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# Some Properties of Fuzzy Evidence Graph

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***Abstract***— A fuzzy evidence graph is a non-empty set V= $\mathcal{P}(X)\setminus\phi$ and E ={(A,B): A⊆ B , A,B∈ V } together with a pair of functions m : V→ [0,1] and ρ : E →[0,1] such that ρ(A,B) =m(A) Λ m(B) . Also $\sum_{A\in V} m(A)$=1.In this paper we introduce some properties of fuzzy evidence graph, a special type of fuzzy digraph, including isomorphism and a subgraph of fuzzy evidence graph called Hasse subgraph.

***Keywords***- *Fuzzy evidence graph, Hasse subgraph*

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## I. Introduction

Fuzzy graphs were introduced to include fuzziness in relations . The first definition of fuzzy graph by Kaufmann was based on Zadeh's fuzzy relations. After that Rosenfield developed the structure of fuzzy graphs. In [3], Sunil Mathew and M.S Sunitha presented basic concepts in fuzzy graph connectivity, which plays a remarkable role in information networks and quality based clustering.
Evidence theory is a special branch of fuzzy measure theory which is based on belief measures and plausibility measures. In [7], we introduced a type of fuzzy digraph called fuzzy evidence graph.
In this paper we introduce some properties of fuzzy evidence graph. We also give applications of fuzzy evidence graph in evidence theory to define total ignorance, commanality function etc.

## II. PRELIMINARIES

Definition 2.1
Let X be a crisp set. A fuzzy evidence graph is a non-empty set V= $\mathcal{P}(X)\setminus\phi$ and E ={(A,B): A⊆ B , A,B∈ V } together with a pair of functions m : V→ [0,1] and ρ : E →[0,1] such that ρ(A,B) =m(A) Λ m(B) . Also $\sum_{A\in V} m(A)$=1.
FEG can be denoted by G=(V,m,ρ) , where m is called the assignment function and ρ is called the edge function.

Definition 2.2
If we know that an element is in the universal set, having no evidence about its location in any subsets of X, then we use the term total ignorance.

Definition 2.3
For each U∈ $\mathcal{P}(X)$, the function defined by Q ( U ) = $\sum_{V/U\subseteq V} m(V)$ is called the commanality function which represents the total portion of belief that can move freely to every point of U.

Definition 2.4
A vertex a of a digraph is said to be reachable from a vertex b if there exist a (a,b) – dipath in it.

Definition 2.5
A digraph is simply connected if the underlying graph is connected.

Definition 2.6
A digraph is strongly connected if for any pair of vertices ,both are reachable from each other.

Definition 2.7
Consider a nonempty set X and V be a collection of subsets of X . The union graph denoted by U (V) of V is a graph with vertex set V and two vertices $v_i$ and $v_j$ ,$i \neq j$ are adjacent if $v_i \cup v_j \in V$.

Definition 2.8
The indegree $d^-(v)$ of a vertex v is the number of edges that have v as its head.
The outdegree $d^+(v)$ of a vertex v is the number of edges that edges that have v as its tail.

Definition 2.9
An ordering ≤ of A is total if any two elements are comparable.

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The corresponding pair (A,≤) is called a totally ordered set.

Definition 2.10
A lattice is a poset in which every elements have a unique least upper bound and a unique greatest lower bound.

## III. MAIN RESULTS

In this section we are discussing some properties of fuzzy evidence graph.

Theorem 3.1

The vertex set V of an FEG is partially ordered with the relation ⊆. That is (V, ⊆) is a poset.

Proof
Set of all vertices of an FEG is the collection of non-empty subsets of the crisp set X. So (V, ⊆) is a poset.

Theorem 3.2
The vertex set (V, ⊆) is a lattice.
Proof
Since power set is a lattice ordered by inclusion with supremum –the union and infimum-the intersection of subsets and V is the nonempty subset of the power set.

Theorem 3.3
Hasse diagram is not an FEG.
Proof
In Hasse diagram we cannot find an edge function $\rho : V \times V \rightarrow [0,1]$ between every pair (A,B) such that A⊆B.

Definition 3.1
A Hasse subgraph of an FEG G=(m, ρ) is a fuzzy graph H=(n, τ) such that n(A) ≤ m(A) for all A and τ(A,B) ≤ ρ(A,B) for all A ,B such that A is the immediate predecessor of B by ⊆ .

Remark 3.1
1.Hasse subgraph is a fuzzy graph but not an fuzzy evidence graph.
2.Even though FEG is not connected we can construct connected Hasse subgraphs (by removing vertices)

Theorem 3.4
A fuzzy evidence graph is evidently connected if every Hasse subgraph is connected

Proof
Assume that every Hasse subgraph is connected.
Then by definition for every A,B , $CONN_G(A,B) > 0$ which ensures the evidently connectedness of FEG
.

Theorem 3.5
If an FEG G is evidently connected then there exist atleast one connected Hasse subgraph .

Proof
Assume that FEG is evidently connected.
Let H be a Hasse subgraph of G. Then by definition for every A,B such that A⊆B $CONN_H(A,B) > 0$.
That is max{Strengths of all paths from A to B } > 0.
ie , max{ min(weights of the weakest edge of the paths from A to B )} >0 ⟹ there exist atleast path from A to B with positive strength.
Hence H is connected.

Definition 3.1
An edge of an FEG is an evidence bridge if it's removal evidently disconnects G.

Proposition 3.1
The following properties hold in a fuzzy evidence graph.
1.Every edge in a FEG is an evidence bridge.
2.FEG can be considered as a simply connected, oriented, weighted digraph.
3.As a weighted digraph, FEG is weakly connected.

Propostion 3.2
Fuzzy evidence graph is simply connected
Proof
Proof is evident from the definition.

Remark 3.2
The underlying graph of fuzzy evidence graph is a union graph.

Remark 3.3
The number of fuzzy evidence graphs with same cardinality of vertex set is infinite ,but the underlying graph is unique up to isomorphism.

Remark 3.4
A fuzzy evidence graph G=(V,m,ρ) with the property that m(X) =1 and m(A) =0 for all A⊆X, where X is the corresponding crisp set , is called evidence ignorance graph.

Definition 3.2
Two fuzzy evidence graphs $G_1=(V_1,m_1,\rho_1)$ and $G_2=(V_2,m_2,\rho_2)$ are said to be similar if there exist a one –one function from $V_1$ onto $V_2$.

Definition 3.3
Two fuzzy evidence graphs $G_1=(V_1,m_1,\rho_1)$ and $G_2=(V_2,m_2,\rho_2)$ are said to be isomorphic if there exist a one-one

function from $V_1$ onto $V_2$ such that $|m_1 - m_2| \leq \frac{1}{2} min\{m_{1,} m_2\}$ for the corresponding vertices.

Remark 3.5
Any two fuzzy evidence graphs with same cardinality are similar.

Remark 3.6
Every isomorphic fuzzy evidence graphs are similar.

Remark 3.7
The underlying graphs of every similar fuzzy evidence graphs are isomorphic.

Definition 3.4
An isomorphism of a fuzzy evidence graph G=(V,m,ρ) with itself is called an automorphism of G.

Proposition 3.3
The number of automorphisms of a fuzzy evidence graph with n elements in the vertex set is uncountable.

Proof
Since m : V→ [0,1], we can find uncountable number of automorphisms.

Results 3.1
1.For a fuzzy evidence graph there exist exactly one vertex having outdegree zero and indegree $2^n$-2, which is the complete vertex.
2.There exist exactly n vertices with with outdegree 1 and indegree 0.

Definition 3.5
Let X be a crisp set. The complement of the fuzzy evidence graph G=(V,m,ρ) is a non-empty set V= $\mathcal{P}(X)\backslash \phi$ together with a pair of functions m : V→ [0,1] and ρ : V x V →[0,1] such that for all A,B∈ V, (A,B) ∈ ρ, whenever A⊆ B and ρ(A,B) < m(A) Λ m(B). Also $\sum_{A \in V} m(A)$=1

Remark 3.8
Complement of a fuzzy evidence graph is a fuzzy graph but not a fuzzy evidence graph.

## IV. CONCLUSION

In this paper we introduced Hasse subgraph and defined some properties of fuzzy evidence graph. Further we proved some theorems based on these properties. Also we define isomorphism between two fuzzy evidence graphs.

## REFERENCES

[1] George J. Klir and Bo Yuan, Fuzzy Sets & Fuzzy Logic Theory And Applications, Pearson [1995] .
[2] John N Mordeson and Premchand S Nair, Fuzzy Graphs and Fuzzy Hypergraphs, Physica-Verlag[2000].
[3] Sunil Mathew and M. S Sunitha, Fuzzy graphs Basics, Concepts and Applications, LAP LAMBERT Academic Publishing.
[4] G.Suresh Singh, Graph Theory,PHI Learning Private Limited[2010]
[5] Frank Harary, Graph Theory, Addison-Wesly Publishing Company[1972]
[6] Karel Hrbacek and Thomas Jech, Introduction to Set Theory[1978]
[7] Mathew Varkey T. K and Sreena T. D , Fuzzy Evidence Graph, Advances in Fuzzy Mathematics, Volume 12, Number 3 (2017), pp.489-498