A Study on Graph Theory of Path Graphs

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ABSTRACT: In a simple graph the G is define as G = (V, E), here V is known as non-empty set of vertices and E is consider as edges. It is the set of unordered combination of unique elements of V. A simple graph has their points of confinement in demonstrating this present reality. Rather, we use multigraphs, which comprise of vertices and undirected edges between these vertices, with various edges between sets of vertices permitted. In this field of diagram hypothesis, a path graph or straight diagram is a graph whose vertices can be recorded in the request $v_1, v_2, \ldots\ldots, v_n$ to such an extent that the edges $\{v_1, v_{i+1}\}$ are the place I = 1, 2, … , n − 1. Proportionally, a way with in any event two vertices is associated and has two terminal (vertices that have degree 1), while all others (assuming any) have degree 2.

The path graph $P_k(G)$ of a diagram G is acquired by depicting the path P_k in G by vertices and joining two vertices when the comparing path in G structure a path P_{k+1} or a cycle C_k.

The path graph $P_k(G)$ of a graph G is obtained by describing the paths P_k in G by vertices and joining two vertices when the corresponding paths P_k in G form a path P_{k+1} or a cycle C_k.

Key words: *Graph, vertices, edges, degree, path.*

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I. INTRODUCTION

From the start, the helpfulness of Euler's thoughts and of "Graph theory" itself we just tackling riddles and in examining games and different entertainments. In 1800s, in any case, individuals started to understand that graphs could be utilized to show numerous things that were of enthusiasm for society.

For example, in 1852 DeMorgan presented the "Four Color Map Conjecture", was a well known issue that was apparently disconnected to graph theory.

The guess expressed that four is the most extreme number of hues required to color any guide where flanking districts are hued in an unexpected way.

This guess can without much of a stretch be expressed as far as chart hypothesis, and numerous analysts used this methodology during the dozen decades that the issue stayed unsolved.

Graph theory field is started in the 20th century as increasingly demonstrating potential outcomes were perceived and the development proceeds. It is intriguing to take note of that as explicit applications have expanded in number and in extension, the hypothesis itself has grown wonderfully also.

When thought about an irrelevant part of topology, graph theory has since a long time ago supported its reality through numerous significant commitments to a wide scope of fields.

Wealthy in intriguing issues and applications, it is presently one of the most examined and quickest developing territories inside discrete arithmetic and software engineering.

Outline of the dissertation:

This dissertation is divided into four chapters.

- In chapter I, we investigate some basic definitions.
- In chapter II, we will discuss the regions of path graphs definitions and their theorems.
- In chapter III, we discuss about the Hamilton path graphs definitions and their theorems.
- In chapter IV, we discuss about the Euler path graphs definitions and their theorems.

CHAPTER – I

PRILIMINARIES

Definition: 1.1

A graph is denoted by G. it is define as G = (V, E), it is a non empty finite set of V. here V is called vertices and

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E is the unordered pairs of unique vertices, this is known as edges.

A point is known as vertex.

A line that uses to join any two points is known as edge.

Definition: 1.2

An edge with indistinguishable ends is known as a loop or Aloop is an edge that interfaces a vertex to itself. In the event that a graph has more than one edge joining some pair of vertices then these edges are called multiple edges**.**

Definition: 1.3

A simple graph in which respective pair of distinct vertices is connected by an edge is called a ***complete graph.***

Definition: 1.4

A simple graph is a graph that does not have more than one edge between any two vertices and no edge begins and finishes at a similar vertex. At the end of the day a simple graph is a diagram without multiple edges and loops.

CHAPTER – II

SOME THEOREMS OF PATH GRAPHS

Definition: 2.1

Let G be a graph. A walk w in G is called a ***path.*** If all its vertices are distinct. (or)

A ***path*** is a sequence of vertices with the property that each vertex in the sequence is adjacent to the vertex next to it. A path that does not repeat vertices is called a ***simple path.***

Theorem: 2.2

In a graph G with vertices u and v, every u-v walk contains a u-v path.

Proof:

Let W be a u-v walk in G.

We Prove that,

This theorem by induction on the length of W.If W is of length 1 or 2.Then it is easy to see that W must be a path.

For the induction hypothesis,

Suppose the result is true for all walks of length less than k.

And suppose W has length k.We say that is,

$$u = w_0, w_1, w_2, \ldots, w_{k-1} = v$$

Where the vertices are not necessarily distinct.

If the vertices are in fact distinct.Then W itself is desired u-v path.If not, then let j be the smallest integer.

Such that,

$u = w_0, \ldots\ldots, w_i, w_{r+1}, \ldots, w_k = v$. This walk has length strictly less than.

Therefore, the induction hypothesis implies that,W_1 contains a u-v path.This means that,W contains a u-v path.

Hence the proof is complete.

CHAPTER - III

HAMILTONIAN PATH GRAPHS

Definition: 3.1

A Hamiltonian path is known as a path of graph between two vertices of graph that visits every vertex precisely once. The subsequent diagram cycle is called Hamiltonian cycle if a Hamiltonian path of exists whose endpoints are contiguous.

Definition: 3.

A graph G known as bipartite if its vertex set can be divided into two sets X and Y so that each edge of G has one end vertex in X and the other in Y. For this situation, X and Y are known as the partite sets.

Theorem: 3.3

If a graph contains no odd cycles than graph is bipartite with minimum two vertices.

Proof:

Let G be a bipartite graph with partite sets X and Y.Let C be a cycle of G and say that C is $v_1, v_2, \ldots\ldots, v_k, v_1$.

Assume without loss of generality that $v_1 \in X$. The nature of bipartite graphs implies then that $v_i \in X$ for all odd i and $v_i \in Y$ for all even i.

Since v_k is adjacent to v_1. It must be that k is even.Hence C is an even cycle.

For the reverse direction of the theorem,Let G be a graph of order at least two such that G contains no odd cycles.

Without loss of generality,

We can assume that G is connected for if not, we could treat each of its connected components separately.

Let v be a vertex of G.Define the set X to be

$$X = \{x \in V(G) / \textit{the shortest path from } x \textit{ to } v \textit{ has even length}\}, Y = V(G) \backslash X.$$

Now let x and x' be vertices of X and Suppose that,x and x' are adjacent.If x=v then the shortest path from v to x' has length one.

But this implies that, $x' \in Y$ a contradiction.So, it must be that $x \neq v$ and by a similar argument, $x' \neq v$

Let P_1 be a path from v to x of shortest length (a shortest v-x path). Let P_2 be a shortest $v - x'$ path.

Say that P_1 is $v = v_0, v_1, \ldots\ldots, v_{2k} = x$ and P_2 is $v = w_0, w_1, \ldots\ldots, w_{2t} = x'$ The path P_1 and P_2 certainly have v in common. Let v' be a vertex on both paths such that the $v' - x$ path. Call it P_1' and $v' - x'$ path.Call it P_2', have only the vertex v' in common.

Essentially, v' is the "last" vertex common to P_1 and P_2 .It must be that P_1' and P_2' are shortest $v' - x$ and $v' - x'$ paths, respectively.

It must be that $v' = v_i = w_i$ for some i.But since x and x' are adjacent.

$$v_i, v_{i+1}, \ldots\ldots\ldots, v_{2k}, w_{2t}, w_{2t-1}, \ldots\ldots, w_i$$

is a cycle of length $(2k - i) + (2t - i) + 1$

Which is odd, and that is a contradiction.Thus, no two vertices in X are adjacent to each other.

A similar argument shows that no two vertices in Y are adjacent to each other.Therefore, G is bipartite with partite sets X and Y.

II. CONCLUSION

The idea of this thesis was to find out the path graphs. Then we proved some important definitions and properties of path graphs. There are various problems connected with the work done here, worth investigating.

This work have taken up a problem to define the Hamiltonian and Euler path graphs for the path graphs obtained by means of the graphs operations on paths and cycles.

To derive similar results in the context of the other variants of path is an open area of research.The interested reader is encouraged to refer to them and the references in them for more details

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