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Optimal Specially Structured N X 2 Flow Shop Scheduling To Minimize Total Waiting Time of Jobs Including Job Block Concept with Processing Time Separated From Set up Time

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ABSTRACT: In the present state of affairs the current engineering and manufacturing built- up units are facing mishmash of problems in a lot of aspects such as man power, machining time, raw material, electricity and customer's constraints. The flow-shop scheduling is one of the most significant manufacturing behaviors particularly in manufacturing planning. The creation of every time admirable schedules has verified to be enormously complicated. This paper involves the fortitude of the order of processing of m jobs on 2 machines. This paper proposes the specially structured Flow Shop Scheduling problem separated from set up time assuming that maximum of the equivalent processing time on first machine is less than or equal to the minimum of equivalent processing time on second machine with the objective of getting the optimal sequence of jobs for total waiting time of jobs using the heuristic algorithm by taking two of the jobs as a group job. The proposed technique is followed by numerical example.

KEY WORDS: *Waiting time of jobs, Job Block, Set up time, Flow shop Scheduling, Processing time.*

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I. Introduction

Transfer lines have long been recognized as the most proficient method of producing goods in a high volume/ high variety and mid volume/ mid variety manufacturing, though have always been played with difficulties. A lot of the production problems are attributable to problems in the scheduling function: not having the sources when they are needed, not having apparatus available when it is needed, by means of surplus inventory to hide problems, inflexibility and lack of awareness. Scheduling conceivably defined as the problem of deciding when to implement a given set of activities, subject to chronological constraints and resources capacities, with the intention of optimize some function. The flow shop contains **m** different machines arranged in series on which a set of **n** jobs are to be processed. The common scheduling problem for a usual flow shop gives rise to $\mathbf{(n!)^m}$ possible schedules. With the aim to reduce the number of possible schedules it is logical to take for granted that all the jobs share the same processing order on every machine. Efforts in the past have been made by researchers to lessen this number of possible schedules as much as achievable without compromising on optimality condition.

This paper presents a solution methodology in a flow shop scheduling problem for minimization the waiting time of jobs specifically defined as the sum of the times of all the jobs which was devoted in waiting for their turn on both of the machines.

II. Literature review

Johnson [1] has proved that in a 2 machine flow shop problem an optimal schedule for minimizing the total elapsed time can be constructed. It was verified later that *m* machine flow shop scheduling problem (FSSP) is robustly NP- hard for $m \geq 3$. Solution methods for flow shop scheduling range from heuristics developed by Palmer [2], Campbell et al.[3] and Dannenbring [4] to more complex techniques such as branch and bound [5], tabu search [6, 7]. Maggu P. L. et al.[8] introduced the concept of equivalent jobs for job block by taking two of the jobs as a group job. Yoshida et al. [9] explain two stage production scheduling by taking the set up time separated from processing time. Nawaz et al. [10] proposed that a job with longer total processing time should have higher priority in the sequence. Singh T.P. et al. [13] considered the problem associated with group job restrictions in a flow shop which engross independent set- up time and transportation time. Further Gupta D. [15] simplified the problem of minimization of Rental Cost in Two Stage Flow Shop Scheduling Problem, in which Setup Time was separated from Processing Time and each associated with probabilities including Job Block Criteria. Another approach to study the specially structured three stage flow shop scheduling to minimize the rental cost is presented by Gupta D. et al.[17]

Recently Gupta D. et al.[19] studied optimality for waiting time of jobs in which processing times are associated with probabilities. This study was further extended by including job block concept and taking set up time separated from set up time [20, 21]. The problem discussed here has noteworthy use of conjectural results in process industries or in the circumstances when the objective is to minimize the total waiting time of jobs. The present paper is an extension made by Gupta D. et al. [19, 20, 21] in the sense that we have taken into consideration the set up time and the job block criterion.

PRACTICAL SITUATION

Industrial units play a significant role in the financial growth of a nation. Flow shop scheduling arises in various organizations, service stations, banks, airports etc. In our routine working in factories and manufacturing units different jobs are processed on various machines. In textile industry different types of fabric is shaped using different types of yarn. Here, the maximum equivalent time taken in dying of yarn on first machine is always less than or equal to the minimum equivalent time taken in weaving of yarn on the second machine. The idea of minimizing the waiting time may be an reasonable aspect from Factory /Industry manager's view point when he has minimum time agreement with a commercial party to complete the jobs.

NOTATIONS

J_k: Sequence obtained by applying the algorithm proposed.

R_k: Processing time of k^{th} job on machine R

S_k: Processing time of k^{th} job on machine S.

R_k': Equivalent processing time of k^{th} job on machine R.

S_k': Equivalent processing time of k^{th} job on machine S.

t_{Rk}: Time for set up of k^{th} job on machine R.

t_{Sk}: Time for set up of k^{th} job on machine S.

T_{aS}: The completion time of job a on machine S.

W_γ: Waiting time of job γ.

W: Total waiting time of all the jobs.

PROBLEM FORMULATION

Assume that R and S are two machines processing m jobs in the order R S. R_k and S_k are the respective processing times and t_{Rk} and t_{Sk} are the respective set up times of the k^{th} job on machines R & S. Our intention is to find an optimal sequence $\{J_k\}$ of jobs minimizing the total waiting time of all jobs. Equivalent processing times of k^{th} job on machine R & S are defined as

$R_k' = R_k - t_{Sk}$, $S_k' = S_k - t_{Rk}$ satisfying processing times structural relationship

Max $R_k' \leq$ Min S_k', p and q are any jobs amongst the given m jobs such that job p occurs before job q in the order of job block (p, q), the equivalent job β is defined as (p, q)

TABLE 1: MATRIX FORM OF THE MATHEMATICAL MODEL OF THE PROBLEM

Job	Machine R		Machine S	
I	R_k	t_{Rk}	S_k	t_{Sk}
1.	R_1	t_{R1}	S_1	t_{S1}
2.	R_2	t_{R2}	S_2	t_{S2}
3.	R_3	t_{R3}	S_3	t_{S3}
.	. .	. .	. .	. .
m.	R_m	t_{Rm}	S_m	t_{Sm}

ASSUMPTIONS

In the given flow shop scheduling the following assumptions are made

1) There are *m* number of jobs (I) and two machines (R & S).
2) The order of sequence of operations in both of the machines is the same.
3) Jobs are independent to each other.
4) It is given to sequence r jobs $j_1, j_2 \ldots \ldots j_r$ as a block or group job in the order $(j_1, j_2 \ldots \ldots j_r)$ showing priority of job j_1 over j_2 etc.
5) Machines break down interval, transportation time is not considered for calculating waiting time.
6) Pre- emption is not allowed i.e. jobs are not being split clearly, once a job is started on a machine, the process on that machine can't be stopped unless the job is completed.

Lemma1. Assuming two machines R, S are processing m jobs in order R S with no passing permitted. Let R_k and S_k are the processing times of job k ($k = 1,2,3, \ldots \ldots, m$) on each machine correspondingly presumptuous their respective set up times t_{Rk} and t_{Sk}. Equivalent processing times of k^{th} job on machine R & S are defined as $R_k' = R_k - t_{Sk}$, $S_k' = S_k - t_{Rk}$ satisfying processing times structural relationship Max $R_k' \leq$ Min S_k' then for the m job sequence S: $\gamma_1, \gamma_2, \gamma_3, \ldots \ldots \ldots \gamma_m$

$T_{\gamma_m S} = R'_{\gamma_1} + S'_{\gamma_1} + S'_{\gamma_2} \ldots + S'_{\gamma_m}$

Where T_{aS} is the completion time of job a on machine S.

Proof. Applying mathematical Induction hypothesis on m:

Assuming the statement $P(m): T_{\gamma_m S} = R'_{\gamma_1} + S'_{\gamma_1} + S'_{\gamma_2} \ldots + S'_{\gamma_m}$

$T_{\gamma_1 R} = R'_{\gamma_1}$
$T_{\gamma_1 S} = R'_{\gamma_1} + S'_{\gamma_1}$
Hence for m= 1 the statement P(1) is true.
Let for m = k, the statement P(k) be true, i.e.,
$T_{\gamma_k S} = R'_{\gamma_1} + S'_{\gamma_1} + S'_{\gamma_2} \dots + S'_{\gamma_k}$
Now,
$T_{\gamma_{k+1} S} = \mathrm{Max}\left(T_{\gamma_{k+1} R}, T_{\beta_k S}\right) + S'_{\gamma_{k+1}}$
As Max $R'_k \le$ Min S'_k
Hence $T_{\gamma_{k+1} S} = R'_{\gamma_1} + S'_{\gamma_1} + S'_{\gamma_2} \dots + S'_{\gamma_k} + S'_{\gamma_{k+1}}$
Hence for n = k + 1 the statement P(k + 1) holds true.
Since P(m) is true for m = 1, m = k,
m = k + 1, and k being arbitrary. Hence P(m): $T_{\gamma_m S} = R'_{\gamma_1} + S'_{\gamma_1} + S'_{\gamma_2} \dots + S'_{\gamma_m}$ is true.

Lemma2. With the same notations as that of Lemma1, for m- job sequence S: $\gamma_1, \gamma_2, \gamma_3, \dots \dots, \gamma_k, \dots \gamma_m$

$$W_{\gamma_1} = 0$$

$$W_{\gamma_k} = R'_{\gamma_1} + \sum_{r=1}^{k-1} y_{\gamma_r} - R'_{\gamma_k}$$

Where W_{γ_k} is the waiting time of job γ_k for the sequence $(\gamma_1, \gamma_2, \gamma_3, \dots \dots, \dots \gamma_m)$

$y_{\gamma_r} = S'_{\gamma_r} - R'_{\gamma_r}$, $\gamma_r \in (1, 2, 3, \dots., m)$

Proof. $W_{\gamma_1} = 0$

$$W_{\gamma_k} = \mathrm{Max}\left(T_{\gamma_k R}, T_{\gamma_{k-1} S}\right) - T_{\gamma_k R}$$
$$= R'_{\gamma_1} + S'_{\gamma_1} + S'_{\gamma_2} \dots + S'_{\gamma_{k-1}} - R'_{\gamma_1} - R'_{\gamma_2} \dots - R'_{\gamma_k}$$
$$= R'_{\gamma_1} + \sum_{r=1}^{k-1} (S'_{\gamma_r} - R'_{\gamma_r}) - R'_{\gamma_k}$$
$$= R'_{\gamma_1} + \sum_{r=1}^{k-1} (y_{\gamma_r}) - R'_{\gamma_k}$$

Theorem1. Let two machines R, S are processing m jobs in order R S with no passing allowed. Let R_k and S_k are the processing times of job k (i = 1,2,3, ..., m) on each machine respectively assuming their respective set up times t_{Rk} and t_{Sk}. Equivalent processing times are defined as $R'_k = R_k - t_{Sk}$, $S'_k = S_k - t_{Rk}$ satisfying processing times structural relationship
Max $R'_k \le$ Min S'_k then for any m job sequence S: $\gamma_1, \gamma_2, \gamma_3, \dots \dots \dots \gamma_m$ the total waiting time W (say)

$$W = mR'_{\gamma_1} + \sum_{r=1}^{m-1} z_{\gamma_r} - \sum_{k=1}^{m} R'_k$$

$z_{\gamma_r} = (m - r) y_{\gamma_r}$; $\gamma_r \in (1, 2, 3, \dots, m)$

Proof. From Lemma 2 we have

$W_{\gamma_1} = 0$

For k = 2,

$$W_{\gamma_2} = R'_{\gamma_1} + \sum_{r=1}^{1} y_{\gamma_r} - R'_{\gamma_2}$$

For k = 3,

$$W_{\gamma_3} = R'_{\gamma_1} + \sum_{r=1}^{2} y_{\gamma_r} - R'_{\gamma_3}$$

Continuing in this way
For k = m,

$$W_{\gamma_m} = R'_{\gamma_1} + \sum_{r=1}^{m-1} y_{\gamma_r} - R'_{\gamma_m}$$

Hence total waiting time

$$W = \sum_{i=1}^{m} W_{\gamma_i}$$

$$W = mR'_{\gamma_1} + \sum_{r=1}^{m-1} z_{\gamma_r} - \sum_{k=1}^{m} R'_k$$

Where $z_{\gamma_r} = (m - r) y_{\gamma_r}$

Equivalent Job Block Theorem

Theorem 2. In processing a schedule I = (1,2,3, ..., m) of m jobs on two machines R and S in the order R S with no passing allowed. A job k (k = 1,2,3 ..., m) has processing time R_k and S_k on each machine respectively. The job block (p, q) is equivalent to the single job β (called equivalent job β). Now the processing times of job β on the machines R and S are denoted respectively by R_β, S_β are given by

$$R_\beta = R_p + R_q - \min\left(R_q, S_p\right)$$

$$S_\beta = S_p + S_q - \min\left(R_q, S_p\right)$$

The proof of the theorem is given by Maggu P.L. et al. [8].

ALGORITHM

Step 1: Equivalent processing times R'_k and S'_k on machine R & S respectively be calculated in first step as defined in the lemma 1.

Step 2: Take equivalent job $\beta = (p, q)$ and define processing times using equivalent job block theorem and replace the pair of jobs (p, q) in this order by the single job.

Fill up the values in the following table:

Job	Machine R	Machine S	
I	R'_k	S'_k	$y_k = S'_k - R'_k$
1.	R'_1	S'_1	y_1
2.	R'_2	S'_2	y_2
3.	R'_3	S'_3	y_3
. .	.	.	.
β.	R'_β	S'_β	y_β
. .	. .	. .	. .
p.	R'_p	S'_p	y_p

Step 3: Assemble the jobs in increasing order of y_i.

Assuming the sequence found be $(\gamma_1, \gamma_2, \gamma_3, \dots \dots \dots \gamma_p)$

Step 4: Locate $\min\{ R'_k \}$

For the following two possibilities

$R'_{\gamma_1} = \min\{ R'_k \}$, Schedule according to step 3 is the required optimal sequence

$R'_{\gamma_1} \neq \min\{ R'_k \}$ move on to step 5

Step 5: Consider the different sequence of jobs $J_1, J_{2,} J_3, \dots \dots, J_p$. Where J_1 is the sequence obtained in step 3, Sequence $J_m (m = 2,3, \dots \dots, p)$ can be obtained by placing m^{th} job in the sequence J_1 to the first position and rest of the sequence remaining same.

Step 6: Calculate the entries for the following table

TABLE 2

Job	Machine R	Machine S		$z_{ki} = (m - i)y_k$				
I	R'_i	S'_i	y_k	i = 1	i= 2	i = 3	...	i = n-1
1.	R'_1	S'_1	y_1	z_{11}	z_{12}	z_{13}	…	$z_{1\ m-1}$
2.	R'_2	S'_2	y_2	z_{21}	z_{22}	z_{23}	…	$z_{2\ m-1}$
3.	R'_3	S'_3	y_3	z_{31}	z_{32}	z_{33}	…	$z_{3\ m-1}$
.	.	.	.	.	.	.	.	.
m.	R'_m	S'_m	y_m	z_{m1}	z_{m2}	z_{m3}	…	$z_{m\ m-1}$

Step 7: Compute the total waiting time W for all the sequences $J_1, J_{2,} J_3, \dots \dots, J_p$ using the following formula:

$$W = mR'_b + \sum_{k=1}^{m-1} z_{ak} - \sum_{k=1}^{m} R'_k$$

R'_b = Equivalent processing time of the first job on machine R in sequence J_m

The sequence with minimum total waiting time is the required optimal sequence.

NUMERICAL ILLUSTRATION

Assume 6 jobs 1, 2, 3, 4, 5, 6 has to be processed on two machines R & S with processing times R_k and S_k and set up times t_{Rk} and t_{Sk} respectively

TABLE 3: PROCESSING TIME MATRIX

Job	Machine R		Machine S	
I	R_k	t_{Rk}	S_k	t_{Sk}
1.	6	2	10	1
2.	5	4	11	3
3.	8	5	15	4
4.	5	1	7	3
5.	6	2	12	4
6.	7	1	12	3

Our objective is to obtain optimal string, minimizing the total waiting time for the jobs.

Solution

As per step 1- Equivalent processing time R'_k & S'_k on machine R & S given in the following table

`TABLE 4: EQUIVALENT PROCESSING TIME MATRIX

Job	Machine R	Machine S
I	R'_k	S'_k
1.	5	8
2.	2	7
3.	4	10
4.	2	6
5.	2	10
6.	4	11

Max $R'_k \le$ Min S'_k

As per step 2- Take equivalent job $\beta = (2, 5)$. Calculating the processing times for equivalent job β

$R'_\beta = 2, S'_\beta = 15$

Job	Machine R	Machine S	
I	R'_k	S'_k	$y_k = S'_k - R'_k$
1.	5	8	3
β.	2	15	13
3.	4	10	6
4.	2	6	4
6.	4	11	7

As per step 3- Arrange the jobs in increasing order of y_i i.e. the sequence found be $1, 4, 3, 6, \beta$.

As per step 4- $\text{Min}\{R'_k\} = 2 \neq R'_1$

As per step 5- Consider the following different sequences of jobs

$J_1: 1, 4, 3, 6, \beta$; $J_2: 4, 1, 3, 6, \beta$; $J_3: 3, 1, 4, 6, \beta$; $J_4: 6, 1, 4, 3, \beta$; $J_5: \beta, 1, 4, 3, 6$

As per step 6- Fill up the values in the following table

TABLE 5

Job	Machine R	Machine S		$z_{ki} = (6 - i)y_k$				
I	R'_k	S'_k	$y_k = S'_k - R'_k$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i=5$
1.	5	8	3	15	12	9	6	3
2.	2	7	5	25	20	15	10	5
3.	4	10	6	30	24	18	12	6
4.	2	6	4	20	16	12	8	4
5.	2	10	8	40	32	24	16	8
6.	4	11	7	35	28	21	14	7

As per step 7- The total waiting time for the sequences obtained in step 5 can be calculated

Here, $\sum_{i=1}^{5} R'_k = 19$

For the sequence $J_1: 1, 4, 3, 6, \beta$ or $J_1: 1, 4, 3, 6, 2, 5$

Total waiting time $W = 79$

For the sequence $J_2: 4, 1, 3, 6, \beta$ or $J_2: 4, 1, 3, 6, 2, 5$

Total waiting time $W = 62$

For the sequence $J_3: 3, 1, 4, 6, \beta$ or $J_3: 3, 1, 4, 6, 2, 5$

Total waiting time $W = 78$

For the sequence $J_4: 6, 1, 4, 3, \beta$ or $J_4: 6, 1, 4, 3, 2, 5$

Total waiting time $W = 81$

For the sequence $J_5: \beta, 1, 4, 3, 6$ or $J_5: 2, 5, 1, 4, 3, 6$

Total waiting time $W = 73$

Hence schedule $J_2: 4, 1, 3, 6, 2, 5$ is the required schedule with minimum total waiting time.

III. Conclusion

The present study deals with the flow shop scheduling problem with the main idea to minimize the total waiting time of jobs. However it may increase the other costs like machine idle cost or penalty cost of the jobs, yet the idea of minimizing the waiting time may be an economical aspect from Factory /Industry manager's view point when he has minimum time contract with a commercial party to complete the jobs. The work can be extended by introducing various parameters like transportation time, break down interval etc.

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