Numerical Solution to Ninth order Non- Linear Differential Equation Using the Ninth Degree B-Spline Collocation Method

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Abstract: In this paper, Collocation method using recursive form of Ninth degree B-spline functions as basis is developed and employed to find the numerical solution for ninth order boundary value problems. Non linear boundary value problem is considered to test the performance, stability and accuracy of the present developed method.

Keywords *: B-Spline, Collocation, Recursive, Linear differential equation ,*

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1. INTRODUCTION

This paper is concerned with the numerical solution of ninth order linear boundary value problem by using ninth degree B-spline collocation solution.

The ninth order linear differential equation with boundary conditions is given as

$$P0(x)\frac{d^9U}{dx^9}+P_1(x)\frac{d^8U}{dx^8}+P_2(x)\frac{d^7U}{dx^7}+P_3(x)\frac{d^6U}{dx^6}+P_4(x)\frac{d^5U}{dx^5}+P_5(x)\frac{d^4U}{dx^4}+P_6(x)\frac{d^3U}{dx^3}+P_7(x)\frac{d^2U}{dx^2}+P_8(x)\frac{dU}{dx}+P_9(x)U=Q(x)$$

$$x\in(a,b), \qquad (1)$$

with the boundary conditions

$$U(a)=d1, U(b)=d2 \quad U'(a)=d3, U'(b)=d4, U''(a)=d5, U''(b)=d6, U'''(a)=d7, U'''(b)=d8, U^{iv}(b)=d9 \qquad (2)$$

where $a,b,d_1,d_2,d_3,d_4,d_5,d_6,d_7,d8,d9$ **are constants.** $P_0(x), P_1(x), P_2(x), P_3(x), P_4(x), P_5(x), P_6(x), P_7(x), P_8(x), P_9(x), Q(x)$ **are function of** x.

Different methods have been developed by many authors to solve the ninth order boundary value problems. Homotopy perturbation method was applied to obtain the solution for ninth and tenth order boundary value problems by Tauseef and Ahmet [1]. Samir presented spectral collocation method to solve ninth order boundary value problems [2].

In this paper, recursive form of ninth degree B-spline is employed as basis function in collocation method to solve the ninth order boundary value problems of the type (1)-(2).

2. DESCRIPTION OF METHOD

The solution domain $a \le x \le b$ is partitioned into a mesh of uniform length $h = x_{j+1} - x_j$, where j=0,1,2,…, N-1,N . Such that $a= x_0 < x_1 < x_2 \ldots\ldots < x_{n-1} < x_n=b$.

In the ninth degree B-spline collocation method the approximate solution is written as the linear combination of ninth degree B-spline basis functions for the approximation space under consideration. The proposed numerical solution for solving Eq. (1) using the collocation method with ninth degree B-spline is to find an approximation solution $U^h(x)$ to the exact solution $U(x)$ in the form:

$$U^h(x)=\sum_{i=-9}^{n+9} C_i N_{i,p}(x) \qquad (3)$$

where C_i's are constants to be determined from the boundary conditions and collocation from the differential equation.

A zero degree and other than zero degree B-spline basis functions [3, 4] are defined at x_i recursively over the knot vector space $X = \{x_1, x_2, x_3 \ldots\ldots x_{n-1}, x_n\}$ as

i) if $p = 0$

$N_{i,p}(x) = 1$ if $x \in (x_i, x_{i+i})$ $N_{i,p}(x) = 0$ if $x \notin (x_i, x_{i+i})$

ii) if $p \geq 1$

$$N_{i,p}(x) = \frac{x - x_i}{x_{i+p} - x_i} N_{i,p-1}(x) + \frac{x_{i+p+1} - x}{x_{i+p+1} - x_{i+1}} N_{i+1,p-1}(x)$$

………. (4)

where p is the degree of the B-spline basis function and x is the parameter belongs to X .When evaluating these functions, ratios of the form 0/0 are defined as zero.

Derivatives of B-splines

If p=9, we have

$$N'_{i,p}(x) = \frac{x - x_i}{x_{i+p} - x_i} N'_{i,p-1}(x) + \frac{N_{i,p-1}(x)}{x_{i+p} - x_i} + \frac{x_{i+p+1} - x}{x_{i+p+1} - x_{i+1}} N'_{i+1,p-1}(x) - \frac{N_{i+1,p-1}(x)}{x_{i+p+1} - x_{i+1}}$$

$$N^{ix}{}_{i,p}(x) = 9 \frac{N^{viii}{}_{i,p-1}(x)}{x_{i+p} - x_i} - 9 \frac{N^{viii}{}_{i+1,p-1}(x)}{x_{i+p+1} - x_{i+1}} \quad \ldots\ldots\ldots \quad (5)$$

$$(U^h)^{ix}(x) = \sum_{i=-9}^{n+9} C_i N^{ix}{}_{i,p}(x) \quad \ldots\ldots\ldots\ldots \quad (6)$$

The x_i's are known as nodes, the nodes are treated as knots in collocation B-spline method where the B-spline basis functions are defined and these nodes are used to make the residue equal to zero to determine unknowns C_i's in (3).Nine extra knots are taken into consideration besides the domain of problem to maintain the partition of unity when evaluating the ninth degree B-spline basis functions at the nodes which are within the considered domain.

Substituting the equations (3) to (6) in equation (1) for U (x) and derivatives of U (x).Then system of (n+1) linear equations are obtained in (n+9) constants. Applying the boundary conditions to equation (2), eight more equations are generated in constants. Finally, we have (n+10) equations in (n+10) constants.

Solving the system of equations for constants and substituting these constants in equation (3) then assumed solution becomes the known approximation solution for equation (1) at corresponding the collocation points.

This is implemented using the Matlab programming.

3. NUMERICAL EXPERIMENT

A non linear ninth order differential equation with boundary conditions[5] is considered to test the performance of the proposed method.

$$\frac{d^9 y}{dx^9} - y^2 \frac{dy}{dx} = \cos^3 x$$

with the boundary conditions $y(0) = 0$ $y(1) = \sin 1$ $y'(0) = 1$ $y'(1) = \cos 1$ $y''(0) = 0$ $y''(1) = -\sin 1$ $y'''(0) = -1$ $y'''(1) = \cos 1$ $y^{iv}(1) = 0$

The exact solution for the considered example is $y = \sin x$

In this section, the considered nonlinear problem is solved by the present proposed method and exact solution. Both the solution are tabulated in table1 and relative errors at these points are shown graphically in figure1

Table1 Comparison of numerical results with the exact values

x	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
Bspline solution	0.00	.0998	.1987	.2955	.3894	.4794	.5646	.6442	.7174	.7833
Exact solution	0	.0998	.1987	.2955	.3894	.4794	.5646	.6442	.7174	.7833

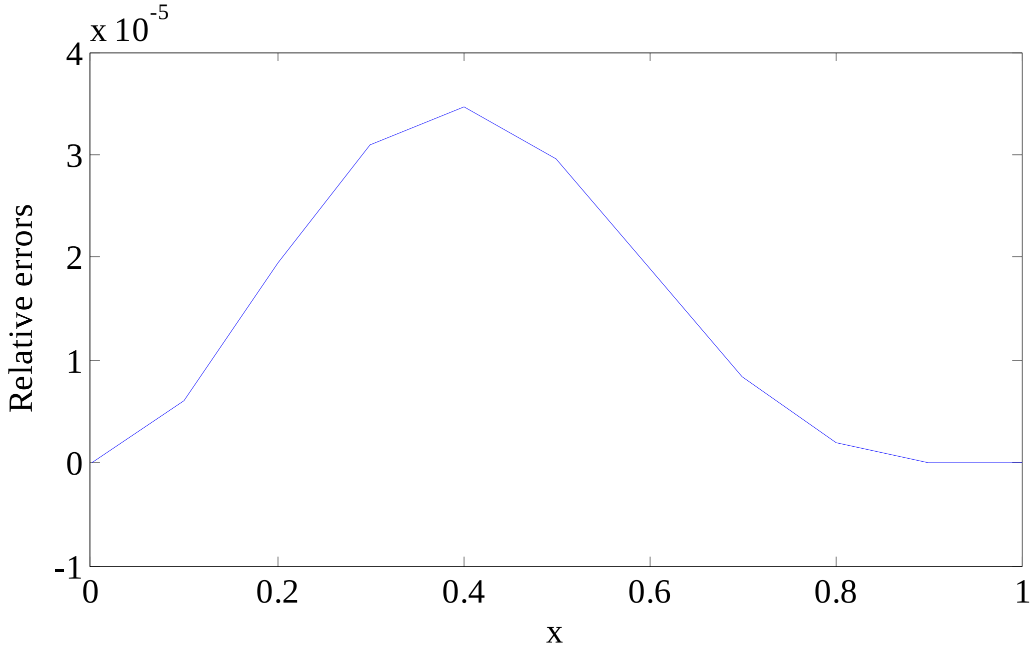

Figure1

Conclusion

In this paper, proposed collocation method by using the ninth degree B-spline as basis function is applied to non-linear ninth order linear differential equations with boundary conditions problem. It is observed that obtained values are very clse to the exact values and further absolute relative errors are very less at the nodes.This shows that the proposed method is effective and useful to find the numerical solutions for no-linear ninth order linear differential equation with boundary value problems.

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