Decision Making Support Systems with VIKOR Method For Supply Chain Management Problems

V. Thiagarasu
Associate Professor of Computer Science
Gobi Arts & Science College
Gobichettipalayam, India

V. Rengaraj
Assistant Professor in Computer Science
Thanthai Hans Roever College
Perambalur, India
vr.msu2013@gmail.com

Abstract—In this work scientific and simple calculation method for manufacturer's decision-makers to choose the most ideal supplier in the Supply Chain Management problem has been provided. As a fundamental decision-making for manufacturers, the quality of supplier performance not only affects the downstream business, but also determines the success of the whole supply chain. Therefore, choosing suitable suppliers in the supply chain becomes a key strategic step; it directly impacts the benefit for manufacturers. This paper deals with the supplier selection problem based on VIKOR algorithm (Vlse Kriterijumska Optimizacija Kompromisno Resenje) which is a compromise multiple criteria decision making approach with entropy method which gives the weights to indicators. The VIKOR algorithm deals with the conflicts between indicators based on certain way to sort the scheme and choose the best scheme. A numerical example is proposed to illustrate the effectiveness of this algorithm. However, Sensitivity Analysis for the weighting vectors is performed to make the result of evaluations more objective and accurate.

Keywords-*VIKOR; MADM; Sensitivity Analysis; Entropy; DMSS.*

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I. INTRODUCTION

In multiple attribute decision making (MADM) problem, a decision maker (DM) has to choose the best alternative that satisfies the evaluation criteria among a set of candidate solutions. It is generally hard to find an alternative that meets all the criteria simultaneously, so a good compromise solution is preferred. The VIKOR (The compromise solution method, also known as the VIKOR - VIšekriterijumsko KOmpromisno Rangiranje) method was developed for multi-criteria optimization of complex systems. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. It introduces the multi-criteria ranking index based on the particular measure of ''closeness" to the ''ideal" solution. To deal with the uncertainty and vagueness from humans' subjective perception and experience in decision process, this paper presents an evaluation model based on deterministic data, fuzzy numbers, interval numbers and linguistic terms. In this research work, VIKOR method will be extended to develop a methodology for solving MADM problems together with some Data Mining techniques for an efficient DMSS (Decision Making Support System). What we basically suggest in this study is to extend the VIKOR method with four main types of information (deterministic data, fuzzy numbers, interval numbers and linguistic terms) in decision-making matrix for solving multiple attribute decision making problems.

Decision support system (DSS) is seen as building blocks that offers the best combination of computational power, value for money and significantly offers efficiency in certain decision making problem solving. Based on these building blocks, modern DSS applications comprise of integrated resources working together which are model base, database or knowledge base, algorithms, user interface and control mechanisms used to support certain decision problem. There are many application areas suitable for DSS which include academic advising, water resource planning, direct mailing decisions, e-sourcing, tendering decisions and many more. DSS has a vast field of research scopes which are categorized as model management, design, multi-criteria decision making (MCDM), implementation, organization science, cognitive science, and group DSS (GDSS). DSS also has direct relation with Human Computer Interaction (HCI) and Database Management System (DBMS). MCDM constitutes an advanced field of research that is dedicated to the development and implementation of DSS tools and methodologies to handle complex decision problems involving multiple criteria, goals or objectives of conflicting nature. MCDM is broadly classified into two categories which are Multiple Attribute Decision Making (MADM) and Multiple Objective Decision Making (MODM). MADM methods are used for selecting single most preferred alternative or short listing a limited number of alternatives, while MODM methods are used for designing a problem involving an infinite number of alternatives implicitly defined by mathematical constraints. Evaluation of a problem in DSS can either be done by a single decision Maker (DM) or a group of decision makers (DMs). If it involves a single DM, the DSS is called Single DSS (SDSS) and if a group of DMs are involved, the term group DSS (GDSS) is used. GDSS comprises a large body of research and it remains an active area of investigation. A GDSS in web-based environment is a computerized system that makes use of model base and database/knowledge base which delivers decision support information or decision support tools to a group of DMs/users using a web browser such as Netscape Navigator or Internet Explorer.

Group decision is usually understood as aggregating different individual preferences on a given set of alternatives to a single collective preference. It is assumed that the individuals participating in making a group decision face the same common problem and are all interested in finding a solution. A group decision situation involves multiple actors (decision makers), each with different skills, experience and knowledge relating to different aspects (criteria) of the problem. In a correct method for synthesizing group decisions, the competence of the different actors to the different professional fields has also to be taken into account. We assume that each actor considers the same sets of alternatives and criteria. It is also assumed that there is a special actor with authority for establishing consensus rules and determining voting powers to the group members on the different criteria. Many researchers call this entity the Supra Decision Maker (SDM). The final decision is derived by aggregating (synthesizing) the opinions of the group members according to the rules and priorities defined by the SDM.

Some values of the multi attribute decision models are often subjective. The weights of the criteria and the scoring values of the alternatives against the subjective (judgmental) criteria contain always some uncertainties. It is therefore an important question how the final ranking or the ranking values of the alternatives is sensitive to the changes of some input parameters of the decision model. The simplest case is when the value of the weight of a single criterion is allowed to vary. For additive multi attribute models, the ranking values of the alternatives are simple linear functions of this single variable and attractive graphical tools can be applied to present a simple sensitivity analysis to a user. For a wide class of multi attribute decision models there are different methods to determine the stability intervals or regions for the weights of different criteria. These consist of the values that the weights of one or more criteria can take without altering the results given by the initial set of weights, all other weights being kept constant. In this work we have concentrated on Decision Making problems based on VIKOR method together with entropy method and sensitivity analysis.

II. THE GENERAL VIKOR METHOD

Multi-criteria optimization is the process of determining the best feasible solution according to the established criteria (representing different effects). Practical problems are often characterized by several non-commensurable and conflicting criteria and there may be no solution satisfying all criteria simultaneously. Thus, the solution is a set of non-inferior solutions, or a compromise solution according to the decision maker's preferences. The compromise solution was established by Zeleny, (1982) for a problem with conflicting criteria and it can help the decision makers to reach a final solution. In classical MADM methods, the ratings and the weights of the criteria are known precisely, whereas in the real world, in an imprecise and uncertain environment, it is an unrealistic assumption that the knowledge and representation of a decision maker or expert are so precise. For example, human judgment including preferences is often vague and decision maker (DM) cannot estimate his preference with exact numerical values. In these situations, determining the exact value of the attributes is difficult or impossible. So, to describe and treat imprecise and uncertain elements present in a decision problem, fuzzy approaches and linguistic terms are frequently used. In the

works of linguistic terms decision making, linguistic terms are assumed to be with known by fuzzy linguistic membership function. However, in reality to a decision maker it is not always easy to specify the membership function in an inexact environment. At least in some of the cases, the use of interval numbers may serve the purpose better. An interval number can be thought as an extension of the concept of a real number, however, in decision problems its use is not much attended as it merits (Hwang & Yoon, 1981). Recently, some authors have extended TOPSIS and VIKOR method to solve decision making problems with interval data. According to a comparative analysis of VIKOR and TOPSIS written by Opricovic and Tzeng (2002; 2003; 2004; 2007), VIKOR and TOPSIS methods use different aggregation functions and different normalization methods. TOPSIS method is based on the principle that the optimal point should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). Therefore, this method is suitable for cautious (risk avoider) decision maker(s), because the decision maker(s) might like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible. Besides, computing the optimal point in the VIKOR is based on the particular measure of ''closeness'' to the PIS. Therefore, it is suitable for those situations in which the decision maker wants to have maximum profit and the risk of the decisions is less important for him/her. Therefore, in this paper, VIKOR method was extended to develop a methodology for solving MADM problems. What we basically suggest in this study is to extend the VIKOR method with four main types of information (deterministic data, fuzzy numbers, interval numbers and linguistic terms) in decision-making matrix for solving multiple attribute decision making problems. To validate the application of the model and to examine its effectiveness, the proposed extension methodology was used for deriving preference order of open pit mines equipment. The selection of equipment for mining applications is not a well-defined process and because it involves the interaction of several subjective factors or criteria, decisions are often complicated and may even embody contradictions. Various types of cost model have been proposed for application to the selection of mining equipment.

A. VIKOR method

Decision-making problem is the process of finding the best option from all of the feasible alternatives. In almost all such problems, the multiplicity of criteria for judging the alternatives is pervasive. For many such problems, the DM wants to solve a multiple attribute decision making (MADM) problem (Hwang & Yoon, 1981). A MADM problem can be concisely expressed in matrix format as:

$$
A_m \begin{array}{c} C_n \\ \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \end{array}
$$

where $A_1, A_2, \dots, A_m$ are possible alternatives among which decision makers have to choose, $C_1, C_2, \dots, C_n$ are criteria with which alternative performance are measured, x_{ij} is the rating of

alternative A_i with respect to criterion C_j. The foundation for compromise solution was established by Zeleny (1982) and later advocated by Opricovic & Tzeng (2002, 2003, 2004, 2007). The compromise solution is a feasible solution that is the closest to the ideal solution, and a compromise means an agreement established by mutual concession. The compromise solution method, also known as the VIKOR (VIšekriterijumsko KOmpromisno Rangiranje) method was introduced as one applicable technique to implement within MADM. The multiple attribute merit for compromise ranking was developed from the Lp-metric used in the compromise programming method (Zeleny, 1982). The main procedure of the VIKOR method is described below:

Step 1: The first step is to determine the objective, and to identify the pertinent evaluation attributes. Also determine the best, i.e., f_j^+ and the worst, f_j^-, values of all attributes.

Step 2: Calculate the values of S_i and R_i:

$$S_i = \sum_{j=1}^{M} w_j \left([f_j^+ - f_{ij}] / [f_j^+ - f_j^-] \right)$$

$$R_i = \underset{j}{Max} \left\{ w_j \left([f_j^+ - f_{ij}] / [f_j^+ - f_j^-] \right) \right\}, j = 1, 2, ..., M.$$

Step 3: Calculate the values of Q_i :

$$Q_i = v \frac{(S_i - S^*)}{(S^- - S^*)} + (1 - v) \frac{(R_i - R^*)}{(R^- - R^*)}$$

where $S^* = \underset{i}{\min} S_i$; $S^- = \underset{i}{\max} S_i$

where S^* is the maximum value of S_i, and S^- the minimum value of S_i; R^* is the maximum value of R_i, and R^- is the minimum value of R_i . v is introduced as weight of the strategy of 'the majority of attributes'. Usually, the value of v is taken as 0.5. However, v can take any value from 0 to 1.

Step 4: Arrange the alternatives in the descending order, according to the values of Q_i. Similarly, arrange the alternatives according to the values of S_i and R_i separately. Thus, three ranking lists can be obtained. The compromise ranking list for a given v is obtained by ranking with Q_i measures. The best alternative, ranked by Q_i, is the one with the minimum value of Q_i.

Step 5: For given attribute weights, propose a compromise solution, alternative A1, which is the best ranked by the measure Q, if the following two conditions are satisfied:
Condition 1: 'Acceptable advantage'
$Q(A_2) - Q(A_1) \geq (1/(N-1))$.

A_2 is the second-best alternative in the ranking by Q.

Condition 2: 'Acceptable stability in decision making'. Alternative A_1 must also be the best ranked by S and/or R. This compromise solution is stable within a decision-making process, which could be: 'voting by majority rule'

(when $v > 0.5$ is needed) or 'by consensus' (when $v = 0.5$) or 'with veto'
(when $v < 0.5$).

If one of the conditions is not satisfied, then a set of compromise
solutions is proposed, which consists of:
– Alternatives A_1 and A_2 if only condition 2 is not satisfied.
– Alternatives $A_1, A_2, . . ., A_m$ if condition 1 is not satisfied; A_m is determined by the relation $Q(A_m) - Q(A_1) < (1/(N-1))$ for maximum M (the positions of these alternatives are ''in closeness'').

VIKOR is a helpful tool in MADM, particularly in a situation where the decision maker is not able, or does not know how to express preference at the beginning of system design. The obtained compromise solution could be accepted by the decision makers because it provides a maximum 'group utility' (represented by S⁻) of the 'majority' and a minimum of individual regret (represented by R⁻) of the 'opponent' (Opricovic & Tzeng, 2002, 2003, 2004, 2007).

III. SUPPLIER SELECTION PROBLEM WITH THE APPLICATION OF VIKOR METHOD AND SENSITIVITY ANALYSIS

In recent years, with the rapid development of IT industry, the aggravation of severe competition, the ceaseless changes of market demand, manufacturers face severe challenges of reducing the cost, decreasing the storage, improving the quality and service, enhancing customer satisfaction, shortening the delivery date, raising efficiency, and heightening the competitive awareness. If manufacturers can both operate internal resources and integrate external resource, they can ensure their competitive advantages for survival and development in the fiercely competitive environment. So manufacturers have to adjust the logistic process driven by customers' services and implement supply chain management (SCM), a new management model to reduce cost and improve service, which adapts to social, economic and technological environments in the new era. Several criteria have been identified for supplier selection, such as supplier's credit and reputation, product price, delivery date, the net price, quality, capacity and communication systems, historical supplier performance and so forth. Supplier, as the object of enterprise purchasing activities, it directly determines the quality of the raw material and parts purchased by the manufacture, and the supplier selection is one of the essential steps in supply chain design. Since selecting the right suppliers considerably shrinks the purchasing cost and improves competitiveness, the supplier selection process is known as the most significant act of a purchasing department. Furthermore, a good decision-making method of supplier selection is quite necessary. So in this work, we use VIKOR algorithm with entropy method to select suppliers.

A. The basic principle of VIKOR algorithm

The VIKOR algorithm was proposed by Opricovic in1998, which is a multi-attribute decision making method for complex system based on ideal point method. The basic view

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of VIKOR is determining positive-ideal solution and negative-ideal solution in the first place. The positive-ideal solution is the best value of alternatives under assessment criteria, and the negative-ideal solution is the worst value of alternatives under assessment c z riteria. Finally, arrange the priority of the schemes according to the proximity of the altenatives assessed value to the ideal schemes. In comprehensive evaluation, VIKOR adopted L_p-metric aggregate function:

$$L_{pj} = \left\{ \sum_{i=1}^{n} \left[w_i \left(f_i^* - f_{ij} \right) / \left(f_i^* - f_i^- \right) p \right] \right\}^{\frac{1}{p}}$$

In this function $1 \le p \le \infty$; j=1,2,...,n, the variable J represents the number of alternatives is indicated as a_j , f_{ij} is the evaluation value of the ith criterion for alternative a_j ; the measure L_{pj} means the distance between alternative a$_j$ and positive-ideal solution. With this programming method the VIKOR algorithm maximizing the group utility, so the compromise solution can be accepted by decision-makers.

Ranking by VIKOR may produce different values of criteria weights, criteria weights impact compromise solution. The VIKOR method determines the weight stability intervals, using the methodology presented in Opricovic (2002; 2003; 2004; 2007). The compromise solution obtained with initial weights (w$_i$=1,2,...,n) will be replaced if the value of weight is not within the stability interval. The analysis of weight stability intervals for a single criterion is performed for all criterion function, with the (given) same initial values of weights. In this way the preference stability of an obtained compromise solution may be analyzed using the VIKOR program.

B. The calculation step of VIKOR algorithm

Step1: Calcuate each indicator's positive- ideal solution's value f$_1$* and negative-ideal solution's value f$_1$-, i=1,2,...,n.

$$f_i^* = \left[\left(\max_j f_{ij} \mid i \in I_1 \right), \left(\min_j f_{ij} \mid i \in I_2 \right) \right], \; f_i^- = \left[\left(\min_j f_{ij} \mid i \in I_1 \right), \left(\max_j f_{ij} \mid i \in I_2 \right) \right].$$

I$_1$ is a benefit type indicator set, I$_2$ is a cost type indicator set.

Step2: Calculate the values of S$_i$ and R$_i$, j=1,2,...,J, S$_i$ is the optimal solution of schemes comprehensive evaluation, R$_i$ is the most inferior solution of schemes comprehensive evaluation.

$$S_i = \sum_i w_i \left(f_i^* - f_{ij} \right) / \left(f_i^* - f_i^- \right), \quad R_i = \max \left[w_i \left(f_i^* - f_{ij} \right) / \left(f_i^* - f_i^- \right) \right].$$

In the function, w$_i$ are weights of each indicator, meaning the relative importance among the indicators. The weights of each indicator are determine by entropy method.

Step3: Calculate Q$_j$: The value of interest ratio brought by scheme, j=1,2,...,J.

$$Q_j = \gamma \left(S_i - S^* \right) / \left(S^- - S^* \right) + (1 - \gamma) \left(R_j - R^* \right) / \left(R^- - R^* \right),$$

Where

$$s^* = \min_j s_j; \; s^- = \max_j s_j; \; R^* = \min_j R_j; \; R^- = \max_j R_j.$$

γ represents the weights of "the majority of criteria" strategy or the largest groups utility value, here we define the value v=0.5.

Step4: According to S, R and Q separately to rank the schemes, we get 3 rank tables.

Step5: If the following two conditions are met simultaneously, then the scheme with minimum value of Q in ranking is considered the optimal compromise scheme, C1 is accepted advantage.

$$Q(a^{(2)}) - Q(a^{(1)}) \ge 1/(m-1).$$

a$^{(2)}$ is the suboptimal scheme in the rank tables according to Q. C2 is the acceptable stability in decision-making process. a$^{(1)}$ is the optimal solution in S or R rank tables with Q ranking has been set simultaneously. This compromise solution is stable in decision-making process it may differ as v> 0.5, decision making will be according to majority criteria; when v≅ 0.5 the selection will consider to overall and individuals evaluation; when v< 0.5, veto the scheme set. Here, v is the weight of the decision making strategy " the majority of criteria" (or "the maximum group utility"). If one of the above two conditions is not satisfied, we will get a compromise solution set, including:

(1) If the condition C2 is not satisfied then a$^{(1)}$ and a$^{(2)}$ schemes are both compromise solution.

(2) If the condition C1 is not satisfied, we will get schemes a$^{(1)}$, a$^{(2)}$,…, a$^{(r),}$ a$^{(r)}$ is determined by the relation Q(a$^{(r)}$)-Q(a$^{(1)}$)≥1/(m-1) for maximum (the positions of these alternatives are "in closeness").

The following theorem depicts changes in the weights of attributes:

Theorem:
In the MADM model, if the weight of the pth attributes, changes Δp, then the weight of other attributes change by Δ_j where,

$$\Delta_j = \frac{\Delta_p . w_j}{w_p - 1}; j = 1, 2, \dots k, j \ne p.$$

Proof: If new weights of attributes are w$_j$', and new weights of pth change as,

$$w_p' = w_p + \Delta_p \rightarrow (1)$$

Then, the new weight of the other attributes would changes as
$$w_j' = w_j + \Delta_j; j = 1, 2, \dots k, j \ne p \rightarrow (2)$$
The sum of the weight must be 1 then,

$$\sum_{j=1}^{k} w_j' = \sum_{j=1}^{k} w_j + \sum_{j=1}^{k} \Delta_j$$

$$\sum_{j=1}^{k} w_j' - \sum_{j=1}^{k} w_j = \sum_{j=1}^{k} \Delta_j$$

$$\sum_{j=1}^{k} \Delta_j = 0 \qquad \rightarrow (3)$$

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Here we have,

$$w_p' = w_p + \Delta_p \rightarrow (a)$$

$$\sum_{j=1}^{k} w_j' = \sum_{j=1}^{k} w_j + \sum_{j=1}^{k} \Delta_p, \; j=1,2,\dots k, \; j \neq p \rightarrow (b)$$

$(a)+(b)$ gives,

$$(w_p' - w_p) + \sum_{j=1}^{k} w_j + \sum_{j=1}^{k} w_j = \Delta_p + \sum_{j=1}^{k} \Delta_j$$

$$0 + 1 - 1 = \Delta_p + \sum_{j=1}^{k} \Delta_j$$

$$\Delta_p = -\sum_{j=1}^{k} \Delta_j \rightarrow (4)$$

$$\Delta_j = \frac{\Delta_p . w_j}{w_p - 1}; \; j=1,2,\dots k, \; j \neq p \rightarrow (5)$$

$$\Delta_p = -\sum_{j=1}^{k} \Delta_j \qquad (\text{from}(4))$$

since

$$-\Delta_p = \sum_{j=1}^{k} \Delta_j = \sum_{j=1}^{k} \frac{\Delta_p . w_j}{w_p - 1} = \frac{\Delta_p}{w_p - 1} \sum_{j=1}^{k} w_j = \frac{\Delta_p}{w_p - 1}(1 - w_p) = -\Delta_p \rightarrow (6)$$

$$-\Delta_p = -\frac{\Delta_p}{(w_p - 1)}(1 - w_p) \rightarrow (A)$$

Now we have to find the value of $\sum_{j=1}^{k} w_j$

$$\Delta_j = \frac{\Delta_p . w_j}{w_p - 1}$$

$$\Delta_j (w_p - 1) = \Delta_p . w_j$$

$$\Delta_j (w_p - 1) = -\sum_{j=1}^{k} \Delta_j . w_j \qquad (\text{from}(4))$$

$$(w_p - 1) = -\sum_{j=1}^{k} w_j \qquad \rightarrow (6)$$

sub (6) in (A)

$$-\Delta_p = -\frac{\Delta_p}{(w_p - 1)}(1 - w_p) = -\frac{\Delta_p}{(w_p - 1)}\{-(w_p - 1)\} = -\Delta_p \qquad \rightarrow (7)$$

Main Result:

In a MADM problem, if the weight of the pth attribute changes from w_p to w_p' as:

$$w_p' = w_p + \Delta_p' \qquad \rightarrow (8)$$

Then the weight of other attribute would change as,

$$w_j' = w_j + \Delta_j = w_j + \frac{\Delta_p . w_j}{w_p - 1} = \frac{w_j(w_p - 1) + \Delta_p . w_j}{w_p - 1} = \frac{-(1 - w_p - \Delta_p).w_j}{-(w_p - 1)}$$

$$w_j' = \frac{-(1 - w_p - \Delta_p).w_j}{-(w_p - 1)} = \frac{(1 - w_p - \Delta_p).w_j}{(w_p - 1)} \qquad \rightarrow (9)$$

$$w_j' = \frac{(1 - w_p')}{(1 - w_p)} . w_j \qquad \rightarrow (10), \quad j=1,2,\dots,k, \; j \neq p \quad (\text{from}(8))$$

Then new vector for weights of attributes would be
$w^{t'} = (w_1', w_2', \dots w_k')$ that,

$$w_j' = \begin{cases} w_j + \Delta_p, & if \quad j = p \\ \dfrac{(1 - w_p')}{(1 - w_p)} . w_j, & j=1,2,\dots k, \; j \neq p \end{cases}$$

Here, $\quad w_p' = w_p + \Delta_p$

$$\Rightarrow \begin{cases} \text{if } w_p' > w_p \;\; \Rightarrow \;\; w_j' < w_j \\ \text{if } w_p' < w_p \;\; \Rightarrow \;\; w_j' > w_j \end{cases}, \; j=1,2,\dots,k, \; j \neq p \rightarrow (11)$$

The sum of new weights of attributes in (11) is 1.
Hence,

$$\sum_{j=1}^{k} W_j' = \sum_{j=1, j \neq p}^{k} w_j' + w_p' = \sum_{j=1, j \neq p}^{k} \frac{(1 - w_p - \Delta_p)w_j}{(w_p - 1)} + (w_p + \Delta_p) \quad (\text{from}(9)\&(10))$$

$$= \frac{((1 - w_p - \Delta_p)w_j)}{(w_p - 1)} . \sum_{j=1, j \neq p}^{k} (w_j + w_p + \Delta_p) = \frac{(1 - w_p - \Delta_p)}{(w_p - 1)}(w_p - 1)$$

hence $\quad \sum_{j=1}^{k} w_j' = 1 \rightarrow (12)$

Corollary:

In the new vector of weights that is obtained by (11) the weights ratio is the same (exception of the pth attribute) because new weights for attributes (exception of the pth attribute) is obtained by multiplying the constant

$$\frac{1 - w_p - \Delta_p}{1 - w_p}$$

to the old weight .Then the ratio of new weight of attribute c_i to new weight of attribute c_j for i , $j=1,2,\dots.k, \; j \neq p$ is the same to ratio of old ones .That is

$$\frac{w_i'}{w_j'} = \frac{w_i}{w_j}, \; i, \; j=1,2,\dots.k, \; j \neq p \rightarrow (13)$$

C. The step of entropy method to determine the weight of each indicators

Entropy was originally a thermodynamic concept, first introduced into information theory by Shannon. It has been widely used in the engineering, socioeconomic and other fields. According to the basic principles of information theory, information is a measure of systems ordered degree, and the entropy is a measure of systems disorder degree.

Step 1: Calculate p_{ij} (the ith schemes j^{th} indicators values proportion). $P_{ij} = \dfrac{r_{ij}}{\sum_{j=1}^{m} r_{ij}}$,

r_{ij} is the ith schemes j^{th} indicators value.

Step 2: Calculate the jth indicators entropy value e_j,

$$e_j = -k \sum_{i=1}^{m} P_{ij} \ln(P_{ij})$$

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$$k = \frac{1}{\ln m} \, , \quad m \text{ is the number of assessment schemes.}$$

Step 3: Calculate weight w_j (j^{th} indicators weight).

$$w_j = \frac{(1 - e_j)}{\sum_{j=1}^{n}(1 - e_j)}$$

n is the number of indicators, and $0 \le w_j \le 1$, $\sum_{j=1}^{n} w_j = 1$

In entropy method, the smaller the indicators entropy value e_j is, the bigger the variation extent of assessment value of indicators is, the more the amount of information provided, the greater the role of the indicator in the comprehensive evaluation, the higher its weight should be.

IV. Supplier Selection Problem–VIKOR Method: Numerical Illustration

Before To illustrate the VIKOR method, we discuss a simplified problem as follows, ABC Mechanical company is a core enterprises in the supply chain. It faces a problem of supplier selection. There are four suppliers S_i, (i=1,2,3,4) selected as alternatives against five attributes Q_j, (j=1,2,3,4,5). The four attributes are product quality, service quality, delivery time, reputation risk & price. Q_2, Q_3, Q_4 are benefit attributes the greater value being better, Q_1, Q_5 are cost attributes the smaller value being better.

VIKOR algorithm with entropy method:
Step 1: Invite experts to rate the suppliers according to the evaluation criteria. The indicators values are shown in table1.

Table-1: The value of indicators

Suppliers	Reputation risk	Service quality	Delivery time	Product quality	price
S_1	90	90	70	55	95
S_2	80	85	85	90	90
S_3	80	85	90	69	85
S_4	60	80	85	83	85

Step 2: Assign weights to each inndicators to entropy method which is introduced each indicators weights,
W={0.3814, 0.0309, 0.1186, 0.3762, 0.0412}

Step 3: Make indicators value being dimensionless, establish decision-making matrix,

The cost type indicators:

$$V_{ij} = \left(\frac{\min x_{ij}}{x_{ij}} \right)$$

The benefit type indicators:

$$V_{ij} = \left(\frac{x_{ij}}{\max x_{ij}} \right)$$

Hence we have the following decision-matrix:

$$v = \begin{pmatrix} 0.6667 & 1.0000 & 0.7778 & 0.6154 & 0.8947 \\ 0.7500 & 0.9444 & 0.9444 & 1.0000 & 0.9444 \\ 0.7500 & 0.9444 & 1.0000 & 0.7692 & 1.0000 \\ 1.0000 & 0.8889 & 0.9444 & 0.9230 & 1.0000 \end{pmatrix}$$

Step 4: The positive-ideal solutions are, $f^* = (1,1,1,1)$
The negative-ideal solutions are,

$$f^- = (0.6667, \ 0.8889, \ 0.7778, \ 0.6154, \ 0.8947)$$

Step 5: Calculate each suppliers S,R,Q value

$$S_i = \sum_{i=1}^{n} (w_i(f_i * - f_{ij}) / (f_i * - f_i^-)) \, ,$$

$S_1 = 0.9173, \ S_2 = 0.3519, \ S_3 = 0.5217, \ S_4 = 0.1359.$

And $R_i = \max \left[w_i \left(f_i^* - f_{ij} \right) / \left(f_i^* - f_i^- \right) \right]$

$R_1 = 0.3814, \ R_2 = 0.2861, \ R_3 = 0.2863, \ R_4 = 0.0753.$

And then,

$$Q_j = v \left(S_i - S^* \right) / \left(S^- - S^* \right) + (1-v)\left(R_j - R^* \right) / \left(R^- - R^* \right),$$

Where $S^* = \min S_j$; $S^- = \max S_j$; $R^* = \min R_j$; $R^- = \max R_j$,
$S^* = 0.1359$, $S^- = 0.9173$, $R^* = 0.0753$, $R^- = 0.3813$.

$$Q_1 = v \left(S_1 - S^* \right) / \left(S^- - S^* \right) + (1-v)\left(R_1 - R^* \right) / \left(R^- - R^* \right),$$

$Q_1 = 1$, $Q_2 = 0.4826$, $Q_3 = 0.5850$, $Q_4 = 0$.

The evaluation value of each supplier

	S_1	S_2	S_3	S_4
S	0.9173	0.3519	0.5119	0.1359
R	0.3813	0.2861	0.2863	0.0753
Q	1	0.4826	0.5850	0

Step 6: Rank suppliers in ascending order

Rank the suppliers by VIKOR

	S_1	S_2	S_3	S_4
S	4	2	3	1
R	4	2	3	1
Q	4	2	3	1

According to $Q(a^{(2)}) - Q(a^{(1)}) \ge 1/(m-1)$.
(where $Q(a^{(2)})$ is the suboptimal scheme in Q rank table and Q's VIKOR evaluation value),

We can get, $Q(S_2) - Q(S_4) = 0.4826 - 0$
$= 0.4826 \ge 1/(4-1)$ (here, m=4)
$= 0.4826 > 1/3$.

S_1 is the optimal solution in S or R rank tables with Q ranking has been set simultaneously. So the final ranking result according to VIKOR algorithm method is $S_4 > S_2 > S_3 > S_1$.

∴ The best supplier is S_4.

V. SENSITIVITY ANALYSIS FOR THE VIKOR METHOD-NUMERICAL ILLUSTRATION

Now we assume that, $\Delta_2 = 0.0517$.

$$w_2' = w_2 + \Delta_2 = 0.0309 + 0.0517 = 0.0826.$$

$$\begin{aligned} w_j' &= \left((1 - w_2')/(1 - w_2)\right) w_j \\ &= \left((1 - 0.0826)/(1 - 0.0309)\right) w_j \\ &= (0.9174)/(0.9691) w_j \\ &= 0.9466 \; w_j \end{aligned}$$

0.9466 * 0.3814 = 0.3610
0.9466 * 0.0309 = 0.0826
0.9466 * 0.1186 = 0.1123
0.9466 * 0.3762 = 0.3561
0.9466 * 0.0412 = 0.0389

w'=(0.3610, 0.0826, 0.1123, 0.3561, 0.03899)

Proceeding Step1 to Step4, as in previous chapter, then we have,

*Step 5:*Calculate each suppliers S, R, Q value:

$$S_i = \sum_{i=1}^{n} (w_i(f_i * - f_{ij})/(f_i * - f_i^-))$$

$S_1 = 0.8684$, $S_2 = 0.36083$, $S_3 = 0.52584$, $S_4 = 0.18199$.

And $R_i = \max\left[w_i\left(f_i^* - f_{ij}\right)/\left(f_i^* - f_i^-\right) \right]$

$R_1 = 0.3610$, $R_2 = 0.27077$, $R_3 = 0.2708$, $R_4 = 0.0826$.

And then,

$$Q_j = \gamma\left(S_i - S^*\right)/\left(S^- - S^*\right) + (1 - \gamma)\left(R_j - R^*\right)/\left(R^- - R^*\right),$$

Where S^*=min S_j; S^- =max S_j; R^*=min R_j;R^- =max R_j,

S^*=0.18199, S^- =0.8684; R^*=0.0826, R^- =0.3610

$$Q_1 = \gamma\left(S_1 - S^*\right)/\left(S^- - S^*\right) + (1 - \gamma)\left(R_1 - R^*\right)/\left(R^- - R^*\right),$$

Q_1=1, Q_2=0.4683, Q_3=0.5885, Q_4=0.

The evaluation value of each supplier

	S_1	S_2	S_3	S_4
S	0.8684	0.36083	0.52584	0.18199
R	0.3610	0.27077	0.2708	0.0826
Q	1	0.4683	0.5885	0

*Step 6:*Rank suppliers in ascending order

Rank the suppliers by VIKOR

	S_1	S_2	S_3	S_4
S	4	2	3	1
R	4	2	3	1
Q	4	2	3	1

According to $Q(a^{(2)}) - Q(a^{(1)}) \geq 1/(m-1)$,
(where $Q(a^{(2)})$ is the suboptimal scheme in Q rank table and Q's VIKOR evaluation value),
We can get, $Q(S_2) - Q(S_4) = 0.4683 - 0$
$= 0.4683 \geq 1/(4-1)$ (here, m=4)
$= 0.4683 > 1/3$.

S_1 is the optimal solution in S or R rank tables with Q ranking has been set simultaneously. So the final ranking result according VIKOR algorithm and entropy method is $S_4 > S_2 > S_3 > S_1$.
∴ The best supplier is S_4.

VI. SUPPLIER SELECTION PROBLEM – VIKOR METHOD COMBINED WITH ENTROPY METHOD: NUMERICAL ILLUSTRATION

Entropy method:
To find weights we use the entropy method.
*Step 1:*Calculate p_{ij}

$$P_{ij} = \frac{r_{ij}}{\sum\limits_{j=1}^{m} r_{ij}},$$

$$\therefore p_{ij} = \begin{pmatrix} 0.2903 & 0.2647 & 0.2121 & 0.1851 & 0.2676 \\ 0.2580 & 0.2500 & 0.2575 & 0.3030 & 0.2535 \\ 0.2580 & 0.2500 & 0.2727 & 0.2323 & 0.2394 \\ 0.1935 & 0.2352 & 0.2575 & 0.2794 & 0.2394 \end{pmatrix}$$

*Step 2:*Calculate the entropy value e_j,

$$e_j = -k \sum_{i=1}^{m} P_{ij} \; \ln(P_{ij}), \; k = \frac{1}{\ln(m)}$$

here m=4, k =1/ln m =1/ln 4, $k = 0.72135$

Now we have to calculate the value of $\sum\limits_{r=1}^{m} p_{ij} \ln p_{ij}$

Calculation for each columns:

$\sum\limits_{r=1}^{m} p_{i1} \ln p_{i1} = -1.37594,$ $\sum\limits_{r=1}^{m} p_{i2} \ln p_{i2} = -1.38538,$ $\sum\limits_{r=1}^{m} p_{i3} \ln p_{i3} = -1.38196,$

$\sum\limits_{r=1}^{m} p_{i4} \ln p_{i4} = -1.36938,$ $\sum\limits_{r=1}^{m} p_{i5} \ln p_{i5} = -1.38516.$

Now we calculate the entropy value e_j,

$$e_j = -k \sum_{i=1}^{m} P_{ij} \; \ln(P_{ij}),$$

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$e_1 = (-1.37594)(-0.72135) = 0.99253$
$e_2 = (-1.38538)(-0.72135) = 0.99934$
$e_3 = (-1.38196)(-0.72135) = 0.99687$
$e_4 = (-1.36938)(-0.72135) = 0.98780$
$e_5 = (-1.38510)(-0.72135) = 0.99918$

Now we have to calculate the weights

$$w_j = \frac{(1-e_j)}{\sum_{j=1}^{n}(1-e_j)}$$

w_1=0.3077, w_2=0.0272, w_3=0.1289, w_4=0.5026, w_5=0.0336.
It can be easily seen that $\sum w_j = 1$
∴ w'=(0.3077, 0.0272, 0.1289, 0.5026, 0.0336)
Proceeding Step1 to Step 4 as in the previous illustrations, then we have,
*Step 5:*Calculate each suppliers S, R, Q value:

$$S_i = \sum_{i=1}^{n}(w_i(f_i * - f_{ij})/(f_i * - f_i^-))$$

S_1=0.9728, S_2=0.4541, S_3=0.54602, S_4=0.16007.

And $R_i = \max\left[w_i\left(f_i^* - f_{ij}\right)/\left(f_i^* - f_i^-\right)\right]$

R_1=0.5026, R_2=0.2308, R_3=0.30161, R_4=0.10062.
And   then,

$$Q_j = \gamma\left(S_i - S^*\right)/\left(S^- - S^*\right)+(1-\gamma)\left(R_j - R^*\right)/\left(R^- - R^*\right),$$

Where S^*=min S_j; S^-=max S_j; R^*=min R_j; R^-=max R_j,
S^*=0.16007; S^-=0.9728; R^*=0.10062; R^-=0.5026
Q_1=v$(S_1-S^*)/(S^--S^*)+(1-v)(R_1-R^*)/(R^--R^*)$,
Q_1=1, Q_2=0.34281, Q_3=0.48744, Q_4=0.

The evaluation value of each supplier

	S_1	S_2	S_3	S_4
S	0.9728	0.45410	0.54602	0.16007
R	0.5026	0.2308	0.30161	0.10062
Q	1	0.34281	0.48744	0

*Step 6:*Rank suppliers in ascending order

Rank the suppliers by VIKOR

	S_1	S_2	S_3	S_4
S	4	2	3	1
R	4	2	3	1
Q	4	2	3	1

According to $Q(a^{(2)})-Q(a^{(1)})\geq 1/(m-1)$,
(where $Q(a^{(2)})$ is the suboptimal scheme in Q rank table and Q's VIKOR evaluation value),
We can get, $Q(S_2)-Q(S_4)$ = 0.3428-0
                   = 0.3428 ≥ 1/(4-1)   (here, m=4)
                   = 0.3428 > 1/3.

S_1 is the optimal solution in S or R rank tables with Q ranking has been set simultaneously. So the final ranking result according VIKOR algorithm and entropy method is $S_4 > S_2 > S_3 > S_1$.

∴ The best supplier is S_4.

Table-1: Comparison of the three methods

METHOD	RANKING OF ALTERNATIVES
Method-1: Using VIKOR method	$S_4 > S_2 > S_3 > S_1$
Method-2: Using VIKOR method with SENSİTİVİTY analysis	$S_4 > S_2 > S_3 > S_1$
Method-3: Using VIKOR method with ENTROPY	$S_4 > S_2 > S_3 > S_1$

VII. CONCLUSION

The proposed research work has concentrated on issues and complexities in applying VIKOR method to real world problems like supplier selection problems in supply chain management. The general VIKOR method, Sensitivity analysis for VIKOR method was proposed and new algorithm was proposed for Multiple Attribute Decision Making efficiently. Various concepts and techniques related to Decision Making were presented in detail. A road map to Multiple Attribute Decision Making (MADM) was presented in detail. The development, leading from simple to concrete MADM techniques was the major concern of the work. Then the procedure for a general VIKOR method is discussed. In general the VIKOR method is known for its compromise kind of solutions. Then, a case study with the theory of selecting the best supplier in a supply chain management is analyzed with the help of the proposed algorithm of VIKOR method extended with a sensitivity analysis with changes taking place in weighting vector is presented. A numerical illustration is presented utilizing the VIKOR method for supplier selection problem. Finally a new solution method for VIKOR method combined with a sensitivity analysis was proposed for possible changes occurring between the weighting vectors with the same numerical illustration presented in previous sections. Also a numerical illustration was presented to explain the new proposed algorithm of VIKOR method with Entropy method.

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