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Two Warehouse Inventory Model for Deteriorating Products with Stock Dependent Demand and Shortages

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Abstract::- In this paper a deterministic inventory model for two warehouses has been developed. In two warehouses the first is the owned warehouse with a fixed capacity of W units and the other one is rented warehouse with unlimited stocking capacity. The deterioration occurs in both the warehouses. First the demand is fulfilled from the inventory in rented warehouse and after thatthe inventory in owned warehouse has been used. The shortages are allowed in owned warehouse only and the excess demand is partially backlogged. For the generality of the model we presented the equations for total cost of the system. A numerical example and sensitivity analysis with respect to different associated parameters has also been presented to illustrate the model.

Keywords: *Two warehouse, Inventory, Deterioration, Stock dependent demand, Shortages, Partial backlogging*

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Introduction:

In today's era of globalization a glamorous display of stock level attracts a lot of customers. A large amount of displayed stock level increases the demand rate. Hence it can be said that the demand rate of any product will be a function of displayed stock level. Very few researchers developed the inventory models with stock dependent selling rate. Baker and Urban (1988) were the first to develop such kind of inventory model. After this Datta and Pal (1990) modified the model of Baker and Urban (1988) by introducing that the selling rate of any product increases with the stock level upto a fix level of inventory and after that it becomes constant. Sarkar et al. (1997) developedan economic order quantity inventorymodel with inventory-level dependent demand and deterioration of products. Ouyang et al. (2003) presented an inventory model for deteriorating items with stock dependent demand under the conditionsof inflation and time-value of money. Kumar and Singh (2009) introduced a twowarehouse inventory model with stock dependent demand for deteriorating items with shortages. Singh and Sharma (2014) presented an optimal trade credit policy for perishable items deeming imperfect production and stock dependent demand.

Traditionally the inventory models developed for deteriorating products are considered with single warehouse but due to many different reasons such as offered concession in bulk purchasing, a higher transportation cost etc. the businessmen are forced to purchase the more quantity then their requirement. In such cases the capacity of available showroom will not enough and an additional space is required in terms of rented warehouse. Recently many authors developed the different inventory models for two warehouses. Such type of model was first discussed by Hartley (1976). After that Sarma (1983) presented a deterministic inventory model with twolevels of storage and an optimum release rule.But this model was developed with no shortages.Bhunia and Maiti (1994) discussed a two warehouse inventory model for a linear trend in demand, but all of these models are developed for non deteriorating items. Yang (2004) provided a two warehouse inventory model with constant demand and occurring shortages under inflationary environment.Tayal et al. (2014) presented a production inventory problem for deteriorating items with space restriction. In this paper it is shown that due to a fix capacity of owned warehouse the extra ordered quantity at the time of arrival of stock is returned to supplier with a penalty cost. Tayal et al. (2015)also presented an inventory model for deteriorating items with seasonal products and an option of an alternative market. This paper is developed for seasonal products and it is shown that after the completion of season in primary market the remaining stock is transferred to any secondary location, where it is required at that time.

Most of all above mentioned models developed for two warehouses are proposed for non deteriorating products but deterioration is that factor which occurs normally in all products in any form. So to consider the models without the assumption of deterioration rate will not reflect the reality. Ghare and Schrader (1963) were the first to introduce the deterioration in inventory modelling. In this model they considered that the products deteriorate at a constant rate. After this Covert and Philip (1973) developed an inventory model for weibull rate of deterioration. This model of Covert and Philip (1973) was extended by Chakrabarti et al. (1998) for trended demand and occurring shortages. Singh and Singh (2007) came forward with an inventory model for deteriorating products with ramp type demand and partial backlogging of occurring shortages. Tayal et al. (2014) introduced a two echelon supply chain model for deteriorating items with effectiveinvestment in preservation technology. With the help of preservation technology the deterioration rate can be reduced up to a certain limit. Tayal et al. (2015) also presented an inventory model for those products which maintain its quality for a period of time and then it begins to deteriorate. This model was developed for time dependent holding cost and exponential demand rate. Singh et al. (2016) developed an inventory model of deteriorating products with preservation technology and trade credit period for stock level dependent demand rate.

Shortages and partial backlogging is also a very important factor to be considered in the development of inventory models. In traditional inventory models shortages are not allowed. After some time researchers introduced shortages in their model but they assumed it completely lost or completely backlogged. But except some cases the occurring demand during shortages will not be completely lost/completely backlogged. Some impatient customers make their purchases from others and some permanent customers come back for their demand. Singh and Singh (2008) developed a two-warehouse partial backlogging inventory model for perishable products having exponential demand.Tayal et al. (2014) presented a multi item inventory model for deteriorating items with expiration date and allowable shortages.Khurana et al. (2015) developed a supply chain production inventory model for deteriorationg product with stock dependent demandunder inflationary environment and partial backlogging. Tayal et al. (2016) introduced an integrated production inventory model for perishable products with trade credit period and investment in preservation technology and partial backlogging of occurring shortages.

Xu et al (2017) considered a two-warehouse inventory problem for deteriorating items with a constant demand rate over a finite time horizon. A modified first-in-first-out dispatching policy is first proposed, and a new two-warehouse inventory model based on this dispatching policy is developed.

In the present paper we have developed a two warehouse inventory model for deteriorating products. The shortages are allowed only in owned warehouse and partially backlogged. A numerical example and sensitivity analysis with respect to different system parameters are also shown in the model.

Assumptions:

1. The products considered in this model are deteriorating in nature.
2. The system involves only one item.
3. The demand rate for the products in rented warehouse is constant and for the owned warehouse it is stock level dependent.
4. The entire lot is delivered in single batch at the beginningof each cycle.
5. Shortages are allowed and partially backlogged.
6. The deteriorated items are completely discarded.
7. The owned warehouse has a fix capacity of W units and the remaining quantity if greater than W is transferred to the rented warehouse.
8. The rented warehouse has unlimited capacity.
9. A fixed transportation cost is considered.

Notations:

K deterioration coefficient
W capacity of owned warehouse
t_1 the time at which inventory level becomes zero in rented warehouse
v the time at which inventory level becomes zero in owned warehouse
Q_1 initial ordered quantity
Q_2 backordered quantity during stockout
T cycle time
θ rate of backlogging
c_1 purchasing cost per unit
h_r holding cost per unit in rented warehouse
h_o holding cost per unit in owned warehouse
d deterioration cost per unit
c_2 shortage cost per unit
c_3 lost sale cost per unit

Mathematical Modelling:

In the development of this model we assume that a company purchases Q_1 units out of which W units are stored in owned warehouse and the remaining (Q_1-W) units are transported to rented warehouse. Initially the stock in rented warehouse is consumed and only after that the stock in owned warehouse is used. We have developed this model for two different cases of demand rates.

When the demand is stock level dependent:

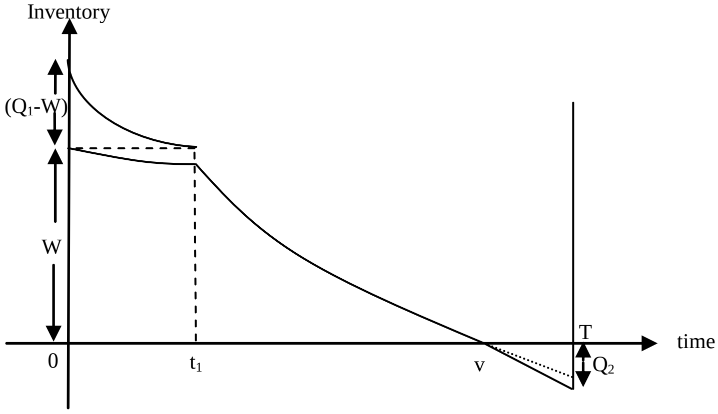

Fig 1: Inventory time graph of two warehouse inventory model

These are the differential equations showing the inventory time behavior of the system:

$$\frac{dI_r(t)}{dt} = -KI_r(t) - \alpha \qquad 0 \le t \le t_1 \qquad (1)$$

with boundary condition $I_r(t_1) = 0$ $\qquad 0 \le t \le t_1 \qquad$ (2)

$$\frac{dI_o(t)}{dt} = -KI_o(t) - (\beta + \gamma I_0(t)) \qquad t_1 \le t \le v \qquad (3)$$

with boundary condition $I_o(0) = W$ and $I_o(v) = 0$

Solving these equations with the help of above mentioned boundary conditions:

$$I_r(t) = \frac{\alpha}{K}(e^{K(t_1 - t)} - 1) \qquad 0 \le t \le t_1 \qquad (4)$$

$$I_o(t) = We^{-Kt} \qquad 0 \le t \le t_1 \qquad (5)$$

$$I_o(t) = \frac{\beta}{(K+\gamma)}(e^{(K+\gamma)(v-t)} - 1) \qquad t_1 \le t \le v \qquad (6)$$

Initially an order of Q_1+Q_2 units is made out of which the Q_2 units are used to meet the backordered quantity and the remaining Q_1 units are stored as the initial stock level for next cycle. Since the owned warehouse has a limited capacity of W units, so if the stock level $Q_1 \ge W$, then the remaining quantity $(Q_1 - W)$ will be stored in rented warehouse.

With the help of equation (5):

$$I_r(0) = (Q_1 - W)$$

$$Q_1 = W + \frac{\alpha}{K}(e^{Kt_1} - 1) \qquad (7)$$

Now the T.A.C. of the system in this case will be:

$$T.A.C. = \frac{1}{T}[P.C. + H.C. + D.C. + T.C. + S.C. + L.S.C.] \qquad (8)$$

The purchasing cost for this model can be calculated as:

Since Q_1 is the initial stock level and Q_2 is the backordered quantity. So the purchasing cost in this case will be:

$$P.C. = (Q_1 + Q_2)c_1$$

Where Q_1 is given by equation (7) and Q_2 can be calculated as follow:

$$Q_2 = \int_v^T \theta\alpha dt$$

$$Q_2 = \alpha\theta(T - v)$$

Then purchasing cost will be given as:

$$P.C. = \{(W + \frac{\alpha}{K}(e^{Kt_1} - 1)) + \alpha\theta(T - v)\}c_1 \qquad (9)$$

The holding cost for the system in this case is given by:

The available stock is stored in both the warehouse so the holding cost will be charged for rented warehouse as well as owned warehouse.

$$H.C_R = h_r \int_0^{t_1} I_r(t)dt$$

$$H.C_R = h_r \frac{\alpha}{K}(\frac{e^{Kt_1} - 1}{K} - t_1) \qquad (10)$$

$$H.C_o = h_o\{\int_0^{t_1} I_o(t)dt + \int_{t_1}^{v} I_o(t)dt\}$$

$$H.C_o = \frac{Wh_0}{K}(1 - e^{-Kt_1}) + h_o \frac{\alpha}{(K+\gamma)}(\frac{e^{(K+\gamma)(v-t_1)} - 1}{(K+\gamma)} + (t_1 - v)) \qquad (11)$$

The deterioration cost of the system in this case can be calculated as follow:

The inventory is stored in R.W. and O.W. so the deterioration will occur in both the warehouse.

$$D.C_R = d(I_r(0) - D[0, t_1])$$

$$D.C_R = d\{\frac{\alpha}{K}(e^{Kt_1} - 1) - \alpha t_1\} \qquad (12)$$

$$D.C_o = d(W - D[t_1, v])$$

$$D.C_o = d[W - \{\alpha v + \frac{\gamma\beta}{(K+\gamma)}(\frac{e^{(K+\gamma)(v-t_1)} - 1}{(K+\gamma)} + (t_1 - v)\}] \qquad (13)$$

Transportation cost will be charged to transfer the stock from rented warehouse to owned warehouse.

$$T.C. = A \qquad (14)$$

The shortage cost will occur during stock out. It can be calculated as follow.

$$S.C. = c_2 \int_v^T \alpha dt$$

$$S.C. = c_2\alpha(T - v) \qquad (15)$$

During stock out some customers make their purchases from any other place. In this situation the occurring demand during stock out is partially backlogged or partially lost. Thus a lost sale cost occur and can be calculated as follows:

$$L.S.C. = c_3 \int_v^T (1 - \theta)\alpha dt$$

$$L.S.C. = c_3(1 - \theta)\alpha(T - v) \qquad (16)$$

When the demand rate is an exponential function of time:

In this case the differential equations showing the inventory level at any time t are given as follow:

$$\frac{dI_r(t)}{dt} = -KI_r(t) - \alpha e^{\beta t} \qquad 0 \le t \le t_1 \qquad (17)$$

$$\frac{dI_o(t)}{dt} = -KI_o(t) \qquad 0 \le t \le t_1 \qquad (18)$$

$$\frac{dI_o(t)}{dt} = -KI_o(t) - \alpha e^{\beta t} \qquad t_1 \le t \le v \qquad (19)$$

with boundary condition $I_r(t_1) = 0$, $I_o(0) = W$ and $I_o(v) = 0$ (20)

Using these boundary conditions the solutions of these equations are given by:

$$I_r(t) = \frac{\alpha}{(\beta + K)}(e^{(\beta+K)t_1} - e^{(\beta+K)t})e^{-Kt} \qquad 0 \le t \le t_1 \qquad (21)$$

$$I_o(t) = We^{-Kt} \qquad 0 \le t \le t_1 \qquad (22)$$

$$I_o(t) = \frac{\alpha}{(\beta + K)}(e^{(\beta+K)v} - e^{(\beta+K)t})e^{-Kt} \qquad t_1 \le t \le v \qquad (23)$$

Initially at t=0, an order of (Q_1+Q_2) units is made, where Q_2 is the backordered quantity against the occurring backlogging during stock out and Q_1 is the initial stock level. The owned warehouse has a fix capacity of W units and the remaining stock will be transferred to rented warehouse.
From equation (22):

$$I_r(0) = (Q_1 - W)$$

$$Q_1 = W + \frac{\alpha}{(\beta + K)}(e^{(\beta+K)t_1} - 1) \qquad (24)$$

Now the T.A.C. of the system in this case will be:

$$T.A.C. = \frac{1}{T}[P.C. + H.C. + D.C. + T.C. + S.C. + L.S.C.] \qquad (25)$$

Purchasing cost can be calculated as follow:

$$P.C. = (Q_1 + Q_2)c_1$$

Here Q_1 is the initial stock level and Q_2 is the backordered quantity. The backordered quantity Q_2can be calculated as follow:

$$Q_2 = \int_v^T \alpha e^{\beta t}\theta dt$$

$$Q_2 = \frac{\alpha\theta}{\beta}(e^{\beta T} - e^{\beta v})$$

$$P.C. = \{(W + \frac{\alpha}{(\beta + K)}(e^{(\beta+K)t_1} - 1)) + \frac{\alpha\theta}{\beta}(e^{\beta T} - e^{\beta v})\}c_1 \qquad (26)$$

Holding cost will be charged in both the warehouse.
The cost for carrying the stock in rented warehouse:

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$$H.C_R = h_r \int_0^{t_1} I_r(t)dt$$

$$H.C_R = h_r \frac{\alpha}{(\beta+K)}(\frac{e^{(\beta+K)t_1}}{K}(1-e^{-Kt_1}) - \frac{e^{\beta t_1}-1}{\beta}) \quad (27)$$

The cost for carrying the stock in owned warehouse:

$$H.C_o = h_o\{\int_0^{t_1} I_o(t)dt + \int_{t_1}^{v} I_o(t)dt\}$$

$$H.C_o = \frac{Wh_0}{K}(1-e^{-Kt_1}) + h_o \frac{\alpha}{(\beta+K)}(\frac{(e^{K(v-t_1)}-1)e^{\beta v}}{K} + \frac{(e^{\beta t_1}-e^{\beta v})}{\beta}) \quad (28)$$

The inventory is stored in both the warehouse, so the deterioration will occur in both the places.
The cost of deterioration in rented warehouse:

$$D.C_R = d(I_r(0) - D[0,t_1])$$

$$D.C_R = d\{\frac{\alpha}{(\beta+K)}(e^{(\beta+K)t_1}-1) - \frac{\alpha}{\beta}(e^{\beta t_1}-1)\} \quad (29)$$

The cost of deterioration in owned warehouse:

$$D.C_o = d(W - D[t_1, v])$$

$$D.C_o = d\{W - \frac{\alpha}{\beta}(e^{\beta v} - e^{\beta t_1})\} \quad (30)$$

Shortage occurs only in owned warehouse, so the shortage cost can be given as:

$$S.C. = c_2 \int_v^T \alpha e^{\beta t} dt$$

$$S.C. = c_2 \frac{\alpha}{\beta}(e^{\beta T} - e^{\beta v}) \quad (31)$$

During stockout only a fraction of occurring demand is backlogged and the remaining demand will be lost. So the cost occurs due to the lost sale can be calculated as follow:

$$L.S.C. = c_3 \int_v^T (1-\theta)\alpha e^{\beta t} dt$$

$$L.S.C. = c_3(1-\theta)\frac{\alpha}{\beta}(e^{\beta T} - e^{\beta v}) \quad (32)$$

The transportation cost for transferring the stock from rented warehouse to owned warehouse.

$$T.C. = A \quad (33)$$

Here T.A.C. is a function of two variables t_1 and v. To optimize this cost we have to find out the optimal value of t_1 and v.

$$\frac{\partial T.A.C.}{\partial t_1} = 0 \text{ and } \frac{\partial T.A.C.}{\partial v} = 0$$

$$\frac{\partial^2 T.A.C.}{\partial t_1^2} > 0, \frac{\partial^2 T.A.C.}{\partial v^2} > 0 \text{ and } (\frac{\partial^2 T.A.C.}{\partial t_1^2})(\frac{\partial^2 T.A.C.}{\partial v^2}) - (\frac{\partial^2 T.A.C.}{\partial t_1 \partial v}) > 0$$

With the help of these equations the optimal solution of t_1, v and T.A.C. can be obtained and below mentioned figure (2) shows that the model is convex.

Numerical Example:

Here a numerical example is given to illustrate the model.

$\alpha = 100\,units,\;\; \beta = 0.02,\;\; \theta = 0.5,\;\; W = 5000\,unit,\;\; c_1 = 15\,rs/unit,\;\; c_2 = 3\,rs/unit,\;\; c_3 = 5\,rs/unit$

$h_1 = 0.6\,rs/unit,\;\; h_2 = 0.4\,rs/unit,\;\; K = 0.001,\;\; d = 16\,rs/unit,\;\; T = 90\,days$

corresponding to these the optimal value come out to be as follow:

$v = 64.6375\,days,\;\; t_1 = 53.1499\,days,\;\; T.A.C. = 6148.86\,rs$

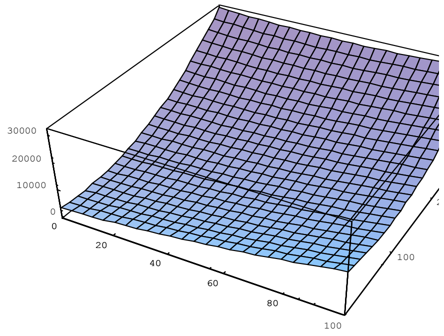

Fig 2: Convexity of the T.A.C. function

Sensitivity Analysis:

Table 1: Variation in T.A.C. with the variation in α:

% variation in α	α	t_1	v	T.A.C.
-20%	80	53.1499	64.6375	5499.75
-15%	85	53.1499	64.6375	5662.03
-10%	90	53.1499	64.6375	5824.31
-5%	95	53.1499	64.6375	5986.58
0%	100	53.1499	64.6375	6148.86
5%	105	53.1499	64.6375	6311.14
10%	110	53.1499	64.6375	6473.41
15%	115	53.1499	64.6375	6635.69
20%	120	53.1499	64.6375	6797.96

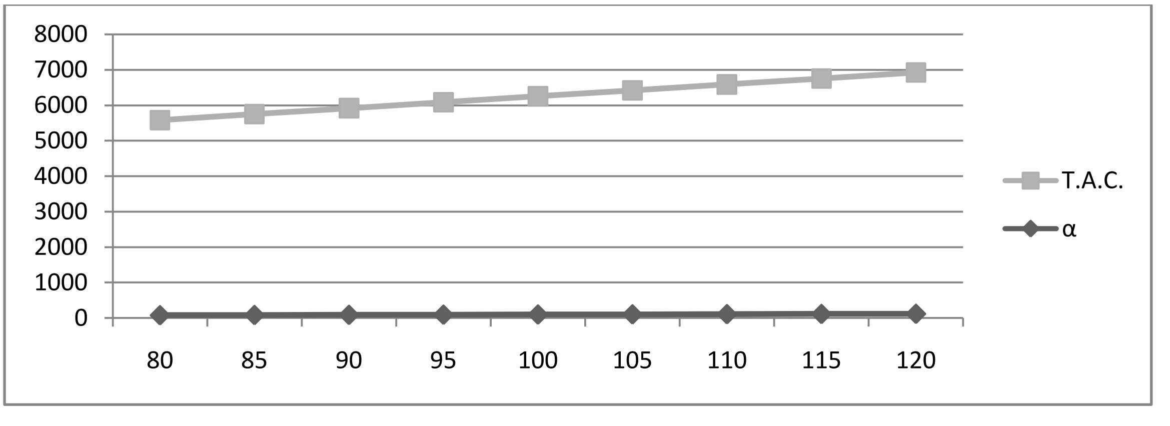

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Table 2: Variation in T.A.C. with the variation in β:

% variation in β	β	t_1	v	T.A.C.
-20%	0.016	53.7627	65.397	6206.75
-15%	0.017	53.5825	65.1736	6189.68
-10%	0.018	53.4223	64.975	6174.53
-5%	0.019	53.2789	64.7974	6161
0%	0.02	53.1499	64.6375	6148.86
5%	0.021	53.0332	64.4928	6137.89
10%	0.022	52.9271	64.3613	6127.93
15%	0.023	52.8302	64.2412	6118.85
20%	0.024	52.7414	64.1311	6110.54

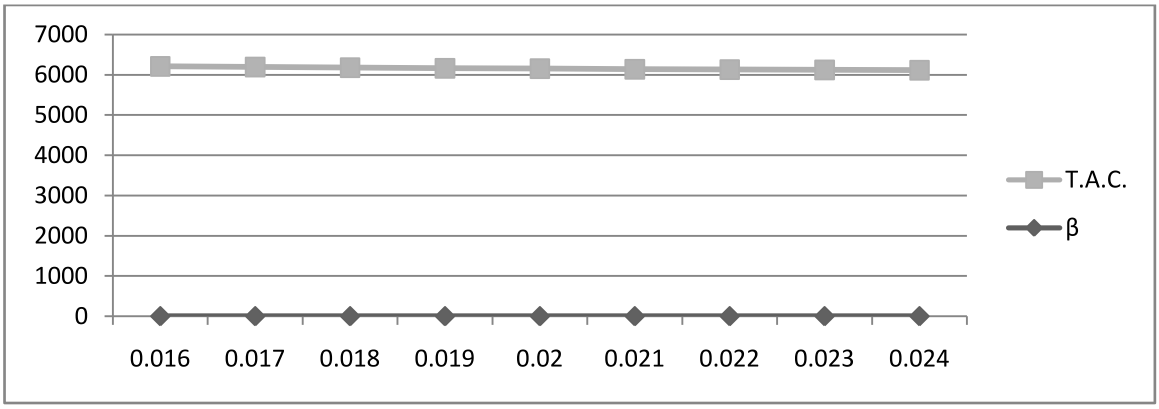

Table 3: Variation in T.A.C. with the variation in θ:

% variation in θ	θ	t_1	v	T.A.C.
-20%	0.4	51.375	62.4375	5944.63
-15%	0.425	51.8187	62.9875	5994.4
-10%	0.45	52.2624	63.5375	6045.88
-5%	0.475	52.7062	64.0875	6097.37
0%	0.5	53.1499	64.6375	6148.86
5%	0.525	53.5937	65.1875	6200.36
10%	0.55	54.0374	65.7374	6251.87
15%	0.575	54.4811	66.2874	6303.37
20%	0.6	54.9249	66.8374	6354.9

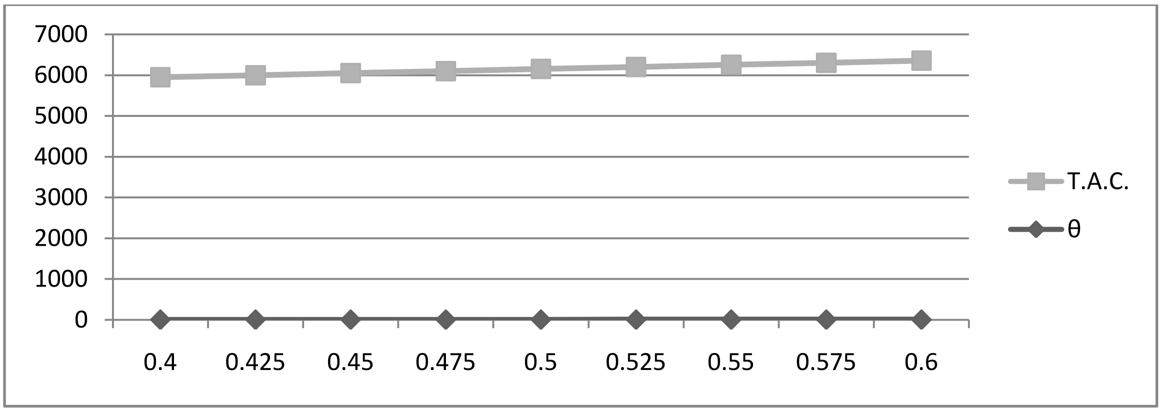

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Table 4: Variation in T.A.C. with the variation in W:

% variation in W	W	t_1	v	T.A.C.
-20%	4000	53.1499	64.6375	5568.19
-15%	4250	53.1499	64.6375	5713.36
-10%	4500	53.1499	64.6375	5858.53
-5%	4750	53.1499	64.6375	6003.69
0%	5000	53.1499	64.6375	6148.86
5%	5250	53.1499	64.6375	6294.03
10%	5500	53.1499	64.6375	6439.19
15%	5750	53.1499	64.6375	6584.36
20%	6000	53.1499	64.6375	6729.53

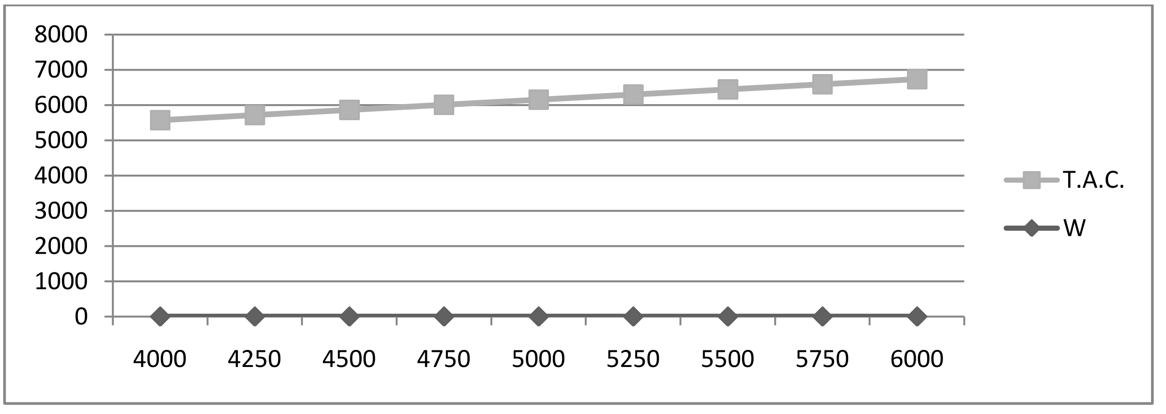


Table 5: Variation in T.A.C. with the variation in K:

% variation in K	K	t_1	v	T.A.C.
-20%	0.0008	52.6597	64.0298	6102.91
-15%	0.00085	52.7822	64.1817	6114.36
-10%	0.0009	52.9048	64.3337	6125.84
-5%	0.00095	53.0274	64.4856	6137.35
0%	0.001	53.1499	64.6375	6148.86
5%	0.00105	53.2725	64.7894	6160.4
10%	0.0011	53.395	64.9413	6171.95
15%	0.00115	53.5176	65.0932	6183.54
20%	0.0012	53.6401	65.2451	6195.13

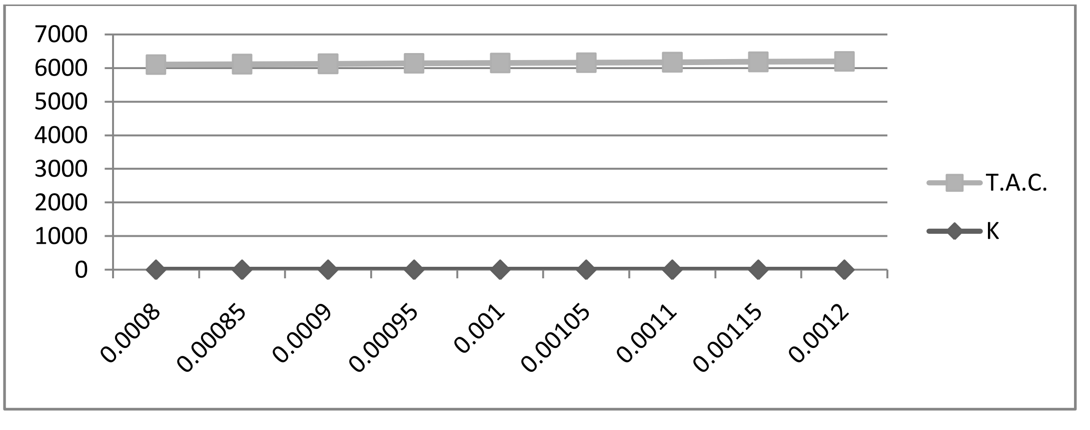
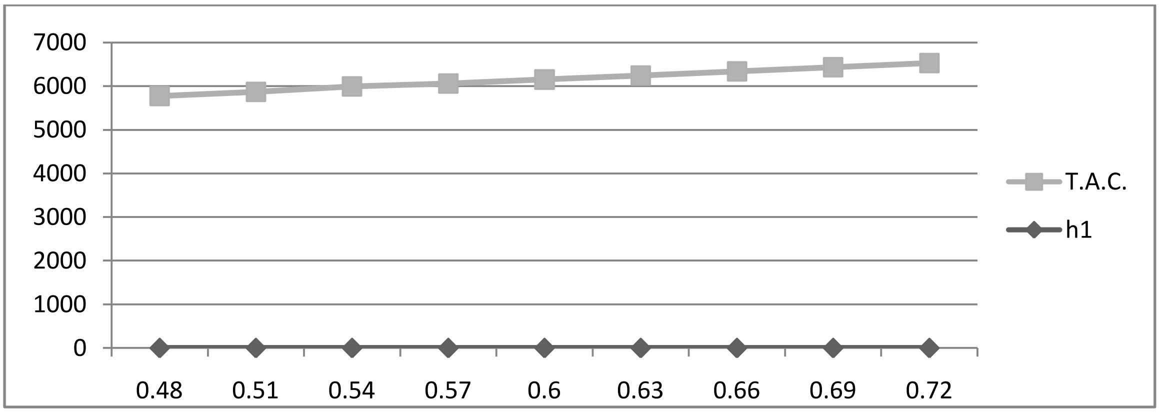


Table 6: Variation in T.A.C. with the variation in h_1:

% variation in h_1	h_1	t_1	v	T.A.C.
-20%	0.48	53.1499	64.6375	5772.2
-15%	0.51	53.1499	64.6375	5866.37
-10%	0.54	53.1499	64.6375	5991.92
-5%	0.57	53.1499	64.6375	6054.7
0%	0.6	53.1499	64.6375	6148.86
5%	0.63	53.1499	64.6375	6243.02
10%	0.66	53.1499	64.6375	6337.19
15%	0.69	53.1499	64.6375	6431.35
20%	0.72	53.1499	64.6375	6525.51



Observations:

A sensitivity analysis is carried out to check the stability of T.A.C. with respect to different associated parameters α, β, θ, W, K and h_1.

1. Table 1 shows the sensitivity of T.A.C. with the variation in demand parameter α and other variables unchanged. It is observed that as the value of α increases the T.A.C. of the system also increases.
2. Table 2 lists the variation in demand parameter β, it is observed from this table that with the increment in β, the T.A.C. of the system decreases.
3. From table 3 we observe the variation in T.A.C. with the variation in backlogging parameter θ and other variables unchanged. It is observed that with the increment in backlogging rate θ, the T.A.C. of the system increases due to the increment in purchasing cost.
4. Table 4 lists the variation in T.A.C. with the variation in warehouse capacity W. It is observed that an increase in warehouse capacity W, results also an increase in T.A.C. of the system.
5. Table 5 shows the variation in T.A.C. with the changes in deterioration parameter K. As the value of deterioration parameter increases, the cost of deteriorated units will increase and it will results an increase in total cost.
6. From table 6 we observe that with the increment in holding cost h_1 , the total cost of the system shows the same effect of increment.

Conclusion:

Most of the researchers ignored the concept of deterioration in the development of two warehouses inventory model. Here in this model the deterioration is considered in both the warehouses. The demand rate for the rented warehouse is constant and for the owned warehouse due to the availability of display it is assumed to be stock level dependent. The model is solved and optimal solution has been find out. The sensitivity of the optimal solution has been checked with respect to different system parameters and model is found to be quite stable. The model can be extended further for time varying holding cost, time value of money and for different rate of time dependent deterioration.

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