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Isomorphism on Complementary Fuzzy Graphs

Mr. B. Senthilkumar
M.Sc., M.Phil., B.Ed.,
Assistant Professor,
Department of Mathematics,
PRIST University, Thanjavur

R. Maria Amala Jothi
M.Phil Scholar,
Department of Mathematics,
PRIST University, Thanjavur

Abstract—In this thesis, the order, size and degree of the nodes of the isomorphic fuzzy graphs are analysed. Isomorphism between fuzzy graphs are examined. Some properties of self-complementary and self-weak complementary fuzzy graphs are reviewed.

Keywords— *fuzzy graphs, weak isomorphism, co weak isomorphism, self-complementary and self-weak complementary fuzzy graphs.*

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I. INTRODUCTION

Graph theory is proved to be tremendously useful in modeling the essential features of systems with finite components. Each object is represented by a vertex and the relationship between them is represented by an edge if the relationship is unordered and by means of a directed edge.

Relationship among the objects need not always be precisely defined criteria; when we think of an imprecise concept, the fuzziness arises. In 1965, L.A. Zadeh. Introduced a mathematical frame work to explain the concept of uncertainty in real life through the publication of a seminal paper. The fuzzy graph introduced by A. Rosenfeld [56] using fuzzy relation, represents the relationship between the objects by precisely indicating the level of the relationship between the objects of the given set. P. Bhattacharya in showed that a fuzzy graph can be associated with a fuzzy group in a natural way as an automorphism group. K.R .Bhutani introduced the concept of weak isomorphism and isomorphism between fuzzy graphs.

The objects are represented by vertices and relations by edges. So the graphs are simply models of relations.When there is vagueness in the description of objects or in its relationships or in both, it is natural to assign a "ISOMORPHISM ON FUZZY GRAPHS".

Generally an undirected graph is a symmetric binary relation on a non- empty vertex set V. A symmetric binary fuzzy relation on a fuzzy subset is defined fuzzy graph (undirected). Rosenfeld considered fuzzy relation on fuzzy sets and developed the theory of fuzzy graphs in 1975.

Although the first definition of fuzzy graphs was given by Kaufman,R.T.Yeh and S.V.Bang have also introduced various connectedness concepts in fuzzy graphs during the same time. The concept of isomorphism was also introduced by R.T.Yeh and S.V.Bang**.**

Zadeh's ideas have found application in computer science, artificial intelligence, decision analysis, information science, system science, control engineering, expert systems, pattern recognition, management science, operation research and robotics.

In this dissertation work,

Chapter I defines some basic concepts in fuzzy graph theory.

Chapter II deals with the concept of isomorphism on fuzzy graphs.

Chapter III discusses the isomorphism between fuzzy graphs and its complement.

Chapter IV illustrates the definition of self-complementary fuzzy graphs and few results regarding self- complementary fuzzy graphs.

Chapter V deals with self-weak complementary fuzzy graphs and its properties.

This dissertation work consists of same basic concepts of fuzzy graphs and isomorphism on fuzzy graphs. This dissertation also explains role played by isomorphism in the concept of self-complementary and self-weak complementary fuzzy graphs.

CHAPTER-I

PRELIMINARIES

Definition 1.1

Let U be a non-empty set, to be called the universal set (or) the universe of discourse (or) simply a domain.

Then by a fuzzy set on U is meant a function, $A : U \to [0,1]$. 'A' is called the **membership function**, A(x) is called the **membership grade** of x.

We also write,

$$A = \{x, A(x) : x \in U\}$$

Definition 1.2

$\sigma$is said to be **fuzzy subset** of τ, written as $\sigma \subseteq \tau$, if $\sigma(x) \leq \tau(x)$ for every $u \in V$. Hereσ and τ be two fuzzy sets of a set V.

Definition 1.3

A **fuzzy graph** $G : (\sigma, \mu)$ is a pair of functions $\sigma : S \to [0,1]$ and $\mu : S \times S \to [0,1]$, we have,

$$\mu(x, y) \leq \sigma(x) \wedge \sigma(y), \forall x, y \in S.$$

Here σ is a subset of a non-empty set S and μ is a symmetric fuzzy relation on σ.

Definition 1.4

A fuzzy graph $H : (\gamma, t)$ is called a **fuzzy subgraph**of $G : (\sigma, \mu)$ if

$$\gamma(v) \leq \sigma(v), \forall v \in S, t(x, y) \leq \mu(x, y), \forall x, y \in S.$$

CHAPTER-II
ISOMORPHISM ON FUZZY GRAPH
Definition 2.1

A **homomorphism of fuzzy graphs** $h : G \to G'$ is a map $h : S \to S'$ which satisfies

$$\sigma(x) \leq \sigma'(h(x)), \forall x \in S \text{ and}$$
$$\mu(x, y) \leq \mu'(h(x), h(y)), \forall x, y \in S.$$

Definition 2.2

An **isomorphism** $h : G \to G'$ is a map $h : S \to S'$ which is a bijective that satisfies

$$\sigma(x) \leq \sigma'(h(x)), \forall x \in S \text{ and}$$
$$\mu(x, y) \leq \mu'(h(x), h(y)), \forall x, y \in S.$$

We denote it as G≅G'

Definition 2.3

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A **weak isomorphism** $h: G \to G'$ is a map $h: S \to S'$ which is a bijective homomorphism that satisfies

$$\sigma(x) \le \sigma'(h(x)), \forall x \in S$$

Definition 2.4

A **co-weak isomorphism** $h: G \to G'$ is a map $h: S \to S'$ which is a bijective homomorphism that satisfies

$$\mu(x, y) \le \mu'(h(x), h(y)), \forall x, y \in S.$$

Theorem: 2.5

If there is a weak isomorphism between G and G' then there is a weak isomorphism between G' and $\bar{G}$.

Proof:

If h is a weak isomorphism between G & G' then, $h: S \to S'$ is a bijective map that satisfies, $h(x) = x, \forall x \in S.$

$$\sigma(x) = \sigma'(h(x)), \forall x \in S \text{ and}$$
$$\mu(x, y) \le \mu'(h(x), h(y)), \forall x, y \in S$$

As $h^{-1}: S \to S'$ is also bijective for every x' in S' satisfies, $h^{-1}(x') = x', \forall x \in S.$ then

Equation (a) $\Rightarrow \sigma'(x') = \sigma(h^{-1}(x')), \forall x' \in S'$

And using the equations a and b in the complement of a fuzzy graph

$$(i.e)\overline{\mu}(u, v) = \sigma(u) \wedge \sigma(v) - \mu(u, v)$$
$$\Rightarrow \overline{\mu}(h^{-1}(x), (h^{-1}(y))) = \sigma(x) \wedge \sigma(y) - \mu(x, y) \qquad \text{---------(c)}$$

We have, $\mu(x, y) \le \mu'(h(x), h(y)), \forall x, y \in S$

$$\Rightarrow \sigma(x) \wedge \sigma(y) - \mu(x, y) \ge \sigma(x) \wedge \sigma(y) - \overline{\mu}(h(x), h(y)), \forall x, y \in S$$

From the above result,

$$C \Rightarrow \overline{\mu}(h^{-1}(x'), (h^{-1}(y'))) \ge \sigma(x) \wedge \sigma(x) - \overline{\mu}(h(x), h(y)), \forall x, y \in S$$
$$\Rightarrow \overline{\mu}(h^{-1}(x'), (h^{-1}(y'))) \ge \sigma'(h(x)) \wedge \sigma'(h(y)) - \overline{\mu}(h(x), h(y))$$
$$\Rightarrow \overline{\mu}(h^{-1}(x'), (h^{-1}(y'))) \ge \sigma'(x') \wedge \sigma'(y') - \overline{\mu}(x', y')$$

Thus we have,

$$\overline{\mu}(h^{-1}(x'), (h^{-1}(y'))) \ge \overline{\mu}(x', y'), \forall x', y' \in S'$$
$$(i.e)\overline{\mu'}(x', y') \le \mu'(h^{-1}(x'), h^{-1}(y'))$$

Thus $h: S \to S'$ is a bijective map $\bar{G}'$onto $\bar{G}$ satisfying and

Hence $\bar{G}'$onto $\bar{G}$ are weak isomorphic (i.e) there exists a weak isomorphism between $\bar{G}'$ and $\bar{G}$.

CHAPTER-III

SELF-COMPLEMENT FUZZY GRAPHS

Definition 3.1

A fuzzy graph G is said to be **self-complementary** if $\overline{G} \cong \overline{G'}$.

Theorem 3.2

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Let $G:(\sigma,\mu)$ be a fuzzy graph. If $\mu(u,v)=\frac{1}{2}(\sigma(u)\wedge\sigma(v)), \forall u,v\in S$, then is self- complementary.

Proof:

Let $G:(\sigma,\mu)$ be a fuzzy graph such that

$$\mu(u,v)=\frac{1}{2}(\sigma(u)\wedge\sigma(v)), \forall u,v\in S$$

Then by the definition of complement we have,

$$\overline{\mu}(u,v)=\sigma(u)\wedge\sigma(v)-\mu(u,v)$$

$$=\sigma(u)\wedge\sigma(v)-\frac{1}{2}(\sigma(u)\wedge\sigma(v))$$

$$=\frac{1}{2}(\sigma(u)\wedge\sigma(v)), \forall u,v\in S$$

$$\overline{\mu}(u,v)=\mu(u,v), \forall u,v\in S$$

Therefore G is isomorphic to $\bar{G}$ under the identity map on S.

Thus if $\mu(u,v)=\frac{1}{2}(\sigma(u)\wedge\sigma(v)), \forall u,v\in S$

Then $\overline{G}\cong\overline{G'}$

Hence the theorem.

II. CONCLUSION

In this thesis, isomorphism between fuzzy graphs is proved to be an equivalence relation and weak isomorphism is proved to be a partial order relation. Similarly it is expected that coweak isomorphism can be proved to be a partial order relation. A necessary and then a sufficient condition for a fuzzy graph to be self weakcomplementary are reviewed. The results considered may be used to study about several fuzzy graph invariants.

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