Simulative Investigations on Log-Normal Turbulence Model for Free Space Optical Communication

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Abstract---The log-normal probability-density function is based on the following paradigm for the optical field after propagation through atmospheric turbulence, a field with minimised coherence that follows Rice-Nakagami statistics is modulated by a multiplicative factors those obeys log-normal statistics. The bigger eddies in the turbulent medium produce the log-normal statistics and as well as the smaller ones produce the Gaussian statistics. By investigating this model all the parameters required by the density function could be calculated by means of physical parameters such as turbulence strength inner scale, and propagation configuration. The heuristic density function is reliable with available data at low and at high turbulence levels.

Keywords---*Free Space Optics, Irradiance, Log-Normal, Probability Density Function, Turbulence,*

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I. INTRODUCTION

In general the probability-density function of optical scintillations will be log normal or very nearly so, after propagation through weak turbulence in the atmosphere. The hypothetical argument for this distribution is based on the method of smooth perturbations. In this rough calculation the effect of turbulence is to perturb the propagating wave by a large number of independent, multiplicative events and a central-limittheorem disagreement direct to the log-normal distribution. The data seem to concur relatively well with this prediction. In extreme conditions of weak turbulence, conservative perturbation theory should be valid i.e. by applying the central-limit theorem to this expansion leads to a prediction that the optical amplitude should obey Rice-Nakagami statistics' rather than log-normal ones [1]. Though the RiceNakagami distribution approaches the log-normal one as the variance becomes very small, that is the condition required by the Born approximation. This give the impression to resolve the conflict between the two expansions. Actually, The probability-density function after propagation through stronger turbulence or over greater distances is not understood deeply. Some of the early measurements suggested that the fluctuations remained nearly log normal even in strong turbulence. It has been predicted that the density function of irradiance should approach a negative exponential in the limit of infinite turbulence. The disagreement is that the field at the spectator is the sum of contributions from many independent scattering paths and should therefore have a circular Gaussian probability-density function according to the central limit theorem. Which shows that a Rayleigh density function of the optical amplitude and a negative exponential density function of irradiance [1]. But, if each contribution to the sum is a log-normal variate the sum could remain nearly log normal under rather high turbulence levels because of the slow central-limit-theorem convergence of log-normal variates. The extreme turbulence levels that produce Rayleigh amplitude statistics are not common and under typical strong-turbulence conditions these statistics do not appear to be precisely log normal [3]. This realization has led to a number of proposed density functions that are combinations of these. Although this considered a distribution consisting of an elliptical Gaussian field and a log-normal modulation factor. This is shown to provide a good description of radio propagation through the ionosphere.

II. LOG-NORMAL TURBULENCE MODEL

In describing the pdf of the irradiance fluctuation in a turbulent atmosphere, the beam is first represented by its constituent electric field $\vec{E}$ By employing Maxwell's electro-magnetic equations for the case of a spatially variant dielectric like the atmosphere, the following expression is derived [2]

$$\nabla^2\vec{E} + k^2 n_{as}^2\vec{E} + 2\nabla\left[\vec{E}.\vec{\nabla}In\left(n_{as}\right)\right] = 0 \qquad (1)$$

where the wave number $k = 2\pi/\lambda$, and the vector gradient operator

$\vec{\nabla} = (\partial/\partial x)i + (\partial/\partial y)j + (\partial/\partial z)k$ with i, j and k being the unit vectors along the x, y and z axes, respectively The last term on the left-hand side of Equation 3.97 represents the turbulence-induced depolarization of the wave In a weak atmospheric turbulence regime, which is haracterized by single scattering event, the wave depolarization is negligible [80,93,94] In fact, it has been shown both theoretically [2] and experimentally [3] that the depolarization is insignificant even for strong turbulence conditions Equation 1 then reduces to

$$\nabla^2 \vec{E} + k^2 n_{as}^2 \vec{E} = 0 \qquad (2)$$

The position vector will henceforth be denoted by r and $\vec{E}$ represented by $E(r)$ for convenience In solving this last equation, Another approach introduced a Gaussian complex variable $\Psi(r)$ defined as the natural logarithm of the propagating field $E(r)$, and termed it as the Rytov transformation That is,

$$\Psi(r) = In\left[E(r)\right] \qquad (3)$$

The Rytov approach is also based on a fundamental assumption that the turbulence is weak and that it is characterized by single scattering process By invoking the Rytov transformation (3), and equating the mean refractive index of the channel $n0$, to unity, Equation 2 transforms to the following Riccati equation whose solution already exists

$$\nabla^2 \Psi + (\nabla \Psi)^2 + k^2\left(1 + n_{as_1}\right)^2 = 0 \qquad (4)$$

The next stage involves breaking $\Psi(r)$ down to its free-space form $\Psi0(r)$, and its turbulence-induced departure form is represented by $\Psi1(r)$ This is done via the smooth perturbing method, which in effect implies that $\Psi(r) = 0(r) + \Psi1(r)$ Combining this with the Rytov change of variable (4) results in the following

$$\Psi_1(r) = \Psi(r) - \Psi_0(r) \qquad (5)$$

$$\Psi_1(r) = In\left[E(r)\right] - In\left[\frac{E(r)}{E_0(r)}\right] \qquad (6)$$

where the electric field and its free-space (without turbulence) form $E0(r)$ are by definition given as

$$E(r) = A(r)exp\left(i\phi(r)\right) \qquad (7)$$

$$E_0(r) = A_0(r)exp\left(i\phi_0(r)\right) \qquad (8)$$

where $A(r)$ and $\phi(r)$, and $A_0(r)$ and $\phi_0(r)$ represent the amplitude and phase of the actual field with and without atmospheric turbulence, respectively These transformations can then be used to arrive at the solution of Equation 1 which describes the behaviour of a field in weak atmospheric turbulence In finding the irradiance fluctuation statistical distribution, first combine Equations 3 and 4 to arrive at the turbulence-induced field amplitude fluctuation given below as [4]

$$\Psi_1(r) = In\left[\frac{A(r)}{A_0(r)}\right] + i\left[\phi(r) - \phi_0(r)\right] = x + iS \qquad (9)$$

Since $\Psi_1(r)$ is Gaussian, it follows therefore that, x is the Gaussian distributed log-amplitude fluctuation, and similarly S is the Gaussian distributed phase fluctuation of the field By concentrating only on the field amplitude, however, the pdf of x is thus

$$p(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} exp\left\{-\frac{\left(x - E[x]\right)^2}{2\sigma_x^2}\right\} \qquad (10)$$

where $E[x]$ denotes the expectation of x and σ_x^2 is the log-amplitude variance, commonly referred to as the Rytov parameter. Accordingly, the σ_x^2 which characterizes the extent of field amplitude fluctuation in atmospheric turbulence is related to the index of refraction structure parameter, the horizontal distance Lp, travelled by the optical field/radiation by the following equations

$$\sigma_x^2 = 0.56k^{7/6} \int_0^{L_p} C_n^2(x)\left(L_p - x\right)^{5/6} \qquad (11)$$

And

$$\sigma_x^2 = 0.56k^{7/6} \int_0^{L_p} C_n^2(x)(x/L)^{5/6}\left(L_p - x\right)^{5/6} dx \qquad (12)$$

For a field propagating horizontally through the turbulent medium, as is the case in most terrestrial applications, the refractive index structure parameter C_n^2, is constant, and the log irradiance variance for a plane wave becomes [5]

$$\sigma_l^2 = 1.23 C_n^2 k^{7/6} L_2^{11/6} \qquad (13)$$

The field irradiance (intensity) in the turbulent medium is $I = |A(r)|^2$ .while the intensity in free-space (no turbulence) is given by $I_0 = |A_0(r)|^2$, the log-intensity is then given by

$$l = log_e\left|\frac{A(r)}{A_0(r)}\right|^2 = 2x \qquad (14)$$

$$I = I_0\, exp(l) \qquad (15)$$

To obtain the irradiance pdf, invoke the transformation of variable p $p(I) = p(x)\left|\frac{dx}{dl}\right|$ to arrive at the log-normal distribution function given by

$$p(I)=\frac{1}{\sqrt{2\pi\sigma_l^2}}\frac{1}{I}exp\left\{-\frac{\left(In(I/I_0)-E[l]\right)^2}{2\sigma_l^2}\right\}I\geq 0 \quad (16)$$

In the region of weak fluctuations, the statistics of the irradiance fluctuations have been experimentally found to obey the log-normal distribution. From Equation 16 the log-intensity variance $\sigma_l^2=4\sigma_x^2$ and the mean log intensity $E[l]=2E[X]$. It follows that $E[\exp(l)] = E[I/I_0]=1$ since there is no energy loss during the turbulence-induced scattering process and, as such, $E[I] = I_0$ The expectation, $E[l]$, is obtained by invoking the standard Equation 17 which is valid for any real-valued Gaussian random variable [2] An expression for $E[l]$ is then obtained as illustrated in the following steps.

$$E\left[exp(az)\right]=exp(aE[z]+0.5a^2\sigma_z^2 \quad (17)$$

$$1=exp\left(E[l]+05\sigma_l^2\right) \quad (18)$$

Hence

$$E[l]=-\frac{\sigma_l^2}{2} \quad (19)$$

$$\sigma_l^2=E\left[I^2\right]-E[I]^2=I_0^2\left\{E\left[exp(2l)\right]-E\left[exp(l)^2\right]\right\} \quad (20)$$

By applying Equation 15 into Equation 19 and substituting for $E[l]$, the intensity variance
is obtained as [6]

$$\sigma_l^2=I_0^2\left[exp\left(\sigma_l^2\right)-1\right] \quad (21)$$

The normalized variance of intensity, often referred to as the scintillation index (*S.I.*), is thus

$$S.I.=\sigma_N^2=\frac{\sigma_l^2}{I_0^2}=exp\left(\sigma_l^2\right)-1 \quad (22)$$

III. RESULT AND DISCUSSION

The log-normal pdf is plotted in Figure 1 for different values of log-irradiance variance σ_l^2. As the value of σ_l^2 increases, the distribution becomes more skewed with longer tails in the infinity direction. This denotes the extent of fluctuation of the irradiance as the channel inhomogeneity increases. After obtaining the pdf of the irradiance fluctuation, it is also paramount to derive an expression for the variance of the irradiance fluctuation σ_l^2, which characterizes the strength of irradiance fluctuation.

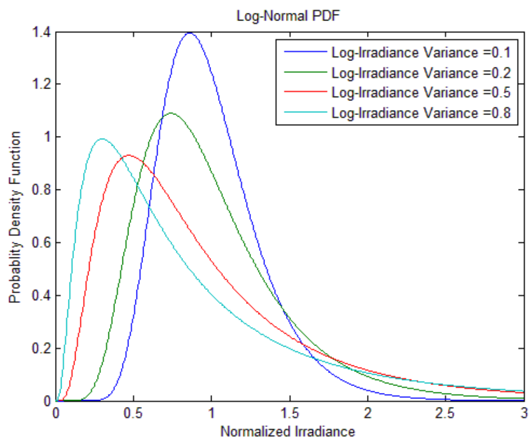

Fig. 1 Log-normal pdf with E[I] = 1 for a range of log irradiance variance σ_l^2.

IV. CONCLUSION

In weak turbulence the predominance of data proposes that the density function of intensity should be nearly log normal. The initiates that function does appear nearly log normal in this regime even if less than half of the variance is due to the log-normal modulation. The predicted intensity variance by this theory is nearly identical to the accepted value. In strong turbulence, the density function should be measured by using large data sets so that these low probability tails can be observed.

REFERENCES

[1] S. Hranilovic, Wireless Optical Communication Systems, New York:USA: Springer, 2005.
[2] G R Osche, Optical Detection Theory for Laser Applications New Jersey: Wiley, 2002.
[3] N. D. Chatzidiamantis, D. S. Michalopoulos, E. E. Kriezis, G. K. Karagiannidis, R. Schober, "Relay selection protocols for relay-assisted free-space optical systems", J. Opt. Commun. Netw., vol. 5, no. 1, pp. 92-103, Jan. 2013.
[4] R. Boluda-Ruiz, A. Garcia-Zambrana, C. Castillo-Vazquez, B. Castillo-Vazquez, "Adaptive selective relaying in cooperative free-space optical systems over atmospheric turbulence and misalignment fading channels", Opt. Express, vol. 22, no. 13, pp. 16629-16644, Jun. 2014.
[5] P. Puri, P. Garg, M. Aggarwal, "Analysis of spectrally efficient two-way relay assisted free space optical systems in atmospheric turbulence with path loss", Int. J. Commun. Syst, May 2014.
[6] M. A. Khalighi, M. Uysal, "Survey on free space optical communication: a communication theory perspective", IEEE Commun. Surv. Tut., vol. 16, no. 4, pp. 2231-2258, Jun. 2014.