# Analysis of the Different Medicinal Leaf with Fractal Dimension

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**Abstract**: Fractal analysis has been applied to describe various aspects connected with the complexity of plant morphology. In this work we determined the fractal dimension of leaves from various species of Peepal leaf, Castrol oil leaf, papaya leaf in order to characterize the structure/architecture of these leaves. The present study deals with the analysis of leaf shapes in terms of fractal geometry with medicinal leaves using the techniques of Image Processing. In this work we determined the fractal dimension of different leaves. The results are very informative.

Key Words: Image Processing, Peepal Leaf, Betel Leaf medicinal

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# **1. INTRODUCTION**

Obviously, some part of this huge data volume comes from digitization of analog data and important portion of this digitized analog data is digitized historical data including images. Ancient images are quite important and valuable resources in the digital humanities. Because through this data, digital humanity scientists can get a rich playground for their research. So, digitization and distribution of the documents in the form of digital image will have a great impact and will accelerate the ongoing research in digital humanities.

Fractals are complex geometric figures made up of small scale and large-scale structures that resemble one another. Generally, there are two types: geometric (regular) and nongeometric (irregular). A geometric fractal consists of large and small structures that resemble precise duplication of each other. In irregular fractals, there are also large and small structures, but they do not resemble to each other. Instead, the structures are geometrically related. Irregular fractals have many patterns in nature. Fractal geometry offers simple descriptions of some elaborate fern shapes[1]. Structures that grow by continually repeating simple growth rules are suitable model for L-systems and fractal analysis [2],[3],[4] . Talking about fractals we usually think of the fractal dimension. Topology is a branch of mathematics which has essentially been developed in 20th century. It deals with questions of form and shape from a qualitative point of view. Two of its basic notions are "dimension" and "homeomorphism". Topology deals with the way shapes can be pulled and distorted in a space that behaves like rubber [5].

At the turn of the last century it was one of the major problems in mathematics to determine what dimension means and which properties it has. And since then mathematicians have come up with some ten different notions of dimension: topological dimension, Hausdorff dimension, fractal dimension, self-similarity dimension, box-counting dimension, capacity dimension, information dimension, Euclidean dimension, and more. Some of them, however, make sense in certain situations, but not at all in others, where alternative definitions are more helpful. Sometimes they all make sense and are the same. In general, many researches mainly focus on only three of these dimensions, namely self-similarity dimension, compass dimension (also called divider dimension) and box-counting dimension. And also, of these three notions of dimension the box-counting dimension has the most applications in science. The reason for its dominance lies in the easy and automatic computability by machine. It is straightforward to count boxes and to maintain statistics allowing dimension calculation. The program can be carried out for shapes with and without self similarity [6].

Fractal geometry has found many applications in the sciences in the last few decades. Examples include the classification and analysis of dynamical systems, modeling of diffusion processes in statistical mechanics, classifying surface roughness, analyzing crack propagation in solids, and studying the spread of forest fires and infectious diseases, to list a very few. Study done by Bayırlı [7] calculated the fractal dimension of manganese dendrites that formed on surface of magnesite ore. These kinds of studies and applications have been becoming very popular in biological science and ecology as well [8], [9] analyzed more than 300 leaves from 10 tree species and reported several classical biometric descriptors as well as 16 fractal dimension features on digitized leaf silhouettes. It has been point out that properly defined fractal dimension based features may be used to discriminate

between species, especially when used together with other measures. Because of this, they can be utilized in computer identification systems and for taxonomical purposes [9].

**Introduction to medicinal plants:** India is a country known for ancient scripts, the number system ,invention of zero..and vedas.l Medicines in India are used by about 60 per cent of the world's population. These are not only used for primary health care not just in rural areas in developing countries, but also in developed countries as well where modern medicines are predominantly used. While the traditional medicines are derived from medicinal plants, minerals, and organic matter, the herbal drugs are prepared from medicinal plants only.

Use of plants as a source of medicine has been an ancient practice and is an important component of the health care system in India. In the Indian systems of medicine, most practitioners formulate and dispense their own recipes, hence this requires proper documentation and research. In west also the use of herbal medicines is growing with approximately 40 per cent of population reporting use of herb to treat medical diseases within the past year. General Public, academic and government interest in traditional medicines is growing rapidly due to the increase side effects of the adverse drug reactions and cost factor of the modern system of medicine.

## Fractal Key issues

- The fractal concept has improved our understanding of many phenomena in biomedical sciences.
- Recently, a fractal globule model describing the 3D configuration of the cell nucleus has been introduced.
- In histology and cytology, fractal characteristics of chromatin have been described during the last 20 years.
- Fractal features, such as the fractal dimension or the goodness-of-fit of the regression curve can easily be estimated in digitalized microscopic images.
- During carcinogenesis and tumor progression, the fractal dimension (FD) of chromatin usually increases.
- In various tumors, an increased FD is an unfavorable prognostic facture.
- The goodness-of-fit of the regression line may predict outcome, with a closer fit to the ideal fractal structure indicating a better prognosis.
- Accumulating genetic and epigenetic alterations cause progressive chromatin remodeling.
- A more complex chromatin structure, less heterochromatin and a less perfect self-organization of the nucleus are expected in more aggressive neoplasias.

#### **2. RELATED WORK**

The review article [10] emphasizes that the increasing number of applications of fractal theory in the environmental sciences reflects the recognized Importance of spatial and temporal scale to the study of ecological systems and processes. In this paper, we summarize the various algorithms that have been developed for estimating the fractal dimension of such natural phenomena as landscapes, soils, plant root systems, paths of foraging animals, and so forth. We also discuss the potential utility and limitations of a fractal approach, and outline how fractals have been used in ecology. The review article [11] on Recognition of plant leaf images is an important and difficult task. Extracting the texture feature of leaf images becomes the key to solve this problem in recent years. Considering some wavelet methods only focus on low-frequency sub-bands of images and some fractal dimension methods using a single exponent also cannot identify the images well, a novel wavelet fractal feature based approach for plant

leaf images recognition is proposed. Firstly, the preprocessed leaf images are pyramid decomposed with 5/3 lifting wavelet transform and sub images are obtained. Then fractal dimensions of each sub images

are calculated to be the wavelet fractal feature of leaf images. Finally back propagation artificial neural network is used to classify plant leaf images. The experimental results show that the proposed method can improve the performance for plant image recognition compared with methods using only wavelet or fractal dimension.

In [12] a review article on Fractal dimensions of leaves from Cercis canadensis L., Robinia pseudoacacia L., Amelanchier arborea (F.Michx.) Fernald, Prunus persica (L.) Batsch, Quercus alba L., Carpinus caroliniana Walter, Ficus carica L., Morus rubra L., Platanus orientalis L., and Ulmus rubra Muhl. were calculated. The values were then confirmed and compared by those obtained from box-counting method and the exponent values of density correlation function (first time in the literature). It is now proposed for the first time that there is a relationship between a fractal dimension of the leaf and a surface density of the image and was concluded that together with other measures, the fractal dimensions with surface density function could be used as a new approach to taxonomical study of plants.

In [13] studies on A novel method for detecting rice leaf disease using image processing technique called fractal dimension and chaos theory is proposed in this paper. The analysis of a diseased leaf is carried out according to its image pattern and fractal dimension, and especially boxcounting ratio calculation, and chaos, are applied to be able to identify the disease pattern's self-similarity and to recreate the fractal. The image's self-similarity is the disease infected one which is same as when it is fully infected. This method is proposed as preliminary information for the development of an early detection system or for developing knowledge based expert system or decision support system.

In [14] a study on Organisms supports continual exchange with the environment so that they maintain in a state far from their thermodynamic equilibrium. The plants maintain themselves under low entropy conditions, a necessary prerequisite to life. The concept of fractal dimension to describe structures, which look the same at all length scales, was first proposed by Mandelbrot Objects are usually referred to as self-similar to indicate their scale-invariant structure. The common characteristic of such fractal objects is that their length depends on the length scale used to measure it, and the fractal dimension tells us the precise nature of this dependence. Estimation of fractal dimension of leaf shape was recently performed form various authors. We estimated Fractal Dimension of different kinds of leaves looking at their inner structure until to the cellular nucleus. The results of the applied methodology resulted rather

satisfactory so that in following papers we will apply it to investigation of plant structures under different experimental conditions as plant stress and per oxidation

In [15] a review article Fractal analysis has been to describe various Applied Aspects connected with the Morphology of plant complexity. Tomato leaves and we have multiple sinuses Suggested That this complexity is related to the necessity of light penetration through the plant. The eggplant leaves have Between fractal dimension of the medium pepper and tomatoes. This Means That this Needs assure eggplant structure of light and temperature. From this it That Follows Higher complexity of tomatoes Need to Develop by comparison with the other studied species.

In [16] a review article the effect of noise on fractal dimension of digital images has been tested. Since fractal dimension is a measure of texture which is property of neighbourhood, it is interesting to check how noise affects the values of fractal dimension. For this purpose, three standard digital images have been used and Gaussian noise, salt and pepper noise and speckle noise have been applied to these images to generate noisy images. The fractal dimension values of actual images and noisy images have been estimated and compared. Various aspects related to the estimation of fractal dimension of digital images are discussed for noisy and non-noisy images. Since variation of noise makes sense for a local window in which fractal dimension is estimated, it is required to observe the noise effect in the window.

Some of the other works include ([17] to [21]).

#### **3. PROBLEM SPECIFICATION**

In the available literature some authors predict that canny method could be preferred as one of the best edge detection methods.

Based on this the statement, the present analysis is carried out to check the efficiency of the canny method

by taking the same different medicinal leaves.

While conducting the experiment the following procedure adapted.

Canny Edge Detection Algorithm.

The algorithm runs in 5 separate steps:

1. Smoothing: Blurring of the image to remove noise.

2. Finding gradients: The edges should be marked where the gradients of the image has large magnitudes.

3. Non-maximum suppression: Only local maxima should be marked as edges.

4. Double thresholding: Potential edges are determined by thresholding.

5. Edge tracking by hysteresis: Final edges are determined by suppressing all edges that are not connected to a very certain (strong) edge.

The main objective of the present study is to make a detailed analysis of different Indian Medicinal leaves by using the techniques of image processing methodologies. Different samples are taken and the experiments are conducted.

Edges characterize boundaries and are therefore a problem of fundamental importance in image processing. Edges in images are areas with strong intensity contrasts – a jump in intensity from one pixel to the next. Edge detecting an image significantly reduces the amount of data and filters out useless information, while preserving the important structural properties in an image. Canny edge detection algorithm is also known as the optimal edge detector. Canny's intentions were to enhance the many edge detectors in the image

. • The first criterion should have low error rate and filter out unwanted information while the useful information preserve.

• The second criterion is to keep the lower variation as possible between the original image and the processed image.

• Third criterion removes multiple responses to an edge.

Good detection accuracy:

• minimize the probability of false positives (detecting spurious edges caused by noise),

• false negatives (missing real edges) – Good localization:

• edges must be detected as close as possible to the true edges. – Single response constraint:

• minimize the number of local maxima around the true edge (i.e. detector must return single point for each true edge point)

The four steps of edge detection (1) Smoothing: suppress as much noise as possible, without destroying the true edges. (2) Enhancement: apply a filter to enhance the quality of the edges in the image (sharpening). (3) Detection: determine which edge pixels should be discarded as noise and which should be retained (usually, thresholding provides the criterion used for detection). (4) Localization: determine the exact location of an edge (sub-pixel resolution might be required for some applications, that is, estimate the location of an edge to better than the spacing between pixels). Edge thinning and linking are usually required in this step

edge(I,'approxcanny') detect edges using the approximate Canny method. The 'approxcanny' method is an approximate version of the Canny edge detection algorithm that provides faster execution time at the expense of less precise detection. For the approxcanny method, floating point images are expected to be normalized in the range [0 1].

Scalar value that specifies the standard deviation of the Gaussian filter. The default is sqrt(2). edge chooses the size of the filter automatically, based on sigma.

The Canny method applies two thresholds to the gradient: a high threshold for low edge sensitivity and a low threshold for high edge sensitivity. edge starts with the low sensitivity result and then grows it to include connected edge pixels from the high sensitivity result. This helps fill in gaps in the detected edges.

In all cases, edge chooses the default threshold heuristically, depending on the input data. The best way to vary the threshold is to run edge once, capturing the calculated threshold as the second output argument. Then, starting from the value calculated by edge, adjust the threshold higher (fewer edge pixels) or lower (more edge pixels).

# 4. METHODOLOGY

In order to make a detailed analysis of the leaves, the codes are written in Matlab 7.50 Version.

In order to determine the perimeter of a leaf, the outline of the leaf needed to be identified. After the image was converted into a binary image, an edge detector (using the Sobel approximation to the derivative, returning points where the gradient of the image is maximum) and boundary maker (tracing the exterior boundaries of objects) were used to create an image that is completely black with a pixel wide white outline of the leaf. This process did not ensure a continuous boundary thus a morphing application was used to "bridge" pixel gaps in the outline. With noted pixel locations outlining the leaf, "rulers" of standard measure were created by locating pixels "x" apart and the distance between these pixels was accumulated to determine the perimeter. The last distance that closed the outline was either calculated as stated or was determined by finding the distance between the last "x" away point and the first point used and added to the accumulation of distances to determine the perimeter. Vlcek and Cheung (1986) state that "Mandelbrot defined the fractal dimension, D, to be an inverse of a least-squares estimate of the slope of a line the equation: given in log curve segment or ruler, and (N  $\Box$ ) is the total length of the curve, and (1-D) is the slope of the line. Therefore the fractal dimension is found by subtracting the slope of the line from 1.

# 5. EXPERIMENTS AND RESULTS

## Perimeter method:

The perimeter method was used on leaves found on plants and trees. The color images, regardless of size, were converted to grayscale. With the use of several Matlab image processing functions, the outline of the leaf was obtained; white outline on black background. Another imaging procedure was used to obtain the pixels of the outline.

After plotting the outline of the leaf, the user is prompted to locate a pixel on the outline by using the outline figure and pixel information. For this particular example, the pixel (379, 186) was found to have a value of 1, "white" pixel, therefore on the outline. The point selected is noted in blue and the outline is "traced" in green westward from the initial point, indicating the continuous pixels identified as part of the outline.

The outline pixels are in a p x 2 array, where p is the number of "points", pixels, used the make the outline of the leaf.

The first and last "points" are the same, to enclose the leaf. The "ruler" is set to 50 pixels such that the distance from the first point (initial point) to the 51st point is calculated. The  $51^{st}$  point becomes the "initial" point and the distance from this initial point to the 51st point (the 101st point in the original list) is calculated, and so on. All these distances are accumulated. If the last initial point is not the last point on the list, the distance between the last initial point and the last point is found and added to the accumulated distances.

Column one of "data" contains the "ruler" sizes (50, 42, 33, 25, 16, 10 pixel count) while column two contains the perimeter the leaf using such rulers. The log of perimeter versus log of ruler size is plotted. The slope of the best fit line is subtracted from 1 to obtain the fractal dimension.

The experiments to analyze the leaves of medicinal plants are conducted by using Matlab (Version 7.5). In

Order to study the fractal dimension, number of pixels, Hibiscus leaves considered. The results are conducted different methods and presented in Tables 1 to 8, and Graphs 1 to 8 respectively.

The procedure adapt is getting values displayed in table 1 to table 8.

Same procedure is employed for the other method and results are presented in Tables 1 to 8 and Graphs 1 to 8 respectively.

# Figure 1 RGB & Gray Color Leaf



Table 1: Peepal Leaf

Sl.	Canny Method			
NO	Da	ata	Lambda	Distance
1	0.0500	1.3312	3.9120	7.1938
2	0.0420	1.3416	3.7377	7.2016
3	0.0330	1.3316	3.4965	7.1941
4	0.0250	1.3667	3.2189	7.2202
5	0.0160	1.3791	2.7726	7.2292
6	0.0100	1.3884	2.3026	7.2359

Number of Pixels: 1276 Fractal dimension: 1.0282





#### Table 2: Hisbiscus Leaf

Sl.				
INO	Da	ata	Lambda	Distance
1	0.0500	1.2029	3.9120	7.0925
2	0.0420	1.1494	3.7377	7.0470
3	0.0330	1.1843	3.4965	7.0769
4	0.0250	1.2268	3.2189	7.1121
5	0.0160	1.2343	2.7726	7.1182
6	0.0100	1.2898	2.3026	7.1622

Number of Pixels: 1213 Fractal dimension: 1.0561

#### Graph 2



#### Table 3: Karpooravalli Leaf

S1.		Canny	/ Method		
No	D	Data Lambda			
1	0.0500	2.7618	3.9120	7.9237	
2	0.0420	2.7768	3.7377	7.9290	
3	0.0330	2.8015	3.4965	7.9379	
4	0.0250	2.8104	3.2189	7.9411	
5	0.0160	2.8257	2.7726	7.9465	
6	0.0100	2.8498	2.3026	7.9550	

Number of Pixels: 2608 Fractal dimension: 1.0183





Table 4: Papaya Leaf

Sl.	Canny Method			
NO	Da	ata	Lambda	Distance
1	0.0500	1.2804	3.9120	7.1549
2	0.0420	1.3162	3.7377	7.1825
3	0.0330	1.3910	3.4965	7.2378
4	0.0250	1.4705	3.2189	7.2934
5	0.0160	1.5278	2.7726	7.3316
6	0.0100	1.5980	2.3026	7.3765

Number of Pixels: 1504 Fractal dimension: 1.1387

# Graph 4



 Table 5: Eucalyptus Leaf

Sl.	Canny Method				
INO	Da	ata	Lambda	Distance	
1	0.0500	4.9985	3.9120	8.5169	
2	0.0420	5.0038	3.7377	8.5180	
3	0.0330	4.9986	3.4965	8.5169	
4	0.0250	5.0399	3.2189	8.5251	
5	0.0160	5.0496	2.7726	8.5271	
6	0.0100	5.0602	2.3026	8.5292	

Number of Pixels: 4837 Fractal dimension: 1.0085





# Table 6: Betel Leaf

Sl.	Canny Method			
INO	Data		Lambda	Distance
1	0.0500	2.4427	3.9120	7.8009
2	0.0420	2.4525	3.7377	7.8048
3	0.0330	2.4545	3.4965	7.8048
4	0.0250	2.4772	3.2189	7.8149
5	0.0160	2.4807	2.7726	7.8163
6	0.0100	2.4933	2.3026	7.8214

Number of Pixels: 2238 Fractal dimension: 1.0126

## Graph 6



Table	7:	Castor	Oil	Leaf
	•••	0.0001	~	

Sl.	Canny Method			
INO	Γ	Data	Lambda	Distance
1	0.0500	5.550619	3.9120	6.3191
2	0.0420	6.487412	3.7377	6.4750
3	0.0330	5.895710	3.4965	6.3794
4	0.0250	6.212994	3.2189	6.4318
5	0.0160	6.971708	2.7726	6.5470
6	0.0100	7.327527	2.3026	6.5968

Number of Pixels: 718 Fractal dimension: 1.1483





# Table 8: Grape Leaf

Sl.	Canny Method				
INO	]	Data	Lambda	Distanc	
				e	
1	0.0500	354.0910	3.9120	5.8696	
2	0.0420	372.6178	3.7377	5.9206	
3	0.0330	416.2696	3.4965	6.0313	
4	0.0250	416.2696	3.2189	5.9807	
5	0.0160	472.4011	2.7726	6.1578	
6	0.0100	498.2344	2.3026	6.2111	

Number of Pixels: 474 Fractal dimension: 1.2096

# Graph 8



# Table 9

Sl.No	Name of the Leaf	Canny Method	
		Number	Fractal
		of Pixels	Value
01	Peepal Leaf	1276	1.0282
02	Hibiscus Leaf	1213	1.0561
03	Karpooravalli Leaf	2608	1.0183
04	Papaya Leaf	1504	1.1387
05	Eucalyptus Leaf	4837	1.0085
06	Betel Leaf	2238	1.0126
07	Castor Oil Leaf	718	1.1483
08	Grape Leaf	474	1.2096

#### CONCLUSION

Leaf veins are an important aspect of leaf structure and responsible for both the mechanical support of leaves and long distance of transport of water, nutrients and photo assimilates.

In this paper we pointed out the importance of fractal dimension of different medicinal leaves by using image processing techniques. Our results showed the different fractal dimension value with different irregular shapes in the leaves. The dimensional value of a fractal dimension on a plane is always between one and two.

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