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A Study on Euclidean Space and Affine Space

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Abstract: In this thesis, a Euclidean space is a finite-dimensional real vector space with an inner product.But when we analysed deeply, they are quite different as discussed in this paper. The objects of study in advanced calculus are differentiable functions of several variables.This approach also has the advantage of easily allowing the generalization of geometry to Euclidean space of more than three dimensions.

Keywords — Euclidean vector space, Affine Transform, inner product, Euclidean geometry

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I. INTRODUCTION

Euclidean space is the fundamental space of geometry. Originally, this was the three-dimensional space of Euclidean geometry, but, in modern mathematics, there are Euclidean space of any non-negative integer dimension, including the three-dimensional space and the Euclidean plane (dimension two).

It has been introduced by the Ancient Greek mathematician Euclid of Alexandria, and the qualifier Euclidean has been added for distinguishing it from other spaces that are considered in physics and modern mathemati cs.

Their great innovation was to prove all properties of the space (theorems) by starting from a few fundamental properties, called postulates, which either were considered as evidences (for example, there is exactly one straight line passing through two points), or seemed impossible to prove (parallel postulate). After the introduction at the end of 19th century of non-Euclidean geometries, the old postulates have been formalized for defining Euclidean spaces through an axiomatic theory. Another definition of Euclidean spaces by mean of vector spaces and linear algebra has been shown to be equivalent to the axiomatic definitions. This is this definition that is more commonly used in modern mathematics, and detailed in this article.

For all definitions, Euclidean spaces consist of points, which are defined only by the properties that they must have for forming a Euclidean space. There is essentially only one Euclidean space of each dimension; that is, all Euclidean spaces of a given dimension are isomorphic. Therefore, in many cases, it is possible to work with a specific Euclidean space. isomorphism from a Euclidean space associates to each point a n-tuple of real numbers, which locate them in the Euclidean space and are called Cartesian coordinates.

Euclidean space can, as one possible choice of representation, be modelled using Cartesian coordinatesEuclidean spaces

have finite dimension. In geometry, an affine spaces which preserves points, straight lines and planes.

We shall study the **“Euclidean space and affine space”** is interesting topics in geometry and develops necessary general preliminaries for the study of followings.

Chapter I deal with some basic definitions of “Euclidean space of vector space ”.

Chapter II introduce the concept of “Euclidean space” and necessary definitions and theorems.

Chapter III introduce “the concept of affine space” and necessary definition and theorems and also provide a conclusion and Bibliography.

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II. CHAPTER – II

PRELIMINARY DEFINITIONS AND THEOREMS

DEFINITION:2.1

An inner product or a dot product on a vector space V is a map

$\langle\ ,\ \rangle$:V x V→R satisfying the following properties :For x,y,$z\epsilon$ V a nd αϵR,

i. $\langle x,x\rangle \geq 0$ and $\langle x,x\rangle = 0 \Leftrightarrow x = 0$
ii. $\langle x,y\rangle = \langle y,x\rangle$
iii. $\langle x+z,y\rangle = \langle x,y\rangle + \langle z,y\rangle$ and $\langle x,y+z\rangle = \langle x,y\rangle + \langle x,z\rangle$
iv. $\langle \alpha x,y\rangle = \alpha\langle x,y\rangle$

(V,$\langle , \rangle$) is called an **inner product space**.

In particular , the pair (R^n, •) is called the Euclidean n-space.

DEFINITION:2.2

A Euclidean space is defined a fixedsymmetric bilinear form is positive definite.

The vector space itself will be denoted as a rule by L, and the fixed symmetricbilinear form will be denoted by (x, y). Such an expression is also called the inner product of the vectors x and y.

A Euclidean space is a real vector space L in which to every pair of vectors xand y there corresponds a real number (x, y) such that the following conditions are satisfied :

i. $(x_1 + x_2, y) = (x_1, y) + (x_2, y)$ for all vectors $x_1, x_2, y \in L$.
ii. $(\alpha x, y) = \alpha(x, y)$ for all vectors $x, y \in L$ and real number α.
iii. $(x, y) = (y, x)$ for all vectors $x, y \in L$.
iv. $(x, x) > 0$ for $x \neq 0$.

THEOREM: 2.1

For arbitrary vectors x and y in a Euclidean space, the following

inequality holds:

$(x, y) \leq |x| \cdot |y|$.

PROOF:

If one of the vectors x, y is equal to zero, then the inequality is obvious,

and is reduced to the equality 0 = 0.

Now suppose that neither vector is the null vector.

In this case,

let us denote by αy the orthogonal projection of the vector x onto the line$\langle y \rangle$.

(Given a vector $e \neq 0$, every $x \epsilon L$ can be expressed in the form

$x = \alpha e + y$, $(e,y) = 0$ for some scalar α and vector $y \epsilon L$.)

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The relationship x = αy + z,

Where (y, z) = 0.

From this we obtain the equality

$$(x, y) = (\alpha y + z, y)$$
$$= (\alpha y, y)$$
$$= \alpha|y|^2 .$$

This means that

$$|(x, y)| = |\alpha| \cdot |y|^2$$
$$= |\alpha y| \cdot |y|.$$

the inequality

$$|\alpha y| \leq |x| \qquad [\because |x| \geq |\alpha e|]$$

and consequently,

$$|(x, y)| \leq |x| \cdot |y|.$$

Hence the theorem .

III. CHAPTER-III

AFFINE SPACE
DEFINITION: 3.1

A sequence $\{x_n\}$ is said to be **Cauchy's sequence** if given $\in > 0$ there exists $N \in z^+$ such that $\|x_n - x_m\| < \in$, $\forall n, m \in N$

DEFINITION: 3.2

An **affine subspace** is a non empty set such that for all x,y ϵ S,the line joining x and y also lies in S. This means that if x,y ϵ S,then t x + (1- t) y ϵ S for all t ϵ R.

Where, t ϵ R is arbitrary,

S need not to be a vector space.

THEOREM : 3.1

A non empty subset S of V is an affine space if and only if it is of the form v + w for some v ϵ V and a vector subspace W of v.

PROOF :

NECESSARY PART :

Let W be a vector subspace of V.

Let v ϵ V be a fixed , then the set S: = v + w is an affine space. For let x, y ϵ S be arbitrary.

Then , x = v + w_1 and y = v + w_2 for some w_i ϵ W .

The line joining x and y is the set 1(x, y) := { t x + (1 –t) y | t ϵ R }. Let

z ϵ 1(x, y)

To show that z ϵ S. For this , to show that z is of the form v + w for some w ϵ W. Since, z ϵ 1(x, y) ,z = t x + (1 –t) y for some t ϵ R.

Consider that,

$$Z = t\,(v+w_1) + (1 - t)\,(v+w_2)$$

$$= (t + 1 - t)\,v + t\,w_1 + (1 - t)\,w_2$$

$$= v + w$$

Where, w: $= t\,w_1 + (1-t)\,w_2$

Since, W is a vector subspace and w ϵ W. Thus, z is of the form v + w for some w ϵ W and hence z ϵ S.

Therefore, the necessary part is proved.

SUFFICIENT PART:

Consider S = v + W Where v ϵ S. Which is written in the form v = v +0 ϵ v + W, then S = v + W implies that

$$W = S - v$$

This suggest the following approach.

Fix v ϵ S. Consider W : = S –v

To show that W is a vector subspace. Clearly,0 ϵ W , as 0 = v – v ϵ S-v

If w_i ϵ W, to show that $w_1 + w_2$ ϵ W. Which can be written in the form of

$w_1 = x - v$ and $w_2 = y - v$ for some x, y ϵ S.

Now, $w_1 + w_2 = (x - v) + (y - v)$

$$= x + y - v - v$$

This will be in s, when x + y-v ϵ S for x, y, v ϵ S.

Consider, $w_1 + w_2 = z - v$ for some z ϵ S, that is it lies in W. To show that, if w ϵ W and α ϵ R, then α w ϵ W.

Let w = x – v for x ϵ S.

Then, α w = α x – α v

$$= \alpha x - (\alpha - 1)\, v$$

$$= \alpha x + (1 - \alpha)\, v - v$$

Since, S is affine and x, v ϵ S which show that, α x + (1 – α) v ϵ S.

Thus, the displayed equation shows that α w = z – v.

Where, z = α x + (1 – α) v ϵ S. Hence, α w ϵ W.

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W is a vector subspace is proved.

IV. CONCLUSIONS

To set the stage for the study, the Euclidean space as a vector space endowed with the dot product is defined. To aid visualizing points in the Euclidean space, the notion of a vector is introduced Euclidean motions, mappings preserving the Euclidean distance, are briefly discussed. In this thesis, we discussed how to implement different types of Affine Tranform class, which allows the functions to be combined in any order without much coding.

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