Lossless Transmission Lines Terminated by Linear and Nonlinear RLC-Loads

Vasil G. Angelov
Department of Mathematics
University of Mining and Geology
1700 Sofia, Bulgaria
e-mail: angelov@mgu.bg

Marin M. Hristov
Department of Microelectronics
Technical University of Sofia
Sofia, Bulgaria
e-mail:mhristov@ecad.tu-sofia.bg

***Abstract*—**Here we consider lossless transmission lines terminated by a circuit consisting of linear and nonlinear RGCL-loads. First we overcome a difficulty caused by nonlinear boundary conditions utilizing Kirchhoff's laws. The problem arising is to choose as many equations as are the unknown functions. Then we formulate the mixed problem for the hyperbolic system, reduce the mixed problem to an initial value problem on the boundary and obtain a neutral system with respect to new variables. Further on we prove an existence-uniqueness theorem for periodic solution in lossless case.

Keywords*-lossless transmission lines; linear and nonlinear loads; mixed problem for hyperbolic system;fixed point method; periodic solution.*

*****

I. INTRODUCTION

The theory of transmission lines is a rapidly evolving area due to numerous applications[1]- [10]. The main purpose of the present paper is to investigate a transmission line terminated by a particular circuit consisting of linear and nonlinear *RGCL*-loads given in [11]-[16] and shown in Fig.1.

Here weconsider just lossless transmission lines using the method developed in our previous monograph [17]. The loads are in general nonlinear, but the theory developed could be applied to the linear loads too. In Section 2 we derive boundary conditions utilizing Kirchhoff's laws and formulate the mixed problem for lossless transmission lines. We wantto emphasize thatthe main difficulty by solving the mixed problem in question is to choose the unknown functions in such a way that their number equals the number of equations obtained from Kirchhoff's laws. In Section 3we reduce the mixed problem to an initial value problem on the boundary. Then in Section 4 by suitable operators we find conditions guaranteeing existence-uniqueness of a periodic solution. In Section 5 we analyze the

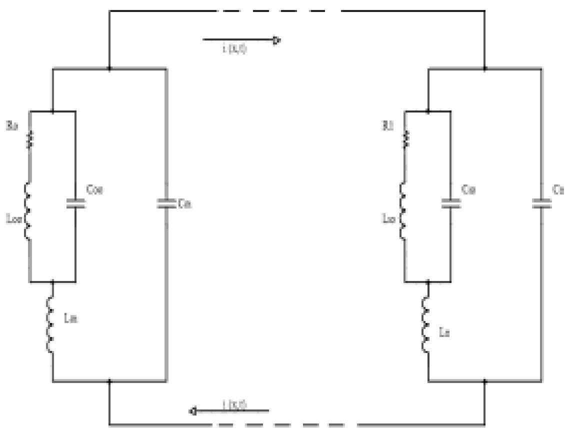

Figure 1. Lossless transmission lineterminated by a circuit consisting of linear and nonlinear loads

arising nonlinearities. In Section 6 we give an operatorpresentation of the periodic problem and prove the main result concerning an existence-uniqueness of periodic solution of the neutral system obtained.We use the fixed point method which allows us to obtain successive approximations tending to the solution. In Section 7 using numerical example we demonstrate how to apply our method to particular problems. We proceed from the Telegrapher equations

$$\partial u(x,t)/\partial x + L\partial i(x,t)/\partial t = 0,$$
$$\partial i(x,t)/\partial x + C\partial u(x,t)/\partial t = 0$$

or

$$\begin{bmatrix}\partial u(x,t)/\partial t\\ \partial i(x,t)/\partial t\end{bmatrix} = \begin{bmatrix}0 & -1/C\\ -1/L & 0\end{bmatrix}\begin{bmatrix}\partial u(x,t)/\partial x\\ \partial i(x,t)/\partial x\end{bmatrix}.$$

By the transformation

$$u(x,t) = U(x,t)/2 + I(x,t)/2,$$
$$i(x,t) = U(x,t)/(2Z_0) - I(x,t)/(2Z_0).$$

where $Z_0 = \sqrt{L/C}$ and $U(x,t)$, $I(x,t)$ are new variables, we reduce the above system in diagonal form

$$\begin{pmatrix}\partial U/\partial t\\ \partial I/\partial t\end{pmatrix} + \begin{pmatrix}1/\sqrt{CL} & 0\\ 0 & -1/\sqrt{CL}\end{pmatrix}\begin{pmatrix}\partial U/\partial x\\ \partial I/\partial x\end{pmatrix} = 0.$$

II. DERIVATION OF THE BOUNDARY CONDITIONS AND MIXED PROBLEM FORMULATION

In this section we derive the boundary conditions having considered Kirchhoff's laws. Let us recall that Λ is the length of the transmission line, $v = 1/\sqrt{LC}$ is the speed of propagation and $T = \Lambda/(1/\sqrt{LC}) = \Lambda\sqrt{LC}$,where L is per unit-length inductance and C – per unit-length capacitance of the line. We assume that R_0 , L_{01} and C_{01} are linear loads, that is, $u_{R_0}(t) = R_0 i_{R_0}(t),$

$$u_{L_{01}}(t) = dL_{01}(i_{L_{01}})/dt =$$
$$= [dL_{01}(i_{L_{01}})/(di_{L_{01}})]di_{L_{01}}(t)/dt = L_{01}di_{L_{01}}(t)/dt,$$
$$i_{C_{01}}(t) = dC_{01}(u_{C_{01}})/dt =$$
$$= [dC_{01}(u_{C_{01}})/du_{C_{01}}]du_{C_{01}}(t)/dt = C_{01}du_{C_{01}}(t)/dt$$

while $C_{00} = C_{00}(u_{C_{00}})$ and $L_{00} = L_{00}(i_{L_{00}})$ are nonlinear ones which implies

$$i_{C00}(t) = [dC_{00}(u_{C00})/du_{C00}]du_{C00}(t)/dt,$$
$$u_{L00}(t) = [dL_{00}(i_{L00})/di_{L00}]di_{L00}(t)/dt.$$

We notice that $u_{C01}(t) = u(0,t),\ i_{R0}(t) = i_{L00}(t),\ i_{C00}(t) = i_{L01}(t)$.

The Kirchhoff's Current Law gives:

$$i_{L00} + i_{C00} + i_{C01} = -i(0,t) \Leftrightarrow$$
$$i_{L00} + i_{C00} + C_{01}du_{C01}(t)/dt = -i(0,t) \Leftrightarrow \quad (1)$$
$$i_{L00} + i_{C00} + C_{01}du(0,t)/dt = -i(0,t).$$

Similarly, the Kirchhoff's Voltage Law gives

$$u_{R_0} + u_{L_{00}} + u_{L_{01}} = u(0,t), \quad (2)$$

$$u_{R0} + u_{L00} = u_{C00} \Leftrightarrow R_0 i_{L00} + [dL_{00}/di_{L00}]di_{L00}/dt = u_{C00}, \quad (3)$$

$$u_{L01} + u_{C00} = u(0,t) \Leftrightarrow L_{01}di_{L00}/dt + u_{C00} = u(0,t). \quad (4)$$

Finally, from (1)-(4) we obtain the system

$$C_{01}du(0,t)/dt = -i(0,t) - i_{L00}(t) - i_{C00}(t)$$
$$[dL_{00}(i_{L00})/di_{L00}]di_{L00}(t)/dt = C_{00}^{-1}(i_{C00}(t)) - R_0 i_{L00}(t)$$
$$L_{01}\frac{di_{L_{01}}(t)}{dt} = u(0,t) - C_{00}^{-1}(i_{C_{00}}(t))$$
$$u_{R0} + u_{L00} + u_{L01} = u(0,t).$$

Having in mindthat $i_{C_{00}}(t) = i_{L_{01}}(t)$ we obtain

$$C_{01}du(0,t)/dt = -i(0,t) - i_{L00}(t) - i_{C00}(t)$$
$$[dL_{00}(i_{L00})/di_{L00}]di_{L00}(t)/dt = C_{00}^{-1}(i_{C00}(t)) - R_0 i_{L00}(t) \quad (5)$$
$$L_{01}di_{C00}(t)/dt = u(0,t) - C_{00}^{-1}(i_{C00}(t)).$$

In the above system the unknown functions are

$$u(0,t), i(0,t),\ i_{C00}(t),\ i_{L00}(t).$$

Similar reasoning leads to boundary conditions for the right-hand side of the line:

$$C_{11}du(\Lambda,t)/dt = i(\Lambda,t) - i_{L10}(t) - i_{C10}(t)$$
$$[dL_{10}(i_{L10})/di_{L10}]di_{L10}(t)/dt = C_{10}^{-1}(i_{C10}(t)) - R_1 i_{L10}(t) \quad (6)$$
$$L_{11}di_{C10}(t)/dt = u(\Lambda,t) - C_{10}^{-1}(i_{C10}(t))$$

with unknown functions $u(\Lambda,t),\ i(\Lambda,t),\ i_{C10}(t),\ i_{L10}(t)$.

Remark 1. In the above equations we have to assume that $C_{00}^{-1}(.),\ C_{10}^{-1}(.)$ exist. This problem will be considered below.

Now we are able to formulate the mixed problem for the hyperbolic transmission line equations: to find a solution $(u(x,t), i(x,t))$ of the hyperbolic system

$$\partial u(x,t)/\partial x + L\partial i(x,t)/\partial t = 0,$$
$$\partial i(x,t)/\partial x + C\partial u(x,t)/\partial t = 0$$

for $(x,t) \in \Pi = \{(x,t) \in R^2 : 0 \le x \le \Lambda,\ t \ge 0\}$, satisfying the initial conditions

$$u(x,0) = u_0(x),\ i(x,0) = i_0(x) \text{ for } x \in [0,\Lambda] \quad (7)$$

and the boundary conditions for $x = 0$

$$C_{01}du(0,t)/dt = -i(0,t) - i_{L00}(t) - i_{C00}(t)$$
$$[dL_{00}(i_{L00})/di_{L00}]di_{L00}(t)/dt = C_{00}^{-1}(i_{C00}(t)) - R_0 i_{L00}(t) \quad (8)$$
$$L_{01}di_{C00}(t)/dt = u(0,t) - C_{00}^{-1}(i_{C00}(t))$$

and for $x = \Lambda$

$$C_{11}du(\Lambda,t)/dt = i(\Lambda,t) - i_{L10}(t) - i_{C10}(t)$$
$$\frac{dL_{10}(i_{L_{10}})}{di_{L_{10}}}\frac{di_{L_{10}}(t)}{dt} = C_{10}^{-1}(i_{C_{10}}(t)) - R_1 i_{L_{10}}(t) \quad (9)$$
$$L_{11}di_{C10}(t)/dt = u(\Lambda,t) - C_{10}^{-1}(i_{C10}(t)).$$

III. Reducing the Mixed Problem to an Initial Value Problem on the Boundary

Here we formulate a mixed problem for the system with respect to new variables $U(x,t), I(x,t)$ (see [17]):

$$\partial U/\partial t + [1/\sqrt{LC}]\partial U/\partial x = 0$$
$$\partial I/\partial t - [1/\sqrt{LC}]\partial I/\partial x = 0. \quad (10)$$

To find $U(x,t),\ I(x,t)$ satisfying (10) and initial conditions

$$U(x,0) = u(x,0) + Z_0\, i(x,0) = u_0(x) + Z_0\, i_0(x) \equiv U_0(x),$$
$$I(x,0) = u(x,0) - Z_0\, i(x,0) = u_0(x) - Z_0\, i_0(x) \equiv I_0(x) \quad (11)$$

where as usually we use the denotation $Z_0 = \sqrt{L/C}$.

To obtain new boundary conditions we take into account that

$$u(0,t) = [U(0,t) + I(0,t)]/2,$$
$$i(0,t) = [U(0,t) - I(0,t)]/(2Z_0)$$

and

$$u(\Lambda,t) = [U(\Lambda,t) + I(\Lambda,t)]/2,$$
$$i(\Lambda,t) = [U(\Lambda,t) - I(\Lambda,t)](2Z_0)$$

and substitute the above expressions into (8) and (9):

$$C_{01}d([U(0,t)/2] + [I(0,t)/2])/dt =$$
$$= -i_{L00}(t) - i_{C00}(t) - [(U(0,t) - I(0,t))]/2Z_0$$
$$[dL_{00}(i_{L00})/di_{L00}]di_{L00}(t)/dt = C_{00}^{-1}(i_{C00}(t)) - R_0 i_{L00}(t)$$
$$L_{01}di_{C00}(t)/dt = -C_{00}^{-1}(i_{C00}(t)) + [U(0,t) + I(0,t)]/2 \quad (12)$$
$$C_{11}d([U(\Lambda,t)/2] + [I(\Lambda,t)/2)]dt =$$
$$= -i_{L10}(t) - i_{C10}(t) + [U(\Lambda,t) - I(\Lambda,t)]/2Z_0$$
$$[dL_{10}(i_{L10})/di_{L10}]di_{L10}(t)/dt = C_{10}^{-1}(i_{C10}(t)) - R_1 i_{L10}(t)$$
$$L_{11}di_{C10}(t)/dt = -C_{10}^{-1}(i_{C10}(t)) + [U(\Lambda,t) + I(\Lambda,t)]/2.$$

Integrating(10)along the characteristics we obtain

$$U(0,t) = U(\Lambda,t+T),\ I(0,t+T) = I(\Lambda,t)$$

which implies

$$U(0,t-T) = U(\Lambda,t),\ I(0,t) = I(\Lambda,t-T).$$

Let us suppose that the unknown functions are

$$U(0,t) \equiv U(t),\ I(\Lambda,t) \equiv I(t),\ i_{L_{00}}(t),\ i_{C_{00}}(t),\ i_{L_{10}}(t),\ i_{C_{10}}(t)$$

and then we have to solve the following system consisting of six differential equations with constant delays:

$$dU(t)dt = -[U(t) + I(t-T)]/(C_{01}Z_0) -$$
$$-2(i_{L00}(t) + i_{C00}(t))/(C_{01}) - dI(t-T)/dt$$
$$dI(t)/dt = [U(t-T) - I(t)]/(C_{11}Z_0) -$$
$$-2(i_{L10}(t) + i_{C10}(t))/(C_{11}) - dU(t-T)/dt$$

$$[dL_{00}(i_{L00})/di_{L00}]di_{L00}(t)/dt = C_{00}^{-1}(i_{C00}(t)) - R_0 i_{L00}(t)$$
$$di_{C00}(t)/dt = (U(t) + I(t-T) - 2C_{00}^{-1}(i_{C00}(t)))/2L_{01} \quad (13)$$
$$[dL_{10}(i_{L10})/di_{L10}]di_{L10}(t)/dt = C_{10}^{-1}(i_{C10}(t)) - R_1 i_{L10}(t)$$
$$di_{C10}(t)/dt = [U(t-T) + I(t) - 2C_{10}^{-1}(i_{C10}(t))]/2L_{11}.$$

So we obtain a neutral system of differential equations with retarded arguments.

IV. Estimates of the Arising Nonlinearities and Introducing Function Spaces

We consider nonlinear capacitances (cf. [1]-[10]):

$$C_{p0}(u) = c_{p0}/\sqrt[h]{1-u/\Phi_{p0}} = c_{p0}\sqrt[h]{\Phi_{p0}}\left(\Phi_{p0}-u\right)^{-1/h},$$

where

$$c_{p0} > 0, \Phi_{p0} > 0,\ h \in [2,3];\ (p = 0,1)$$

are constants and $|u| \le \phi_0 < \min\{\Phi_{00}, \Phi_{10}\}$. We have to find an interval where $C_{p0}(u)$ has inverse function. We have

$$dC_{p0}(u)/du = c_{p0}\sqrt[h]{\Phi_{p0}}\left(\Phi_{p0}-u\right)^{-(1+h)/h}/h,$$
$$d^2C_{p0}(u)/du^2 = (h+1)c_{p0}\sqrt[h]{\Phi_{p0}}\left(\Phi_{p0}-u\right)^{-(1+2h)/h}/h^2.$$

Assumptions (C): $|u| \le \varphi_0 < \Phi_{p0}$.

The minimal value of $dC_{p0}(u)/du$ is

$$\min\{dC_{p0}(u)/du : |u| \le \varphi_0\} = dC_{p0}(-\varphi_0)/du =$$
$$= c_{p0}\sqrt[h]{\Phi_{p0}}\left(\Phi_{p0}+\varphi_0\right)^{-(1+h)/h}/h = \hat{C}_{p0} > 0, (p = 0,1)$$

and then

$$0 < q_{\min} \equiv c_{p0}/\sqrt[h]{\left(1+\varphi_0/\Phi_{p0}\right)} \le q = C_{p0}(u) =$$
$$= c_{p0}/\sqrt[h]{\left(1-u/\Phi_{p0}\right)} \le c_{p0}/\sqrt[h]{\left(1-\varphi_0/\Phi_{p0}\right)} \equiv q_{\max}.$$

Consequently $C_{p0}(u)$ is monotone increasing and possesses a unique inverse function. Its explicit form can be obtained from the equation

$$q = c_{p0}/\sqrt[h]{1-u/\Phi_{p0}} \Rightarrow u = \Phi_{p0}\left[1-\left(c_{p0}/q\right)^h\right]$$

and $u = C_{p0}^{-1}(q) : [q_{\min}, q_{\max}] \to [-\varphi_0, \varphi_0]$,

$$dC_{p0}^{-1}(q)/dq = \Phi_{p0}d\left[1-\left(c_{p0}/q\right)^h\right]/dq = \Phi_{p0}hc_{p0}^h/q^{h+1};$$
$$\left|dC_{p0}^{-1}(q)/dq\right| \le \Phi_{p0}hc_{p0}^h/q^{h+1} \le \Phi_{p0}h\left(1+\varphi_0/\Phi_{p0}\right)^{(h+1)/h}/c_{p0}.$$

We have also

$$\left|dC_{p0}(u)/du\right| \le c_{p0}\sqrt[h]{\Phi_{p0}}\left(\Phi_{p0}-\varphi_0\right)^{-(1+h)/h}/h \equiv M_p,$$
$$\left|d^2C_{p0}(u)/du^2\right| \le$$
$$\le c_{p0}\sqrt[h]{\Phi_{p0}}\left(\Phi_{p0}-\varphi_0\right)^{-(1+2h)/h}(h+1)/h^2 \equiv H_p\ (p = 0,1).$$

For $L_{p0}(i) = \sum_{n=1}^{m} l_n^{(p)} i^n$ we get

$$dL_{p0}(i)/di = \sum_{n=1}^{m} nl_n^{(p0)}i^{n-1}; d^2L_{p0}(i)/di^2 = \sum_{n=2}^{m}(n+1)nl_n^{(p0)}i^{n-2},$$
$$\left|dL_{p0}(i)/di\right| \le \sum_{n=2}^{m}(n+1)nl_n^{(p0)}|i|^{n-2} \le \sum_{n=2}^{m}(n+1)nl_n^{(p0)}|i_0|^{n-2}.$$

Assumptions (L): $|i| \le i_0 \Rightarrow$

$$dL_{p0}(i)/di = \sum_{n=1}^{m} nl_n^{(p0)}i^{n-1} \ge \hat{L}_{p0} > 0\ (p = 0,1).$$

Let $T = mT_0$ for some $m \in N$, T_0 is a period of the functions forming the function space where we seek a solution of the above system.

For the initial functions we assume

(IN): $U_0(.),\ I_0(.) \in C_{T_0}^1[0,T]$, $|U_0(t)| \le U_0 e^{\mu(t-kT_0)}$, $|I_0(t)| \le I_0 e^{\mu(t-kT_0)} (k = 0,1,2,\dots,m-1)$.

Assumptions (U): $e^{\mu T_0}(U_0 + I_0)/2 \le \varphi_0$.

It follows

$$|u(0,t)| \le (|U(t)| + |I(t-T)|)/2 \le (U_0 e^{\mu(t-kT_0)} +$$
$$+ I_0 e^{\mu(t-T-kT_0)})/2 \le e^{\mu T_0}(U_0 + I_0 e^{-\mu T})/2 \le \varphi_0;$$
$$|u(\Lambda,t)| \le (|U(t-T)| + |I(t)|)/2 \le (U_0 e^{\mu(t-T-kT_0)} +$$
$$+ I_0 e^{\mu(t-kT_0)})/2 \le e^{\mu T_0}(U_0 e^{-\mu T} + I_0)/2 \le \varphi_0.$$

Denote by $C_{T_0}^1[T,2T]$ the set of T_0-periodic smooth functions, where $mT_0 = T$ for some positive integer m. Introduce the function sets we look for periodic solution

$$M_U = \left\{u \in C_{T_0}^1[T,2T] : |U(t)| \le U_0 e^{\mu(t-T-kT_0)}\right\},$$
$$M_I = \left\{u \in C_{T_0}^1[T,2T] : |I(t)| \le_0 e^{\mu(t-T-kT_0)}\right\},$$
$$M_{L_0} = \left\{i_{L00}(t) \in C_{T_0}^1[T,2T\} : |i_{L00}(t)| \le I_{L_0} e^{\mu(t-T-kT_0)}\right\},$$
$$M_{C_0} = \left\{i_{C00}(t) \in C_{T_0}^1[T,2T\} : |i_{C00}(t)| \le I_{C_0} e^{\mu(t-T-kT_0)}\right\},$$
$$M_{L_1} = \left\{i_{L10}(t) \in C_{T_0}^1[T,2T\} : |i_{L10}(t)| \le I_{L_1} e^{\mu(t-T-kT_0)}\right\},$$

where $t \in [T + kT_0, T + (k+1)T_0]$, $k = 0,1,2,\dots,m-1$, I_{R_0}, U_0, I_{R_1}, I_0, T_0, μ are positive constants (chosen below) and $\mu T_0 = \mu_0 = \text{const}$.

Introduce the metrics

$$\rho_k(U,\bar{U}) = \max\left\{|U(t) - \bar{U}(t)|e^{-\mu(t-kT_0-T)} : t \in [T + kT_0, T + (k+1)T_0]\right\},$$
$$\rho_k(I,\bar{I}) = \max\left\{|I(t) - \bar{I}(t)|e^{-\mu(t-kT_0-T)} : t \in [T + kT_0, T + (k+1)T_0]\right\},$$
$$\rho_k(i_{L00},\bar{i}_{L00}) =$$
$$= \max\left\{|i_{L00}(t) - \bar{i}_{L00}(t)|e^{-\mu(t-T-kT_0)} : t \in [T + kT_0, T + (k+1)T_0]\right\},$$
$$\rho_k(i_{C00},\bar{i}_{C00}) =$$
$$= \max\left\{|i_{C00}(t) - \bar{i}_{C00}(t)|e^{-\mu(t-T-kT_0)} : t \in [T + kT_0, T + (k+1)T_0]\right\},$$
$$\rho_k(i_{L10},\bar{i}_{L10}) =$$
$$= \max\left\{|i_{L10}(t) - \bar{i}_{L10}(t)|e^{-\mu(t-T-kT_0)} : t \in [T + kT_0, T + (k+1)T_0]\right\},$$
$$\rho_k(i_{C10},\bar{i}_{C10}) =$$
$$= \max\left\{|i_{C10}(t) - \bar{i}_{C10}(t)|e^{-\mu(t-T-kT_0)} : t \in [T + kT_0, T + (k+1)T_0]\right\}$$
$$(k = 0,1,2,\dots,m-1).$$

The set $M_U \times M_I \times M_{L_0} \times M_{C_0} \times M_{L_1} \times M_{C_1}$ turns out into a complete uniform space with countable set of metrics $k = 0,1,2,\dots$ (cf. [17]):

$$\rho_k\left((U,I,i_{L00},i_{C00},i_{L10},i_{C10}),(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00},\bar{i}_{L10},\bar{i}_{C10})\right) =$$

 ISSN: 2321-8169
1341– 1352

$$= \max\{\rho_k(U,\bar{U}), \rho_k(I,\bar{I}), \rho_k(i_{L00},\bar{i}_{L00}), \rho_k(i_{C00},\bar{i}_{C00}), \rho_k(i_{L10},\bar{i}_{L10}), \rho_k(i_{C10},\bar{i}_{C10})\}.$$

V. Operator Presentation of the Periodic Problem

Now we formulate the main problem: to find a T_0-periodic solution $(U,I,i_{L_{00}},i_{C_{00}},i_{L_{10}},i_{C_{10}})$ of the system (13) on the interval $[T,2T]$ coinciding with prescribed T_0-periodic initial functions $U_0(t), I_0(t)$ on the initial interval $[0,T]$:

$$U(t)=U_0(t),\ dU(t)/dt = dU_0(t)/dt,\ t\in[0,T];$$
$$I(t)=I_0(t),\ dI(t)/dt = dI_0(t)/dt,\ t\in[0,T]$$
$$U_0(T)=0,\ I_0(T)=0;$$
$$i_{L00}(T)=0,\ i_{C00}(T)=0,\ i_{L10}(T)=0,\ i_{C10}(T)=0.$$

Remark 2. As in [17] one can shift the initial functions $u_0(x), i_0(x)$ defined on the interval $[0,\Lambda]$ along the characteristic to the interval $[0,T]$.

The main difficulty is to define a suitable operator whose fixed point is a solution sought. We define the operator in the following way:

$$B=(B_U(t), B_I(t), B_{i_{L_{00}}}(t), B_{i_{C_{00}}}(t), B_{i_{L_{10}}}(t), B_{i_{C_{10}}}(t))$$

such that on every interval $[T+kT_0, T+(k+1)T_0]\ (k=0,1,2,\ldots)$ its components are defined by the formulas

$$B_U^{(k)}(U,I,i_{L00},i_{C00})(t):= = \int_{T+kT_0}^{t} F_U(U,I,i_{L00},i_{C00})(s)ds - \frac{t-T-kT_0}{T_0}\int_{T+kT_0}^{T+(k+1)T_0} F_U(U,I,i_{L00},i_{C00})(s)ds$$

$$B_I^{(k)}(U,I,i_{L10},i_{C10})(t):= = \int_{T+kT_0}^{t} F_I(U,I,i_{L10},i_{C10})(s)ds - \frac{t-T-kT_0}{T_0}\int_{T+kT_0}^{T+(k+1)T_0} F_I(U,I,i_{L10},i_{C10})(s)ds$$

$$B_{i_{L00}}^{(k)}(i_{L00},i_{C00})(t):= \qquad (14)$$
$$= \int_{T+kT_0}^{t} F_{i_{L00}}(i_{L00},i_{C00})(s)ds - \frac{t-T-kT_0}{T_0}\int_{T+kT_0}^{T+(k+1)T_0} F_{i_{L00}}(i_{L00},i_{C00})(s)ds$$

$$B_{i_{C00}}^{(k)}(U,I,i_{C00})(t):= = \int_{T+kT_0}^{t} F_{i_{C00}}(U,I,i_{C00})(s)ds - \frac{t-T-kT_0}{T_0}\int_{T+kT_0}^{T+(k+1)T_0} F_{i_{C00}}(U,I,i_{C00})(s)ds$$

$$B_{i_{L10}}^{(k)}(i_{L10},i_{C10})(t):= = \int_{T+kT_0}^{t} F_{i_{L10}}(i_{L10},i_{C10})(s)ds - \frac{t-T-kT_0}{T_0}\int_{T+kT_0}^{T+(k+1)T_0} F_{i_{L10}}(i_{L10},i_{C10})(s)ds$$

$$B_{i_{C10}}^{(k)}(U,I,i_{C10})(t):= = \int_{T+kT_0}^{t} F_{i_{C10}}(U,I,i_{C10})(s)ds - \frac{t-T-kT_0}{T_0}\int_{T+kT_0}^{T+(k+1)T_0} F_{i_{C10}}(U,I,i_{C10})(s)ds,$$

where

$$F_U(U,I,i_{L00},i_{C00})(t)=[dI(t-T)/dt]- -[U(t)+I(t-T)](C_{01}Z_0)-2[i_{L00}(t)-i_{C00}(t)]/C_{01},$$
$$F_I(U,I,i_{L10},i_{C10})(t)=-[dU(t-T)/dt]+ +[U(t-T)-I(t)](C_{11}Z_0)-2[i_{L10}(t)-i_{C10}(t)]/C_{11},$$
$$F_{i_{L00}}(i_{L00},i_{C00})(t)=[C_{00}^{-1}(i_{C00}(t))-R_0 i_{L00}(t)]/[dL_{00}(i_{L00})/di_{L00}],$$
$$F_{i_{C00}}(U,I,i_{C00})(t)=[U(t)+I(t-T)-2C_{00}^{-1}(i_{C00}(t))]/2L_{01},$$
$$F_{i_{L10}}(i_{L10},i_{C10})(t)\equiv[C_{10}^{-1}(i_{C10}(t))-R_1 i_{L10}(t)]/[dL_{10}(i_{L10})/di_{L10}],$$
$$F_{i_{C_{10}}}(U,I,i_{C_{10}})(t)\equiv[U(t-T)+I(t)-2C_{10}^{-1}(i_{C_{10}}(t))]/2L_{11}.$$

and $U(t-T), I(t-T), t\in[T,2T]$ are replaced by the initial functions $U_0(t), I_0(t), t\in[0,T]$.

Lemma 1. If assumptions **(L), (C)**, **(IN), (U)** are satisfied and $(U,I,i_{L_{00}},i_{C_{00}},i_{L_{10}},i_{C_{10}})\in M_U\times M_I\times M_{L_0}\times M_{C_0}\times M_{L_1}\times M_{C_1}$ then the functions

$$B_U(t), B_I(t), B_{i_{L_{00}}}(t), B_{i_{C_{00}}}(t), B_{i_{L_{10}}}(t), B_{i_{C_{10}}}(t)$$

are T_0-periodic.

Proof: Since $F_U(U,I,i_{L00},i_{C00})(t)$ is T_0-periodic function we have

$$\int_t^{t+T_0} F_U(U,I,i_{L_{00}},i_{C_{00}})(s)ds = \int_{T+kT_0}^{T+kT_0+T_0} F_U(U,I,i_{L_{00}},i_{C_{00}})(s)ds.$$

For $t+T_0\in[T+(k+1)T_0, T+(k+2)T_0]$ we have $t\in[T+(k+1)T_0, T+(k+1)T_0]$ and it is easy to verify that

$$B_U^{(k)}(U,I,i_{L00},i_{C00})(t-T_0)=B_U^{(k+1)}(U,I,i_{L00},i_{C00})(t).$$

Similarly we obtain the same relations for the other components of the operator *B*.

Lemma 1 is thus proved.

Lemma 2. If $(U,I,i_{L_{00}},i_{C_{00}},i_{L_{10}},i_{C_{10}})\in M_U\times M_I\times M_{L_0}\times M_{C_0}\times M_{L_1}\times M_{C_1}$ then the functions

$$(B_U(t), B_I(t), B_{i_{L_{00}}}(t), B_{i_{C_{00}}}(t), B_{i_{L_{10}}}(t), B_{i_{C_{10}}}(t))\in(C_{T_0}^1[T,2T])^6.$$

Proof: We first prove that functions (14) are continuously differentiable. Indeed, the continuity follows from

$$B_U^{(k)}(U,I,i_{L00},i_{C00})(T+(k+1)T_0) = \int_{T+kT_0}^{T+(k+1)T_0} F_U(U,I,i_{L00},i_{C00})(s) - \frac{T+(k+1)T_0-T-kT_0}{T_0}\int_{T+kT_0}^{T+(k+1)T_0} F_U(U,I,i_{L00},i_{C00})(s)ds = 0,$$

$$B_U^{(k)}(U,I,i_{L00},i_{C00})(T+(k+1)T_0):= \int_{T+(k+1)T_0}^{T+(k+1)T_0} F_U(U,I,i_{L00},i_{C00})(s)ds - \frac{T+(k+1)T_0-T-(k+1)T_0}{T_0}\int_{T+(k+1)T_0}^{T+(k+2)T_0} F_U(U,I,i_{L00},i_{C00})(s)ds = 0,$$

$$B_U^{(0)}(i_{R_0L_0},U,i_{R_1L_1},I)(T) = = \int_T^T F_U(U,I,i_{L00},i_{C00})(s)ds - \frac{T-T}{T_0}\int_T^{T+T_0} F_U(U,I,i_{L00},i_{C00})(s)ds = 0,$$

$$B_U^{(0)}(U,I,i_{L_{00}},i_{C_{00}})(T)=U_0(T)=0,$$

$$B_U^{(k)}\left(U,I,i_{L_{00}},i_{C_{00}}\right)(T+(k+1)T_0)=\int_{T+kT_0}^{T+(k+1)T_0}F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(s)ds-$$

$$-\left([T+(k+1)T_0-T-kT_0]/T_0\right)\int_{T+kT_0}^{T+(k+1)T_0}F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(s)ds=0,$$

$$B_U^{(k+1)}\left(U,I,i_{L_{00}},i_{C_{00}}\right)(T+(k+1)T_0)=\int_{T+(k+1)T_0}^{T+(k+1)T_0}F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(s)ds-$$

$$-\left([T+(k+1)T_0-T-(k+1)T_0]/T_0\right)\int_{T+(k+1)T_0}^{T+(k+2)T_0}F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(s)ds=0.$$

For the rest components we can proceed in a similar way.
The differentiability follows from

$$dB_U^{(k)}\left(U,I,i_{L_{00}},i_{C_{00}}\right)(T+(k+1)T_0)/dt=$$

$$=F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(T+(k+1)T_0)-(1/T_0)\int_{T+kT_0}^{T+(k+1)T_0}F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(s)ds=$$

$$=F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(T+(k+1)T_0)-(1/T_0)\int_{T+(k+1)T_0}^{T+(k+2)T_0}F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(s)ds=$$

$$=(T+(k+1)T_0)dB_U^{(k+1)}\left(U,I,i_{L_{00}},i_{C_{00}}\right)/dt.$$

Lemma 2 is thus proved.

Lemma 3. The periodic problem (13) has a solution $(U,I,i_{L_{00}},i_{C_{00}},i_{L_{10}},i_{C_{10}})\in M_U\times M_I\times M_{L_0}\times M_{C_0}\times M_{L_1}\times M_{C_1}$ iff the operator B has a fixed point $(U,I,i_{L00},i_{C00},i_{L10},i_{C10})\in M_U\times M_I\times M_{L0}\times M_{C0}\times M_{L_1}\times M_{C_1}$, that is,

$$U=B_U\left(U,I,i_{L00},i_{C00}\right),\ I=B_I\left(U,I,i_{L10},i_{C10}\right),$$

$$i_{L00}=B_{i_{L00}}\left(i_{L00},i_{C00}\right),\ i_{C00}=B_{i_{C00}}\left(U,I,i_{C00}\right),$$

$$i_{L_{10}}=B_{i_{L_{10}}}\left(i_{L_{10}},i_{C_{10}}\right),i_{C_{10}}=B_{i_{C_{10}}}\left(U,I,i_{C_{10}}\right).$$

Proof: Let $(U,I,i_{L00},i_{C00},i_{L10},i_{C10})\in M_U\times M_I\times M_{L0}\times\times M_{C_0}\times M_{L_1}\times M_{C_1}$ be a T_0-periodic solution of (13). Then integrating the first equation of (13) we have:

$$U(t)=\int_{T+kT_0}^{t}F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(s)ds\ \Rightarrow$$

$$0=U(T+(k+1)T_0)=\int_{T+kT_0}^{T+(k+1)T_0}F_U\left(U,I,i_{L00},i_{C00}\right)(s)ds\Rightarrow$$

$$\int_{T+kT_0}^{T+(k+1)T_0}F_U\left(U,I,i_{L00},i_{C00}\right)(s)ds=0.$$

Therefore

$$U(t)=B_U^{(k)}\left(U,I,i_{L_{00}},i_{C_{00}}\right)(t)=\int_{T+kT_0}^{t}F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(s)ds-$$

$$-\left[(t-T-kT_0)/T_0\right]\int_{T+kT_0}^{T+(k+1)T_0}F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(s)ds.$$

Analogous reasoning leads to the seeking conclusion.This means $(U,I,i_{L00},i_{C00},i_{L10},i_{C10})\in M_U\times M_I\times M_{L0}\times M_{C0}$ is a fixed point of *B*.

Conversely, let B have a fixed point $(U,I,i_{L00},i_{C00},i_{L10},i_{C10})\in M_U\times M_I\times M_{L0}\times M_{C0}\times M_{L_1}\times M_{C_1}$.

Then

$$\left|\int_{T+kT_0}^{T+(k+1)T_0}F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(s)ds\right|\le$$

$$\le\left|\int_{T+kT_0}^{T+(k+1)T_0}\left[-dI(s-T)/dt-\left(U(s)-I(s-T)-2Z_0(i_{L_{00}}(s)+i_{C_{00}}(s))\right)/C_{01}Z_0\right]ds\right|\le$$

$$\le\left|-I(T+(k+1)T_0-T)+I(T+kT_0-T)\right|+$$

$$+\left(1/C_{01}Z_0\right)\left(\int_{T+kT_0}^{T+(k+1)T_0}\left|U(s)\right|ds+\int_{T+kT_0}^{T+(k+1)T_0}\left|I(s-T)\right|ds\right)+$$

$$+\left(2/C_{01}\right)\left(\int_{T+kT_0}^{T+(k+1)T_0}\left|i_{L00}(s)\right|ds+\int_{T+kT_0}^{T+(k+1)T_0}\left|i_{C00}(s)\right|ds\right)\le$$

$$\le U_0+I_0e^{-\mu T}+2Z_0(I_{L0}+I_{C0})/(C_{01}Z_0)\int_{T+kT_0}^{T+(k+1)T_0}e^{\mu(s-T-kT_0)}ds\le$$

$$\le(e^{\mu T_0}-1)\left(U_0+I_0e^{-\mu T}+2Z_0(2I_{L0}+2I_{C0})/\mu C_{01}Z_0\right);$$

$$\left|\int_{T+kT_0}^{T+(k+1)T_0}F_I\left(U,I,i_{L_{10}},i_{C_{10}}\right)(s)ds\right|\le$$

$$\le\left|\int_{T+kT_0}^{T+(k+1)T_0}\left[-dU(s-T)/dt-\left(U(s-t)-I(s)-2Z_0(i_{L_{10}}(s)+i_{C_{10}}(s))\right)/C_{01}Z_0\right]ds\right|\le$$

$$\le\left(U_0e^{-\mu T}+I_0+2Z_0(I_{L_0}+I_{C_0})/\mu C_{11}Z_0\right)\int_{T+kT_0}^{T+(k+1)T_0}e^{\mu(s-T-kT_0)}ds\le$$

$$\le(e^{\mu T_0}-1)\left(U_0e^{-\mu T}+I_0+2Z_0(I_{L_0}+I_{C_0})/\mu C_{11}Z_0\right);$$

$$\left|\int_{T+kT_0}^{T+(k+1)T_0}F_{i_{L00}}\left(i_{L_{00}},i_{C_{00}}\right)(s)ds\right|\le\left|\int_{T+kT_0}^{T+(k+1)T_0}\frac{C_{00}^{-1}(i_{C_{00}}(s))-R_0i_{L_{00}}(s)}{dL_{00}(i_{L_{00}})/di_{L_{00}}}ds\right|\le$$

$$\le(\varphi_0+R_0I_{L_0})/\hat{L}_{00}\left|\int_{T+kT_0}^{T+(k+1)T_0}e^{\mu(s-T-kT_0)}ds\right|\le(\varphi_0+R_0I_{L_0})(\varphi_0+R_0I_{L_0})/\mu\hat{L}_{00};$$

$$\left|\int_{T+kT_0}^{T+(k+1)T_0}F_{i_{C00}}\left(U,I,i_{C_{00}}\right)(s)ds\right|\le$$

$$\le\left|\int_{T+kT_0}^{T+(k+1)T_0}\left[\left(U(s)+I(s-T)-2C_{00}^{-1}(i_{C_{00}}(s))\right)/2L_{01}\right]ds\right|\le$$

ISSN: 2321-8169
1341– 1352

$$\left|\int_{T+kT_0}^{T+(k+1)T_0} F_{i_{C00}}\left(U,I,i_{C_{00}}\right)(s)ds\right| \le$$

$$\le \left|\int_{T+kT_0}^{T+(k+1)T_0}\left[\left(U(s)+I(s-T)-2C_{00}^{-1}(i_{C_{00}}(s))\right)/2L_{01}\right]ds\right| \le$$

$$\le \left|\int_{T+kT_0}^{T+(k+1)T_0}\left(e^{\mu(s-T-kT_0)}(U_0+I_0e^{-\mu T}+2\varphi_0)/2L_{01}\right)ds\right| \le$$

$$\le (e^{\mu T_0}-1)\left((U_0+I_0e^{-\mu T}+2\varphi_0)/2\mu L_{01}\right);$$

$$\left|\int_{T+kT_0}^{T+(k+1)T_0} F_{i_{L10}}\left(i_{L10},i_{C10}\right)(s)ds\right| \le$$

$$\le \left|\int_{T+kT_0}^{T+(k+1)T_0}\frac{C_{10}^{-1}(i_{C10}(s))-R_1 i_{L10}(s)}{dL_{10}(i_{L10})/di_{L10}}ds\right| \le \frac{\varphi_0+R_1 I_{L_1}}{\hat{L}_{10}}\,\frac{e^{\mu T_0}-1}{\mu};$$

$$\left|\int_{T+kT_0}^{T+(k+1)T_0} F_{i_{C10}}\left(U,I,i_{C_{10}}\right)(s)ds\right| \le$$

$$\le \left|\int_{T+kT_0}^{T+(k+1)T_0}\left(U(s-T)+I(s)+2C_{10}^{-1}(i_{C_{10}}(s))\right)/(2L_{11})ds\right| \le$$

$$\le (e^{\mu T_0}-1)(U_0e^{-\mu T}+I_0+2\varphi_0)/(2\mu L_{11}).$$

If we assume $\left|\int_{T+kT_0}^{T+(k+1)T_0} F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(s)ds\right| \ne 0$ in view of $\lim\limits_{\mu\to\infty}\left|\int_{T+kT_0}^{T+(k+1)T_0} F_U\left(U,I,i_{L00},i_{C00}\right)(s)ds\right| = 0$ we obtain a contradiction.

Consequently, $\int_{T+kT_0}^{T+(k+1)T_0} F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(s)ds = 0$.

In a similar way we obtain

$$\int_{T+kT_0}^{T+(k+1)T_0} F_I\left(U,I,i_{L10},i_{C10}\right)(s)ds = 0,$$

$$\int_{T+kT_0}^{T+(k+1)T_0} F_{i_{L00}}\left(i_{L_{00}},i_{C_{00}}\right)(s)ds = 0, \quad \int_{T+kT_0}^{T+(k+1)T_0} F_{i_{C00}}\left(U,I,i_{C_{00}}\right)(s)ds = 0,$$

$$\int_{T+kT_0}^{T+(k+1)T_0} F_{i_{L10}}\left(i_{L10},i_{C10}\right)(s)ds = 0, \quad \int_{T+kT_0}^{T+(k+1)T_0} F_{i_{C_{10}}}\left(U,I,i_{C_{10}}\right)(s)ds = 0.$$

Therefore

$$U(t) = \int_{T+kT_0}^{t} F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(s)ds,\quad I(t) = \int_{T+kT_0}^{t} F_I\left(U,I,i_{L10},i_{C10}\right)(s)ds,$$

$$i_{L_{00}}(t) = \int_{T+kT_0}^{t} F_{i_{L_{00}}}\left(i_{L_{00}},i_{C_{00}}\right)(s)ds,\quad i_{C_{00}}(t) = \int_{T+kT_0}^{t} F_{i_{C_{00}}}\left(U,I,i_{C_{00}}\right)(s)ds,$$

$$i_{L_{10}}(t) = \int_{T+kT_0}^{t} F_{i_{L_{10}}}\left(i_{L_{10}},i_{C_{10}}\right)(s)ds,\quad i_{C_{10}}(t) := \int_{T+kT_0}^{t} F_{i_{C_{10}}}\left(U,I,i_{C_{10}}\right)(s)ds.$$

Differentiating the last equalities we conclude that (13) has T_0-periodic solution.

Lemma 3 is thus proved.

VI. Main Result

Here we formulate the main result for existence-uniqueness of a periodic solution.

Theorem 1. Let conditions **(U)**, **(L)**, **(C)** and **(IN)** be fulfilled. Then for sufficiently large $\mu > 0$ and $\mu T_0 = \text{const}$ there exists a unique T_0-periodic solution of (13).

Proof: We show that B maps $M_U \times M_I \times M_{L_0} \times M_{C_0} \times \times M_{L_1} \times M_{C_1}$ into itself. Indeed, for the first component we get

$$\left|B_U^{(k)}\left(U,I,i_{L_{00}},i_{C_{00}}\right)(t)\right| \le \left|\int_{T+kT_0}^{t} F_U\left(U,I,i_{L_{00}},i_{C_{00}}\right)(s)ds\right| +$$

$$+\left|\frac{t-T-kT_0}{T_0}\right|\left|\int_{T+kT_0}^{T+(k+1)T_0} F_U\left(U,I,i_{L00},i_{C00}\right)(s)ds\right| \le$$

$$\le \left|\int_{T+kT_0}^{t}\left[-dI(s-T)/dt-\left(U(s)-I(s-t)-2Z_0(i_{L_{10}}(s)+i_{C_{10}}(s))\right)/C_{01}Z_0\right]ds\right| +$$

$$+(e^{\mu T_0}-1)\left(U_0+I_0e^{-\mu T}+2Z_0(I_{L_0}+I_{C_0})\right)/\mu C_{01}Z_0 \le$$

$$\le \left|-I(t-T)\right| + \left(1/C_{01}Z_0\right)\left(\int_{T+kT_0}^{t}|U(s)|ds + \int_{T+kT_0}^{t}|I(s-T)|ds\right) +$$

$$+\left(2/C_{01}\right)\left(\int_{T+kT_0}^{t}\left|i_{L_{00}}(s)\right|ds + \int_{T+kT_0}^{t}\left|i_{C_{00}}(s)\right|ds\right) +$$

$$+(e^{\mu T_0}-1)\left(U_0+I_0e^{-\mu T}+2Z_0(I_{L_0}+I_{C_0})\right)/\mu C_{01}Z_0 \le$$

$$\le I_0e^{-\mu T}e^{\mu(t-T-kT_0)} + \left[\left(U_0+I_0e^{-\mu T}+2Z_0(I_{L_0}+I_{C_0})\right)/C_{01}Z_0\right]\int_{T+kT_0}^{t} e^{\mu(s-T-kT_0)}ds +$$

$$+(e^{\mu T_0}-1)\left(U_0+I_0e^{-\mu T}+2Z_0(I_{L_0}+I_{C_0})\right)/\mu C_{01}Z_0 \le$$

$$\le I_0e^{-\mu T}e^{\mu(t-T-kT_0)} +$$

$$+(e^{\mu(s-T-kT_0)}-1)\left(U_0+I_0e^{-\mu T}+2Z_0(I_{L_0}+I_{C_0})\right)/\mu C_{01}Z_0 +$$

$$+(e^{\mu T_0}-1)\left(U_0+I_0e^{-\mu T}+2Z_0(I_{L_0}+I_{C_0})\right)/\mu C_{01}Z_0 \le$$

$$\le e^{\mu(t-T-kT_0)}\left[I_0e^{-\mu T}+e^{\mu T_0}\left(U_0+I_0e^{-\mu T}+2Z_0(I_{L_0}+I_{C_0})\right)/\mu\right] \le$$

$$\le e^{\mu(t-T-kT_0)}U_0;$$

$$\left|B_I^{(k)}\left(U,I,i_{L_{10}},i_{C_{10}}\right)(t)\right| \le \left|\int_{T+kT_0}^{t} F_I\left(U,I,i_{L_{10}},i_{C_{10}}\right)(s)ds\right| +$$

$$+\left|(t-T-kT_0)/T_0\right|\left|\int_{T+kT_0}^{T+(k+1)T_0} F_I\left(U,I,i_{L_{10}},i_{C_{10}}\right)(s)ds\right| \le$$

 ISSN: 2321-8169
1341– 1352

$$\leq \left| \int_{T+kT_0}^{t} \left[-dU(s-T)/dt - \left(U(s-t) - I(s) - 2Z_0(i_{L_{10}}(s) + i_{C_{10}}(s)) \right) / C_{01}Z_0 \right] ds \right| +$$

$$+ (e^{\mu T_0} - 1)\left(U_0 e^{-\mu T} + I_0 + 2Z_0(I_{L_1} + I_{C_1}) \right) / \mu C_{11} Z_0 \leq$$

$$\leq U_0 e^{-\mu T} e^{\mu(t-T-kT_0)} + \left(U_0 e^{-\mu T} + I_0 + 2Z_0(I_{L_1} + I_{C_1}) \right) / (C_{11}Z_0) \int_{T+kT_0}^{t} e^{\mu(s-T-kT_0)} ds +$$

$$+ (e^{\mu T_0} - 1)\left(U_0 e^{-\mu T} + I_0 + 2Z_0(I_{L_1} + I_{C_1}) \right) / \mu C_{11} Z_0 \leq$$

$$\leq e^{\mu(t-T-kT_0)} \left[U_0 e^{-\mu T} + e^{\mu T_0} \left(U_0 e^{-\mu T} + I_0 + 2Z_0(I_{L_1} + I_{C_1}) \right) / (\mu C_{11} Z_0) \right] \leq$$

$$\leq e^{\mu(t-T-kT_0)} I_0;$$

$$\left| B^{(k)}_{i_{L_{00}}} \left(i_{L_{00}}, i_{C_{00}} \right)(t) \right| \leq \left| \int_{T+kT_0}^{t} \frac{C_{00}^{-1}(i_{C_{00}}(s)) - R_0 i_{L_{00}}(s)}{dL_{00}(i_{L_{00}})/di_{L_{00}}} ds \right| +$$

$$+ \left| (t-T-kT_0)/T_0 \right| \left| \int_{T+kT_0}^{T+(k+1)T_0} \frac{C_{00}^{-1}(i_{C_{00}}(s)) - R_0 i_{L_{00}}(s)}{dL_{00}(i_{L_{00}})/di_{L_{00}}} ds \right| \leq$$

$$\leq \left| \int_{T+kT_0}^{t} e^{-\mu(s-T-kT_0)} \left(\varphi_0 + R_0 I_{L_0} \right) / (\hat{L}_{00}) ds \right| +$$

$$+ \left| \int_{T+kT_0}^{T+(k+1)T_0} e^{-\mu(s-T-kT_0)} \left(\varphi_0 + R_0 I_{L_0} \right) / (\hat{L}_{00}) ds \right| \leq$$

$$\leq 2(\varphi_0 + R_0 I_{L_0}) / (\hat{L}_{00}) \left| \int_{T+kT_0}^{t} e^{-\mu(s-T-kT_0)} ds \right| \leq$$

$$\leq 2(e^{-\mu(t-T-kT_0)} - 1)(\varphi_0 + R_0 I_{L_0}) / \mu \hat{L}_{00} \leq$$

$$\leq 2e^{-\mu(t-T-kT_0)} (\varphi_0 + R_0 I_{L_0}) / \mu \hat{L}_{00} \leq I_{L_0} e^{-\mu(t-T-kT_0)};$$

$$\left| B^{(k)}_{i_{C_{00}}} \left(U, I, i_{C_{00}} \right)(t) \right| \leq$$

$$\leq \left| \int_{T+kT_0}^{t} \left(U(s) + I(s-T) - 2C_{00}^{-1}(i_{C_{00}}(s)) \right) / (2L_{01}) ds \right| +$$

$$+ \left| (t-T-kT_0)/T_0 \right| \left| \int_{T+kT_0}^{T+(k+1)T_0} \left(U(s) + I(s-T) - 2C_{00}^{-1}(i_{C_{00}}(s)) \right) ds \right| \leq$$

$$\leq (e^{\mu(t-T-kT_0)} - 1)(U_0 + I_0 e^{-\mu T} + 2\varphi_0) / (2\mu L_{01}) +$$

$$+ (e^{\mu T_0} - 1)(U_0 + I_0 e^{-\mu T} + 2\varphi_0) / (2\mu L_{01}) \leq$$

$$\leq e^{\mu(t-T-kT_0)} e^{\mu T_0} (U_0 + I_0 e^{-\mu T} + 2\varphi_0) / (2\mu L_{01}) \leq e^{\mu(t-T-kT_0)} I_{C_0};$$

$$\left| B^{(k)}_{i_{L_{10}}} \left(i_{L_{10}}, i_{C_{10}} \right)(t) \right| \leq \left| \int_{T+kT_0}^{t} \frac{C_{10}^{-1}(i_{C_{10}}(s)) - R_1 i_{L_{10}}(s)}{dL_{10}(i_{L_{10}})/di_{L_{10}}} ds \right| +$$

$$+ \left| (t-T-kT_0)/T_0 \right| \left| \int_{T+kT_0}^{T+(k+1)T_0} \frac{C_{10}^{-1}(i_{C_{10}}(s)) - R_1 i_{L_{10}}(s)}{dL_{10}(i_{L_{10}})/di_{L_{10}}} ds \right| \leq$$

$$\leq \left| \int_{T+kT_0}^{t} e^{-\mu(s-T-kT_0)} \left(\varphi_0 + R_1 I_{L_1} \right) / (\hat{L}_{10}) ds \right| +$$

$$+ \left| \int_{T+kT_0}^{T+(k+1)T_0} e^{-\mu(s-T-kT_0)} \left(\varphi_0 + R_1 I_{L_1} \right) / (\hat{L}_{10}) ds \right| \leq$$

$$\leq \left[2(\varphi_0 + R_1 I_{L_1}) / \hat{L}_{10} \right] \left| \int_{T+kT_0}^{t} e^{-\mu(s-T-kT_0)} ds \right| \leq$$

$$\leq 2(\varphi_0 + R_1 I_{L_1})(e^{-\mu(t-T-kT_0)} - 1) / \mu \hat{L}_{10} \leq I_{L_1} e^{-\mu(t-T-kT_0)};$$

$$\left| B^{(k)}_{i_{C_{10}}} \left(U, I, i_{C_{10}} \right)(t) \right| \leq$$

$$\leq \left| \int_{T+kT_0}^{t} \left(\left(U(s-T) + I(s) + 2C_{10}^{-1}(i_{C_{10}}(s)) \right) / 2L_{11} \right) ds \right| +$$

$$+ \left| (t-T-kT_0)/T_0 \right| \left| \int_{T+kT_0}^{T+(k+1)T_0} \left(\left(U(s-T) + I(s) + 2C_{10}^{-1}(i_{C_{10}}(s)) \right) / 2L_{11} \right) ds \right| \leq$$

$$\leq \left(U_0 e^{-\mu T} + I_0 + 2\varphi_0 / 2L_{11} \right) \int_{T+kT_0}^{t} e^{\mu(s-T-kT_0)} ds +$$

$$+ \left(\left(U_0 e^{-\mu T} + I_0 + 2\varphi_0 \right) / 2L_{11} \right) \int_{T+kT_0}^{T+(k+1)T_0} e^{\mu(s-T-kT_0)} ds \leq$$

$$\leq e^{\mu(t-T-kT_0)} e^{\mu T_0} (U_0 e^{-\mu T} + I_0 + 2\varphi_0) / 2\mu L_{11} \leq e^{\mu(t-T-kT_0)} I_{C_1}.$$

Consequently the operator Bmaps $M_U \times M_I \times M_{L_0} \times M_{C_0} \times M_{L_1} \times M_{C_1}$ into itself.

It remains to show that B is contractive operator. Indeed,

$$\left| B_U^{(k)} \left(U, I, i_{L_{00}}, i_{C_{00}} \right)(t) - B_U^{(k)} \left(\bar{U}, \bar{I}, \bar{i}_{L_{00}}, \bar{i}_{C_{00}} \right)(t) \right| \leq$$

$$\leq \left| \int_{T+kT_0}^{t} \left(F_U \left(U, I, i_{L_{00}}, i_{C_{00}} \right)(s) - F_U \left(\bar{U}, \bar{I}, \bar{i}_{L_{00}}, \bar{i}_{C_{00}} \right)(s) \right) ds \right| +$$

$$+ \left| (t-T-kT_0)/T_0 \right| \left| \int_{T+kT_0}^{T+(k+1)T_0} \left(F_U \left(U, I, i_{L_{00}}, i_{C_{00}} \right)(s) - F_U \left(\bar{U}, \bar{I}, \bar{i}_{L_{00}}, \bar{i}_{C_{00}} \right)(s) \right) ds \right| \leq$$

$$\leq \left| \int_{T+kT_0}^{t} \left(dI(s-T)/dt - d\bar{I}(s-T)/dt \right) ds \right| +$$

$$+ C_{01}Z_0 \left(\int_{T+kT_0}^{t} \left| U(s) - \bar{U}(s) \right| ds + \int_{T+kT_0}^{t} \left| I(s-T) - \bar{I}(s-T) \right| ds \right) +$$

$$+ (2/C_{01}) \left(\int_{T+kT_0}^{t} \left| i_{L_{00}}(s) - \bar{i}_{L_{00}}(s) \right| ds + \int_{T+kT_0}^{t} \left| i_{C_{00}}(s) - \bar{i}_{C_{00}}(s) \right| ds \right) +$$

$$+ \left| \int_{T+kT_0}^{T+(k+1)T_0} \left(dI(s-T)/dt - d\bar{I}(s-T)/dt \right) ds \right| +$$

$$+ C_{01}Z_0 \left(\int_{T+kT_0}^{T+(k+1)T_0} \left| U(s) - \bar{U}(s) \right| ds + \int_{T+kT_0}^{T+(k+1)T_0} \left| I(s-T) - \bar{I}(s-T) \right| ds \right) +$$

ISSN: 2321-8169
1341– 1352

$$+(2/C_{01})\left(\int_{T+kT_0}^{T+(k+1)T_0}\left|i_{L00}(s)-\bar{i}_{L00}(s)\right|ds+\int_{T+kT_0}^{T+(k+1)T_0}\left|i_{C00}(s)-\bar{i}_{C00}(s)\right|ds\right)\le$$

$$\le\left|I(t-T)-\bar{I}(t-T)\right|+\int_{T+kT_0}^{t}e^{\mu(s-T-kT_0)}ds\times$$

$$\times\left(\rho_k(U,\bar{U})+\rho_k(I,\bar{I})e^{-\mu T}+2Z_0\left(\rho_k(i_{L_{00}},\bar{i}_{L_{00}})+\rho_k(i_{C_{00}},\bar{i}_{C_{00}})\right)\right)/(C_{01}Z_0)+$$

$$+\left|I((k+1)T_0)-I(kT_0)-\bar{I}((k+1)T_0)+\bar{I}(kT_0)\right|+$$

$$+\int_{T+kT_0}^{T+(k+1)T_0}e^{\mu(s-T-kT_0)}ds\times$$

$$\times\left(\rho_k(U,\bar{U})+\rho_k(I,\bar{I})e^{-\mu T}+2Z_0\left(\rho_k(i_{L_{00}},\bar{i}_{L_{00}})+\rho_k(i_{C_{00}},\bar{i}_{C_{00}})\right)\right)/(C_{01}Z_0)\le$$

$$\le e^{\mu(t-T-kT_0)}\rho_k(I,\bar{I})e^{-\mu T}+\left[(e^{\mu(t-T-kT_0)}-1)/(\mu C_{01}Z_0)\right]\times$$

$$\times\left(\rho_k(U,\bar{U})+\rho_k(I,\bar{I})e^{-\mu T}+2Z_0\left(\rho_k(i_{L_{00}},\bar{i}_{L_{00}})+\rho_k(i_{C_{00}},\bar{i}_{C_{00}})\right)\right)+$$

$$+\left(e^{\mu T_0}-1\right)\left(\rho_k(U,\bar{U})+\rho_k(I,\bar{I})e^{-\mu T}+2Z_0\left(\rho_k(i_{L_{00}},\bar{i}_{L_{00}})+\rho_k(i_{C_{00}},\bar{i}_{C_{00}})\right)\right)/(\mu C_{01}Z_0)\le$$

$$\le e^{\mu(t-T-kT0)}\left[e^{-\mu T}+e^{\mu T0}(1+e^{-\mu T}+4Z_0)/\mu Z_0\right]\times$$

$$\times\rho_k\left((U,I,i_{L00},i_{C00},i_{L10},i_{C10}),(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00},\bar{i}_{L10},\bar{i}_{C10})\right)\equiv$$

$$\equiv e^{\mu(t-T-kT0)}K_U\rho_k\left((U,I,i_{L00},i_{C00},i_{L10},i_{C10}),(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00},\bar{i}_{L10},\bar{i}_{C10})\right)$$

It follows

$$\rho_k\left(B_U^{(k)}(U,I,i_{L00},i_{C00}),B_U^{(k)}(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00})\right)\le$$

$$\le K_U\rho_k\left((U,I,i_{L00},i_{C00},i_{L10},i_{C10}),(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00},\bar{i}_{L10},\bar{i}_{C10})\right).$$

For the second component we have

$$\left|B_I^{(k)}(U,I,i_{L_{10}},i_{C_{10}})(t)-B_I^{(k)}(\bar{U},\bar{I},\bar{i}_{L_{10}},\bar{i}_{C_{10}})(t)\right|\le$$

$$\le\left|\int_{T+kT_0}^{t}\left(F_I(U,I,i_{L_{10}},i_{C_{10}})(s)-F_I(\bar{U},\bar{I},\bar{i}_{L_{10}},\bar{i}_{C_{10}})(s)\right)ds\right|+$$

$$+\left|(t-T-kT_0)/T_0\right|\left|\int_{T+kT_0}^{T+(k+1)T_0}\left(F_I(U,I,i_{L_{10}},i_{C_{10}})(s)-F_I(\bar{U},\bar{I},\bar{i}_{L_{10}},\bar{i}_{C_{10}})(s)\right)ds\right|\le$$

$$\le\left|\int_{T+kT_0}^{t}\left(-dU(s-T)/ds+d\bar{U}(s-T)/ds\right)ds\right|+$$

$$+\left(1/C_{11}Z_0\right)\left(\int_{T+kT_0}^{t}\left|U(s-T)-\bar{U}(s-T)\right|ds+\int_{T+kT_0}^{t}\left|I(s)-\bar{I}(s)\right|ds\right)+$$

$$+(2/C_{11})\left(\int_{T+kT}^{t}\left|i_{L_{10}}(s)-\bar{i}_{L_{10}}(s)\right|ds+\int_{T+kT_0}^{t}\left|i_{C_{10}}(s)-\bar{i}_{C_{10}}(s)\right|ds\right)+$$

$$+\left|\int_{T+kT_0}^{T+(k+1)T_0}\left(-dU(s-T)/ds+d\bar{U}(s-T)/ds\right)ds\right|+$$

$$\left(1/C_{11}Z_0\right)\left(\int_{T+kT_0}^{T+(k+1)T_0}\left|U(s-T)-\bar{U}(s-T)\right|ds+\int_{T+kT_0}^{T+(k+1)T_0}\left|I(s)-\bar{I}(s)\right|ds\right)+$$

$$+(2/C_{11})\left(\int_{T+kT}^{T+(k+1)T_0}\left|i_{L_{10}}(s)-\bar{i}_{L_{10}}(s)\right|ds+\int_{T+kT_0}^{T+(k+1)T_0}\left|i_{C_{10}}(s)-\bar{i}_{C_{10}}(s)\right|ds\right)\le$$

$$\le\left|U(t-T)-\bar{U}(t-T)\right|+\int_{T+kT}^{t}e^{\mu(s-T-kT_0)}ds\times$$

$$\times\left(\rho_k(U,\bar{U})e^{-\mu T}+\rho_k(I,\bar{I})+2Z_0(\rho_k(i_{L_{10}},\bar{i}_{L_{10}})+\rho_k(i_{C_{10}},\bar{i}_{C_{10}})\right)/(C_{11}Z_0)+$$

$$+\int_{T+kT}^{T+(k+1)T_0}e^{\mu(s-T-kT_0)}ds\le$$

$$\le e^{\mu(t-T-kT0)}\rho_k(U,\bar{U})e^{-\mu T}+(e^{\mu(t-T-kT0)}-1)/\mu C_{11}Z_0\times$$

$$\times\left(\rho_k(U,\bar{U})e^{-\mu T}+\rho_k(I,\bar{I})+2Z_0(\rho_k(i_{L10},\bar{i}_{L10})+\rho_k(i_{C10},\bar{i}_{C10}))\right)+$$

$$+\left(\rho_k(U,\bar{U})e^{-\mu T}+\rho_k(I,\bar{I})+2Z_0(\rho_k(i_{L10},\bar{i}_{L10})+\rho_k(i_{C10},\bar{i}_{C10}))\right)\pm$$

$$\times(e^{\mu T_0}-1)/\mu C_{11}Z_0\le$$

$$\le e^{\mu(t-T-kT0)}\left[e^{-\mu T}+e^{\mu T0}\left(e^{-\mu T}+1+4Z_0\right)/\mu Z_0C_{11}\right]\times$$

$$\times\rho_k\left((U,I,i_{L00},i_{C00},i_{L10},i_{C10}),(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00},\bar{i}_{L10},\bar{i}_{C10})\right)\equiv$$

$$\equiv e^{\mu(t-T-kT0)}K_I\,\rho_k\left((U,I,i_{L00},i_{C00},i_{L10},i_{C10}),(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00},\bar{i}_{L10},\bar{i}_{C10})\right).$$

It follows

$$\rho_k\left(B_I^{(k)}(U,I,i_{L10},i_{C10}),B_I^{(k)}(\bar{U},\bar{I},\bar{i}_{L10},\bar{i}_{C10})\right)\le$$

$$\le K_I\,\rho_k\left((U,I,i_{L00},i_{C00},i_{L10},i_{C10}),(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00},\bar{i}_{L10},\bar{i}_{C10})\right).$$

For the third component we obtain

$$\left|B_{i_{L_{00}}}^{(k)}(i_{L_{00}},i_{C_{00}})(t)-B_{i_{L_{00}}}^{(k)}(\bar{i}_{L_{00}},\bar{i}_{C_{00}})(t)\right|\le$$

$$\le\int_{T+kT_0}^{t}\left|\frac{C_{00}^{-1}(i_{C00}(s))-R_0i_{L00}(s)}{dL_{00}(i_{L00})/di_{L00}}-\frac{C_{00}^{-1}(\bar{i}_{C00}(s))-R_0\bar{i}_{L00}(s)}{dL_{00}(\bar{i}_{L00})/di_{L00}}\right|ds+$$

$$+\int_{T+kT_0}^{T+(k+1)T0}\left|\frac{C_{00}^{-1}(i_{C00}(s))-R_0i_{L00}(s)}{dL_{00}(i_{L00})/di_{L00}}-\frac{C_{00}^{-1}(\bar{i}_{C00}(s))-R_0\bar{i}_{L00}(s)}{dL_{00}(\bar{i}_{L00})/di_{L00}}\right|ds\le$$

$$\le\int_{T+kT_0}^{t}\left|\frac{C_{00}^{-1}(i_{C00}(s))-R_0i_{L00}(s)}{dL_{00}(i_{L00})/di_{L00}}-\frac{C_{00}^{-1}(\bar{i}_{C00}(s))-R_0\bar{i}_{L00}(s)}{dL_{00}(i_{L00})/di_{L00}}\right|+$$

$$+\left|\frac{C_{00}^{-1}(\bar{i}_{C00}(s))-R_0\bar{i}_{L00}(s)}{dL_{00}(i_{L00})/di_{L00}}-\frac{C_{00}^{-1}(\bar{i}_{C00}(s))-R_0\bar{i}_{L00}(s)}{dL_{00}(\bar{i}_{L00})/di_{L00}}\right|ds+$$

$$+\int_{T+kT_0}^{T+(k+1)T0}\left|\frac{C_{00}^{-1}(i_{C00}(s))-R_0i_{L00}(s)}{dL_{00}(i_{L00})/di_{L00}}-\frac{C_{00}^{-1}(\bar{i}_{C00}(s))-R_0\bar{i}_{L00}(s)}{dL_{00}(i_{L00})/di_{L00}}\right|+$$

$$+\left|\frac{C_{00}^{-1}(\bar{i}_{C00}(s))-R_0\bar{i}_{L00}(s)}{dL_{00}(i_{L00})/di_{L00}}-\frac{C_{00}^{-1}(\bar{i}_{C00}(s))-R_0\bar{i}_{L00}(s)}{dL_{00}(\bar{i}_{L00})/di_{L00}}\right|ds\le$$

$$\le\frac{1}{\hat{L}_{00}}\int_{T+kT_0}^{t}\left|C_{00}^{-1}(i_{C00}(s))-C_{00}^{-1}(\bar{i}_{C00}(s))\right|ds+\frac{R_0}{\hat{L}_{00}}\int_{T+kT_0}^{t}\left|i_{L00}(s)-\bar{i}_{L00}(s)\right|ds+$$

$$+\varphi_0\int_{T+kT_0}^{t}e^{\mu(s-T-kT_0)}\left|di_{L_{00}}/dL_{00}(i_{L_{00}})-di_{L_{00}}/dL_{00}(\bar{i}_{L_{00}})\right|ds+$$

$$+\left(1/\hat{L}_{00}\right)\int_{T+kT_0}^{T+(k+1)T_0}\left|C_{00}^{-1}(i_{C_{00}}(s))-C_{00}^{-1}(\bar{i}_{C_{00}}(s))\right|ds+$$

$$+(R_0/\hat{L}_{00})\int_{T+kT0}^{T+(k+1)T0}\left|i_{L00}(s)-\bar{i}_{L00}(s)\right|ds+$$

ISSN: 2321-8169
1341– 1352

$$+\varphi_0 \int_{T+kT_0}^{T+(k+1)T_0} e^{\mu(s-T-kT_0)} \left| di_{L_{00}} / dL_{00}(i_{L_{00}}) - di_{L_{00}} / dL_{00}(\bar{i}_{L_{00}}) \right| ds \le$$

$$\le \left(\Phi_{00} h(1+\varphi_0/\Phi_{00})^{(h+1)/h}\right)/(c_{00}\hat{L}_{00}) \int_{T+kT_0}^{t} \left| i_{C_{00}}(s) - \bar{i}_{C_{00}}(s) \right| ds +$$

$$+\left(R_0/\hat{L}_{00}\right) \int_{T+kT_0}^{t} \left| i_{L_{00}}(s) - \bar{i}_{L_{00}}(s) \right| ds +$$

$$+\varphi_0 \int_{T+kT_0}^{t} \left| \frac{d^2 L_{00}(i_{L00})/di_{L00}^2}{(dL_{00}(i_{L00})/di_{L00})^2} \right| \left| i_{L00}(s) - \bar{i}_{L00}(s) \right| ds +$$

$$+\left(\Phi_{00} h(1+\varphi_0/\Phi_{00})^{(h+1)/h}\right)/(c_{00}\hat{L}_{00}) \int_{T+kT_0}^{T+(k+1)T_0} \left| i_{C_{00}}(s) - \bar{i}_{C_{00}}(s) \right| ds +$$

$$+\left(R_0/\hat{L}_{00}\right) \int_{T+kT_0}^{T+(k+1)T_0} \left| i_{L_{00}}(s) - \bar{i}_{L_{00}}(s) \right| ds +$$

$$+\varphi_0 \int_{T+kT_0}^{T+(k+1)T_0} \left| \frac{d^2 L_{00}(i_{L00})/di_{L00}^2}{(dL_{00}(i_{L00})/di_{L00})^2} \right| \left| i_{L00}(s) - \bar{i}_{L00}(s) \right| ds \le$$

$$\le \left(\Phi_{00} h(1+\varphi_0/\Phi_{00})^{(h+1)/h}\right) \rho_k(i_{C_{00}}, \bar{i}_{C_{00}})/(c_{00}\hat{L}_{00}) \int_{T+kT_0}^{t} e^{\mu(s-T-kT_0)} ds +$$

$$+\frac{R_0}{\hat{L}_{00}} \rho_k\left(i_{L_{00}}, \bar{i}_{L_{00}}\right) \int_{T+kT_0}^{t} e^{\mu(s-T-kT_0)} ds +$$

$$+\varphi_0 \left[\sum_{n=2}^{m} (n+1) n l_n^{(00)} |i_0|^{n-2} / \hat{L}_{00}^2 \right] \int_{T+kT_0}^{t} e^{\mu(s-T-kT_0)} ds\, \rho_k\left(i_{L00}, \bar{i}_{L00}\right) +$$

$$+\left(\Phi_{00} h(1+\varphi_0/\Phi_{00})^{(h+1)/h}\right) \rho_k(i_{C_{00}}, \bar{i}_{C_{00}})/(c_{00}\hat{L}_{00}) \int_{T+kT_0}^{T+(k+1)T_0} e^{\mu(s-T-kT_0)} ds +$$

$$+\left(R_0/\hat{L}_{00}\right) \rho_k\left(i_{L_{00}}, \bar{i}_{L_{00}}\right) \int_{T+kT_0}^{T+(k+1)T_0} e^{\mu(s-T-kT_0)} ds +$$

$$+\varphi_0 \left[\sum_{n=2}^{m} (n+1) n l_n^{(00)} |i_0|^{n-2} / \hat{L}_{00}^2 \right] \int_{T+kT_0}^{T+(k+1)T_0} e^{\mu(s-T-kT_0)} ds\, \rho_k\left(i_{L00}, \bar{i}_{L00}\right) \le$$

$$\le e^{\mu(t-T-kT_0)} \rho_k\left((U, I, i_{L00}, i_{C00}, i_{L10}, i_{C10}), (\bar{U}, \bar{I}, \bar{i}_{L00}, \bar{i}_{C00}, \bar{i}_{L10}, \bar{i}_{C10})\right) \times$$

$$\left[\left(\left[\Phi_{00} h(1+\varphi_0/\Phi_{p0})^{(h+1)/h}\right]/(\hat{L}_{00} c_{p0}) + (R_0/\hat{L}_{00}) + \right.\right.$$

$$\left.+\varphi_0 \sum_{n=2}^{m} (n+1) n l_n^{(00)} |i_0|^{n-2} / \hat{L}_{00}^2 \right)/\mu +$$

$$+(e^{\mu T_0} - 1)\left(\left[\Phi_{00} h(1+\varphi_0/\Phi_{00})^{(h+1/h)}\right]/(\hat{L}_{00} c_{00}) + (R_0/\hat{L}_{00}) + \right.$$

$$\left.\left. +\varphi_0 \sum_{n=2}^{m} (n+1) n l_n^{(00)} |i_0|^{n-2} / \hat{L}_{00}^2 \right)/\mu \right] \le$$

$$\le e^{\mu(t-T-kT_0)} \rho_k\left((U, I, i_{L00}, i_{C00}, i_{L10}, i_{C10}), (\bar{U}, \bar{I}, \bar{i}_{L00}, \bar{i}_{C00}, \bar{i}_{L10}, \bar{i}_{C10})\right) \times$$

$$\times e^{\mu T_0} \left[\left[\Phi_{00} h(1+\varphi_0/\Phi_{00})^{(h+1)/h} / c_{00}\right] + R_0 + \right.$$

$$\left. +\varphi_0 \sum_{n=2}^{m} (n+1) n l_n^{(00)} |i_0|^{n-2} / \hat{L}_{00} \right] / \mu \hat{L}_{00} \equiv$$

$$\equiv e^{\mu(t-T-kT_0)} K_{I_{L0}} \rho_k\left((U, I, i_{L00}, i_{C00}, i_{L10}, i_{C10}), (\bar{U}, \bar{I}, \bar{i}_{L00}, \bar{i}_{C00}, \bar{i}_{L10}, \bar{i}_{C10})\right).$$

It follows

$$\rho_k\left(B_{i_{L00}}^{(k)}\left(i_{L00}, i_{C00}\right), B_{i_{L00}}^{(k)}\left(\bar{i}_{L00}, \bar{i}_{C00}\right)\right) \le$$

$$\le K_{I_{L0}} \rho_k\left((U, I, i_{L00}, i_{C00}, i_{L10}, i_{C10}), (\bar{U}, \bar{I}, \bar{i}_{L00}, \bar{i}_{C00}, \bar{i}_{L10}, \bar{i}_{C10})\right).$$

For the fourth component we have

$$\left| B_{i_{C00}}^{(k)}\left(U, I, i_{C_{00}}\right)(t) - B_{i_{C00}}^{(k)}\left(\bar{U}, \bar{I}, \bar{i}_{C_{00}}\right)(t) \right| \le$$

$$\le \left| \int_{T+kT_0}^{t} \left(F_{i_{C00}}\left(U, I, i_{C_{00}}\right)(s) - F_{i_{C00}}\left(\bar{U}, \bar{I}, \bar{i}_{C_{00}}\right)(s) \right) ds \right| +$$

$$+\left| (t-T-kT_0)/T_0 \right| \left| \int_{T+kT_0}^{T+(k+1)T_0} \left(F_{i_{C00}}\left(U, I, i_{C_{00}}\right)(s) - F_{i_{C00}}\left(\bar{U}, \bar{I}, \bar{i}_{C_{00}}\right)(s) \right) ds \right| \le$$

$$\le \left| \int_{T+kT_0}^{t} \left(\frac{U(s) - \bar{U}(s) + I(s-T) - \bar{I}(s-T)}{2L_{01}} - \frac{C_{00}^{-1}(i_{C_{00}}(s)) - C_{00}^{-1}(\bar{i}_{C_{00}}(s))}{L_{01}} \right) ds \right| +$$

$$+\left| \int_{T+kT_0}^{T+(k+1)T_0} \left(\frac{U(s) - \bar{U}(s) + I(s-T) - \bar{I}(s-T)}{2L_{01}} - \frac{C_{00}^{-1}(i_{C_{00}}(s)) - C_{00}^{-1}(\bar{i}_{C_{00}}(s))}{L_{01}} \right) ds \right| \le$$

$$\le \left(\rho_k(U, \bar{U}) + \rho_k(I, \bar{I}) e^{-\mu T}\right)/(2L_{01}) \int_{T+kT_0}^{t} e^{\mu(s-T-kT_0)} ds +$$

$$+\left(\Phi_{00} h(1+\varphi_0/\Phi_{00})^{(h+1)/h}\right) \rho_k(i_{C_{00}}, \bar{i}_{C_{00}})/(L_{01} c_{00}) \int_{T+kT_0}^{t} e^{\mu(s-T-kT_0)} ds +$$

$$+\left(\rho_k(U, \bar{U}) + \rho_k(I, \bar{I}) e^{-\mu T}\right)/(2L_{01}) \int_{T+kT_0}^{T+(k+1)T_0} e^{\mu(s-T-kT_0)} ds +$$

$$+\left(\Phi_{00} h(1+\varphi_0/\Phi_{00})^{(h+1)/h}\right) \rho_k(i_{C_{00}}, \bar{i}_{C_{00}})/(L_{01} c_{00}) \int_{T+kT_0}^{T+(k+1)T_0} e^{\mu(s-T-kT_0)} ds \le$$

$$\le \rho_k\left((U, I, i_{L00}, i_{C00}, i_{L10}, i_{C10}), (\bar{U}, \bar{I}, \bar{i}_{L00}, \bar{i}_{C00}, \bar{i}_{L10}, \bar{i}_{C10})\right) \times$$

$$\times\left(\left(e^{\mu(t-T-kT_0)} - 1\right)\left((1+e^{-\mu T}) c_{00} + 2\Phi_{00} h(1+\phi_0/\Phi_{00})^{(h+1)/h}\right)/(2\mu c_{00} L_{01}) + \right.$$

$$\left. +\left(e^{\mu T_0} - 1\right)\left((1+e^{-\mu T}) c_{00} + 2\Phi_{00} h(1+\phi_0/\Phi_{00})^{(h+1)/h}\right)/(2\mu c_{00} L_{01})\right] \le$$

$$\le e^{\mu(t-T-kT_0)} \rho_k\left((U, I, i_{L_{00}}, i_{C_{00}}, i_{L_{10}}, i_{C_{10}}), (\bar{U}, \bar{I}, \bar{i}_{L_{00}}, \bar{i}_{C_{00}}, \bar{i}_{L_{10}}, \bar{i}_{C_{10}})\right) \times$$

$$\times e^{\mu T_0} \left((1+e^{-\mu T}) c_{00} + 2\Phi_{00} h(1+\phi_0/\Phi_{00})^{(h+1)/h}\right)/(2\mu L_{01} c_{00}) \equiv$$

$$\equiv e^{\mu(t-T-kT_0)} K_{I_{C_0}} \times$$

$$\times \rho_k\left((U, I, i_{L_{00}}, i_{C_{00}}, i_{L_{10}}, i_{C_{10}}), (\bar{U}, \bar{I}, \bar{i}_{L_{00}}, \bar{i}_{C_{00}}, \bar{i}_{L_{10}}, \bar{i}_{C_{10}})\right).$$

It follows

$$\rho_k\left(B_{i_{C00}}^{(k)}\left(U, I, i_{C_{00}}\right), B_{i_{C00}}^{(k)}\left(\bar{U}, \bar{I}, \bar{i}_{C_{00}}\right)\right) \le$$

$$\le K_{I_{C0}} \rho_k\left((U, I, i_{L00}, i_{C00}, i_{L10}, i_{C10}), (\bar{U}, \bar{I}, \bar{i}_{L00}, \bar{i}_{C00}, \bar{i}_{L10}, \bar{i}_{C10})\right).$$

Finally

$$\left| B_{i_{L_{10}}}^{(k)}\left(i_{L_{10}}, i_{C_{10}}\right)(t) - B_{i_{L_{10}}}^{(k)}\left(\bar{i}_{L_{10}}, \bar{i}_{C_{10}}\right)(t) \right| \le$$

$$\le \left| \int_{T+kT_0}^{t} \left(F_{i_{L_{10}}}\left(i_{L_{10}}, i_{C_{10}}\right)(s) - F_{i_{L_{10}}}\left(\bar{i}_{L_{10}}, \bar{i}_{C_{10}}\right)(s) \right) ds \right| +$$

$$+\left|(t-T-kT_0)/T_0\right|\left|\int_{T+kT_0}^{T+(k+1)T_0}\left(F_{i_{L_{10}}}\left(i_{L_{10}},i_{C_{10}}\right)(s)-F_{i_{L_{10}}}\left(\bar{i}_{L_{10}},\bar{i}_{C_{10}}\right)(s)\right)ds\right|\le$$

$$\le\int_{T+kT_0}^{t}\left|\frac{C_{10}^{-1}(i_{C_{10}}(s))-R_1 i_{L_{10}}(s)}{dL_{10}(i_{L_{10}})/di_{L_{10}}}-\frac{C_{10}^{-1}(\bar{i}_{C_{10}}(s))-R_1\bar{i}_{L_{10}}(s)}{dL_{10}(\bar{i}_{L_{10}})/di_{L_{10}}}\right|ds+$$

$$+\int_{T+kT_0}^{T+(k+1)T_0}\left|\frac{C_{10}^{-1}(i_{C_{10}}(s))-R_1 i_{L_{10}}(s)}{dL_{10}(i_{L_{10}})/di_{L_{10}}}-\frac{C_{10}^{-1}(\bar{i}_{C_{10}}(s))-R_1 i_{L_{10}}(s)}{dL_{10}(\bar{i}_{L_{10}})/di_{L_{10}}}\right|ds\le$$

$$\le\Phi_{10}h\left(1+\varphi_0/\Phi_{10}\right)^{(h+1)/h}/(c_{10}\hat{L}_{10})\int_{T+kT_0}^{t}\left|i_{C_{10}}(s)-\bar{i}_{C_{10}}(s)\right|ds+$$

$$+R_1/\hat{L}_{10}\int_{T+kT_0}^{t}\left|i_{L_{10}}(s)-\bar{i}_{L_{10}}(s)\right|ds+\left(\varphi_0+R_1I_{L_1}\right)\times$$

$$\times\int_{T+kT_0}^{t}e^{\mu(s-T-kT_0)}\left|\frac{d^2L_{10}(i_{L10})/di_{L10}^2}{\left(dL_{10}(i_{L10})/di_{L10}\right)^2}\right|\left|i_{L10}(s)-\bar{i}_{L10}(s)\right|ds+$$

$$+\Phi_{10}h\left(1+\varphi_0/\Phi_{10}\right)^{(h+1)/h}/(c_{10}\hat{L}_{10})\int_{T+kT_0}^{T+(k+1)T_0}\left|i_{C_{10}}(s)-\bar{i}_{C_{10}}(s)\right|ds+$$

$$+R_1/\hat{L}_{10}\int_{T+kT_0}^{T+(k+1)T_0}\left|i_{L_{10}}(s)-\bar{i}_{L_{10}}(s)\right|ds+\left(\varphi_0+R_1I_{L_1}\right)\times$$

$$\times\int_{T+kT_0}^{T+(k+1)T_0}e^{\mu(s-T-kT_0)}\left|\frac{d^2L_{10}(i_{L10})/di_{L10}^2}{\left(dL_{10}(i_{L10})/di_{L10}\right)^2}\right|\left|i_{L10}(s)-\bar{i}_{L10}(s)\right|ds\le$$

$$\le\Phi_{10}h\left(1+\varphi_0/\Phi_{10}\right)^{(h+1)/h}/(c_{10}\hat{L}_{10})\int_{T+kT_0}^{t}\left|i_{C_{10}}(s)-\bar{i}_{C_{10}}(s)\right|ds+$$

$$+R_1/\hat{L}_{10}\int_{T+kT_0}^{t}\left|i_{L_{10}}(s)-\bar{i}_{L_{10}}(s)\right|ds+\left(\varphi_0+R_1I_{L_1}\right)\times$$

$$\times\left[\sum_{n=2}^{m}(n+1)nl_n^{(10)}\left|i_0\right|^{n-2}/\hat{L}_{10}^2\right]\int_{T+kT_0}^{t}e^{\mu(s-T-kT_0)}\left|i_{L10}(s)-\bar{i}_{L10}(s)\right|ds+$$

$$+\Phi_{10}h\left(1+\varphi_0/\Phi_{10}\right)^{(h+1)/h}/(c_{10}\hat{L}_{10})\int_{T+kT_0}^{T+(k+1)T_0}\left|i_{C_{10}}(s)-\bar{i}_{C_{10}}(s)\right|ds+$$

$$+R_1/\hat{L}_{10}\int_{T+kT_0}^{T+(k+1)T_0}\left|i_{L_{10}}(s)-\bar{i}_{L_{10}}(s)\right|ds+\left(\varphi_0+R_1I_{L_1}\right)\times$$

$$\times\left[\sum_{n=2}^{m}(n+1)nl_n^{(10)}\left|i_0\right|^{n-2}/\hat{L}_{10}^2\right]\int_{T+kT_0}^{T+(k+1)T_0}\left|i_{L10}(s)-\bar{i}_{L10}(s)\right|ds\le$$

$$\le\Phi_{10}h\left(1+\varphi_0/\Phi_{10}\right)^{(h+1)/h}/(c_{10}\hat{L}_{10})\rho_k(i_{C_{10}},\bar{i}_{C_{10}})\int_{T+kT_0}^{t}e^{\mu(s-T-kT_0)}ds+$$

$$+R_1/\hat{L}_{10}\rho_k(i_{L_{10}},\bar{i}_{L_{10}})\int_{T+kT_0}^{t}e^{\mu(s-T-kT_0)}ds+\left(\varphi_0+R_1I_{L_1}\right)\times$$

$$\times\left[\sum_{n=2}^{m}(n+1)nl_n^{(10)}\left|i_0\right|^{n-2}/\hat{L}_{10}^2\right]\rho_k(i_{L10},\bar{i}_{L10})\int_{T+kT_0}^{t}e^{\mu(s-T-kT_0)}ds+$$

$$+\Phi_{10}h\left(1+\varphi_0/\Phi_{10}\right)^{(h+1)/h}/(c_{10}\hat{L}_{10})\rho_k(i_{C_{10}},\bar{i}_{C_{10}})\int_{T+kT_0}^{T+(k+1)T_0}e^{\mu(s-T-kT_0)}ds+$$

$$+R_1/\hat{L}_{10}\rho_k(i_{L_{10}},\bar{i}_{L_{10}})\int_{T+kT_0}^{T+(k+1)T_0}e^{\mu(s-T-kT_0)}ds+\left(\varphi_0+R_1I_{L_1}\right)\times$$

$$\times\left[\sum_{n=2}^{m}(n+1)nl_n^{(10)}\left|i_0\right|^{n-2}/\hat{L}_{10}^2\right]\rho_k(i_{L10},\bar{i}_{L10})\int_{T+kT_0}^{T+(k+1)T_0}e^{\mu(s-T-kT_0)}ds\le$$

$$\le e^{\mu(t-T-kT_0)}\rho_k\left((U,I,i_{L00},i_{C00},i_{L10},i_{C10}),(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00},\bar{i}_{L10},\bar{i}_{C10})\right)\cdot$$

$$\left[\Phi_{10}h\left(1+\varphi_0/\Phi_{10}\right)^{(h+1)/h}/(\mu\hat{L}_{10}c_{10})+\right.$$

$$+\left(\varphi_0+R_1I_{L_1}\right)/(\mu\hat{L}_{10}^2)\sum_{n=2}^{m}(n+1)nl_n^{(10)}\left|i_0\right|^{n-2}+$$

$$+R_1(1+\mu)/\mu\hat{L}_{10}+\left[(e^{\mu T_0}-1)/\mu\right]\left[\Phi_{10}h\left(1+\varphi_0/\Phi_{10}\right)^{(h+1)/h}/(\hat{L}_{10}c_{10})+\right.$$

$$\left.\left.+\left(\left(\varphi_0+R_1I_{L_1}\right)\left[\sum_{n=2}^{m}(n+1)nl_n^{(10)}\left|i_0\right|^{n-2}/\hat{L}_{10}^2\right]\right)\right]\right]\le$$

$$\le\rho_k\left((U,I,i_{L00},i_{C00},i_{L10},i_{C10}),(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00},\bar{i}_{L10},\bar{i}_{C10})\right)\times$$

$$\times e^{\mu(t-T-kT_0)}e^{\mu T_0}\left[\left(\Phi_{10}h\left(1+\varphi_0/\Phi_{10}\right)^{(h+1)/h}/c_{10}\right)+R+\right.$$

$$\left.+\left(\varphi_0+R_1I_{L_1}\right)\left[\sum_{n=2}^{m}(n+1)nl_n^{(10)}\left|i_0\right|^{n-2}/\hat{L}_{10}^2\right]\right)/\mu\hat{L}_{10}\equiv$$

$$\equiv e^{\mu(t-T-kT_0)}\times K_{I_{L_1}}\rho_k\left((U,I,i_{L00},i_{C00},i_{L10},i_{C10}),(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00},\bar{i}_{L10},\bar{i}_{C10})\right).$$

It follows

$$\rho_k\left(B^{(k)}_{i_{L10}}\left(i_{L10},i_{C10}\right),B^{(k)}_{i_{L10}}\left(\bar{i}_{L10},\bar{i}_{C10}\right)\right)\le$$

$$\le K_{I_{L_1}}\rho_k\left((U,I,i_{L00},i_{C00},i_{L10},i_{C10}),(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00},\bar{i}_{L10},\bar{i}_{C10})\right).$$

Further we have

$$\left|B^{(k)}_{i_{C10}}\left(U,I,i_{C_{10}}\right)(t)-B^{(k)}_{i_{C10}}\left(\bar{U},\bar{I},\bar{i}_{C_{10}}\right)(t)\right|\le$$

$$\le\left|\int_{T+kT_0}^{t}\left(F_{i_{C_{10}}}\left(U,I,i_{C_{10}}\right)(s)-F_{i_{C_{10}}}\left(\bar{U},\bar{I},\bar{i}_{C_{10}}\right)(s)\right)ds\right|+$$

$$+\left|(t-T-kT_0)/T_0\right|\left|\int_{T+kT_0}^{T+(k+1)T_0}\left(F_{i_{C_{10}}}\left(U,I,i_{C_{10}}\right)(s)-F_{i_{C_{10}}}\left(\bar{U},\bar{I},\bar{i}_{C_{10}}\right)(s)\right)ds\right|\le$$

$$\le(1/2L_{11})\left(\int_{T+kT_0}^{t}\left|U(s-T)-\bar{U}(s-T)\right|ds+\int_{T+kT_0}^{t}\left|I(s)-\bar{I}(s)\right|ds\right)+$$

$$+\int_{T+kT_0}^{t}\left|C_{10}^{-1}(i_{C_{10}}(s))-C_{10}^{-1}(\bar{i}_{C_{10}}(s))\right|/(L_{11})ds+$$

$$+(1/2L_{11})\left(\int_{T+kT_0}^{T+(k+1)T_0}\left|U(s-T)-\bar{U}(s-T)\right|ds+\int_{T+kT_0}^{T+(k+1)T_0}\left|I(s)-\bar{I}(s)\right|ds\right)+$$

$$+(1/L_{11})\int_{T+kT_0}^{T+(k+1)T_0}\left|C_{10}^{-1}(i_{C_{10}}(s))-C_{10}^{-1}(\bar{i}_{C_{10}}(s))\right|ds\le$$

$$\le (1/2L_{11})\left(\int_{T+kT_0}^{t}\left|U(s-T)-\bar{U}(s-T)\right|ds+\int_{T+kT_0}^{t}\left|I(s)-\bar{I}(s)\right|ds\right)+$$

$$+\Phi_{10}h(1+\varphi_0/\Phi_{10})^{(h+1)/h}/(L_{11}c_{10})\int_{T+kT_0}^{t}\left|i_{C_{10}}(s)-\bar{i}_{C_{10}}(s)\right|ds+$$

$$+(1/2L_{11})\left(\int_{T+kT_0}^{T+(k+1)T_0}\left|U(s-T)-\bar{U}(s-T)\right|ds+\int_{T+kT_0}^{T+(k+1)T_0}\left|I(s)-\bar{I}(s)\right|ds\right)+$$

$$+\Phi_{10}h(1+\varphi_0/\Phi_{10})^{(h+1)/h}/(L_{11}c_{10})\int_{T+kT_0}^{T+(k+1)T_0}\left|i_{C_{10}}(s)-\bar{i}_{C_{10}}(s)\right|ds\le$$

$$\le\left(c_{10}(\rho_k(U,\bar{U})e^{-\mu T}+\rho_k(I,\bar{I}))+2\Phi_{10}h(1+\varphi_0/\Phi_{10})^{(h+1)/h}\rho_k(i_{C_{10}},\bar{i}_{C_{10}})\right)\times$$

$$\times\left(\int_{T+kT_0}^{t}e^{\mu(s-T-kT_0)}ds+\int_{T+kT_0}^{T+(k+1)T_0}e^{\mu(s-T-kT_0)}ds\right)/(2L_{11}c_{10})\le$$

$$\le\left(c_{10}(e^{-\mu T}+1)+2\Phi_{10}h(1+\varphi_0/\Phi_{10})^{(h+1)/h}\right)\left(e^{\mu(t-T-kT_0)}-1\right)\times$$

$$\times\rho_k\left((U,I,i_{L_{00}},i_{C_{00}},i_{L_{10}},i_{C_{10}}),(\bar{U},\bar{I},\bar{i}_{L_{00}},\bar{i}_{C_{00}},\bar{i}_{L_{10}},\bar{i}_{C_{10}})\right)/(\mu L_{11}c_{10})+$$

$$+\left(c_{10}(e^{-\mu T}+1)+2\Phi_{10}h(1+\varphi_0/\Phi_{10})^{(h+1)/h}\right)\left(e^{\mu T_0}-1\right)\times$$

$$\times\rho_k\left((U,I,i_{L_{00}},i_{C_{00}},i_{L_{10}},i_{C_{10}}),(\bar{U},\bar{I},\bar{i}_{L_{00}},\bar{i}_{C_{00}},\bar{i}_{L_{10}},\bar{i}_{C_{10}})\right)/(\mu L_{11}c_{10})\le$$

$$\le e^{\mu(t-T-kT_0)}e^{\mu T_0}\left(c_{10}(e^{-\mu T}+1)+2\Phi_{10}h(1+\varphi_0/\Phi_{10})^{(h+1)/h}\right)\times$$

$$\times\rho_k\left((U,I,i_{L_{00}},i_{C_{00}},i_{L_{10}},i_{C_{10}}),(\bar{U},\bar{I},\bar{i}_{L_{00}},\bar{i}_{C_{00}},\bar{i}_{L_{10}},\bar{i}_{C_{10}})\right)/(\mu L_{11}c_{10})\equiv$$

$$\equiv e^{\mu(t-T-kT_0)}\times$$

$$\times K_{C_1}\rho_k\left((U,I,i_{L00},i_{C00},i_{L10},i_{C10}),(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00},\bar{i}_{L10},\bar{i}_{C10})\right).$$

It follows

$$\rho_k\left(B^{(k)}_{i_{C10}}\left(U,I,i_{C_{10}}\right),B^{(k)}_{i_{C10}}\left(\bar{U},\bar{I},\bar{i}_{C_{10}}\right)\right)\le$$

$$\le K_{I_{C_1}}\rho_k\left((U,I,i_{L00},i_{C00},i_{L10},i_{C10}),(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00},\bar{i}_{L10},\bar{i}_{C10})\right).$$

Denoting by $K=\max\left\{K_U,K_I,K_{I_{L_0}},K_{I_{C_0}},K_{I_{L_1}},K_{I_{C_1}}\right\}<1$ and $\bar{B}_U=B_U\left(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00}\right),\ldots,\bar{B}_{i_{C10}}=B_{i_{C10}}\left(\bar{U},\bar{I},\bar{i}_{C_{10}}\right)$ we obtain

$$\rho_k\left(\left(B_U,B_I,B_{i_{L00}},B_{i_{C00}},B_{i_{L10}},B_{i_{C10}}\right),\left(\bar{B}_U,\bar{B}_I,\bar{B}_{i_{L00}},\bar{B}_{i_{C00}},\bar{B}_{i_{L10}},\bar{B}_{i_{C10}}\right)\right)\le$$

$$\le K\rho_k\left((U,I,i_{L00},i_{C00},i_{L10},i_{C10}),(\bar{U},\bar{I},\bar{i}_{L00},\bar{i}_{C00},\bar{i}_{L10},\bar{i}_{C10})\right).$$

Therefore Bhas a unique fixed point which is a periodic solution of (13).

Theorem 1 is thus proved.

VII. NUMERICAL EXAMPLE

Here we collect all inequalities guaranteeing the existence-uniqueness result:

$$e^{\mu_0}(U_0+I_0)/2\le\varphi_0;\ \ 2(\varphi_0+R_0I_{L_0})/(\mu\hat{L}_{00})\le I_{L_0};\ \ 2(\varphi_0+R_1I_{L_1})/\mu\hat{L}_{10}\le I_{L_1};$$

$$I_0e^{-\mu T}+e^{\mu T_0}\left([(U_0+I_0e^{-\mu T})/Z_0]+2I_{L_0}+2I_{C_0}\right)/\mu C_{01}\le U_0;$$

$$U_0e^{-\mu T}+e^{\mu T_0}\left([(U_0e^{-\mu T}+I_0)/Z_0]+2I_{L_1}+2I_{C_1}\right)/\mu C_{11}\le I_0;$$

$$;\ \ e^{\mu T_0}\left([(U_0+I_0e^{-\mu T})/2]+\varphi_0\right)/\mu L_{01}\le I_{C_0};$$

$$;\ \ e^{\mu T_0}\left([(U_0e^{-\mu T}+I_0)/2]+\varphi_0\right)/\mu L_{11}\le I_{C_1};$$

$$K_U=e^{-\mu T}+e^{\mu T_0}\left([(1+e^{-\mu T})/Z_0]+4\right)/\mu C_{01}<1;$$

$$K_I=e^{-\mu T}+e^{\mu T_0}\left([(1+e^{-\mu T})/Z_0]+4\right)/\mu C_{11}<1;$$

$$K_{I_{L_0}}=e^{\mu T_0}\left[(\Phi_{00}h/c_{00})[(\Phi_{00}+\varphi_0)/\Phi_{00}]^{(h+1)/h}+R_0+\right.$$

$$\left.+(\varphi_0/\hat{L}_{00})\sum_{n=2}^{m}(n+1)nl_n^{(00)}\left|i_0\right|^{n-2}\right]/\mu\hat{L}_{00}<1;$$

$$K_{I_{C_0}}=e^{\mu T_0}\left[[(1+e^{-\mu T})/2]+\right.$$

$$\left.+(\Phi_{00}h/c_{00})[(\Phi_{00}+\varphi_0)/\Phi_{00}]^{(h+1)/h}\right]/\mu\hat{L}_{01}<1;$$

$$K_{I_{L_1}}=e^{\mu T_0}\left[\{\Phi_{10}h[(\Phi_{10}+\varphi_0)/\Phi_{10}]^{(h+1)/h}/c_{10}\}+R_1+\right.$$

$$\left.+(\varphi_0/\hat{L}_{10})\sum_{n=2}^{m}(n+1)nl_n^{(10)}\left|i_0\right|^{n-2}\right]/\mu\hat{L}_{10}<1;$$

$$K_{I_{C_1}}=e^{\mu T_0}\left[[(e^{-\mu T}+1)/2]+\right.$$

$$\left.+\{\Phi_{10}h[(\Phi_{10}+\varphi_0)/\Phi_{10}]^{(h+1)/h}/c_{10}\}\right]/\mu\hat{L}_{11}<1.$$

For a transmission line with length $\Lambda=10m$; $C=80\,pF/m$ $L=0{,}45\,\mu H/m$; $\sqrt{LC}=6.10^{-9}$, $Z_0=\sqrt{L/C}=75\Omega$; $v=1/\left(6.10^{-9}\right)=1{,}66.10^8$. Then $T=\Lambda\sqrt{LC}=6.10^{-8}$sec .Let us check the propagation of waves with $\lambda_0=(1/6)10^{-3}m$. We have $f_0=1/\left(\lambda_0\sqrt{LC}\right)=10^{12}\,Hz\Rightarrow$ $T_0=1/f_0=10^{-12}$. We choose $\mu=10^{12}$, then $\mu T_0=\mu_0=1$ and $T=6.10^{-8}.2.10^{12}T_0=12.10^4.T_0\Rightarrow m=12.10^4$; $e^{-6.10^4}\approx 0$; $\mu T=10^{12}.6.10^{-8}=6.10^4$.

Let us take

$C_{00}(u)=C_{10}(u)=c_{00}/\sqrt{1-(u/\Phi_{00})}=c_{10}\sqrt{\Phi_{10}}/\sqrt{\Phi_{10}-u}$, where $h=2$; $R_0=R_1=35\Omega$; $c_{00}=c_{10}=50\,pF=5.10^{-11}F$; $\varphi_0=0{,}2$; and $\Phi_{00}=\Phi_{10}=0{,}4\,V\Rightarrow$; $U_0<\varphi_0<0{,}4$; $C_{01}=C_{11}=50.10^{-12}F$ .We choose $L_{00}(i)=L_{10}(i)=3i-(1/12)i^3$. Then $dL_{00}(i)/di=3-0{,}25i^2$. For $i_0=1$ we obtain $3-0{,}25i^2>3-0{,}25=2{,}75$ and then $\sum_{n=2}^{3}(n+1)nl_n^{(00)}\left|i_0\right|^{n-2}=0{,}5$; $\hat{L}_{00}=\hat{L}_{10}=2{,}75$. If we choose $U_0=I_0=I_{L_0}=I_{C_0}=I_{L_1}=I_{C_1}$ and replace φ_0 by U_0 or I_0 in the above inequalities we get $e(U_0+I_0)/2\le 0{,}2$; $c\le I_{C_0}$;

$$(e/10^{12}.50.10^{-12})\left((U_0/75)+2I_{L_0}+2I_{C_0}\right)\le U_0;$$

$$(e/10^{12}.50.10^{-12})\left((I_0/75)+2I_{L_1}+2I_{C_1}\right)\le I_0;$$

$$2(U_0+35I_{L_0})/(10^{12}.2{,}75)\le I_{L_0};\ \ 2(I_0+35I_{L_{10}})/(10^{12}.2{,}75)\le I_{L_1};$$

$$e\left(I_0+I_0/2\right)/(10^{12}.2{,}75)\le I_{C_1}$$

or

$$U_0\le 0{,}2/e;\ \ 2(1+35)/(10^{12}.2{,}75)\le 1;\ \ e\left(4+1/75\right)/(10^{12}.50.10^{-12})\le 1;$$

$$e\left(4+1/75\right)/(10^{12}.50.10^{-12})\le 1;\ e\left(1+1/2\right)/(10^{12}.2{,}75)\le 1;$$

$$2(1+35)/(10^{12}.2{,}75)\le 1;\ \ e\left(1+1/2\right)/(10^{12}.2{,}75)\le 1;$$

$$K_U=K_I=4e/(10^{12}.50.10^{-12})+1/75=0{,}218<1;$$

 ISSN: 2321-8169
1341– 1352

$$K_{I_{L_0}} = K_{I_{L_1}} = \frac{2,72}{2,75}\left[\frac{0,8(1,5)^{3/2}}{10^{12}.5.10^{-11}} + \frac{35}{10^{12}} + \frac{0,2.0,5}{10^{12}.2,75}\right] \approx 0,03 < 1;$$

$$K_{I_{C_0}} = K_{I_{C_1}} = \frac{e}{2,75}\frac{8(1,5)^{3/2}}{500} \approx 0,03 < 1. \text{ Then } K = 0,218 < 1.$$

VIII. Conclusion

The solution can be obtained by successive approximations beginning with simple functions. Indeed,

$$U^{(n+1)}(t) = -I^{(n)}(t-T) + \int_{T+kT_0}^{t}\left(\frac{I^{(n)}(s-T)-U^{(n)}(s)}{C_{01}Z_0} - \frac{2i_{L_{00}}^{(n)}(s)+2i_{C_{00}}^{(n)}(s)}{C_{01}}\right)ds -$$

$$-(t-T-kT_0)/T_0 \int_{T+kT_0}^{T+(k+1)T_0}\left(\frac{I^{(n)}(s-T)-U^{(n)}(s)}{C_{01}Z_0} - \frac{2i_{L_{00}}^{(n)}(s)+2i_{C_{00}}^{(n)}(s)}{C_{01}}\right)ds,$$

$$I^{(n+1)}(t) = -U^{(n)}(t-T) + \int_{T+kT_0}^{t}\left(\frac{U^{(n)}(s-T)-I^{(n)}(s)}{C_{11}Z_0} - \frac{2i_{L_{10}}^{(n)}(s)+2i_{C_{10}}^{(n)}(s)}{C_{11}}\right)ds -$$

$$-(t-T-kT_0)/T_0 \int_{T+kT_0}^{T+(k+1)T_0}\left(\frac{U^{(n)}(s-T)-I^{(n)}(s)}{C_{11}Z_0} - \frac{2i_{L_{10}}^{(n)}(s)+2i_{C_{10}}^{(n)}(s)}{C_{11}}\right)ds,$$

$$i_{L_{00}}^{(n+1)}(t) = \int_{T+kT_0}^{t}\frac{C_{00}^{-1}(i_{C_{00}}^{(n)}(s)) - R_0 i_{L_{00}}^{(n)}(s)}{dL_{00}(i_{L_{00}}^{(n)})/di_{L_{00}}}ds -$$

$$-(t-T-kT_0)/T_0 \int_{T+kT_0}^{T+(k+1)T_0}\frac{C_{00}^{-1}(i_{C_{00}}^{(n)}(s)) - R_0 i_{L_{00}}^{(n)}(s)}{dL_{00}(i_{L_{00}}^{(n)})/di_{L_{00}}}ds,$$

$$i_{C_{00}}^{(n+1)}(t) = (1/2L_{01})\int_{T+kT_0}^{t}\left[U^{(n)}(s) + I^{(n)}(s-T) - 2C_{00}^{-1}(i_{C_{00}}^{(n)}(s))\right]ds -$$

$$-(t-T-kT_0)/(2L_{01}T_0)\int_{T+kT_0}^{T+(k+1)T_0}\left(U^{(n)}(s) + I^{(n)}(s-T) - 2C_{00}^{-1}(i_{C_{00}}^{(n)}(s))\right)ds,$$

$$i_{L_{10}}^{(n+1)}(t) = \int_{T+kT_0}^{t}\frac{C_{10}^{-1}(i_{C_{10}}^{(n)}(s)) - R_1 i_{L_{10}}^{(n)}(s)}{dL_{10}(i_{L_{10}}^{(n)})/di_{L_{10}}}ds -$$

$$-(t-T-kT_0)/T_0\int_{T+kT_0}^{T+(k+1)T_0}\frac{C_{10}^{-1}(i_{C_{10}}^{(n)}(s)) - R_1 i_{L_{10}}^{(n)}(s)}{dL_{10}(i_{L_{10}}^{(n)})/di_{L_{10}}}ds,$$

$$i_{C_{10}}^{(n+1)}(t) = \int_{T+kT_0}^{t}\left(\frac{U^{(n)}(s-T)+I^{(n)}(s)}{2L_{11}} - \frac{C_{10}^{-1}(i_{C_{10}}^{(n)}(s))}{L_{11}}\right)ds -$$

$$-(t-T-kT_0)/T_0\int_{T+kT_0}^{T+(k+1)T_0}\left(\frac{U^{(n)}(s-T)+I^{(n)}(s)}{2L_{11}} - \frac{C_{10}^{-1}(i_{C_{10}}^{(n)}(s))}{L_{11}}\right)ds,$$

where

$$\rho_k\left(\left(U^{(n+1)}, I^{(n+1)},\ldots\right),\left(\overline{U}^{(n)}, \bar{I}^{(n)},\ldots\right)\right) \le$$

$$\le (0,218)^n \rho_k\left(\left(U^{(0)}, I^{(0)},\ldots\right),\left(\overline{U}^{(1)}, \bar{I}^{(1)},\ldots\right)\right)/(1-0,218),\ (k=0,1,2,\ldots)$$

References

[1] L. De Broglie, Problemes de Propagations Guidees des ondes Electromagnetiques, Gauthier-Villars,Paris, 1941.

[2] C. A. Holt, Introduction to Electromagnetic Fields and Waves, J. Wiley &Sons, New York, 1963.

[3] E. C. Jordan and K. G. Balmain, Electromagnetic Waves and Radiating Systems, Prentice-Hall Inc., 1968.

[4] S. Ramo, J. R. Whinnery, and T. Duzer, Fields and Waves in Communication Electronics, J.Wiley & Sons, Inc., New York,1994.

[5] L. O. Chua, C. A. Desoer, and E. S. Kuh, Linear and Nonlinear Circuits, McGraw-Hill Book Company, New York, USA, 1987.

[6] L. O. Chua and Pen-Min Lin, Machine Analysis of Electronic Circuits. Energy, Moscow, 1980.

[7] S. Rosenstark, Transmission Lines in Computer Engineering, Mc Grow-Hill, New York, 1994.

[8] S. A. Maas, Nonlinear Microwave and RF Circuits, 2nd ed., Artech House, Inc., Boston London, 2003.

[9] D. K. Misra, Radio-Frequency and Microwave Communication Circuits. Analysis and Design, 2-nd ed., University of Wisconsin-Milwaukee. J. Wiley & Sons, Inc., 2004.

[10] G. Miano and A. Maffucci, Transmission Lines and Lumped Circuits, 2-nd ed., Academic Press, New York, 2001, 2010.

[11] F. Martín, Artificial Transmission Lines for RF and Microwave Applications, Wiley Series in Microwave & Optical Engineering, 2015.

[12] R. Singh, Circuit Theory and Transmission Lines, McGraw Hill Education, 2013.

[13] V. Makwana and B. Bhalja, Transmission Line Protection Using Digital Technology (Energy Systems in Electrical Engineering), 1st ed., 2016 Edition, Springer, 2016.

[14] Dr. S. Ruikar, Electromagnetics and Transmission Lines, Nirali Prakashan, 2016.

[15] R. A. Lundquist, Transmission Line Construction: Methods and Costs, Forgotten Books, 2015.

[16] S. Kalaga and P. Yenumula, Design of Electrical Transmission Lines: Structures and Foundations, CRC Press, 2016.

[17] V. G. Angelov, A Method for Analysis of Transmission Lines Terminated by Nonlinear Loads, Nova Science, New York, 2014.

Acknowledgement. This work was supported by contract DUNK 01/03-12.2009 funded by National Science Fund of the Ministry of Education, Youth and Science of Bulgaria.