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A Semi-Total Domination Number of a Graph

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Abstract: This thesis work on the two parameters that is very important domination parameters, one parameter is known as domination number and other parameter is called as total domination number. S is defined as a set of vertices in a graph G. We characterize a set S of vertices in a graph G with no segregated vertices to be a semitotal overwhelming arrangement of G in the event that it is a ruling arrangement of G and furthermore every vertex in S is inside separation 2 of another vertex of S. The semitotal domination number, indicated by is the base cardinality of a semitotal ruling arrangement of G. We demonstrate that on the off chance that G is an associated graph on n ≥ 4 vertices, at that point and we describe the trees and diagrams of least degree 2 arriving at this bound.

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I. INTRODUCTION

In this paper we investigate a parameter that is a reinforcing of control yet an unwinding of both total domination and pitifully associated domination. These ideas are characterized as pursues. A dominating set of a graph G is a set S of vertices of G with the end goal that each vertex in is contiguous at any rate one vertex in S.

The domination number of G, defined by $\gamma(G)$ is the minimum cardinality of a dominating set. A weakly connected dominating set, abbreviated WCD-set, is a dominating set S with the property that the graph hS_{iw} is connected, where hS_{iw} has vertex set S and its neighbours, and edge set those edges incident with S.

Thus, for any WCD-set of cardinality at least 2, every vertex in S must be distance at most two from a vertex of S. The weakly connected domination number of G, denoted by $\gamma w(G)$ is the minimum cardinality of a WCD-set.

A total dominating set, abbreviated TD-set, of G is a set S of vertices of G such that every vertex in V (G) is adjacent to at least one vertex in S. The total domination number of G, it is defined by $\gamma t(G)$ and it is the minimum cardinality of a TD-set.

We characterize here a set S of vertices in a graph G with no separated vertices to be a semitotal dominating set, curtailed semi-TD-set, of G on the off chance that it is a dominating set of G. At that point, each vertex in S is inside separation 2 of another vertex of S. The semitotal domination number, signified by is the base cardinality of a semi-TD-set.

A semi-TD-set of G of cardinality $\gamma t^2(G)$ we call a $\gamma t^2(G)$ set. For the rest of this paper, we assume that graphs have no isolated vertices. Every semi-TD-set is also a dominating set. Similarly, every TD-set also every WCD-set of cardinality at least 2, is a semi-TD-set.

In this way the below the semitotal domination number is bounded by the domination number and above by both the total domination number and the maximum of 2 and the weakly connected domination number, If G is a graph with no isolated vertices, then

$$\gamma(G) \le \gamma t^2(G) \le \gamma t(G) \max(\gamma w(G),2)$$

We proceed as follows, we provide bounds on the parameter and we show that if G is a connected graph on n ≥ 4 vertices, then $\gamma t^2(G) \le n/2$ and weshall characterize the trees, graphs of minimum degree 2 achieving this bound. We provide more bounds and we discuss the computational complexity of the parameter.

II. TERMINOLOGY AND NOTATION

Let G be a graph with vertex set V = V (G) of order n = |V | and edge set E.

The degree of vertex denote by v in G by dG(v), we 2 denote the maximum (minimum) degree among the vertices of G by Δ(G) (δ(G), respectively), and we call a vertex of degree one a leaf.

The open neighbourhood of v is the set NG(v) = { u ∈ V | uv∈ E } and the closed neighbourhood of v is NG[v] = {v} ∪ NG(v).

For a set S ⊆V, its open neighbourhood is the set NG(S) = S v∈SNG(v), while its closed neighbourhood is the set NG[S] = S ∪ NG(S).

If the graph G is clear from the context, we omit the subscript G. For subsets X, Y ⊆V, we denote the set of edges that join a vertex of X and a vertex of Y by [X, Y].

In this dissertation work,

Chapter I refreshes some basic concepts and preliminaries in graph theory.

Chapter II deals with the concept of a semi-total domination number of a graphand their properties.

Chapter III discusses a semi-total block graph &total block graph.

Chapter IV illustrates the block-point tree of a graph.

This dissertation work consists of same basic concepts of a semi-total domination number of a graphs. This dissertation also explains the concept of a semi-total block graph &total block graph.

CHAPTER – I
PRELIMINARIES

Definition: 1.1

A chart G = (V, E) is a limited non-void set V of items assembled vertices with a set E of unordered sets of unmistakable vertices called edges.

A point is known as a ***vertex.***

A line that join any two points is known as an ***edge***.

Definition: 1.2

A subgraph G and a subset S of the vertex set, the subgraph of G prompted by S and it is signified <S>. It is the subgraph with vertex set S and with edge set {uv\u,vϵS and uvϵE(G)}.

In this way, <S> contains all vertices of S and all edges of G whose end vertices are both in S.

At the end of the day,

A diagram H is asubgraphof a chart G. On the off chance that V(H)≤V(G) and E(H)≤E(G).

For this situation we compose H≤G, and we state that G contains H. In a diagram where the vertices and edges are unlabeled, we state that H≤G if the vertices could be named.

In such a way that,

V(H) ≤V(G) and E(H) ≤E(G).

Definition: 1.3

A ***simple graph*** is a graph this does not have more than one edge between any two vertices and no edge begins and finishes at the equivalent vertex. A diagram without self-circle and parallel edges is known as a simple graph**.**

CHAPTER – II
A SEMI-TOTAL DOMINATION NUMBER OF A GRAPH

The semitotal domination number of a path and a cycle is straightforward to compute.

Lemma: 2.1

If G has n vertices and maximum degree Δ, then $\gamma t^2(G) \geq 2n/(2\Delta+1)$.

Proof:

Let G = (V, E) and

Let S be any semi-TD-set in G.

For each vertex u ∈V

Let f: V → [0, 1] be defined by f(u) = 1/k, where k = |N[u] ∩ S|.

If u is dominated by k vertices of S.

Then f(u) = 1/k. Let v ∈ S.

If every vertex in N[v] is dominated by v but by no other vertex of S.

Which is a contradiction.

Hence there is at least one vertex v_0 in N[v]

That is adjacent to a vertex of S different from v.

We note that,

$$f(v_0) \leq 1/2.$$

The amount of domination done by v.

Which is therefore,

X u∈N[v] f(u) ≤ f(v 0) + X u∈N[v]\{v 0} f(u)

≤ dG(v) + 1/2

≤ Δ + 1/2

and so v effectively dominates at most Δ + 1/2 vertices.

The total amount of domination done by S is therefore at most |S|(Δ + 1/2).

Since S is 3 a dominating set in G.

We therefore have that,

n ≤ |S|(Δ + 1/2) and

so $\gamma t^2(G) = |S| \geq 2n/(2\Delta+1)$.

Hence the proof.

CHAPTER-III
A SEMITOTALBLOCK GRAPH&TOTAL BLOCKGRAPH

In this chapter, we obtain some properties of these graphs and a characterization of semitotal-block graphs whose semitotal-block graphs and the total block graphs are isomorphic.

3.1 Semitotal Block graphs:

Definition: 3.1.1

The **semitotal-block graph** $T_b(G)$ of a graph G as the graphs whose set of points is the union of the set of points and blocks of G and in which two points are adjacent if and only if the corresponding points of G are adjacent or the corresponding member are incident.

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We start with a few preliminary results.If v is a cut point of a graph G, then it is also a cut point of $T_b(G)$ and conversely.

A graph G and its semitotal-block graph $T_b(G)$ have the same number of cut points and blocks.

If G is a connected graph, then $T_b(G)$ is also a connected graph and conversely, For any nontrivial complete graph k_p, $T_p(k_p)=k_{p+1}$.

Theorem: 3.1.1

If G is a connected graph with p points of q lines and if b_i is the number of blocks to which point v_i belongs in G, then the semitotal block graph $T_b(G)$ has

$$\sum_{i=1}^{p} bi+1 \text{ points and } q+\sum_{i=1}^{p} bi$$

lines.

Proof:

It is shown (kulli, 1976) that if G is a connected graph with p points and b_i is the number of blocks to which point v_i belongs in G.

then $b_p(G)$ has $\sum_{i=1}^{p} bi+1$ points and since the graphs $b_p(G)$ and $T_b(G)$ have the same number points.

$$\therefore T_b(G) \text{ has } \sum_{i=1}^{p} bi+1 \text{ points.}$$

The number of lines of $T_b(G)$ is the sum of the number of lines in G and in $b_p(G)$. It is proved (kulli, 1976) that the number of lines in $b_p(G)$ is $\sum_{i=1}^{p} bi.$

$$\therefore \text{The number of lines in } T_b(G) \text{ is } q+\sum_{i=1}^{p} bi.$$

Hence the proof.

III. CONCLUSION

Graphs can be utilized to think about the structure of World Wide Web. We can think about whether two PCs are associated by a correspondence connection utilizing Graphs models of PC arrange.

We can use Graphs to plan assessments and dole out stations to TV slot.

Diagrams are utilized as models to speak to the challenge of various species in a biological specialty, in processing the quantity of various mixes of flights between two urban communities in a carrier arrange, in finding the quantity of hues expected to shading the areas of a guide.

In this task, numerous limits on are achieved and its careful qualities for some standard diagrams are found. Its associations with other parameter.

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