ISSN: 2321-8169
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Characterizations of Prime and Minimal Prime Ideals of Hyperlattices

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Abstract: In this paper, we define an ideal and prime ideal of hyperlattices (hyper join and hyper meet operation) .We obtain results on ideals of lattice in the sense of hyperlattice .We prove some results on prime ideals of hyperlattices. A result analogous to separation theorem is obtained for hyperlattices in respect of prime ideals. Further, we extend the classical result of Nachbin for hyperlattices. Also we furnish some characterizations of minimal prime ideals of hyperlattices.

Key words: *Hyperlattice, Ideal, Prime ideal, Minimal prime ideals*

Mathematics Subject Classifications: 06B10, 06B75.

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1. Introduction:

Hyperstructures were introduced in 1934 by a French mathematician, Marty 1934 at 8th congress of Scandinavian mathematics [9] and plays a central role in the theory of algebraic hyperstructures .Since then this theory has enjoined a rapid development [9,10,15,14,2,3,4]. In its general aspects, the connections with classical algebraic structures and various applications (In geometry, topology, combinatorics, theory of binary relations, theory of fuzzy and rough sets, probability theory ,cryptography and codes theory, automata theory ….and so on) have been investigated in [12]. In particular, hyperlattices were introduced by Mittas and Konstantinidou in [9]. In [5] Rahnamai-Bhargi studied ideal and prime ideal by considering join as hyper operation. The main goal of this paper is to study prime ideals and minimal prime ideals of hyperlattices and to draw several conclusions and we prove analogue of stone's theorem for hyperlattices and also prove classical Nachbin theorem. In the last section we give characterizations of minimal prime ideal and prove the theorem, If L is an ideal of L. Then a prime ideal P containing J is a minimal prime ideal containing J if and only if for each $x \in P$ there is $y \in L\backslash P$ such that $x \otimes y \subseteq J$.

2. Preliminaries:

We recall here some definitions and propositions on hyperlattices from [1] and we establish some results which we need for the development of this chapter.

Definition 2.1: Let L be a non-empty set with two binary operations $\wedge$ and $\vee$. If for all $x, y, z \in L$, the following conditions are satisfied:

i) $x \wedge y = x$, $x \vee y = x$

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ii) $x \wedge y = y \wedge x$, $x \vee y = y \vee x$

iii) $(x \wedge y) \wedge z = x \wedge (y \wedge z)$, $(x \vee y) \vee z = x \vee (y \vee z)$

iv) $(x \wedge y) \vee x = x$, $(x \vee y) \wedge x = x$ then we call $(L, \wedge, \vee)$ is a Lattice.

Definition 2.2: Let H be a non empty set. Let $P(H)$ be the power set of H, $P^*(H)=P(H) - \{\emptyset\}$.The hyper operation "$o$" on H is a map $H\times H \to P^*(H)$ such that for all $x, y, z \in H$ for all $X,Y,Z \in P^*(H)$, we have that $X \, o \, Y \in P^*(H)$, $z \, o \, X = \bigcup_{x \in X,} zox$, $X \, o \, z = \bigcup_{x \in X} xoz$, $X \, o \, Y = \bigcup_{x \in X, y \in Y} xoy$.

Following definition of hyperlattice is from [8]

Definition 2.3: Let H be a non empty set and $\oplus : H \times H \to P^*(H)$ be a hyper operation, and $P^*(H) = P(H) - \{\emptyset\}$ and $\otimes : H \times H \to P^*(H)$ be an operation. Then $(H, \oplus, \otimes)$ is a hyperlattice. if for all $x, y, z \in H$:

i) $x \in x \oplus x$, $x \in x \otimes x$;

ii) $x \oplus y = y \oplus x$, $x \otimes y = y \otimes x$;

iii) $(x \oplus y) \oplus z = x \oplus (y \oplus z)$; $(x \otimes y) \otimes z = x \otimes (y \otimes z)$;

iv) $x \in (x \otimes (x \oplus y))$, $x \in (x \oplus (x \otimes y))$.

Where for all non empty subsets A and B of L, $A \otimes B = U\{x \otimes y / x \in A, y \in B\}$, $A \oplus B = U\{x \oplus y / x \in A, y \in B\}$.

Following M.Konstantinidou and J.Mittas [9], we define a hyperlattice as a set H on which a hyperoperation $\oplus$ and an operation $\otimes$ are defined which satisfy the following axioms

1. $a \in a \oplus a$, $a \otimes a = a$
2. $a \oplus b = b \oplus a$, $a \otimes b = b \otimes a$
3. $(a \oplus b) \oplus c = a \oplus (b \oplus c)$,$(a \otimes b) \otimes c = a \otimes (b \otimes c)$.
4. $a \in (a \otimes (a \oplus b)) \cap (a \oplus (a \otimes b))$
5. $a \in a \oplus a$ implies that $b = a \otimes b$.

Throughout this paper, we refer definition 2.3 for hyperlattice.

Definition 2.4: A hyperlattice $(L, \oplus, \otimes)$ is said to be distributive if for each $x, y, z \in L$:

$x \otimes (y \oplus z) = (x \otimes y) \oplus (x \otimes z)$.

Proposition 2.5: *Let $(L, \oplus, \otimes)$ be a hyperlattice. Then the following holds:*

(1) $A \subseteq A \otimes A$, $A \subseteq A \oplus A$

(2) $A_1 \otimes A_2 = A_2 \otimes A_1$, $A_1 \oplus A_2 = A_2 \oplus A_1$

(3) $(A_1 \otimes A_2) \otimes A_3 = A_1 \otimes (A_2 \otimes A_3)$, $(A_1 \oplus A_2) \oplus A_3 = A_1 \oplus (A_2 \oplus A_3)$;

(4) $A_1 \subseteq A_1 \otimes (A_1 \oplus A_2)$, $A_1 \subseteq A_1 \oplus (A_1 \otimes A_2)$.

Example 2.6: Let $L = \{a, b\}$, $\oplus$ and $\otimes$ be two hyperoperations defined on L as follows.

$\otimes$	**a**	**b**
a	{a,b}	{b}
b	{b}	{b}

$\oplus$	**a**	**b**
a	{ a,b}	{a,b}
b	{a,b}	{b}

It can be verified that $\oplus$ and $\otimes$ satisfy (i) to (iv) of hyperlattice and therefore $(L, \oplus, \otimes)$ is a hyperlattice. For any element x and any subset S of a hyperlattice L, x $\oplus$ S means the set $\cup\{x \oplus a \mid a \in S\}$ and $x \otimes S$ we mean the set $\cup\{x \otimes a \mid a \in S\}$.

Example 2.7: Let $L = \{x, y\}$

$\otimes$	**x**	**y**
x	{x, y}	{y}
y	{y}	{y}

$\oplus$	**x**	**y**
x	{ x}	{y}
y	{x, y}	{y}

(L, $\oplus$, $\otimes$) is **not** a hyperlattice since $x \in x \otimes (x \oplus y) = x \otimes \{y\} = \{y\}$.

Example 2.8: Let $L = \{x_1, x_2, x_3, x_4\}$

$\oplus$	x_1	x_2	x_3	x_4
x_1	$\{x_1, x_2\}$	$\{x_2\}$	$\{x_3, x_4\}$	$\{x_4\}$
x_2	$\{x_2\}$	$\{x_1, x_2\}$	$\{x_4\}$	$\{x_3, x_4\}$
x_3	$\{x_3, x_4\}$	$\{x_4\}$	$\{x_3, x_4\}$	$\{x_4\}$
x_4	$\{x_4\}$	$\{x_3, x_4\}$	$\{x_4\}$	$\{x_3, x_4\}$

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$\otimes$	x_1	x_2	x_3	x_4
x_1	$\{x_1\}$	$\{x_1, x_2\}$	$\{x_1\}$	$\{x_1\}$
x_2	$\{x_1\}$	$\{x_2\}$	$\{x_1\}$	$\{x_1, x_2\}$
x_3	$\{x_1\}$	$\{x_1\}$	$\{x_3\}$	$\{x_3\}$
x_4	$\{x_1\}$	$\{x_1, x_2\}$	$\{x_3\}$	$\{x_3, x_4\}$

$(H, \otimes, \oplus)$ is a hyperlattice.

Definition 2.9: Let $(L, \otimes, \oplus)$ be a hyperlattice. A nonempty subset A of L is called a subhyperlattice of L if $(A, \otimes, \oplus)$ is itself a hyperlattice.

It is easy to see that a nonempty subset A of $(L, \otimes, \oplus)$ is a subhyperlattice of L if and only if A holds: for all $a, b \in A$, $a \otimes b \subseteq A$, $a \oplus b \subseteq A$. That is to say, A is a

Subhyperlattice of $(L, \otimes, \oplus)$ if and only if $A \otimes A \subseteq A$, $A \oplus A \subseteq A$.

Now we consider following example.

Example 2.10: Let $(L, \oplus, \otimes)$ be a lattice .Define the hyperoperations $\oplus$ and

$\otimes$ on L as follows:

$\otimes$	x	y
x	$\{x\}$	$\{y\}$
y	$\{y\}$	$\{y\}$

$\oplus$	x	y
x	$\{x\}$	$\{x\}$
y	$\{x\}$	$\{y\}$

$x \otimes y = \{x \wedge y\}$, $x \oplus y = \{x \vee y\}$, $x \in x \otimes (x \oplus y) = x \otimes \{x\} = \{x\}$, $y \in y \otimes (x \oplus y) = y \otimes \{x\} = \{y\}$ $(L, \otimes, \oplus)$

forms a Hyperlattice. From the above example every lattice is a hyperlattice.

Proposition 2.11: Every lattice is Hyperlattice but converse may not be true.

Proof: Let $(L, \otimes, \oplus)$ is a lattice. It is sufficient to prove properties (i) and (iv) of hyperlattice. By property (i) of lattice $x = x \oplus x$, Clearly x must be element of $x \oplus x$ and by property (iv) of lattice $x = x \otimes (x \oplus y)$ that is $x \otimes (x \oplus y)$ contains element x. Therefore, $x \in (x \otimes (x \oplus y))$. Similarly we can prove for $\otimes$. ∎

But Converse is not true. For this, consider the hyperlattice as shown in following table.

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$\oplus$	0	x	y	1
0	L	{x,1}	{y,1}	{ 1}
x	{x,1}	{x,1}	{ 1}	{ 1}
y	{y,1}	{ 1}	{y,1}	{ 1}
1	{ 1}	{ 1}	{ 1}	{ 1}

$\otimes$	0	x	y	1
0	{0}	{0}	{0}	{0}
x	{0}	{x}	{0}	{x}
y	{0}	{0}	{y}	{y}
1	{0}	{x}	{y}	{1}

Similarly, associative property can be easily verified. Therefore, $(L, \oplus, \otimes)$ which is a hyperlattice.

$x \oplus x = \{x, 1\} \neq x$. Therefore, Idempotent law is not satisfied.

Therefore $(L, \oplus, \otimes)$ is not a lattice.

Now from the above example , it is clear that every hyperlattice may not be a lattice.

3. Ideal of hyperlattice

Definition 3.1: Let $(L, \otimes, \oplus)$ be a hyperlattice and let A be a non-empty subset of L

1. A is called an *ideal* of L if for all $a,b \in A$ *and* $x \in L$

   i) $a \oplus b \subseteq A$

   ii) $a \otimes x \subseteq A$

2. A is called a *filter* of L if for all a, b $\in$ A and x $\in$ L

   i) $a \otimes b \subseteq A$

   ii) $a \oplus x \subseteq A$

Obviously, a subhyperlattice A of $(L, \otimes, \oplus)$ is a ideal of L if and only if $A \otimes L \subseteq A$. Similarly a subhyperlattice A of $(L, \otimes, \oplus)$ is a filter of L if and only if $A \oplus L \subseteq A$.

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Proposition 3.2: *Let $(L, \otimes, \oplus)$ be a hyperlattice and let A be a non empty subset of L. Then the following conditions are equivalent.*

1. *A is an ideal of $(L, \otimes, \oplus)$*
2. *$a \oplus b \subseteq A$, and $a \otimes x \subseteq A$ for all $a,b \in A$ and $x \in L$*
3. *$A \oplus A \subseteq A$ and $A \otimes L \subseteq A$.*

Similarly, the following conditions are equivalent

1. *A is a filter of $(L, \otimes, \oplus)$*
2. $a \otimes b \subseteq A$, and $a \oplus x \subseteq A$ for all $a,b \in A$ and $x \in L$
3. $A \otimes A \subseteq A$ and $A \oplus L \subseteq A$.

Example 3.3: Let $(L, \otimes, \oplus)$ be a hyperlattice.

$\otimes$	**0**	x_1	x_2	**1**
0	{0}	{0}	{0}	{0}
x_1	$\{0, x_1\}$	$\{0, x_1\}$	$\{x_1\}$	$\{x_1\}$
x_2	{0}	$\{x_1\}$	$\{0, x_2\}$	$\{x_2\}$
1	{0}	$\{x_1\}$	$\{x_2\}$	{1}

$\oplus$	**0**	x_1	x_2	**1**
0	{0}	$\{x_1, x_2\}$	$\{x_2\}$	{1}
x_1	$\{x_1, x_2, 1\}$	$\{x_1\}$	$\{x_2, 1\}$	{1}
x_2	$\{x_2, 1\}$	$\{x_2, 1\}$	$\{x_2\}$	{1}
1	{1}	{1}	{1}	{1}

I= $\{0, x_1, x_2\}$ is an ideal of Hyperlattice L.

Proposition 3.4: *Let A_1 and A_2 be two ideals of hyperlattice $(L, \otimes, \oplus)$ such that $A_1 \cap A_2 \neq \emptyset$ then $A_1 \cap A_2$ is also an ideal of hyperlattice.*

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Proof: Suppose A_1 and A_2 be ideals of hyperlattice L with $A_1 \cap A_2 \neq \emptyset$. For all $a_1, a_2 \in A_1 \cap A_2$, then $a_1 \in A_1 \cap A_2$, and $a_2 \in A_1 \cap A_2$. This implies $a_1 \oplus a_2 \subseteq A_1 \cap A_2$. For all a $\in A_1 \cap A_2$ and for all $x \in L$ implies $a \otimes x \subseteq A_1$, $a \otimes x \subseteq A_2$ .This gives, $a \otimes x \subseteq A_1 \cap A_2$. Therefore $A_1 \cap A_2$ is an ideal of L.

■

Let us consider the following caylay table for an hyperlattice $L = \{0, a, b, c, 1\}$

⊗	**0**	**a**	**b**	**c**	**1**
0	{0}	{0}	{0}	{0}	{0}
a	{0}	{ **a** }	{0,1}	{0}	{ **a** }
b	{0}	{0,1}	{0,b}	{0}	{0,1}
c	{0}	{0}	{0}	{c}	{ **a** }
1	{0}	{ a}	{0,1}	{ **a** }	{1}

⊕	**0**	**a**	**b**	**c**	**1**
0	{0}	{1}	{1}	{1}	{1}
a	{1}	{ a}	{ a,b,c}	{c}	{ 1}
b	{1}	{ a,b,c}	{b}	{c}	{1}
c	{1}	{c}	{c}	{c}	{1}
1	{1}	{ 1}	{1}	{c}	{1}

Then $I_1 = \{0, a,1\}$ and $I_2 =\{0,b,1\}$ are the ideal of hyperlattice L. $I_1 \cap I_2 = \{0,1\}$ is also an ideal. and $I_1 \cup I_2 = \{0, a,b, 1\}$ is not an ideal as a $\oplus$ b={a, c} $\not\subseteq I_1 \cup I_2$.

Remark 3.5: Union of two ideals of hyperlattice need not be an ideal.

Proposition 3.6: *Let (L, ⊕ , ⊗) be a lattice .Let I be ideal of lattice of L. Then I is ideal of Hyperlattice L.*

Proof:

Let L be a lattice and I be an ideal of L ($I \subseteq L$). Let $x_1, x_2 \in I \Rightarrow x_1 \oplus x_2 \in I$ and $x_1 \in I$, $x_1 \leq x_2 \Rightarrow x_2 \in I$. But every lattice is hyperlattice.

Therefore L is hyperlattice and $I \subseteq L$. By definition of ideal of lattice ,

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$x_1, x_2 \in I \Rightarrow x_1 \oplus x_2 \in$ I .

Therefore $x_1 \oplus x_2 \subseteq$ I .

By property (ii) of ideal of lattice $x \in I, a \leq x$ and $a \in$ L $\Rightarrow a \in I$. As $a \in I$ and $x \in I$ implies $x \otimes a \subseteq I$.

It is clear that, I is ideal of hyperlattice L.

■

Example 3.7: Let $(L, \wedge, \vee)$ be a lattice. Define the hyperoperations $\otimes$ and $\oplus$ on L as follows: for all $a, b \in L$, $a \otimes b = \{a \wedge b\}$, $a \oplus b = \{a \vee b\}$, then $(L, \otimes, \oplus)$ is a hyperlattice. Every ideal and filter of the lattice $(L, \wedge, \vee)$ are ideal and filter of the hyperlattice $(L, \otimes, \oplus)$, respectively.

Lemma 3.8 : Let $(L, \oplus, \otimes)$ be a distributive hyperlattice. If $p \in L$, then

$(p] = \{a \in L \mid a \in p \otimes a\}$ is an ideal.

Proof: Let $a, b \in (p]$, then $a \in p \otimes a$ and $b \in p \otimes b$. $a \oplus b \subseteq (p \otimes a) \oplus (p \otimes b) \Rightarrow a \oplus b \subseteq p \otimes (a \oplus b)$ .Therefore $a \oplus b \subseteq (p]$ .To prove second property of ideal, Let $a \in (p]$ and $x \in L$ .Then $a \otimes x \subseteq (p \otimes a) \otimes x \Rightarrow a \otimes x \subseteq p \otimes (a \otimes x) \Rightarrow a \otimes x \subseteq (p]$. Therefore $(p]$ is an ideal.

■

Dually, we can prove $[p)$ is a filter.

Lemma 3.9: *Let L be a distributive hyperlattice . If P is an ideal of L and* $a \in L$, *then* $P \oplus (a]$ *is an ideal of L.*

Proof:

Let $x , y \in P \oplus (a]$ *then* $x = p_1 \oplus a$ *and* $y= p_2 \oplus a$, $p_1 , p_2 \in P$. $x \oplus y \subseteq (p_1 \oplus a) \oplus (p_2 \oplus a) \Rightarrow x \oplus y \subseteq (p_1 \oplus p_2) \oplus (a \oplus a) \Rightarrow x \oplus y \subseteq P \oplus b \subseteq P \oplus (a]$ for some $b \in (a]$. Now let $x \in P \oplus (a]$ then $x = p \oplus a$ *and* $q \in L$. $x \otimes q \subseteq (p \oplus a) \otimes q \subseteq (p \otimes q) \oplus (a \otimes q)$. As P is an ideal , $p \in P$ *and* $q \in L$ implies $p \otimes q \subseteq P$ *and* $P \subseteq P \oplus (a]$ and as $a \in (a]$, $q \in L$ by Lemma , *2.3.8* $(a]$ is an ideal .This implies $p \otimes q \subseteq (a]$. Proving $x \otimes q \subseteq P \oplus (a]$.

■

Lemma 3.10: $(x] \cap (y]=(x \otimes y]$

Proof: let $a \in (x] \cap (y] \Rightarrow a \in (x]$ *and* $a \in (y] \Rightarrow a \in x \otimes a$ *and* $a \in y \otimes a \Rightarrow a \otimes a \subseteq (x \otimes a) \otimes (y \otimes a) \Rightarrow a \otimes a \subseteq (x \otimes y) \otimes (a \otimes a) \Rightarrow a \otimes a \subseteq (x \otimes y]$ .But by first property of hyperlattice $a \in a \otimes a$. Therefore $a \in (x \otimes y]$.Conversely, let $a \in (x \otimes y] \Rightarrow a \in (x \otimes y) \otimes a \Rightarrow a \in (x \otimes y) \otimes (a \otimes a)$ as $a \in a \otimes a$ always. $a \in (x \otimes a) \otimes (y \otimes a) \Rightarrow a \in (x \otimes a)$and $a \in (y \otimes a)$.This gives $a \in (x]$ *and* $a \in (y] \Rightarrow a \in (x] \cap (y]$.

■

4 Prime Ideals of Hyperlattices

We define the prime ideals of hyperlattice and definition of prime filter is taken from [9].

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Definition 4.1: Let J and F be respectively a proper ideal and a proper filter of a hyperlattice L .

(i) J is said to be prime if $x , y \in L$ and $x \otimes y \subseteq I$ implies $x \in I$ or $y \in I$.

(ii) F is said to be prime if $x, y \in L$ and $(x \oplus y) \cap F = \emptyset$ implies $x \in F$ or $y \in F$.

Example 4.2: L={x, y} be a hyperlattice .

$\otimes$	**x**	**y**
x	{x,y}	{y}
y	{y}	{y}

$\oplus$	**x**	**y**
x	{x,y}	{x,y}
y	{x,y}	{y}

I= {y} be an ideal. x $\otimes$ y={y} $\subseteq$ I and I is prime ideal as x $\otimes$ y $\subseteq$ I implies x$\notin$ I and $y \in I$.

Definition 4 .3: An ideal M is a maximal ideal in L if it is a maximal element in the set of all ideals of L.

Following definitions are from [2].

Definition 4.4: A hyperlattice $(L, \otimes, \oplus)$ is said to be distributive hyperlattice if $a\otimes (b\oplus c) = (a\otimes b) \oplus (a\otimes c)$ and $a\oplus (b\otimes c)=(a\oplus b) \otimes (a\oplus c)$ holds for every $a ,b, c \in L$.

Definition 4.5: Let $(L, \otimes ,\oplus)$ be a hyperlattice. An element $a \in L$ is said to be an all element of L if $a \in a \oplus x$ and $x \in a \otimes x$ for each $x \in L$. The set of all, **all element** of L, is denoted by I.

Definition 4.6: An element b in a hyperlattice $(L, \otimes, \oplus)$ is said to be a **zero element** of L if $x \in b \oplus x$ and $b \in b \otimes x$ for each $x \in L$. The set of all zero elements of L is denoted by O.

Definition 4.7: A hyperlattice $(L, \otimes, \oplus)$ is said to complemented if for every $a \in L$ there exists elements $a' \in L$, $a_i \in I$, $a_o \in O$ such that $a_i \in a \oplus a'$and $a_o \in a \otimes a'$.

Definition 4.8: A hyperlattice (L,$\otimes$, $\oplus$) with O, I is said to be a hyperboolean algebra if L is distributive and complimented.

Lemma 4.9: *In distributive hyperlattice, Every maximal ideal is prime.*

Proof: Let L be a distributive hyperlattice .Let $a \otimes b \subseteq M$ and $a \notin M$. Then $M \subset M \oplus (a] \subseteq L$.$M$ is maximal .This implies $M \oplus (a] = L$.As $b \in L \Rightarrow b \in M \oplus (a] \Rightarrow b \in \bigcup\{ m_i \oplus a_i\}$ .As $b \in b \otimes b \in b \otimes \bigcup\{ m_i \oplus a_i \}=\bigcup (b\otimes (m \oplus a))= \bigcup((b \otimes m) \oplus (b\otimes a)) \in M$ as M is an ideal and $m\in M$, $b \otimes m \subseteq M$. Therefore $b \in M$. Hence the proof.

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Remark 4.10: Converse of the lemma *4.9* is not true. Following result is by the lemma 4.9.

Corollary 4.11: *In a hyperboolean algebra , Every maximal ideal is prime .*

Lemma 4.12: *Let L be a hyperboolean algebra .Then every prime ideal of L is maximal.*

Proof: Let P be a prime ideal of L and Q be any ideal such that P $\subsetneq$ Q $\subseteq$ L. Since P $\subsetneq$ Q there exist x $\in$ Q such that x $\notin$ P.As L is hyperboolean algebra. L is complemented, There exist $a_o \in L$ and $a_i \in L$ such that $a_i \in x \oplus y$ and $a_o \in x \otimes y$. Let $x \otimes y \subseteq$ P and $x \notin$ P .This implies $y \in P \subset Q$. $x \in$ Q and $y \in Q \Rightarrow x \otimes y \subseteq Q$. $a_o \in Q$. Therefore $Q = L$ .P is maximal ideal of L.

■

Following corollary is by the corollary 4.11 and lemma 4.12.

Corollary 4.13: *In a Boolean algebra the prime ideals and maximal ideals coincides.*

Following definition is an extension from lattice structure to hyperlattice structure.

Definition 4.14 : A sequence of ideals in L , $I_1 \subseteq I_2 \subseteq \ldots\ldots \subseteq I_n \subseteq \ldots\ldots$ *is called an ascending chain of ideals.*

Definition 4.15 : A sequence of ideals in L , $I_1 \supseteq I_2 \supseteq \ldots\ldots \supseteq I_n \supseteq \ldots\ldots$ is called an descending chain of ideals.

A chain is said to be stabilize, if there exist $N \in \mathcal{N}$ such that $I_N = I_{N+K}$ for all $k \in \mathcal{N}$.

Proposition 4.16: *Let L be a hyperlattice .Then following are Equivalent.*

i) Every ascending chain condition (ACC) in L stabilizes.

ii) Hyperlattice L has a maximal element.

Proof: *i)* $\Rightarrow$ ii)

If ii) is false, then there is no maximal element exist in L, So

$\exists\, I_1 \in L$,such that

$I_1 \subset I_2$; for some $I_2 \in L$,

$I_1 \subset I_2 \subset I_3$: for some $I_3 \in L$

So continuing ……..Hence the contradiction. Therefore L has a maximal element.

(ii) $\Rightarrow$ *(i)* .

If *(ii)* holds and $I_1 \subseteq I_2 \subseteq \ldots\ldots\ldots\ldots \subseteq I_n \subseteq \ldots\ldots\ldots$ Then $\{I_1 , I_2,\ldots\ldots,I_n,\ldots\}$ has a maximal element , say I_k, so for every $m \supseteq k$. Therefore $I_m \supseteq I_k \supseteq I_m$,

Hence equality, proving (i) holds.

■

We have the following lemma 4.17 and Theorem 4.19, are extensions of [16] for hyperlattices.

Lemma 4.17: *In a bounded hyperlattice L,*

1) L is distributive

2) For any non-empty subset A $\subseteq$ L the set

$A^O = \{b \in L \mid x \in b \oplus x,\ b \in b \otimes x,\ x \in L\}$ *is an ideal in L and let*

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$A^I = \{a \in L \mid x \in a \oplus x, x \in a \otimes x, x \in L\}$ *is a filter in L.*

3) Every maximal ideal and maximal filter in L is prime.

4) $\cap \{P \mid P$ *is a prime ideal in L,* $a \in P\} = (a]$

5) For any distinct elements a and b in L , there exist a prime ideal P of L containing one of a and b and not containing the other.

Then $(1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4) \Rightarrow (5) \Rightarrow (1)$.

Proof: (1) ⇒ (2) To prove first property , Let $a, b \in A^O$. Then $x \in a \oplus x$, $a \in a \otimes x$ *and* $x \in b \oplus x$ and $b \in b \otimes x$ .This implies $x \oplus x \subseteq (a \oplus x) \oplus (b \oplus x)$ and $a \oplus b \subseteq (a \otimes x) \oplus (b \otimes x) \Rightarrow x \in x \oplus x \subseteq (a \oplus b) \oplus (x \oplus x)$ and $a \oplus b \subseteq (a \oplus b) \otimes x \in (a \oplus b) \otimes x \otimes x$. As $x \oplus x$ is also element of L .Therefore by definition of A^O, $a \oplus b \subseteq A^O$. To prove second property of ideal, let $a \in A^O$ and $y \in L$, Then $x \in a \oplus x$, $a \in a \otimes x$ and $y \in L$ implies $a \otimes y \in (a \otimes x) \otimes y$ and $x \otimes y \in (a \oplus x) \otimes y$ By distributivity , $x \otimes y \in (a \otimes y) \oplus (x \otimes y)$ and By associativity , and $y \in y \otimes y$ implies $a \otimes y \in (a \otimes y) \otimes (x \otimes y)$ .So for any $a \in A^O$ and $y \in L$ gives $a \otimes y \subseteq A^O$. Therefore from property 1 and 2 of ideal $a \oplus b \subseteq A^O$ *and* $a \otimes y \subseteq A^O$. A^O is an ideal. Dually we can prove $A^I = \{a \in L / x \in a \oplus x, x \in a \otimes x, x \in L\}$ is a filter in L.

(2) ⇒ (3) Let (2) holds. There exist $a' \in$ L, $a_i \in$ I and $a_o \in A^O$ such that $a_i \in a \oplus a'$ and $a_o \in a \otimes a'$. Then L is complimented lattice. L is complimented lattice, by Lemma 2.4.11, every maximal ideal is prime. Dually maximal filter is prime.

(3) ⇒ (4) Let (3) holds, Obviously $(a] \subseteq \cap \{P / P$ is a prime ideal in L, $a \in P\}$. Let if possible there exist $b \in \cap \{P$ P is a prime ideal in L, $a \in P\}$ such that $b \notin (a]$.By ACC, there exist a maximal filter say M such that $b \in M$, by assumption M being a prime filter $L \backslash M$ is a prime ideal. By the choice of b, $b \in L \backslash M$, a contradiction. Hence $\cap \{P \mid P$ is a prime ideal in L, $a \in P\} = (a]$.

(4) ⇒ (5) Let $a, b \in L$, such that $a \neq b$ .Therefore $b \notin (a]$ *by (4)*.There exist a prime ideal P containing $(a]$ not containing b. Therefore $a \in P$ and $b \notin P$.

(5) ⇒ (1) Let $a, b, c \in L$ and $(a \otimes b) \oplus (a \otimes c) \subseteq a \otimes (b \oplus c)$ .If $a \otimes (b \oplus c) \subsetneq (a \otimes b) \oplus (a \otimes c)$, then by *(5)*,there exist a prime ideal P in L such that $(a \otimes b) \oplus (a \otimes c) \subseteq P$ and $a \otimes (b \oplus c) \nsubseteq P$. *Then* $(a \otimes b) \subseteq P$ and $(a \otimes c) \subseteq P$ and $a \notin P \Rightarrow b \in P$, $c \in P$ (Since P is prime ideal and $a \notin P$) and hence $b \oplus c \subseteq P$. This leads to a contradiction.$a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)$. ■

Following Corollary can be proved by Lemma 3.8 and lemma 4.17

Corollary 4.18: *For any ideal I of a hyperlattice L,*

$$I = \cap \{P \mid P \text{ is a prime ideal in } L, a \in P\}$$

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Theorem 4.19: *In a hyperlattice L*

1) L is complemented

2) Every prime filter in L is maximal

3) Complement of a maximal ideal in L is a maximal filter.

4) Every prime ideal in L is maximal

5) Complement of a maximal filter in L is a maximal ideal.

Then (1) $\Rightarrow$ (2) $\Rightarrow$(3) $\Rightarrow$ (1) *and* (1) $\Rightarrow$ (4) $\Rightarrow$(5) $\Rightarrow$(1).

Proof: (1) $\Rightarrow$ (2) Let Q be any prime filter in L $(a\oplus b) \cap Q = \emptyset$, this implies $a \in Q$ or $b \in Q$ and $Q \subset F \subseteq L$ for some filter F in L, $x\in F$ *and* $x \notin Q$. Since L is Complemented, x' exist that is $a_o \in x \otimes x' \in L$. Since zero element a_o exist in Q. $x \otimes x' \in Q$ such that $x \notin Q \Rightarrow x' \in Q \Rightarrow x' \in F$. $x \otimes x' \in F$, $a_o \in x \otimes x' \in F \Rightarrow F=L$. F contains complement for each element. This proves that Q is a maximal.

(2) $\Rightarrow$ (3) Let (2) holds. As any maximal ideal is prime (by Lemma 2.4.9) we get L\ M is a maximal filter in L.

(3) $\Rightarrow$ (1) - Let L be not complemented. There does not exist $a' \in L$ and $a_i \in I$, $a_o \in O$ such that $a_i \in a \otimes a'$, $a_o \in a \oplus a'$ and $O \cap I = \emptyset$. There exist maximal ideal M containing O disjoint with I. Therefore L\ M is a maximal filter as $a \notin L\backslash M$, $b \in L\backslash M$ such that $a_o \in a \otimes b$ but then $b \in O \subseteq M \Rightarrow b \in M$, a contradiction. Hence there exist number a in L such that $I \cap O = \emptyset$, L is complemented. ■

Dually we can prove (1)$\Rightarrow$ (4)$\Rightarrow$ (5) $\Rightarrow$ (1).

As an immediate consequence of the theorem, we have

Corollary 4.20: *Let L be a distributive hyperlattice .*

1) L is a Boolean algebra
2) Complement of every maximal filter in L is a maximal ideal.
3) Complement of every maximal ideal in L is a maximal filter

4) Every prime filter in L is maximal

5) Every prime ideal is a maximal ideal.

Then (1) $\Rightarrow$ *(2)* $\Rightarrow$ *(3)* $\Rightarrow$ *(4)* $\Rightarrow$ *(5).*

Now we have the following *Lemma*.

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Lemma 4.21: *Let Q be a non empty proper subset of a hyperlattice L. Then Q is a filter if and only if L\ Q is a prime ideal.*

Proof: Let Q be filter of hyperlattice L. To prove L\ Q is an ideal. Let a , $b \in$ L\ Q. That is a ,$b \notin Q$ .Q *is a filter* $a \oplus$ b $\not\subseteq$ Q. This implies $a \oplus$ b $\subseteq$ L\ Q. Now let $a \in L\backslash Q$, $x \in L$ *and* $x \notin L\backslash Q$. *So* $x \in Q \subseteq L$ implies $a \otimes x \subseteq$ L and $a \otimes x \not\subseteq Q$.*Therefore* $a \otimes x \subseteq$ L\Q. To prove $L\backslash Q$ is a prime ideal. Let $x, y \in L$ such that $x \otimes y \subseteq$ L\ Q. So $x \otimes y \not\subseteq Q$ and hence either $x \notin Q$ *or* $y \notin Q$ as Q is Filter .This implies, either x ∈ L\ Q or y ∈ L\ Q. Therefore L\ Q is a prime ideal. Conversely, let L\ Q be a prime ideal and x, y ∈ Q. Clearly x, y$\notin L\backslash Q$ and hence $x \otimes y \not\subseteq L\backslash Q$ *as* $L\backslash Q$ is a prime ideal. Thus $x \otimes y \subseteq$ Q. Suppose x ∈ Q ⇒ $x \notin L\backslash Q$. *Since* $L\backslash Q$ *is* an ideal , we have $y \notin L\backslash Q$ .Hence $y \in Q$. This implies Q is a filter. ■

Dually we can prove the following Lemma 2.4.22.

Lemma 4.22: Let P be a non empty proper subset of hyperlattice L. Then P is a prime ideal if and only if L\P is a prime filter.

Now we prove Stone's Separation theorem for hyperlattices.

Theorem 4.23: *Let* $(L, \otimes, \oplus)$ *be a distributive hyperlattice. If I and D are an ideal and filter respectively such that* $I \cap D = \emptyset$ *.Then there exists a prime ideal P of L such that* $I \subseteq P$ *and* $P \cap D = \emptyset$*.*

Proof:

Let $\mathcal{F} = \{ A \mid A$ is an ideal of L, $J \subseteq A, A \cap F = \emptyset \}$.Clearly $\mathcal{F}$ satisfies Ascending chain condition. Therefore $\mathcal{F}$ has a maximal element M .Now we prove M is prime . Let a $\otimes b \subseteq M$ *for* a , $b \in L$ *also* a and b does not belongs to M.As $a \notin M \Rightarrow M \subsetneq M \oplus (a] \notin \mathcal{F}$. This implies $M \oplus (a] \cap F \neq \emptyset$ .There exists $x \in M \oplus (a] \cap F$ .And as $b \notin M \Rightarrow M \oplus (b] \cap F \neq \emptyset$. There exists $y \in M \oplus (b] \cap F$ .Therefore $x \otimes y \subseteq (M \oplus (a]) \cap (M \oplus (b]) \subseteq M \oplus ((a] \cap (b]) \subseteq M \oplus (a \otimes b]$ (by Lemma 2.3.12) $a \otimes b \subseteq M$ then $x \otimes y \subseteq M$. *Also,* $x \in F, y \in F$ and F is a filter $x \otimes y \subseteq F$. Therefore $x \otimes y \subseteq M \cap F = \emptyset$.A contradiction , Proving $a \otimes b \subseteq M \Rightarrow a \in M$ *or* $b \in M$. ■

Dually,we have

Theorem 4.24: *Let* $(L, \otimes, \oplus)$ *be a hyperlattice. If J and F are an ideal an filter respectively such that* $J \cap F = \emptyset$ *.Then there exists a prime filter Q of L such that* $F \subseteq Q$ *and* $J \cap Q = \emptyset$*.*

Corollary 4.25: *Let L be a distributive hyperlattice. Let* $a \notin J$*, J is an ideal in L. Then there exist a prime ideal containing J and not containing a.*

Proof:

As $a \notin J$, we get $[a) \cap J = \emptyset$. *If* $x \in [a) \cap J \Rightarrow x \in [a)$ and $x \in J$ which implies $x \in a \oplus x$ and $x \in J$,J is an ideal , implies $a \oplus x \subseteq J$ if and only if $a \in J$, a contradiction. Hence by stone theorem there exist a prime ideal P such that J $\subseteq P$ *and* $[a) \cap P = \emptyset$. *As* $[a) \cap J = \emptyset \Rightarrow a \notin$ P,

This proves the theorem.

■

Following corollary can be proved by theorem 4.24 and corollary 4.25

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Corollary 4.26: *Let L be a distributive hyperlattice. Let a $\notin$ D, J is an ideal in L. Then there exist a prime filter containing F and not containing a.*

Now we prove the following corollary.

Corollary 4.27: *Let $a \neq b$ in L. where L is a distributive hyperlattice .Then there exist a prime ideal containing exactly one of a and b.*

Proof: Let $J= (a]$ and $F= [b)$ .Then $J \cap F=\emptyset$. Hence by stone's theorem there exist a prime ideal P such that $(a] \subseteq P$ and $P \cap [\, b)=\emptyset$.That is $a \in P$ and b $\notin$ P. Therefore there exist a prime ideal $P \subseteq (a]$ and not containing $[b)$.

■

Following corollary can be proved by stone's theorem and corollary 4.26

Corollary 4.28: *Let $a\neq b$ in L. where L is a distributive hyperlattice .Then there exist a prime filter containing exactly one of a and b.*

The following theorem extends the classical result of Nachbin for hyperlattices. This theorem is proved for 0-1 distributive lattice by Pawar and Lokhande [17].

Theorem 4.29 (*Nachbin's theorem***)**: *Let L be a bounded distributive hyperlattice. L is complemented if and only if the set of all prime ideals of L is not ordered.*

Proof: If L is complemented **,** then every prime ideal of L is maximal (by lemma 4.12). Hence the set of all prime ideals of L is unordered. Conversely, Let L be not complemented such that $A^O \cap A^I = \emptyset$.

$$\text{where ,} \quad A^O = \{\, b \in L \mid x \in b \oplus x,\, b \in b \otimes x,\, x \in L\}$$

$$A^I = \{a \in L \mid x \in a \oplus x,\, x \in a \otimes x,\, x \in L\} .$$

As L is distributive, A^I is filter in L (by lemma 4.17) .

consider the filter $F = A^I \oplus [a)$, If $A^O \subseteq$ F ,then $b \in b \otimes a$ for some $b \in A^I$ and $a \in b \oplus a$ for some $a \in [a)$ by definition of filter $[a)$ and $b \in A^I$,Therefore $b \in A^O$. *But then* $b \in A^O \cap A^I = \emptyset$, a contradiction. $A^O \nsubseteq$ F and this will imply that F is a proper filter of L. As $A^O \subseteq$ L , F $\subsetneq$ G $\subseteq$ L. F must be contained in some maximal filter say G of L. Now, define P=L\G. Then P is prime ideal of L (by Lemma 4.21).Therefore F $\cap$ P = $\emptyset$. As $a \in F$ we get $a \notin P$. Consider the ideal $Q = P \oplus (\, a]$. *If* $A^I \subseteq Q$ *, then* $b \in b \oplus a$, by definition of ideal Q for some $b \in P$ and by definition of $(\, a]$, $a \in a \otimes b$, for some $b \in P$, $b \in A^I \subseteq F$ and thus $b \in F \cap P = \emptyset$. a contradiction

Therefore $A^I \nsubseteq$ Q. Q is proper. Q $\subsetneq$ M $\subseteq$ L.L is being distributive hyperlattice; M is prime (by lemma 2.4.9). $a \in M$ and $a \notin P$ shows that $P \subset M$. This not possible as the set of all prime ideals of L is not ordered .Hence for each $a \in L$, $(A^O \cap A^I) \neq \emptyset$ .Hence L is complemented.

■

Theorem 4.30 : *Let J be an ideal of hyperlattice L.A filter M disjoint from J is a maximal filter disjoint from J if and only if for all, $a \notin M$, $\exists$ $b \in M$ s.t $a \otimes b \subseteq J$.*

Proof: Let M be a maximal filter such that it is disjoint from J and $a \notin M$. *Let* $a \otimes b \nsubseteq J$ for all $b \in M$. Consider M_1 $=\{x \in L \,;\, x \in b \otimes x,\, b \in M\}$. Clearly M_1 is filter by the proof of lemma 2.4.17. For any $b \in M$, $b \in b \otimes b$ always,

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which implies $M \subseteq M_1$. *Also* $M_1 \cap J = \emptyset$. If not let, $a \in M_1 \cap J \Rightarrow a \in J$, $a \in M_1$, *So* $a \in b \otimes a$, *and* $b \in$ M $\subseteq L$ and J is an ideal $b \otimes a \subseteq J$, a contradiction. Hence $M_1 \cap J = \emptyset$. *Now* $M \subset M_1$ .which is the contradiction to the maximality of M. Hence there must exist $b \in M$ such that $a \otimes b \subseteq J$. Conversely, if M is not maximal among the filters disjoint from J, then there exist a filter $M_1 \supset M$ and disjoint from J. For any , $a \in M_1 \setminus M$,there exist $b \in M$ such that $a \otimes b \subseteq J$. Hence a , $b \in M_1$, M_1 is a filter , this implies $a \otimes b \subseteq M_1$, a contradiction .Hence M must be a maximal filter disjoint from A.

■

5. Minimal prime ideals of hyperlattice:

Definition 5.1: A prime ideal P of L is said to be minimal prime ideal if there is no prime ideal which is properly contained in P. A prime filter G of L is said to be maximal prime filter if there is no prime filter which is properly contains the filter G.

Lemma 5.2: *Let F be a non empty subset of a hyperlattice L . F is a maximal filter if and only if L \ F is a minimal prime ideal.*

Proof:

Let F be a maximal filter and L\ F is not a minimal prime ideal. Then there exists a prime ideal such that P ⊆ L\ F which implies that F ⊆ L\ P. which contradicts to the maximality of F. Hence L\ F is minimal prime ideal.
Conversely, Let L\ F be a minimal prime ideal and F is not a maximal filter .Thus there exist a proper filter Q such that F ⊆ Q which implies F is maximal filter.

■

The following lemma can be proved dually.

Lemma 5.3: *P is a Minimal prime ideal of L if and only if L \ P is a only maximal prime filter.*

Now we have the following result.

Theorem 5.4: *Every prime ideal of hyperlattice contains a minimal prime ideal.*

Proof: Let P be a prime ideal of *L*. Let *F=L\ P*. Then F is a prime filter .Then by *ACC*. There is a maximal prime filter G in L. $F \subseteq G \Rightarrow L \setminus G \subseteq L \mid F = P$. Therefore (by lemma 5.3) *L-G* is a minimal prime ideal contained in P. ■

Definition 5.5: Let L be a hyperlattice. For A ⊆ L ,

we define $A_J^{\perp} = \{x \in \mathrm{L} \mid x \otimes a \subseteq J \text{ for all } a \in A\}$

Lemma 5.6: $A_J^{\perp} = \{x \in L \mid x \otimes a \subseteq J$ *, for all* $a \in A\}$ *is an ideal.*

Proof: The proof is straight forward. ■

Now we have the following result which is a generalization of theorem 6 in [11].

Theorem 5.7: *Let A be a non empty subset of a hyperlattice L. Then* $A_J^{\perp}$ *is the intersection of all minimal prime ideals not containing A.*

Proof: Let L be a hyperlattice. Let $x \in A_J^{\perp}$, then x $\otimes$ $a \subseteq J$ for all $a \in A$. $p \in X$. since $A \not\subseteq P$, $y \in A$ but $y \notin P$ then x $\otimes$ $y \subseteq J$, J is prime . $J \subseteq P \Rightarrow x \in P$ as P is prime , $x \in X$. Conversely, let x ∈ X , if $x \notin A_J^{\perp}$, $x \otimes y \not\subseteq J$, for $y \in A$. *Let D =[*x $\otimes$ *y).* D is filter disjoint from J. Then by lemma (Filter disjoint from ideal I is contained maximal filter disjoint from I). There is a maximal filter M ⊇ D but disjoint from J. Then by duality of lemma 5.3, $L \setminus M$ is minimal prime ideal containing J. Now $x \notin L \setminus M$, $x \in D \Rightarrow x \in M$. Moreover $A \not\subseteq L \setminus M$ as $y \in A$. $L \setminus M$ is minimal prime ideal. $y \in A$,but $x \otimes y \not\subseteq J$,$y \notin L \setminus M$ .Therefore $A \not\subseteq L \setminus M$ .But $y \in M$ $(y \in D) \Rightarrow y \in L \setminus M$. ∎

Now we have the following result which is a generalization of theorem 3.1 in [13]

Theorem 5.8: *Let L be a hyperlattice. Then the following Statements (1) to (4) are equivalent and any one of them implies (5) ,(6),(7)*

1) L is distributive hyperlattice

2) Every maximal filter of L is prime.

3) If M is a maximal filter of L, L\M is a maximal prime ideal.

4) Every proper filter of L is disjoint from a minimal prime ideal.

5) For each non zero element a of L, there is a minimal prime ideal not containing a

6) For each non zero element a of L, there is a prime ideal not containing a

7) Prime filter contain each non zero element of L .

Proof: (1) ⇒ (2) by lemma 4.17

(2) ⇒ (3) Suppose (2) holds. Let M be any maximal filter of L. By lemma 5.3 (Duality) ,

L\ M is a minimal prime ideal.

(3) ⇒ (4) Suppose (3) holds .Let A be any proper filter of L. A ⊆ M for some maximal filter M. by (3) , L \ M is a minimal prime ideal. Clearly A ∩ (L \ M) = ∅ .

(4) ⇒ (1) Let x, y, z ∈ L such that $(x\otimes y) \oplus (x\otimes z) \subsetneq x\otimes (y\oplus z)$ .Let $((x\otimes y) \oplus (x\otimes z)] = I$ and$[(x\otimes (y\oplus z)) = F$. As $((x\otimes y) \oplus (x\otimes z)) \subset x\otimes (y\oplus z)$ we get I ∩F=∅.By (4), there exist a prime ideal P such that P∩ F=∅. and I ⊆ P $((x\otimes y) \oplus (x\otimes z))=I \subseteq P$ and $(x\otimes (y\oplus z))\cap P = \emptyset$ but then $((x\otimes y) \oplus (x\otimes z)) \subseteq P$ and $[x\otimes (y\oplus z)) \not\subseteq P$ (Since $P\cap F=\emptyset$). Furthermore, If $x \in P$ and $y \oplus z$ is an element of L .Therefore $x\otimes (y\oplus z) \subseteq P$ as P is an ideal, a contradiction. Therefore L is distributive.

(4) ⇒ (5) Suppose (4) holds. Let a be any non zero element of L and Q be a minimal prime ideal of L , by (4), [a) is disjoint from a minimal prime ideal Q. Therefore $a \notin Q$.

(5) ⇒ (6) obvious.

(6)⇒ (7) Suppose (6) holds. Let a be any non zero element of L. By (6), there is a prime ideal A such that a $\notin A$. By duality of lemma 4.21, $L \setminus A$ is a prime filter and clearly $a \in L \setminus A$. ∎

Theorem 5.9: *L be an ideal of L. Then a prime ideal P containing J is a minimal prime ideal containing J if and only if for each* $x \in P$ *there is* $y \in L\backslash P$ *such that* $x \otimes y \subseteq J$.

Proof: Let P be a minimal prime ideal containing J . Let $x \in P$. Suppose for all $y \in L\backslash P$ and $x \otimes y \nsubseteq J$. Set $D = (L \backslash P) \oplus [x)$. Suppose $D \cap J = \emptyset$.Then by corollary 2.4.26 of stones theorem, There is a prime filter Q such that $D \subseteq Q$ and $Q \cap J = \emptyset$. As Q is prime filter. Therefore $L \backslash Q$ is a prime ideal and $J \subseteq L \backslash Q$. Since $L \backslash P \subseteq D \subseteq Q$. We get $L \backslash Q \subseteq P$ and hence $L\backslash Q = P \Rightarrow Q = L \backslash P$. So that $x \in L \backslash P$. This is a contradiction. Therefore $D \cap J \neq \emptyset$. Let $z \in D \cap J$ implies $z \in [(L\backslash P) \oplus [x)] \cap J \Rightarrow z \in (L \backslash P) \oplus [x)$ and $z \in J$. As $z \in (L\backslash P) \oplus [x) \Rightarrow z \in y \otimes a$ where $y \in L\backslash P$ and $a \in [x)$. $z \in y \otimes a$, Since $[x)$ principal filter $\Rightarrow z \in y \otimes (a \oplus x) \Rightarrow z \in (y \otimes a) \oplus (y \otimes x)$. Now let us assume $(y \otimes a) \nsubseteq J$ .If so $y \in L\backslash P$ and as $J \subseteq L\backslash Q \subseteq P$, $y \notin J$.

Moreover $a \in [x) \in P$ .So $x \in P$ for some $x \in J$. Therefore $a \in J$. A contradiction. So $y \otimes a \subseteq J$. Also $z \in J$, $z \in (y \otimes a) \oplus (y \otimes x)$ and $y \otimes a \subseteq J$ implies $y \otimes x \subseteq J$. That is for every $x \in P$ there is $y \notin P$ such that $y \otimes x \subseteq J$. Conversely, Let P be a prime ideal of L containing J such that the given condition holds. Let Q be a prime ideal containing J such that $Q \subseteq P$. Let $x \in P$. Then there is $y \in L\backslash P$ such that $x \otimes y \subseteq J$,as Q containing J. Since Q is prime and $y \notin Q$, implies $x \in Q$. Hence $P \subseteq Q$. $Q = P$. Therefore P is a minimal prime ideal containing J. ■

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