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Stochastic Differential Equation Noise Analysis of Common-Source Amplifier with Capacitive Load

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Abstract—In this paper, we analyse the effect of noise in a common-source amplifier with capacitive load working at high frequencies. Extrinsic noise is analyzed using time domain method employing techniques from stochastic calculus. Stochastic differential equations are used to obtain autocorrelation functions of the output noise voltage and other solution statistics like mean and variance. The analysis leads to important design implications for improved noise characteristics of the common-source amplifier.

Keywords*-common-source amplifier, noise, stochastic differential equation, mean and variance.*

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I. INTRODUCTION

The common-source amplifier is the widely used in analog circuit design. In thispaper, we shall concentrate on the noise analysis of a common-source amplifier with capacitive load. We analyze the effect of the noise signal on the output voltage. Noise can enter the circuit via various paths such as the noise from within the amplifier(intrinsic) and the noise signal which is fed externally(extrinsic).

Circuit noise analysis is traditionally done in frequency domain. The approach is effective in cases where the circuit is linear and time invariant. In this paper we do analysis of extrinsic noise for the common-source amplifier as shown in Fig.1.

For the stochastic model being used in this paper, the external noise is assumed to be a white Gaussian noise process. Although the assumption of a white Gaussian noise is an idealization, it may be justified because of the existence of many random input effects. According to the Central Limit Theorem, when the uncertainty is due to additive effects of many random factors, the probability distribution of such random variables is Gaussian. It may be difficult to isolate and model each factor that produces uncertainty in the circuit analysis. Therefore, the noise sources are assumed to be white with a flat power spectral density(PSD).

In this method, we shall follow a time domain approach based on solving a SDE. The method of SDEs in circuit noise analysis was used in [3] from a circuit simulation point of view. Their approach is based on linearization of SDEs about its simulated deterministic trajectory. In this paper we will use a different approach from which analytical solution to the SDE will be obtained. The analytical solution will take into account the circuit time varying nature and it will be shown that the noise becomes significant at high input signal frequencies. The main aim of our analysis is to observe the effect of noise present in the input signal on the output of the common-source amplifier.

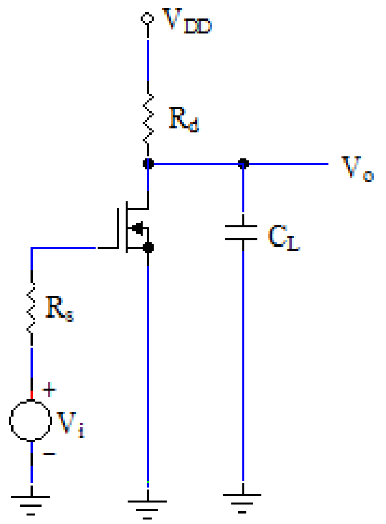

Fig.1. Common-Source Amplifier with capacitive load

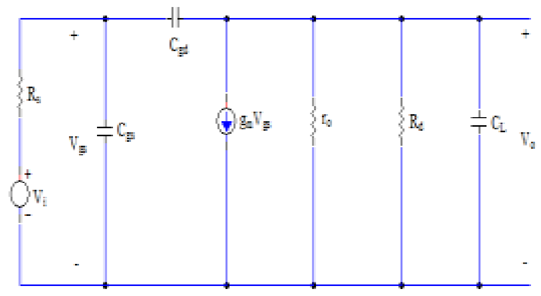

Fig.2. High-Frequency Equivalent Circuit

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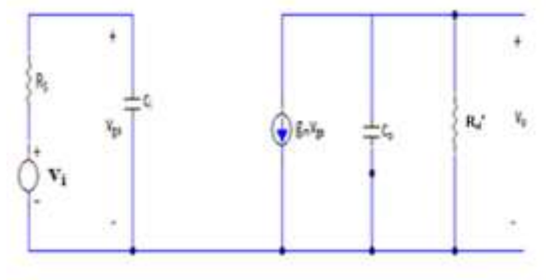

Fig.3. Simplified High-Frequency Equivalent Circuit

II. ANALYSIS OF NOISE VIA SDEs

Consider a common-source amplifier as shown in Fig.1 whose high-frequency equivalent is shown in Fig.2. Using Miller's theorem, we can transfer c_{gd} into input side by $c_i{}' = c_{gd}(1 + g_m R_d{}')$ and into output side by $c_o{}' =$ *cgd(1+gmRd')gmRd'*, which is approximated to *co'=cgd*. Henceforth, we analyze the circuit using SDEs. From the circuit in Fig. 3,

$$\frac{v_i(t) - v_{gs}(t)}{R_s} = c_i \frac{dv_{gs}(t)}{dt}$$

where $c_i = c_{gs} + c_{gd}(1 + g_m R_d{}')$ and $R_d{}' = (r_o R_d)/r_o R_d$. Using some straightforward simplification (1) can be written as

$$\frac{dv_{gs}(t)}{dt} + k_1 v_{gs}(t) = \frac{v_i(t)}{c_i R_s} \quad (1)$$

where $k_1 = \frac{1}{c_i R_s}$ and

$$c_o \frac{dv_o(t)}{dt} + \frac{v_o(t)}{R_d{}'} = -g_m v_{gs}(t) \quad (2)$$

Where $c_0 = (c_L + c_o{}')$

Considering $v_s(t) = \sigma n(t)$, where $n(t)$ represents Gaussian noise process and σ^2 is the magnitude of PSD of input noise process. Substituting $v_s(t) = \sigma n(t)$ in (1), we obtain

$$\frac{dv_{gs}(t)}{dt} + k_1 v_{gs}(t) = \frac{\sigma n(t)}{c_i R_s} (3)$$

First, we multiply both side of (3) with dt, then take expectation both sides. Since the continuous-time white noise process is a generalised function, the solution is rewritten by the replacement $n(t)dt = dW(t)$, where $W(t)$ is Wiener motion process, a continuous, but not differentiable process [4].

$$dE[v_{gs}(t)] + k_1 E[v_{gs}(t)]dt = \frac{E[\sigma dW(t)]}{c_i R_s} \quad (4)$$

Using the fact that $E[\sigma dW(t)] = 0$, (4) results in the following:

$$\frac{dE[v_{gs}(t)]}{dt} + k_1 E[v_{gs}(t)] = 0 \quad (5)$$

The solution of (5) is found out to be

$$E[v_{gs}(t)] = c_1 e^{-k_1 t} \quad (6)$$

where c_1 is a constant whose value depends on the initial circuit conditions. Now, we consider (2) because one of our main purpose is to find the mean of the output due to input noise signal. Simplifying and taking expectation on both sides of (2) we get the following equation for the mean of output

$$\frac{dE[v_o(t)]}{dt} + \frac{E[v_o(t)]}{c_o R_d{}'} = \frac{-g_m E[v_{gs}(t)]}{c_o} \quad (7)$$

The solution to which is

$$E[v_o(t)]e^{k_2 t} = \frac{c_2}{k_2 - k_1} e^{(k_2 - k_1)t} + c_3 (8)$$

Where $k_2 = 1/c_o R_d{}'$ and $c_2 = -g_m c_1/c_o$ and c_3 is constant of integration whose value depends on initial conditions provided. It is evident for initial conditions of $v_o(0) = 0$ and $v_{gs}(0) = 0$ that mean of output voltage is zero.

Next we find the autocorrelation function which will lead us to finding the variance. For the pedagogical reasons, the autocorrelation function is obtained considering initial conditions zero. Rewriting equation (2) and (1)

$$\frac{dv_o(t)}{dt} + k_2 v_o(t) = -\frac{g_m v_{gs}(t)}{c_o} \quad (9)$$

$$\frac{dv_{gs}(t)}{dt} + k_1 v_{gs}(t) = \frac{v_i(t)}{c_i R_s} (10)$$

Now consider (9) at $t = t_2$ with initial conditions $R_{v_0,v_0}(t_1, 0) = E[v_0(t_1)v_0(t_2)]|_{t_2=0} = 0$. Multiplying both side of (9) with $v_o(t_1)$& taking expectation, we obtain

$$\frac{dR_{v_o,v_o}(t_1,t_2)}{dt_2} + k_2 R_{v_o,v_o}(t_1, t_2) = \frac{-g_m R_{v_o,v_{gs}}(t_1,t_2)}{c_0} \quad (11)$$

Again consider (9) at $t = t_1$ with initial conditions $R_{v_0,v_{gs}}(0, t_2) = E[v_0(t_1)v_{gs}(t_2)]|_{t_1=0} = 0$. Multiplying both side of (10) with $v_{gs}(t_2)$& taking expectation, we obtain

$$\frac{dR_{v_o,v_{gs}}(t_1,t_2)}{dt_1} + k_2 R_{v_o,v_{gs}}(t_1, t_2) = \frac{-g_m R_{v_{gs},v_{gs}}(t_1,t_2)}{c_0} (12)$$

Next, we consider (10) at $t = t_1$ with initial conditions $R_{v_{gs},v_{gs}}(0, t_2) = E[v_{gs}(t_1)v_{gs}(t_2)]|_{t_1=0} = 0$. Multiplying both side of (10) with $v_{gs}(t_2)$& taking expectation, we obtain

$$\frac{dR_{v_{gs},v_{gs}}(t_1,t_2)}{dt_1} + k_1 R_{v_{gs},v_{gs}}(t_1, t_2) = \frac{R_{v_i,v_{gs}}(t_1,t_2)}{c_i R_S} \quad (13)$$

Again consider (10) at $t = t_2$ with initial conditions $R_{v_i,v_{gs}}(t_1, 0) = E[v_i(t_1)v_{gs}(t_2)]|_{t_2=0} = 0$. Multiplying both side of (10) with $v_i(t_1)$& taking expectation, we obtain

$$\frac{dR_{v_i,v_{gs}}(t_1,t_2)}{dt_2} + k_1 R_{v_i,v_{gs}}(t_1, t_2) = \frac{R_{v_i,v_i}(t_1,t_2)}{c_i R_S} \quad (14)$$

We need to solve the differential equations (11), (12), (13) and (14) to find out the value of $R_{v_o,v_o}(t_1, t_2)$. Knowing that $R_{v_i,v_i}(t_1, t_2) = \sigma^2 \delta(t_1 - t_2)$, we find the solution of (14) as

$$R_{v_i,v_{gs}}(t_1, t_2) = \frac{\sigma^2}{c_i R_S} e^{k_1(t_1 - t_2)} \ (15)$$

Substituting the value of $R_{v_i,v_{gs}}(t_1,t_2)$ from (15) in (13) and taking the limit of t_1 from 0 to min(t_1,t_2), we obtain the solution of (13) as

$$R_{v_{gs},v_{gs}}(t_1,t_2) = \frac{\sigma^2}{2k_1(R_s c_i)^2}\left(e^{-k_1(t_1-t_2)} - e^{-k_1(t_1+t_2)}\right) \quad (16)$$

Substituting the value of $R_{v_{gs},v_{gs}}(t_1,t_2)$ from (16) in (12) and taking limit of t_1 from 0 to min(t_1,t_2), we obtain the solution of (12) as

$$R_{v_o,v_{gs}}(t_1,t_2) = \frac{k_3}{\frac{1}{R_d{}'} - k_1 c_o}\left(e^{\left(\frac{t_2-t_1}{c_o R_d{}'}\right)} - e^{\left(k_1 t_2 - \frac{t_1}{c_o R_d{}'}\right)} - e^{\left(-2k_1 t_2 + \frac{t_2-t_1}{c_o R_d{}'}\right)} + e^{\left(-k_1 t_2 - \frac{t_1}{c_o R_d{}'}\right)}\right) \quad (17)$$

Where $k_3 = \frac{-g_m \sigma^2}{2k_1(R_s c_i)^2}$. We now substitute the value of $R_{v_o,v_{gs}}(t_1,t_2)$ from (17) in (11) and obtain the autocorrelation function as follows,

$$R_{v_o,v_o}(t_1,t_2) = \frac{-g_m k_3}{\frac{1}{R_d{}'} - k_1 c_o}\left(\left(e^{\frac{t_2-t_1}{c_o R_d{}'}} - e^{\frac{-t_1-t_2}{c_o R_d{}'}}\right)\frac{c_o R_d{}'}{2} + \frac{\left(e^{-2k_1 t_2 + \frac{t_2-t_1}{c_o R_d{}'}} - 2e^{k_1 t_2 - \frac{t_1}{c_o R_d{}'}} + e^{\frac{-t_1-t_2}{c_o R_d{}'}}\right)}{2\left(k_1 + \frac{1}{c_o R_d{}'}\right)} - \frac{\left(e^{-k_1 t_2 - \frac{t_1}{c_o R_d{}'}} - e^{\frac{-t_1-t_2}{c_o R_d{}'}}\right)}{\frac{1}{c_o R_d{}'} - k_1}\right) \quad (18)$$

For $t_1 = t_2 = t$ in (18) we obtain the second moment of output voltage as $E[v_o^2(t)]$ (which is variance in this case),

$$E[v_o^2(t)] = \frac{-g_m k_3}{\frac{1}{R_d{}'} - k_1 c_o}\left(\left(1 - e^{\frac{-2t}{c_o R_d{}'}}\right)\frac{c_o R_d{}'}{2} + \frac{\left(e^{-2k_1 t} - 2e^{\left(k_1 - \frac{1}{c_o R_d{}'}\right)t} + e^{\frac{-2t}{c_o R_d{}'}}\right)}{2\left(k_1 + \frac{1}{c_o R_d{}'}\right)} - \frac{\left(e^{\left(-k_1 - \frac{1}{c_o R_d{}'}\right)t} - e^{\frac{-2t}{c_o R_d{}'}}\right)}{\frac{1}{c_o R_d{}'} - k_1}\right) \quad (19)$$

III. SIMULATION RESULTS

For the simulation of results obtained above, we use the following values for the circuit parameters $R_d = 10k\Omega R_S = 5k\Omega$, $r_o = 44k\Omega$, $\sigma = 0.25$, $c_{gs} = 3pF$, $c_{gd} = 2.8pF$, $c_L = 2pF\, g_m = 0.0016A/V$.

The variation of mean with time is shown in Fig. 4, when initial conditions are nonzero, ($v_{gs}(0) = 0.01V$). If initial conditions are zero the mean is zero all the time.The variation of variance with time is shown in Fig. 5.From Fig.5 it is observed that the variance reaches a constant value of approximately 7.4×10^{-5} after 1μs. The maximum value of variance is approximately 8.2×10^{-5}.

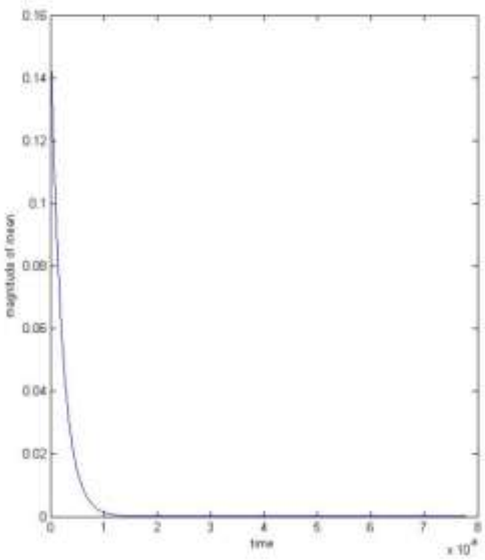

Fig.4. Variation of mean with time

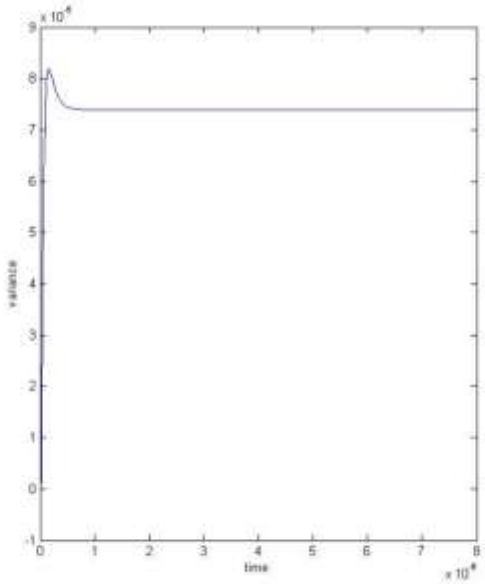

Fig.5. Variation of variance with time

IV. CONCLUSIONS

Noise in common-source amplifier with capacitive load is analyzed using stochastic differential equation. Extrinsic noise is characterized by solving a SDE analytically in time domain. The solution for various solution statistics like mean and variance is obtained which can be used for design process. Suitable design methods which involve changing of device parameters are suggested to aid noise reduction and hence design the amplifier with reduced noise characteristics.

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