ISSN: 2321-8169
696 – 704

Arithmetic Operations on Intuitionistic Hexagonal Fuzzy Numbers Using α Cut

Dr. A. Sahaya Sudha
Assistant Professor, Nirmala College for women
sudha.dass@yahoo.com

M. Revathy
Ph.D Research Scholar , Nirmala College for women
revamaths17@gmail.com

Abstract **-** Presently, the fuzzy set theory has been also developed in a large extent and different variations and simplification. This paper focuses on alpha cuts in intuitionistic hexagonal fuzzy numbers by assuming different alpha values without affecting its originality. We have proposed a new arithmetic operation on alpha - cuts of hexagonal intuitionistic fuzzy numbers. Numerical examples are done to show the efficiency of the study.

Keywords **-** *Fuzzy Numbers, Fuzzy Arithmetic, Hexagonal Fuzzy Numbers , Intuitionistic Hexagonal Fuzzy Numbers, α cuts.*

*****

I. INTRODUCTION

The notion of fuzzy sets was introduced by Zadeh[16]. Fuzzy set theory allows the ongoing assessment of the membership of elements in a set which is described in the interval [0, 1][19]. It can be used in a wide range of domains where information is partial and vague. This fuzzy programming technique is more flexible and allows to find the solutions which are more adequate to the real problem. Fuzzy optimization models reflect real life ambiguity. Some new operators on α cuts of Hexagonal fuzzy numbers (HFNs) are introduced followed by the properties of their arithmetic operations [2, 3, 7, 8, 11, 12]. Stephen Dinagar and Rajesh Kannan [13] introduced the “modified definition” of the Hexagonal Fuzzy number by including conditions for the convexity of the number. Interval arithmetic was optional by means of Zadeh’s extension principle [17, 18]. A fuzzy number is a quantity whose values are inexact, rather than exact as is the case with single-valued numbers. The usual Arithmetic operations on real numbers can be unlimited to the ones defined on Fuzzy numbers. In cases of problem having six different parameters the Triangular or Trapezoidal Fuzzy Numbers are not appropriate to solve them, hence we make use of the HFNs and their operations to solve such problems. Arithmetic operations on hexagonal fuzzy numbers using α cuts were solved by Stephen Dinagar, Hari Narayanan and Kankeyanathan Kannan [14]. Atanassov [1] introduced the concept of Intuitionistic Fuzzy Sets (IFS), which is a overview of the concept of fuzzy set. An IFS has received more interest due to its appearance, because the idea about attribute value is timid. IFS has the benefit of expressing lack of information in the human reasoning and decision process. The concept of an IFS can be viewed as an alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an vague concept by means of a conventional fuzzy sets. In general, the theory of IFS is the generalization of fuzzy sets. Therefore, it is expected that, IFS could be used to simulate human decision-making process and any activities requiring human proficiency and knowledge which are predictably vague or not totally consistent. To Solve Intuitionistic Fuzzy Linear Programming Problem Using Single Step Algorithm was discussed by Nagoorgani A and Ponnalagu K [10]. An application of fuzzy optimization techniques to linear programming problems with multiple objectives has been presented by Zimmermann[20]. Numerous ranking methods have been proposed in literature to rank Intuitionistic Fuzzy numbers [4, 5, 6, 9].

II. PRELIMINARIES

A. Intuitionistic Fuzzy Set[15]

Let X be a nonempty set. An intuitionistic fuzzy set $\tilde{A}^I$ of X is defined as $\tilde{A}^I = \{\langle x, \mu_{\tilde{A}^I}, \gamma_{\tilde{A}^I} \rangle / x \in X\}$ where $\mu_{\tilde{A}^I}(x)$ *and* $\gamma_{\tilde{A}^I}(x)$ are membership and non membership functions such that $\mu_{\tilde{A}^I}(x), \gamma_{\tilde{A}^I}(x) : X \to [0,1]$ and $0 \le \mu_{\tilde{A}^I}(x) + \gamma_{\tilde{A}^I}(x) \le 1$ for all x ε X

 ISSN: 2321-8169
696 – 704

B. Intuitionistic Fuzzy Number[15]

An intuitionistic fuzzy number $\tilde{A}^I$ is

i) an intuitionistic fuzzy subset of the real line,

ii) normal, that is, there is some $x_0 \in R$ such that $\mu_{\tilde{A}}{}^I(x_0) = 1, \gamma_{\tilde{A}}{}^I(x_0) = 0$

iii)convex for the membership function $\mu_{\tilde{A}^I}(x)$, that is, $\mu_{\tilde{A}}{}^I(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}{}^I(x_1), \mu_{\tilde{A}}{}^I(x_2))$, for every $x_1, x_2 \in R$, $\lambda \in [0,1]$,

iv) concave for the membership function $\gamma_{\tilde{A}}{}^I(\lambda x_1 + (1-\lambda)x_2) \geq \min(\gamma_{\tilde{A}}{}^I(x_1), \gamma_{\tilde{A}}{}^I(x_2))$, for every $x_1, x_2 \in R$, $\lambda \in [0,1]$,

C. Hexagonal Intuitionistic Fuzzy Number[15]

A Hexagonal intuitionistic fuzzy number is specified by $\tilde{A}_H{}^I = (a_1, a_2, a_3, a_4, a_5, a_6), (a_1', a_2', a_3, a_4, a_5', a_6')$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_1', a_2', a_5', a_6'$ are real numbers such that $a_1' \leq a_1 \leq a_2' \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_5' \leq a_6 \leq a_6'$ and its membership and non membership are given by

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & for\ a_1 \leq x \leq a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & for\ a_2 \leq x \leq a_3 \\ 1, & for\ a_3 \leq x \leq a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & for\ a_4 \leq x \leq a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & for\ a_5 \leq x \leq a_6 \\ 0, & otherwise \end{cases} \qquad \gamma_{\tilde{A}^I}(x) = \begin{cases} 1-\frac{1}{2}\left(\frac{x-a_1'}{a_2'-a_1'}\right), & for\ a_1' \leq x \leq a_2' \\ \frac{1}{2}\left(\frac{a_3-x}{a_3-a_2'}\right), & for\ a_2' \leq x \leq a_3 \\ 0, & for\ a_3 \leq x \leq a_4 \\ \frac{1}{2}\left(\frac{x-a_4}{a_5'-a_4}\right), & for\ a_4 \leq x \leq a_5' \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_5'}{a_6'-a_5'}\right), & for\ a_5' \leq x \leq a_6' \\ 1, & otherwise \end{cases}$$

D. Hexagonal Intuitionistic Fuzzy Number

A Hexagonal intuitionistic fuzzy number can be defined as $\tilde{A}_H{}^I = (D_1(u), S_1(v), S_2(v), D_2(u)), (D_1'(v), S_1'(u), S_2'(u), D_2'(v))$ for $u \in [0,0.5]$ and $v \in [0.5,1]$, where

i. $D_1(u), S_1'(u)$ is a bounded left continuous non decreasing function over [0,0.5]
ii. $S_1(v), D_1'(v)$ is a bounded left continuous non decreasing function over [0.5,1]
iii. $S_2(v), D_2'(v)$ is a bounded continuous non increasing function over [1,0.5]
iv. $D_2(u), S_2'(u)$ is a bounded left continuous non increasing function over [0.5,0]

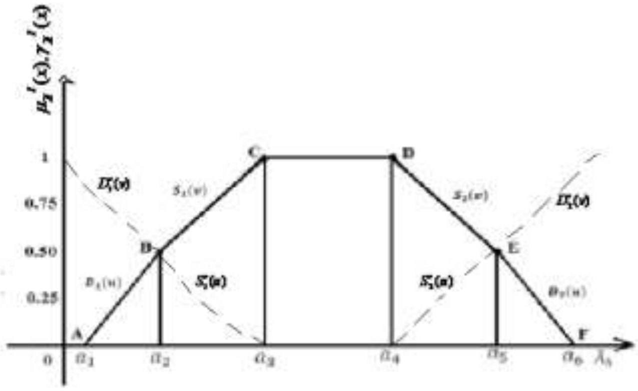

Figure 1: Graph of a Hexagonal Intuitionistic Fuzzy Number

E. Arithmetic Operations On Hexagonal Intuitionistic Fuzzy Numbers[15]

 ISSN: 2321-8169
696 – 704

Let $\tilde{A}_H{}^I = (a_1, a_2, a_3, a_4, a_5, a_6), (a_1', a_2', a_3, a_4, a_5', a_6')$ and $\tilde{B}_H{}^I = (b_1, b_2, b_3, b_4, b_5, b_6), (b_1', b_2', b_3, b_4, b_5', b_6')$ be two HIFNs. Then

$$\tilde{A}_H{}^I + \tilde{B}_H{}^I = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6), \\ (a_1' + b_1', a_2' + b_2', a_3 + b_3, a_4 + b_4, a_5' + b_5', a_6' + b_6')$$

$$\tilde{A}_H{}^I - \tilde{B}_H{}^I = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1), \\ (a_1' - b_6', a_2' - b_5', a_3 - b_4, a_4 - b_3, a_5' - b_2', a_6' - b_1')$$

III. ALPHA CUT IN HEXAGONAL INTUITIONISTIC FUZZY NUMBER

A. α-Cut of Hexagonal Intuitionistic Fuzzy Number

The crisp set A_α called alpha cut is defined as $A_\alpha = \{x \in X \mid \mu_{\tilde{A}_H}(x), \gamma_{\tilde{A}_H}(x) \geq \alpha\}$

$$A_\alpha = \begin{cases} [D_1(u), D_2(u)] & \text{for } \alpha \in [0,0.5) \\ [S_1(v), S_2(v)] & \text{for } \alpha \in [0.5,1] \end{cases}$$

$$\begin{cases} [S_1'(u), S_2'(u)] & \text{for } \alpha \in [0,0.5) \\ [D_1'(v), D_2'(v)] & \text{for } \alpha \in [0.5,1] \end{cases}$$

B. α - Cut Operations

The interval A_α, for $\alpha \in [0,1]$ is obtained as follows:

Consider for membership function

$$D_1(x) = \frac{1}{2}\left(\frac{x - a_1}{a_2 - a_1}\right)$$

Then, $D_1(x) = 2\alpha(a_2 - a_1) + a_1$

Similarly, $D_2(x) = a_6 - 2\alpha(a_6 - a_5)$

This implies $[D_1(x), D_2(x)] = [2\alpha(a_2 - a_1) + a_1, a_6 - 2\alpha(a_6 - a_5)]$

$$S_1(x) = \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2}{a_3 - a_2}\right)$$

Then, $S_1(x) = 2(\alpha - \frac{1}{2})(a_3 - a_2) + a_2$

Similarly, $S_2(x) = 2(1-\alpha)(a_5 - a_4) + a_4$

This implies $[S_1(x), S_2(x)] = 2\left(\alpha - \frac{1}{2}\right)(a_3 - a_2) + a_2,\ 2(1-\alpha)(a_5 - a_4) + a_4$

Consider for non membership function

$$S_1'(x) = \frac{1}{2}\left(\frac{a_3 - x}{a_3 - a_2'}\right)$$

Then, $S_1'(x) = a_3 - 2\alpha(a_3 - a_2')$

Similarly, $S_2'(x) = 2\alpha(a_5' - a_4) + a_4$

This implies $[S_1'(x), S_2'(x)] = [a_3 - 2\alpha(a_3 - a_2'), 2\alpha(a_5' - a_4) + a_4]$

$$D_1'(x) = 1 - \frac{1}{2}\left(\frac{x - a_1'}{a_2' - a_1'}\right)$$

Then, $D_1'(x) = 2(1-\alpha)(a_2' - a_1') + a_1'$

Similarly, $D_2'(x) = 2\left(\alpha - \frac{1}{2}\right)(a_6' - a_5') + a_5'$

This implies $[D_1'(x), D_2'(x)] = 2(1-\alpha)(a_2' - a_1') + a_1',\ 2\left(\alpha - \frac{1}{2}\right)(a_6' - a_5') + a_5'$

Hence,

For membership function: $\begin{cases} 2\alpha(a_2 - a_1) + a_1, a_6 - 2\alpha(a_6 - a_5) & \text{for } \alpha \in [0,0.5) \\ 2\left(\alpha - \frac{1}{2}\right)(a_3 - a_2) + a_2,\ 2(1-\alpha)(a_5 - a_4) + a_4 & \text{for } \alpha \in [0.5,1] \end{cases}$

For non membership function: $\begin{cases} a_3 - 2\alpha(a_3 - a_2'),\ 2\alpha(a_5' - a_4) + a_4 & \text{for } \alpha \in [0,0.5) \\ 2(1-\alpha)(a_2' - a_1') + a_1',\ 2\left(\alpha - \frac{1}{2}\right)(a_6' - a_5') + a_5' & \text{for } \alpha \in [0.5,1] \end{cases}$

C. Property 1

If $\tilde{A}^I = (a_1.a_2, a_3, a_4, a_5, a_6)(a_1'.a_2', a_3, a_4, a_5', a_6')$, $\tilde{B}^I = (b_1, b_2, b_3, b_4, b_5, b_6)(b_1', b_2', b_3, b_4, b_5', b_6')$ are two hexagonal Intuitionistic fuzzy numbers then by using their α cut membership and non membership functions we can obtain the addition of α cut IHFN as

$$\alpha_{\tilde{A}}^{\ I} + \alpha_{\tilde{B}}^{\ I} = \begin{pmatrix} 2\alpha(a_2 - a_1 + b_2 - b_1) + a_1 + b_1, a_6 + b_6 - 2\alpha(a_6 - a_5 + b_6 - b_5) & \text{for } \alpha \in [0,0.5) \\ 2\left(\alpha - \frac{1}{2}\right)(a_3 - a_2 + b_3 - b_2) + a_2 + b_2, 2(1-\alpha)(a_5 - a_4 + b_5 - b_4) + a_4 + b_4 & \text{for } \alpha \in [0.5,1] \end{pmatrix}$$
$$\begin{pmatrix} a_3 + b_3 - 2\alpha(a_3 - a_2' + b_3 - b_2'), 2\alpha(a_5' - a_4 + b_5' - b_4) + a_4 + b_4 & \text{for } \alpha \in [0,0.5) \\ 2(1-\alpha)(a_2' - a_1' + b_2' - b_1') + a_1' + b_1', 2\left(\alpha - \frac{1}{2}\right)(a_6' - a_5' + b_6' - b_5') + a_5' + b_5' & \text{for } \alpha \in [0.5,1] \end{pmatrix}$$

Proof:

For every $\alpha \in [0,1]$,

$$\alpha_{\tilde{A}}^{\ I} = \left(2\alpha(a_2 - a_1) + a_1, 2\left(\alpha - \frac{1}{2}\right)(a_3 - a_2) + a_2, 2(1-\alpha)(a_5 - a_4) + a_4, a_6 - 2\alpha(a_6 - a_5)\right)$$
$$\left(2(1-\alpha)(a_2' - a_1') + a_1', a_3 - 2\alpha(a_3 - a_2'), 2\alpha(a_5' - a_4) + a_4, 2\left(\alpha - \frac{1}{2}\right)(a_6' - a_5') + a_5'\right)$$

where

International Journal on Recent and Innovation Trends in Computing and Communication ISSN: 2321-8169
Volume: 5 Issue: 6 696 – 704

$$\mu_{\tilde{A}}^{I}(x)=\left(2\alpha(a_2-a_1)+a_1,\ 2\left(\alpha-\frac{1}{2}\right)(a_3-a_2)+a_2,\ 2(1-\alpha)(a_5-a_4)+a_4,\ a_6-2\alpha(a_6-a_5)\right)$$

$$\gamma_{\tilde{A}}^{I}(x)=\left(2(1-\alpha)(a_2'-a_1')+a_1', a_3-2\alpha(a_3-a_2'), 2\alpha(a_5'-a_4)+a_4, 2\left(\alpha-\frac{1}{2}\right)(a_6'-a_5')+a_5'\right)$$

$$\alpha_{\tilde{B}}^{I}=\left(2\alpha(b_2-b_1)+b_1, 2\left(\alpha-\frac{1}{2}\right)(b_3-b_2)+b_2, 2(1-\alpha)(b_5-b_4)+b_4, b_6-2\alpha(b_6-b_5)\right)$$
$$\left(2(1-\alpha)(b_2-b_1')+b_1', b_3-2\alpha(b_3-b_2'), 2\alpha(b_5'-b_4)+b_4, 2\left(\alpha-\frac{1}{2}\right)(b_6'-b_5')+b_5'\right)$$

where

$$\mu_{\tilde{B}}^{I}(x)=\left(2\alpha(b_2-b_1)+b_1, 2\left(\alpha-\frac{1}{2}\right)(b_3-b_2)+b_2, 2(1-\alpha)(b_5-b_4)+b_4, b_6-2\alpha(b_6-b_5)\right)$$

$$\gamma_{\tilde{B}}^{I}(x)=\left(2(1-\alpha)(b_2-b_1')+b_1', b_3-2\alpha(b_3-b_2'), 2\alpha(b_5'-b_4)+b_4, 2\left(\alpha-\frac{1}{2}\right)(b_6'-b_5')+b_5'\right)$$

Therefore,

$$\alpha_{\tilde{A}}^{I}+\alpha_{\tilde{B}}^{I}=\begin{pmatrix}2\alpha(a_2-a_1)+a_1+2\alpha(b_2-b_1)+b_1, 2(\alpha-\frac{1}{2})(a_3-a_2)+a_2+2(\alpha-\frac{1}{2})(b_3-b_2)+b_2,\\ 2(1-\alpha)(a_5-a_4)+a_4+2(1-\alpha)(b_5-b_4)+b_4, a_6-2\alpha(a_6-a_5)+b_6-2\alpha(b_6-b_5)\end{pmatrix}$$
$$\begin{pmatrix}2(1-\alpha)(a_2'-a_1')+a_1'+2(1-\alpha)(b_2'-b_1')+b_1', a_3-2\alpha(a_3-a_2')+b_3-2\alpha(b_3-b_2'),\\ 2\alpha(a_5'-a_4)+a_4+2\alpha(b_5'-b_4)+b_4, 2(\alpha-\frac{1}{2})(a_6'-a_5')+b_5'+2(\alpha-\frac{1}{2})(b_6'-b_5')+b_5'\end{pmatrix}$$

$$\alpha_{\tilde{A}}^{I}+\alpha_{\tilde{B}}^{I}=\begin{pmatrix}2\alpha(a_2-a_1+b_2-b_1)+a_1+b_1, a_6+b_6-2\alpha(a_6-a_5+b_6-b_5) & \text{for } \alpha\in[0,0.5)\\ 2\left(\alpha-\frac{1}{2}\right)(a_3-a_2+b_3-b_2)+a_2+b_2, 2(1-\alpha)(a_5-a_4+b_5-b_4)+a_4+b_4 & \text{for } \alpha\in[0.5,1]\end{pmatrix}$$
$$\begin{pmatrix}a_3+b_3-2\alpha(a_3-a_2'+b_3-b_2'), 2\alpha(a_5'-a_4+b_5'-b_4)+a_4+b_4 & \text{for } \alpha\in[0,0.5)\\ 2(1-\alpha)(a_2'-a_1'+b_2'-b_1')+a_1'+b_1', 2\left(\alpha-\frac{1}{2}\right)(a_6'-a_5'+b_6'-b_5')+a_5'+b_5' & \text{for } \alpha\in[0.5,1]\end{pmatrix}$$

Equate the membership function to x,

$$x=2\alpha(a_2-a_1+b_2-b_1)+a_1+b_1,\ x=2\left(\alpha-\frac{1}{2}\right)(a_3-a_2+b_3-b_2)+a_2+b_2,$$

$$x=2(1-\alpha)(a_5-a_4+b_5-b_4)+a_4+b_4 \text{ and } x=a_6+b_6-2\alpha(a_6-a_5+b_6-b_5)$$

We get, $\alpha=\dfrac{x-(a_1+b_1)}{2(a_2+b_2-(a_1+b_1))}$, $\alpha=\dfrac{x-(a_2+b_2)}{2(a_3+b_3-(a_2+b_2))}+\dfrac{1}{2}$,

$$\alpha=\frac{(a_4+b_4)-x}{2(a_5+b_5-(a_4+b_4))}+1 \text{ and } \alpha=\frac{(a_6+b_6)-x}{2(a_6+b_6-(a_5+b_5))}$$

Thus,

$$\mu_{\tilde{A}^I+\tilde{B}^I}(x)=\begin{cases}\dfrac{x-(a_1+b_1)}{2(a_2+b_2-(a_1+b_1))} & a_1+b_1\le x\le a_2+b_2\\ \dfrac{x-(a_2+b_2)}{2(a_3+b_3-(a_2+b_2))}+\dfrac{1}{2} & a_2+b_2\le x\le a_3+b_3\\ 1 & a_3+b_3\le x\le a_4+b_4\\ \dfrac{(a_4+b_4)-x}{2(a_5+b_5-(a_4+b_4))}+1 & a_4+b_4\le x\le a_5+b_5\\ \dfrac{(a_6+b_6)-x}{2(a_6+b_6-(a_5+b_5))} & a_5+b_5\le x\le a_6+b_6\\ 0 & otherwise\end{cases}$$

Similarly for the non membership function

$$x=2(1-\alpha)(a_2'-a_1'+b_2'-b_1')+a_1'+b_1',\ x=a_3+b_3-2\alpha(a_3-a_2'+b_3-b_2'),$$

$$x=2\alpha(a_5'-a_4+b_5'-b_4)+a_4+b_4 \text{ and } x=2\left(\alpha-\frac{1}{2}\right)(a_6'-a_5'+b_6'-b_5')+a_5'+b_5'$$

We get, $\alpha=1+\dfrac{(a_1'+b_1')-x}{2(a_2'+b_2'-(a_1'+b_1'))}$, $\alpha=\dfrac{(a_3+b_3)-x}{2(a_3+b_3-(a_2'+b_2'))}$

$\alpha=\dfrac{x-(a_4+b_4)}{2(a_5'+b_5'-(a_4+b_4))}$ and $\alpha=\dfrac{x-(a_5'+b_5')}{2(a_6'+b_6'-(a_5'+b_5'))}+\dfrac{1}{2}$

Thus,

$$\gamma_{\tilde{A}^I+\tilde{B}^I}(x)=\begin{cases}1+\dfrac{(a_1'+b_1')-x}{2(a_2'+b_2'-(a_1'+b_1'))} & a_1'+b_1'\le x\le a_2'+b_2'\\ \dfrac{(a_3+b_3)-x}{2(a_3+b_3-(a_2'+b_2'))} & a_2'+b_2'\le x\le a_3+b_3\\ 0 & a_3+b_3\le x\le a_4+b_4\\ \dfrac{x-(a_4+b_4)}{2(a_5'+b_5'-(a_4+b_4))} & a_4+b_4\le x\le a_5'+b_5'\\ \dfrac{x-(a_5'+b_5')}{2(a_6'+b_6'-(a_5'+b_5'))}+\dfrac{1}{2} & a_5'+b_5'\le x\le a_6'+b_6'\\ 1 & otherwise\end{cases}$$

So $\tilde{A}^I+\tilde{B}^I$ represented is a hexagonal Intuitionistic fuzzy numbers. Hence $\alpha_{\tilde{A}}{}^I+\alpha_{\tilde{B}}{}^I$ is also a hexagonal Intuitionistic fuzzy number.

D. Property 2

If $\tilde{A}^I=(a_1.a_2,a_3,a_4,a_5,a_6)(a_1'.a_2',a_3,a_4,a_5',a_6')$, $\tilde{B}^I=(b_1,b_2,b_3,b_4,b_5,b_6)(b_1',b_2',b_3,b_4,b_5',b_6')$ are two hexagonal Intuitionistic fuzzy numbers then by using their α cut membership and non membership functions we can obtain the subtraction of α cut IHFN as

 ISSN: 2321-8169
696 – 704

$$\alpha_{\tilde{A}}{}^{I} - \alpha_{\tilde{B}}{}^{I} =$$

$$\begin{pmatrix} 2\alpha(a_2 - a_1 - b_5 + b_6) + a_1 - b_6, a_6 - b_1 - 2\alpha(a_6 - a_5 - b_1 + b_2) & \text{for } \alpha \in [0,0.5) \\ 2\alpha(a_3 - a_2 - b_4 + b_5) + 2(a_2 - b_5) - (a_3 - b_4), -2\alpha(a_5 - a_4 - b_2 + b_3) + 2(a_5 - b_2) - (a_4 - b_3) & \text{for } \alpha \in [0.5,1] \end{pmatrix}$$
$$\begin{pmatrix} a_3 - b_4 - 2\alpha(a_3 - a'_2 - b_4 + b'_5), 2\alpha(a'_5 - a_4 - b'_2 - b_3) + a_4 - b_3 & \text{for } \alpha \in [0,0.5) \\ -2\alpha(a'_2 - a'_1 - b'_5 + b'_6) + 2(a'_2 - b'_5) - (a'_1 - b'_6), 2\alpha(a'_6 - a'_5 - b'_1 + b'_2) + 2(a'_5 - b'_2) - (a'_6 - b'_1) & \text{for } \alpha \in [0.5,1] \end{pmatrix}$$

Proof

The proof is similar to the property 1.

IV. NUMERICAL EXAMPLE

Example 4.1

Let $\tilde{A}$ = [11,13,14,15,16,18] [10,12,14,15,17,19] and $\tilde{B}$ = [10,12,13,14,15,17] [9,11,13,14,16,18] be two HIFNs.
By the arithmetic operation over addition we have,

$\tilde{A} + \tilde{B}$ = [11,13,14,15,16,18] [10,12,14,15,17,19] + [10,12,13,14,15,17] [9,11,13,14,16,18]
= [21,25,27,29,31,35] [19,23,27,29,33,37]

By the new arithmetic operation we have,

$$\alpha_{\tilde{A}}{}^{I} + \alpha_{\tilde{B}}{}^{I} = \begin{pmatrix} 2\alpha(a_2 - a_1 + b_2 - b_1) + a_1 + b_1, a_6 + b_6 - 2\alpha(a_6 - a_5 + b_6 - b_5) & \text{for } \alpha \in [0,0.5) \\ 2\left(\alpha - \frac{1}{2}\right)(a_3 - a_2 + b_3 - b_2) + a_2 + b_2, 2(1-\alpha)(a_5 - a_4 + b_5 - b_4) + a_4 + b_4 & \text{for } \alpha \in [0.5,1] \end{pmatrix}$$
$$\begin{pmatrix} a_3 + b_3 - 2\alpha(a_3 - a'_2 + b_3 - b'_2), 2\alpha(a'_5 - a_4 + b'_5 - b_4) + a_4 + b_4 & \text{for } \alpha \in [0,0.5) \\ 2(1-\alpha)(a'_2 - a'_1 + b'_2 - b'_1) + a'_1 + b'_1, 2\left(\alpha - \frac{1}{2}\right)(a'_6 - a'_5 + b'_6 - b'_5) + a'_5 + b'_5 & \text{for } \alpha \in [0.5,1] \end{pmatrix}$$

$$= \begin{pmatrix} 8\alpha + 21, 35 - 8\alpha & \text{for } \alpha \in [0,0.5) \\ 4\left(\alpha - \frac{1}{2}\right) + 25, 4(1-\alpha) + 29 & \text{for } \alpha \in [0.5,1] \end{pmatrix}$$
$$\begin{pmatrix} 27 - 8\alpha, 8\alpha + 29 & \text{for } \alpha \in [0,0.5) \\ 8(1-\alpha) + 19, 8\left(\alpha - \frac{1}{2}\right) + 33 & \text{for } \alpha \in [0.5,1] \end{pmatrix}$$

When α = 0, $\alpha_{\tilde{A}}{}^{I} + \alpha_{\tilde{B}}{}^{I} = \begin{pmatrix} 21, 35 & \text{for } \alpha \in [0,0.5) \\ 21, 35 & \text{for } \alpha \in [0.5,1] \end{pmatrix} \begin{pmatrix} 27, 29 & \text{for } \alpha \in [0,0.5) \\ 27,29 & \text{for } \alpha \in [0.5,1] \end{pmatrix}$

α = 0.5, $\alpha_{\tilde{A}}{}^{I} + \alpha_{\tilde{B}}{}^{I} = \begin{pmatrix} 25, 31 & \text{for } \alpha \in [0,0.5) \\ 25, 31 & \text{for } \alpha \in [0.5,1] \end{pmatrix} \begin{pmatrix} 23, 33 & \text{for } \alpha \in [0,0.5) \\ 23,33 & \text{for } \alpha \in [0.5,1] \end{pmatrix}$

$$\alpha = 1,\ \alpha_{\tilde{A}}{}^{I} + \alpha_{\tilde{B}}{}^{I} = \begin{pmatrix} 29, 27 & for\ \alpha \in [0,0.5) \\ 29, 27 & for\ \alpha \in [0.5,1] \end{pmatrix} \begin{pmatrix} 19, 37 & for\ \alpha \in [0,0.5) \\ 19, 37 & for\ \alpha \in [0.5,1] \end{pmatrix}$$

since for $\alpha \in [0,0.5)$ and $\alpha \in [0.5,1]$ arithmetic intervals are equal.

Therefore, $\alpha_{\tilde{A}}{}^{I} + \alpha_{\tilde{B}}{}^{I}$ = [21,25,27,29,31,35] [19,23,27,29,33,37]

Hence all the points coincides with the sum of two HIFNs.

Example 4.2

Let $\tilde{A}$ = [11,13,14,15,16,18] [10,12,14,15,17,19] and $\tilde{B}$ = [10,12,13,14,15,17] [9,11,13,14,16,18] be two HIFNs.
By the arithmetic operation over difference we have,

$\tilde{A}$ - $\tilde{B}$ = [11,13,14,15,16,18] [10,12,14,15,17,19] - [10,12,13,14,15,17] [9,11,13,14,16,18]
= [-6,-2,0,2,4,8] [-8,-4,0,2,6,10]

By the new arithmetic operation we have,

$$\alpha_{\tilde{A}}{}^{I} - \alpha_{\tilde{B}}{}^{I} =$$

$$\begin{pmatrix} 2\alpha(a_2 - a_1 - b_5 + b_6) + a_1 - b_6, a_6 - b_1 - 2\alpha(a_6 - a_5 - b_1 + b_2) & for\ \alpha \in [0,0.5) \\ 2\alpha(a_3 - a_2 - b_4 + b_5) + 2(a_2 - b_5) - (a_3 - b_4), -2\alpha(a_5 - a_4 - b_2 + b_3) + 2(a_5 - b_2) - (a_4 - b_3) & for\ \alpha \in [0.5,1] \end{pmatrix}$$
$$\begin{pmatrix} a_3 - b_4 - 2\alpha(a_3 - a_2' - b_4 + b_5'), 2\alpha(a_5' - a_4 - b_2' - b_3) + a_4 - b_3 & for\ \alpha \in [0,0.5) \\ -2\alpha(a_2' - a_1' - b_5' + b_6') + 2(a_2' - b_5') - (a_1' - b_6'), 2\alpha(a_6' - a_5' - b_1' + b_2') + 2(a_5' - b_2') - (a_6' - b_1') & for\ \alpha \in [0.5,1] \end{pmatrix}$$

$$\text{When } \alpha = 0,\ \alpha_{\tilde{A}}{}^{I} - \alpha_{\tilde{B}}{}^{I} = \begin{pmatrix} -6,8 & for\ \alpha \in [0,0.5) \\ -6,8 & for\ \alpha \in [0.5,1] \end{pmatrix} \begin{pmatrix} 0,2 & for\ \alpha \in [0,0.5) \\ 0,2 & for\ \alpha \in [0.5,1] \end{pmatrix}$$

$$\alpha = 0.5,\ \alpha_{\tilde{A}}{}^{I} - \alpha_{\tilde{B}}{}^{I} = \begin{pmatrix} -2,4 & for\ \alpha \in [0,0.5) \\ -2,4 & for\ \alpha \in [0.5,1] \end{pmatrix} \begin{pmatrix} -4,6 & for\ \alpha \in [0,0.5) \\ -4,6 & for\ \alpha \in [0.5,1] \end{pmatrix}$$

$$\alpha = 1,\ \alpha_{\tilde{A}}{}^{I} - \alpha_{\tilde{B}}{}^{I} = \begin{pmatrix} 0,2 & for\ \alpha \in [0,0.5) \\ 0,2 & for\ \alpha \in [0.5,1] \end{pmatrix} \begin{pmatrix} -8,10 & for\ \alpha \in [0,0.5) \\ -8,10 & for\ \alpha \in [0.5,1] \end{pmatrix}$$

since for $\alpha \in [0,0.5)$ and $\alpha \in [0.5,1]$ arithmetic intervals are equal.

Therefore, $\alpha_{\tilde{A}}{}^{I} - \alpha_{\tilde{B}}{}^{I}$ = [-6,-2,0,2,4,8] [-8,-4,0,2,6,10]

Hence all the points coincide with the difference of two HIFNs.

ISSN: 2321-8169
696 – 704

V. Conclusion

In this paper, a hexagonal intuitionistic fuzzy number is utilized to study the arithmetic operations on intuitionistic fuzzy numbers. Moreover, the α cut of the Hexagonal intuitionistic fuzzy number is also studied and the appropriate operations are presented. Further, some important properties were proved using the new proposed arithmetic operations. Numerical example is also solved to prove the property.

References

[1] Atanassov K.T, *Intuitionistic fuzzy sets,* Fuzzy Sets and Systems, 20,(1986), 87-96.
[2] Bansal, A., *Some non linear arithmetic operations on triangular fuzzy numbers (m, α, β)*, Advances in Fuzzy Mathematics, 5, (2010) 147-156.
[3] Bansal, A., *Trapezoidal Fuzzy numbers (a,b,c,d) : Arithmetic behavior*, International Journal of Physical and Mathematical Sciences, (2011).
[4] Chen Y, Li B, *Dynamic multi-attribute decision making model based on triangular intuitionistic fuzzy numbers*, Scientia Iranica B, 18, No. 2(2011), 268-274.
[5] Deng-Feng Li, *A ratio ranking method of Triangular Intuitionic fuzzy number and its application to MADM problems*, Computers Mathematics with Applications, 60 (2010), 1557-1570.
[6] Dipti Dubey, Aparna Mehra, *Linear programming with triangular intuitionistic fuzzy number*, Advances in Intelligent Systems Research, 1, No.1 (2011), 563-569.
[7] Dubois, D. & Prade, H. , *Operations on fuzzy numbers*, International Journal of Systems Science, 9(6), (1978), 613-626.
[8] Klir, G.J. , *Fuzzy arithmetic with requisite constraints*, Fuzzy Sets System, 91, (1997), 165- 175.
[9] Lakshmana Gomathi Nayagam, Venkateshwari G , Geetha Sivaraman, *Modified ranking of intuitionistic fuzzy numbers*, Notes on IFS, 17, No. 1 (2010), 5-22.
[10] Nagoorgani A, Ponnalagu K, *An Approach To Solve Intuitionistic Fuzzy Linear Programming Problem Using Single Step Algorithm*, International Journal of Pure and Applied Mathematics, 86, No. 5, (2013), 819-832.
[11] Rajarajeswari, P., Sahaya Sudha, A. & Karthika, R., *A New Operation on Hexagonal Fuzzy Number*, International Journal of Fuzzy Logic Systems, 3 (3), (2013), 15-26.
[12] Rezvani , S., *Multiplication Operation on Trapezoidal Fuzzy Numbers*, Journal of Physical Sciences, 15, (2011), 17-26.
[13] Stephen Dinagar, D. & Rajesh Kannan, J., *On Fuzzy Inventory Method with Allowable Shortage*, International Journal of Pure and Applied Mathematics, 99 (1), (2015), 65-76.
[14] Stephen Dinagar, Hari Narayanan and Kankeyanathan Kannan, *A Note on Arithmetic Operations of Hexagonal Fuzzy Numbers Using the α Cut Method*, International Journal of Applications of Fuzzy Sets and Artificial Intelligence (ISSN 2241-1240), Vol. 6 (2016), 145 - 162
[15] Thamaraiselvi A, Shanthi R, *On Intutionistic Fuzzy Transportation Problem using Hexagonal Intutionistic Fuzzy Numbers*, International Journal of Fuzzy Logic System, 5, No.1, (2015), 15-28.
[16] Zadeh,L.A., *The concept of a Linguistic variable and its applications to approximate reasoning parts I,II and III,* Inform. Sci.8 (1975) 199-249.
[17] Zadeh, L.A., Fuzzy Sets, Information and Control, 8 (1965), 338-353.
[18] Zadeh, L.A., Fuzzy set as a basis for a theory of possibility, Fuzzy sets and systems, 1 (1978), pp.3-28.
[19] Zimmermann, H. J. (1996), Fuzzy Set Theory and its Applications, Third Edition, Kluwer Academic Publishers, Boston, Massachusetts.
[20] Zimmermann H.J, *Fuzzy programming and Linear programming with several objective functions*, Fuzzy Sets and Systems, 1 (1978), 45-55.