.).,

$\beta(\alpha)=[11+\alpha,14-\alpha],\ \beta(\alpha)=[91+\alpha,94-\alpha],$

$v(\alpha)=[161+\alpha,164-\alpha]$ &

$\theta(\alpha)=[71+\alpha,74-\alpha].$

It is clear that, when $w=w_{\alpha}^{U},\ x=x_{\alpha}^{U},\ y=y_{\alpha}^{U}$ & $z=z_{\alpha}^{U}$, A attains its maximum value and when $w=w_{\alpha}^{L},\ x=x_{\alpha}^{L},\ y=y_{\alpha}^{L}$ & $z=z_{\alpha}^{L}$, A attains its minimum value.From the generated for the given input value of $\bar{\lambda},\ \bar{\beta}, \bar{v},$ & $\bar{\theta}$.

i) For fixed values of $w,\ x$ & $y,\ A$ decreases as z increase.

ii) For fixed values of $x,\ y$ & $z,\ A$ decreases as w increase.

iii) For fixed values of $y,\ z$ & $w,\ A$ decreases as x increase.

iv) For fixed values of $z,\ w$ & $x,\ A$ decreases as y increase.

The smallest value of occurs when w-takes its lower bound.

i.e)., $w=11+\alpha$ and x y and z, take their upper bounds given by $x=94-\alpha,\ and\ y=164-\alpha,\ and\ z=74-\alpha$ respectively. And maximum value of $E(L)$ occurs when $w=14-\alpha,\ x=91+\alpha,\ y=161+\alpha,\ z=71+\alpha.$ If both $E(L)_\alpha^L$ & $E(L)_U^\alpha$ are invertible with respect to $'\alpha'$ then the left shape function $L(A)=[E(L)_\alpha^L]^{-1}$ and right shape function $R(A)=[E(L)_\alpha^U]^{-1}$ can be obtained and from which the membership function $\mu_{\overline{E(L)}}(A)$ can be constructed as:

$$\mu_{\overline{E(L)}}(A)=\begin{cases} L(A), & A_1\le A\le A_2 \\ 1, & A_2\le A\le A_3 \\ R(A), & A_3\le A\le A_4 \end{cases} \qquad (10)$$

The values of A_1, A_2, A_3 & A_4 as obtained from (10) are:

$$\mu_{\overline{E(L)}}(A)=\begin{cases} L(A), & 0.0935\le A\le 0.1011 \\ 1, & 0.1011\le A\le 0.1143 \\ R(A), & 0.1143\le A\le 0.1244 \end{cases}$$

In the same way as we said before we get the following results.

The Virtual time of a Customer in the System

The smallest value of $E(W)$ occurs, when w take its lower bound.

i,e)., $w=11+\alpha$ & y,z & s take their upper bounds. Given by $x=94-\alpha, y=164-\alpha$ & $z=74-\alpha$ respectively. And maximum value of $E(W)$ occurs ,when $w=14-\alpha,\ x=91+\alpha,\ y=161+\alpha$ & $z=71+\alpha.$ if both $E(W)_\alpha^L$ & $(W)_\alpha^U$ are invertible with respect to α then left shape function $L(B)=[(E(W)_\alpha^L]^{-1}$ and right shape function $R(B)=[(E(W)_\alpha^U]^{-1}$ can be obtained from which the membership function $\mu_{\overline{(E(W)}}(B)$ Can be written as:

$$\mu_{\overline{E(W)}}(B)=\begin{cases} L(B), & B_1\le B\le B_2 \\ 1, & B_2\le B\le B_3 \\ R(B), & B_3\le B\le B_4 \end{cases} \quad (11)$$

The values of B_1, B_2, B_3 & B_4 as obtained from (11) are:

$$\mu_{\overline{E(W)}}(B)=\begin{cases} L(B), & 0.2874\le B\le 0.7687 \\ 1, & 0.7687\le B\le 1.2691 \\ R(B), & 1.2691\le B\le 1.8571 \end{cases}$$

The Server is in idle period

The smallest values of P_0 occurs when w-take its lower bound.

i.e)., $w=11+\alpha,$ & $x,y,$ & z take their upper bounds given by $x=94-\alpha, y=164-\alpha$ & $z=74-\alpha$ respectively. And maximum value of P_0 occurs when $w=14-\alpha,\ x=91+\alpha,\ y=161+\alpha$ & $z=71+\alpha.$ If both $(P_0)_\alpha^L$ & $(P_0)_\alpha^U$ are invertible with respect to α then the left shape function $L(C)=[(P_0)_\alpha^L]^{-1}$ and the right shape function $R(C)=[(P_0)_\alpha^U]^{-1}$ Can be obtained from which the membership function $\mu_{\overline{(P_0)}}(C)$ Can be written as:

$$\mu_{\overline{P_0}}(C)=\begin{cases} L(C), & C_1\le C\le C_2 \\ 1, & C_2\le C\le C_3 \\ R(C), & C_3\le C\le C_4 \end{cases} \quad (12)$$

The values of C_1, C_2, C_3 & C_4 as obtained from (12) are

$$\mu_{\overline{P_0}}(C)=\begin{cases} L(C), & 0.2741\le C\le 0.3258 \\ 1, & 0.3258\le C\le 0.3612 \\ R(C), & 0.3612\le C\le 0.4993 \end{cases}$$

The Server is in Regular Busy Period

The smallest value of P_1 occurs, when x take its lower bound.

i,e)., $w=11+\alpha,$ & $x,y,$ & z take their upper bounds given b $x=94-\alpha, y=164-\alpha$ & $z=74-\alpha$ respectively. And maximum value of P_1 occurs when $w=14-\alpha,\ x=91+\alpha\ \ y=161+\alpha$ & $z=71+\alpha.$

If both $(P_1)_\alpha^L$ & $(P_1)_\alpha^U$ are invertible with respect to α then the left shape function $L(D)=[(P_1)_\alpha^L]^{-1}$ and the right shape function $R(D)=[(P_1)_\alpha^U]^{-1}$ can be obtained from which the membership function $\mu_{\overline{(P_1)}}(D)$ Can be written as:

$$\mu_{\overline{P_1}}(D)=\begin{cases} L(D), & D_1\le D\le D_2 \\ 1, & D_2\le D\le D_3 \\ R(D), & D_3\le D\le D_4 \end{cases} \quad (13)$$

The values of D_1, D_2, D_3 & D_4 as obtained from (13) are:

$$\mu_{\overline{P}_1}(D) = \begin{cases} L(D), & 0.8654 \le D \le 0.8896 \\ 1, & 0.8896 \le D \le 0.8925 \\ R(D), & 0.8925 \le D \le 0.9079 \end{cases}$$

Further by fixing the vacation rate by a crisp value $\bar{\theta}$=76.4 and $\bar{\upsilon}$=161.3 taking arrival rate $\bar{\lambda}$=[11,12,13,14], service rate $\overline{\beta}$ =[91,92,93,94] both trapezoidal fuzzy numbers the values of the number of system in the steady state are generated and are plotted in the figure 1, it can be observed that as $\bar{\lambda}$ increases the number of system in steady state increases for the fixed value of the service rate, whereas for fixed value of arrival rate, the number of system in steady state decreases as service rate increases. Similar conclusion can be obtained for the case $\overline{\theta}$ =73.6, $\bar{\upsilon}$=163.8 Again for fixed values of $\overline{\lambda}$=[11,12,13,14], $\overline{\beta}$ =[91,92,93,94] the graphs of the virtual time of a customer in the system are drawn in figure 2 respectively, these figure show that as arrival rate increases that the virtual time also increases, while the virtual time decreases as the service rate increases in both the case.

It is also observed from the data generated that the membership value of the number of system in steady state is 1.7 and the membership value of the mean virtual time 1.85 when the ranges of arrival rate, service rate, and the vacation rate lie in the intervals (12,13.4), (93,93.6), & (161.8,162.4) respectively.

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