 ISSN: 2454-4248
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An Analysis upon Organization of Some Nuclear Mappings and Quasi-Nuclear Mappings

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Abstract: In this paper we sum up the notable outcome which says that the arrangement of semi nuclear maps is nuclear. All the more accurately, we characterize what we call a 2-quasiλ-nuclear guide between normed spaces and we say that the organization of a 2-semi λ-nuclear guide with a semi λ-nuclear guide is a pseudo-λ-nuclear guide. Additionally, we say that a semi λ-nuclear guide is a 2-semi λ-nuclear guide. For a nuclear G_∞-space, we say that a straight guide T between normed spaces is 2-semi λ-nuclear if and only if it is semi λ-nuclear. We present new kind of maps between normed spaces to be specific, p-quasiλ-nuclear guide. We say that the piece of a q-semi λ-nuclear guide ($0 < q \leq 1$) with a p-semi λ-nuclear guide ($0 < p \leq 1$) is a pseudo-λ-nuclear guide. Additionally we say that for a nuclear G_∞-space a direct guide T between normed spaces isp-semi λ-nuclear iff it is q-semi λ-nuclear.

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INTRODUCTION

In this part we sum up the notable outcome which says that the arrangement of semi nuclear maps is nuclear. All the more accurately, we characterize what we call a 2-quasiλ-nuclear guide between normed spaces and we say that the organization of a 2-semi λ-nuclear guide with a semi λ-nuclear guide is a pseudo-λ-nuclear guide. Additionally, we say that a semi λ-nuclear guide is a 2-semi λ-nuclear guide. For a nuclear G_∞-space, we demonstrate that a straight guide T between normed spaces is 2-semi λ-nuclear if and only if it is semi λ-nuclear.

We present new kind of maps between normed spaces to be specific, p-quasiλ-nuclear guide. We say that the piece of a q-semi λ-nuclear guide ($0 < q \leq 1$) with a p-semi λ-nuclear guide ($0 < p \leq 1$) is a pseudo-λ-nuclear guide. Additionally we say that for a nuclear G_∞-space a direct guide T between normed spaces is p-semi λ-nuclear iff it is q-semi λ-nuclear.

We presented the ideas of p-nuclear and p-quasinuclear mappings in Banach spaces. These ideas were as of late stretched out in Miyazaki [4] to (p, g)- nuclear and (p, g)- semi nuclear mappings by utilizing the succession spaces l_p. Then again, these were stretched out in Ceitlin [1] to (Z, p)- nuclear and (Z, p)-semi-nuclear mappings. The object of this paper is to expand these two sorts of ideas to (Z, Λ,)- nuclear and (Z, A)- semi nuclear mappings in Banach spaces by utilizing conceptual succession spaces λ. On the other

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hand that l < p <∞, 1 < g <∞ and is a space of sort Λ for $\hat{q}$. In Area 2, we present the space λ(Z) and consider the double space of λ(Z). Area 3 is dedicated to examining (Z, A)- nuclear mappings and Segment 4 to considering (Z, A)- semi nuclear mappings. We explore Z-nuclear spaces in Area 5.

In this section we demonstrate presence and estimate products for convolution conditions on the spaces of (s;(r,q))(s;(r,q))- semi-nuclear mappings of a given kind and request on a Banach space EE. As unique case this yields products for incomplete differential conditions with steady co-efficients for whole capacities on limited dimensional complex Banach spaces. We additionally demonstrate division hypotheses for (s;m(r,q))(s;m(r,q))- summing elements of a given kind and request that are basic to demonstrate the presence and estimate products.

NUCLEAR MAPPINGS AND SPACES :

Let E is vector space over K and V is a convex, orbited, and spiral subset of E at the point $\{n^{-1}V : n \in N\}$ a 0-neighbourhood base for a locally convex topology $\Im_V$ on E. The Hausdorff t.v.s related with $(E, \Im_V)$ is the remainder space $(E, \Im_V)/p^{-1}(0)$, where p is the gauge of V; this quotient space is normableby the norm $\hat{x} \to \|\hat{x}\| \, p(x)$; where $x \in \hat{x}$. We shall denote by E_V its normed space $\left(E/P^{-1}(0), \| \ \|\right)$, Just introduced, and by $\tilde{E}_V$ its fulfillment, which is a Banach space. If E is avector space and V is a convex, neighbourhood of 0, the topology of the remainder space $E/p^{-1}(0)$ is finer than the topology of E_V. Thus the quotient map (called the cantonical map) is continuous on E into $\tilde{E}_V$. This map will be denoted by ϕ_V.

Dually, if E is a 1.c.s. and $B \neq \phi$ a convex, circled, and bounded subset of E, then

$$E_1 = \bigcup_{n=1}^{\infty} nB$$

is a subspace of E. The measure work p_B in E_1 is immediately observed to be a standard on E_1 . The formed space (E_1, p_B) will henceforth be denoted E_B.

is a subspace of E. The measure work p_B in E_1 is immediately observed to be a standard on E_1 . The formed space (E_1, p_B) will henceforth be denoted E_B.

It is proper that the imbedding map $\psi_B : E_B \to E$ is consistent. Besides, if B is finished in E, then E_B is a Banach space. We at last note that no perplexity can emerge if V = B is a raised, subset of E which is spiral and limited, for this situation the spaces E_V and E_B are identical.

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If U, V are convex, circle, and radial subsets of E with respective gauge functions p,q, and such that $U \subset V$, then $p^{-1}(0) \subset q^{-1}(0)$ and each quivalence class $\tilde{x}$ mod $p^{-1}(0)$ is contained in a unique equivalence class $\tilde{y}$ mod $q^{-1}(0); \tilde{x} \to \tilde{y}$ is a linear map $\phi_{V,U}$, which is called the canonical map of E_U onto E_V. Since $\phi_{V,U}$, is clearly continuous it has a unique continuous extension on $\tilde{E}_U$, into $\tilde{E}_V$ which is again called canonical, and also denoted by $\phi_{V,U}$.

In like manner, if B and C are curves, surrounded, and limited arrangements of an I.c.s E with the end goal that $\phi \neq B \subset C$, at that point $E_B \subset E_C$ and the sanctioned imbedding $\psi_{C,B} : E_b \to E_C$ is continuous. Finally, if U,V,B,C are as before and $\phi_U, \phi_V, \psi_B, \psi_C$ are the canonical maps $E \to \tilde{E}_U, E \to \tilde{E}_V, E_B \to E)$ and $E_C \to E$, we have the rerelations $\phi_V = \phi_{V,U} \circ \phi_U$ and $\psi_C = \psi_{C,B} \circ \psi_B$.

The two strategies for building assistant normed spaces were efficiently utilized by Grothendieck will be amazingly helpful in what follows.

Let E, F be I.c.s and let E_1 be the dual of E. Each element $v \in \tilde{E} \otimes F$ defines a linear map $u \in L(E,F)$ by virtue of

$$x \to u(x) = \sum_{i=1}^{r} f_i(x)_{y_i}$$

if $v = \sum_i^r f_i \otimes y_i$, and $v \to u$ is even an (algebraic) isomorphism of $E_1 \otimes$ F into L (E, F). The mappings $u \in L$ (E, F). , which originates in this fashion from an element $v \in E' \otimes F$, are called persistent maps of limited position. The rank r of u is characterized to be the position of v. The mappings of limited position are exceptionally unique instances of smaller straight maps on E into F. A direct guide u on E onto F is conservative if, for a reasonable 0-neighbourhood U in E, u(U) is a moderately minimized subset of F.

Now, let us assume that E, F are Banach spaces, and let E' be the Banach space which is the solid double of E then the imbedding $v \to u$ is nonstop for the projective topology on E' $\otimes$ F, and the norm topology on L (E, F). If $v \in E' \otimes F$ then

$$\|u\| = \sup \|u(x)\| \leqq \sup \sum_{i=1}^{r} | f_i(x) \| f_i \| \|y_i\| \|x\| \leqq \quad \|x\| \leqq 1$$

for all representation $v = \sum_{i=1}^{r} f_i \otimes y_i$; hence $\|u\| \leqq r_v(v)$, where the standard r is the tensor result of the particular of E and F and its culmination. Since L (E, F) is finished under the standard topology by this rasult the imbedding $v \to u$ has a persistent expansion to E' $\tilde{\otimes}$ F, with values in L (E, F).

The linear maps contained in the range of τ are called nuclear; that is, $u \in L$ (E, F). is nuclear if $u = \tau(v)$ for some $v \in E' \tilde{\otimes} F$.

The meaning of an atomic guide sums up to subjective I.c.s. E, F as follows. A direct map u on E into F is limited if for an appropriate 0-neighbourhood U in E, u (U) is a limited subset of F (for instance, each minimal guide is limited); each limited guide is ceaseless. A limited guide can be disintegrated as follows : Let U be a curve, surrounded by, 0-neighbourhood in E with the end goal that $u(u) \subset B$, where B is convex, circled, and bounded in F; then $u = \psi_B \circ u_0 \circ \phi_U$, were $u_0$is the map in L(E_U,F_B) induced by u. If, in addition, F_B is complete, then u_0 has a continuous extension $u_0 \in L(\tilde{E}_U, F_B)$ for which $u = \psi_B \circ \bar{u}_0 \circ \phi_U$. The definition is now as below:

A straight guide u of a I.c.s. E into another I.c.s. F is atomic if there exists a curve, surrounded by 0-neighbourhood U in E to such an extent that $u(U) \subset B$, where B is limited with $F_B$is complete, and such that the induced mapping u_o is nuclear on $\tilde{E}_U$ into F_B.

It follows on the double that each constant straight guide of limit position is atomic. Moreovere, if u is atomic in L (E, F), there exists a 0-neighbourhood U in E and a limited curve, subset of of F for which $F_B$is complete, such that u is the uniform limit on U of a sequence of maps of finite rank in L(E, F_B). Hence for every $\Im$ –topology on L(E, F), the nuclear maps are contained in the closure of $E' \otimes F$ with this, we obtained the following explicit characterization of nuclear maps.

Theorem (5.1.1) :- A direct guide u $\in$ L(E, F) is atomic if and only if in the event that it is of the sturucture $x \rightarrow u(x) = \sum_{n=1}^{\infty} \lambda_n f_n(x)_{y_n}$, where $\sum_{n=1}^{\infty} |\lambda_n| < \infty$ is an equicontinuous sequence in E', and (y_n) is a sequence contained in a convex, circled and bounded subsets B of F for which F_B is complete.

Proof. The condition is necessary. For, if u in nuclear, then $u = \psi_B \circ \bar{u}_0 \circ \phi_U$, where $\bar{u}_0$ is nuclear in $L(\tilde{E}, F_B)$ being a suitable 0-neighbourhood in E, and B being a suitable bounded subset of F for which F_B is complete, Hence $\bar{u}_0$ originates from an element v of $\left[E_U\right] \tilde{\otimes} F_B$ which is of the form$= \sum_{n=1}^{\infty} \lambda_n h_n \otimes y_n.$, with $\sum_{n=1}^{\infty} |\lambda_n| < +\infty$ and where (h_n) and (y_n) are null sequences in $[E_U]'$ and F_B, respectively. Let us define a sequence (f_n) of linear forms on E by setting $f_n = h_n \circ \phi_U$.Since $(h_n)$is a bounded sequence in $[E_U]'$, the mapping $= \psi_B \circ \tau(v) \circ \phi_U$ is of the from indicated above.

The condition is sufficient. For if u is as indicated in the proposition, let $U = \{x \in E : |f_n(x)| \leqq 1 \, n \in N)$; is convex and circled and is a 0-neighbourhood in E by the equicontinuity of (f_n). Defining $h_n (n \in N)$ by $f_n = h_n \circ \phi_U$ on E_{U_u} and subsequent extension to $\tilde{E}_U$ we obtain $\|h_n\| \leqq 1$ for all n; evidently, $\bar{u}_0$ is the map

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$\hat{x} \to \sum_{n=1}^{\infty} \lambda_n f_n(\hat{x}) y_n$, since $\sum_1^{\infty} |\lambda_n| \|h_n\| \|y_n\|$ converges, the series $\sum_1^{\infty} \lambda_n h_n \otimes y_n$, is absolutely convergent in $[E_U]' \tilde{\otimes} F_B$ and hence defines an element $v \in [E_U]' \tilde{\otimes} F_B$; clearly, $\bar{u}_0 = \tau(v)$, whence u is nuclear.

REMARK. If u is of the form $\sum_1^{\infty} \lambda_n f_n(x) y_n$; we shall find it convenient to write $= \sum_1^{\infty} \lambda_n f_n \otimes y_n$, keeping in mind that u is not, properly speaking, an element of a topological tensor product. It follows then from the first part of the proof that for nuclear u, there exists a representation $u = \sum_1^{\infty} \lambda_n f_n \otimes y_n$ such that (f_n) is a sequence converging to 0 uniformly on a suitable 0-neighbourhood U of E, (y_n) converges to 0 in a suitable Banach space F_B; finally, $\{\lambda_n\} \in l^1$.

COROLLATY 1. Every nuclear map is compact.

Proof. Let $u = \sum_1^{\infty} \lambda_n f_n \otimes y_n$ and let $U = \{x \in |f_n(x)| \leqq 1, n \in N\}$. In view of the preceding remark,, it can be assumed that (y_n) is a null sequence (y_n) in a suitable space F_B and, in additional, that $\sum_{i=1}^{\infty} [\lambda_n] \leqq 1$. It follows that the image u(U) of the 0-neighbourhood U is contained in the closed; convex, circled hull C of the null sequence (y_n) is relatively compact in F_B and F_B is complete, C is compact in F_B and hence a fortori compact in F by the countinuity of $F_B \to F$.

COROLLARY 2. Let E, F, G, H be I.c.s., let u $\in$ L (E, F), let w $\in$ L (G, H), and let v be an atomic guide on F into G. At that point $v \circ u$ and $w \circ v$ (and subsequently $w \circ v \circ u$) are atomic maps.

Proof. It is evident that $v \circ u$ is nuclear, and there exists a convex, circled 0-neighbourhood V in F such that $\overline{v(V)} = B$ is compact in G. Thus B_1 = W(B) is compact in H, hence H_{B_1}, is complete. It is now clear that $w \circ v$ is nuclear in L(F, H).

COROLLARY 3. On the another hand if u $\in$ L(E, F) is atomic, at that point u has an intersting expansion $\bar{u} \in L(\tilde{E}, F)$, , where $\tilde{E}$ is that completion of E, and it is nuclear .

Proof. The first of the expressed properties is shared by u with every single minimized guide on E into F. Actually, if U is a 0-neighbourhood in E to such an extent that $u(U) \subset C$, C is smaller, at that point since u is consistently nonstop, its confinement to U has an exceptional ceaseless augmentation to $\bar{U}$ with values in C, since C is complete. It is immediately clear that this extension is the restriction to $\bar{U}$ of a linear map $\bar{u}$ of $\tilde{E}_E$ into F which is compact, hence continuous; that $\bar{u}$ is nuclear is a direct consequence of the definition of a nuclear map.

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We now characterize an atomic space. A locally curved space E is atomic if there exists a base B of convex, circled 0-neighbourhoods in E such that for each $V \in B$, the canonical mapping $E \to \tilde{E}_{V_y}$ is nuclear.

It is at once clear from this, definition and that a I.c.s. E is nuclear if and only if its completion $\tilde{E}$ is nuclear. The space K_0^d (d is any cardinal) is a first example of a nuclear space; in fact, for any convex, circle 0-neighbouthood V, the space $E_V = \tilde{E}_{V_y}$ is of finite rank and hence nuclear.

Further and all the more intriguing models can be given beneath. If indicates none of the less, that a normed space E can't be atomic except if it is of limited measurement ; for in the event that V is an arched, surrounded by 0-neighbourhood which is limited, at that point $E \to E_V$, is a topological automorphism; hence if $E \to \tilde{E}_V$, is an atomic guide, it is minimal, so, E is limited dimensional. We will have to use for the accompanying elective portrayals of atomic space

Theorem (5.1.2) :-

Give E a chance to be a 1.c.s. The accompanying affirmations are equal:

a) $Eisatomic.$
b) $EachnonstopstraightguideofEintoanyBanachspaceisatomic.$
c) $Eacharched, circumnavigated\ 0neighborhoodUinEcontainsanother, V, with$

$theendgoalthatthesanctioned$map $\tilde{E}_V \to \tilde{E}_U$ is nuclear.

Theorem (5.1.2) :- Let E a chance to be a I .c.s. The accompanying affirmation are equal:

a) E is atomic
b) Each nonstop straight guide of E into any Banach space is atomic.
c) Each arched, sourrounded by 0-neighbourhood U in E contains another V with the end goal that the sanctioned map $\tilde{E}_V \to \tilde{E}_U$ is nuclear.

Proof. (a) $\Rightarrow$ (b): Let F be any Banach space and u $\in$ L (E, F). There exists a covex, circled 0-neighbourhood V in E such that $\phi_V : E \to \tilde{E}_{V_y}$ is nuclear, and such that u (V) is bounded in F. Since $\phi_V(E) = E_V u$ determines a unique $v \in L(\tilde{E}_V, F)$ such that $u = v \circ \phi_V$, and it follows that u is nuclear. (b) $\Rightarrow$ (c) : Let U be any convex, circled 0-neighbourhood in E.

By assumption, the canonical map $\tilde{E}_V \to \tilde{E}_U$ is nuclear, and hence of the form $\phi_U = \sum_1^\infty \lambda_n f_n \otimes y_n$. Let us write $V = U \cap \{x : |f_n(x)| \leq 1; n \in N\}$ then $V \subset U$ convex, circled a 0-neighbourhood by equicontinuity of

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the sequence (f_n). Now each f_n induces a continuous linear from (of norm $\leqq 1$) on $\tilde{E}_V$. Let us denote by h_n its continuous extension to $\tilde{E}_V$: It is now trivial that the canonical map $\phi_{U,V} : \tilde{E}_V \to \tilde{E}_U$ is given by $\sum \lambda_n f_n \otimes y_n$, and hence nuclear.

(c) $\Rightarrow$ (a): If U is a given convex, circled 0-neighbourhood in E, there exists mother, V. such that $\phi_{U,V}$ is nuclear, Since $\phi_U = \phi_{U,V} \circ \phi_V$, it follows that $E \to \tilde{E}_U$ is nuclear, whence E is a nuclear space by definition.

COROLLARY 1. If E is a nuclear space, then $E \to \tilde{E}_{V_y}$ is a nuclear map or every convex, circled neighbourhood V of 0 in E.

For $E \to \tilde{E}_V$ is continuous and $\tilde{E}_V$ is a Banach space.

COROLLARY 2. Each limited subset of an atomic space is precompact.

Proof. If B is an area base of 0 in E comprising of raised, orbited sets, at that point E is isomorphic with a subspace of

$$\prod_{V \in B} \tilde{E}_V$$

by virtue of the mapping $x \to \{\phi_V(x) : V \in \mathrm{B}\}$. This isomorphism carries a bounded set $B \subset E$ into the set $\prod \phi_V(B)$. Now if E is nuclear, each $\phi_V(B)$ is precompact in E_V. Thus the product $\prod \phi_V(B)$ precompact, which proves the assertion. We recall the common usage to understand by $l^p\ (1 \leqq p < +\infty)$ the Banach space of all (real or complex) sequences $x = (x_1, x_2, ...)$ whose p the powers are (absolutely) summable, under the norm $\|x\|_p = \left(\sum |x_n|^p\right)^{1/p}$; l^∞ is the Banach space of bounded sequences with $\|x\|_\infty = \sup_n |x_n|$.

Theorem (5.1.3) :- Let E be an atomic space, let U be a given 0-neighbourhood in E, and let p be a number with the end goal that $1 \leqq p \leqq \infty$.There exists a curved, surrounded 0-neighbourhood $V \subset U$ for which $\tilde{E}_V$, is isomorphic with a subspace of l^{P}.

Proof. We show that there exists a continuous linear map $v \in L(E, l^p)$ such that $v^{-1}(B) \subset U$, where B is the open unit ball of $l^p; V = v^{-1}(B)$ will be the neighbourhood in question. Without any loss of generality, we assume that U is convex and circled. The canonical map ϕ_U is nuclear, hence of the form $\phi_U = \sum \lambda_n f_n \otimes y_n$ where we can assume that

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$\lambda_n > 0 (n \in N), \sum_{n=1}^{\infty} \lambda_n = 1, \|y_n\| = 1$

in $\tilde{E}_U$ $(n \in N)$ and that the sequence $\{f_n\}$ is equicontinuous . Define v by

$$v(x) = (\sqrt[p]{\lambda_1} f_1(x), \sqrt[p]{\lambda_2} f_2(x), \dots .)$$

For all $x \in E$ (set $\sqrt[p]{\lambda_n} = 1$ for all n if $p = \infty$). By the equicontinuity of the sequence $\{f_n\}$ we have $v(x) \in l^p e$ and evidently $v \in (L\, E, l^p)$. Now let $p^{-1} + q^{-1} = 1 (q = 1$ if $p = \infty$ and $q = \infty$ if $p = 1)$ and apply Holder's inequality to $\sum_{n=1}^{\infty} \alpha_n \beta_n$ with $\alpha_n = \sqrt[p]{\lambda_n} f_n(x), \beta_n = \sqrt[q]{\lambda_n}$. Denoting by ‖‖ I the norm in E_U, we obtain

$$\|\phi_U(x)\| = \left\| \sum_{n=1}^{\infty} \lambda_n f_n(x) y_{n,} \right\| \leqq \sum_{1}^{\infty} \lambda_n |f_n(x)| \leqq \|v(x)\|_p,$$

$\tilde{E}_U (n \in N)$ and that the sequence (f_n) is equicontinuous. Let us define v by

$$v(x) = \left(\sqrt[p]{\lambda_1} f_1(x), \sqrt[p]{\lambda_2} f_2(x), \dots . \right)$$

For all $x \in E$ (set $\sqrt[p]{\lambda_n} = 1$ for all n if $p = \infty$). By the equicontinuity of the sequence $\{f_n\}$ we have $v(x) \in l^p$ and evidently $v \in L(E, l^p)$. Now let $p^{-1} + q^{-1} = 1 (q = 1\, if\ p = \infty$ and $q = \infty$ if $p = 1)$ and apply Holder's inequality to with $\alpha_n = \sqrt[p]{\lambda_n} f_n(x), \beta_n = \sqrt[q]{\lambda_n}$. Denoting by $\| \ \|$ the norm in E_U, we obtain

$$\|\phi_U\| = \left\| \sum_{n=1}^{\infty} \lambda_n f_n(x) y_n \right\| \leq \|v(x)\|_p,$$

whence $v^{-1}(x) \subset U$. Letting $V = v^{-1}(x)$, the definition of v implies that E_v is norm isomorphic with v (E); hence $\tilde{E}_V$ is norm isomorphic with, the closed subspace $\overline{v(E)}$ of l^p.

In the three corollaries that follow, let us denote by A a set whose cardinality is the minimal cardinality of a neighbourhood base of 0 in E.

COROLLARY 1. Let E be nuclear, and let $\{E_\alpha : \alpha \in A\}$ be a family of Banach spaces, every one of which is isomorphic with a space $l^p (1 \leqq p \leqq \infty)$. There exist linear maps f_a of E_a into $l^p (1 \leqq p \leqq \infty)$. such that the topology of E is the coarsest topology for which all mappings f_a are continuous.

At the topology of E is the projective topology as for the family $\{(E_\alpha, f_\alpha); \alpha \in A\}$. On the another hand that with p=2, to every component $U_\alpha (\alpha \in A)$ of a 0-neighbourhood base in E, we obtain a base $\{V_\alpha : \alpha \in A\}$ of 0-neighbourhood to such an extent that for each $\alpha \in A, \tilde{E}_\alpha = \tilde{E}_v$ is an Hilbert space. Presently if $\tilde{E}_\alpha$ is a

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Hilbert space, the norm of $\tilde{E}_\alpha$ originates from a positive definite Hermitian form $(\hat{x},\hat{y}) \rightarrow [\hat{x},\hat{y}]_\alpha$ on $\tilde{E}_\alpha \times \tilde{E}_\alpha$; hence if denotes the canonical map $E \rightarrow \tilde{E}_\alpha$ then $((x,y) \rightarrow [\phi_\alpha(x), \phi_\alpha(y)]_\alpha$ is a positive semi-definite Hermitian form on E × E such that $x \rightarrow \left[\phi_\alpha(x), \phi_\alpha(x)^t_\alpha\right]$ is the gauge function p_α of V_α.

COROLLARY 2. In each atomic space E there exists a 0-neighbourhood base $\{V_\alpha : \alpha \in A\}$ to such an extent that for each a $\alpha \in A, \tilde{E}_{V_\alpha}$ is a Hilbert space; consequently the topology E can be produced by a group of semi-standards, every one of which starts from a positive semi-distinct Hermitian structure on $\Im \times$E.

Consolidating this outcome with the development utilized in the confirmation, we acquire a portrayal of atomic space as thick subspaces of projective points of confinement of Hilbert spaces. Subsequently the finishing of an atomic space E is isomorphic with a projective breaking point of Hilbert spaces, and clearly atomic.

COROLLARY 3. Each tota atomic space is isomorphic with the projective of an appropriate family (of cardinality card A) of Hilbert spaces. A Frechet E is atomic if and only if it is the projective furthest reaches of an arrangement of Hilbert spaces.

$$E = \underleftarrow{\lim}\, g_{mn} H_n$$

such that g_{mm} is a nuclear map whenever m < n.

Proof. We have just to demonstrate the second affirmation. In the event that E is an atomic. (F)- space, then there exists a base $\{v_n : n \in N\}$ at 0 which can be supposed diminishing, and to such an extent that each $\tilde{E}_n$ is a Hilbert space and hence we can even suppose that each of the canonical maps $\phi_{Vn,Vn+1} : \tilde{E}_{n+1} \rightarrow \tilde{E}_\alpha$ is nuclear.

The ideal portrayal is at that point got with $H_n = \tilde{E}_n$,, and $g_{mn} = \phi_{V_m,,V_n}(m \leqq n)$.On the other hand, if E is of the structure demonstrated and V is a curved, sourrounded by 0-neighbourhood looked over an appropriate base in E, at that point $E \rightarrow \tilde{E}_V$ can be identified with the projection p of E into a finite product of spaces H_n, say

$$\prod_{k=1}^{m} H_k.$$

Denoting by p_n the projection of E into $H_n(n \in N)$ we have $p = (p_1,....p_m)$; hence $p = (g_{1n} \circ p_n,....g_{mn} \circ p_n)$ for any n > m, which implies that p is nuclear.

The accompanying significant, hypothesis is due to Grothendieck.

Theorem (5.1.4) :- Each subspace and each isolated remainder space of an atomic space is atomic. The result of a self-assertive group of atomic spaces is atomic of the locally curve direct entrity of countable group of atomic space is an atomic space.

Before demonstrating the hypothesis, we note the accompanying quick result:

COROLLARY . The projective further reaches of any group of atomic spaces, and the furthest reaches of a countable gruop of atomic space, are atomic.

Proof :-

$$Let, \quad E = \bigoplus_{i=1}^{\infty} E_i ; (i \in N)$$

1. be nuclear spaces,more, let u by a ceaseless straight guide of E into a given Banach space F If u_i is the limitation of u to the subspace E_i fo E, u, is nonstop and henceforth atomic, guide in this way of the structure

$$u_i = \sum_{i=1}^{\infty} \mu_n^{(i)} h_n^{(i)} \otimes y_{n,i} \ (i \in N).$$

Here we can expect that $\|y_{n,i}\| \leqq 1$ in F for all $(n,i) \in N \times N$, that $\sum_{n=1}^{\infty} \left|\mu_n^{(i)}\right| \leqq i^{-2} (i \in N)$ and that each of the sequences is $\left\{h_n^{(i)}; n \in N\right\}$ equicontinuous on E_i Let U_i be 0-neighbourhood in E_i such that $\left|h_n^{(i)}(x)\right| \leqq 1$ for all $x_i \in U_i$ and all $n \in N$ and define f_n to be the continuous linear form on E which is the extension of $h_n^{(i)}$ to E that vanishes on the complementaty subspace $\bigoplus_{j \neq 1} E_j$. The family $\left(f_n : (n,i) \in N \times N\right)$ is equicontinuous, for if U is the 0 neighbourhood

$\Gamma_i U_i$ in E, 'then $x \in U$ implies $\left|f_n(x)\right| \leqq 1$ for all n and i. Since u can be written as

$$u = \sum_{n,i}^{\infty} \mu_n^{(i)} f_{n,i} \otimes y_n i,$$

it follows that u is atomic.

Furthermore, let u be a constant straight guide of E into a given Banach space F. There exists a 0-neighbourhood V in E with the end goal that u (V) is limited in F, and by meaning of the item topology, V contains a 0-neighbourhood of the structure

$$V_{\alpha_1} \times \ldots V_{\alpha_n} \times \prod_{\beta \neq \alpha_i} E_\beta$$

It follows that u evaporates on the subspace $G = \prod_{\beta \neq \alpha_i} E_\beta$ of E.

Since $E=\prod_{i=1}^{n} E_{\alpha_1} \oplus G$, it remains to show that the restriction of u to $\prod_i E_{\aleph_i}$ is nuclear. But this is clear from the preceding proof, since $\bigoplus_{i=1}^{n} E_{\alpha_i}$ is identical with $\prod_{i=1}^{n} E_{\alpha_i}$.

2. The confirmation of nuclearity for subspaces and remainder spaces will be put together. Let E be an atomic space and let M a chance to be a subspace of E. For each curve, sourrounded by 0-neighbourhood U in E, set $V+M\bigcap U$.We demonstrate that for every V, there exists another such neighbourhood $V_1 \subset V$, such that the canonical map $\tilde{\tilde{M}}_{V_1} \to \tilde{\tilde{M}}_V$ is nuclear. We can assume without loss of generality that $V+M\bigcap U$, where U is such that $\tilde{E}_U$ is a Hilbert space. There exists a 0-neighbourhood $U_1 \subset U$ such that the canonical map $\phi_{U,U_1}: \tilde{\tilde{M}}_{U_1} \to \tilde{M}_U$ is nuclear; let $V_1 = M\bigcap U_1$. Presently it isn't hard to see that $\tilde{E}_{V_1}$ and $\tilde{E}_V$ can be related to closed subspace of $\tilde{E}_{U_1}$ and $\tilde{E}_U$, individually, with the goal that the sanctioned guide ϕ_{V,V_1}, is the restriction of ϕ_{U,U_1} to $\tilde{M}_{V_1}$. But ϕ_{U,U_1} is of the form $\sum_{i=1}^{\infty} \lambda_i f_i \otimes y_i$ with (λ_i) summable (f_i) equicontinuous in $\left[\tilde{E}_{U_1}\right]'$, and $\{y_i\}$ bounded in $\tilde{E}_U$. Denote by p the orthogonal projection of $\tilde{E}_U$ onto $\tilde{M}_V$, let $w_i = py_i$, and denote by g_{ii} the restriction of f_i to $\tilde{M}_{V_1}(i\in N)$. Then $(g_i)(g_1)$ is equicontinuous, (w_i) bounded in $\tilde{M}_{V_1}$, and ϕ_{V,V_1} necessarily of the; from $\sum_{i=1}^{\infty} \lambda_i g_i \otimes w_i$, and hence nuclear.

We utilize a similar example of verification for remainder space Let E be atomic. Let M be a closed subspace of E, F = E/M (topological), and let ϕ be the authoriatative guide $E \to F$. For a given arched, surrounded by 0-neighbourhood V in F, we demonstrate the presence of another, $V_1 \subset V$, such that $\phi_{V,V_1} \tilde{\tilde{F}}_{V_1} \to \tilde{\tilde{F}}_V$, is nuclear. For this we can suppose that $V=\phi(U)$, $\tilde{E}_U$ is a Hilbert space, and $U_1 \subset U$ is such that $\tilde{E}_U$ is a Hilbert space and $\alpha_{U,U_1}: \tilde{E}_{U_1} \to \tilde{E}_U$ is nuclear. The point of the proof consist now in recognizing that $\tilde{\tilde{F}}_V$ can be identified with a quotient space, of $\tilde{E}_{U_L}$. In fact $\tilde{\tilde{F}}_V$ is isomorphic with the space $\tilde{E}_U / L$, where L is the closure of $\phi_U(M)$ in $\tilde{E}_U$. Similarly, setting $V_1=\phi(U_1), F_V$ can be identified with $\tilde{E}_{U_1}/L_1$; where L_1 is the closure of $\phi_{U_1}(M)$ in $\tilde{E}_{U_1}$.

We note further that α, u_1 maps L_1 into L and ϕ_{V,V_1} is nothing else but the map of $\tilde{E}_{U_1}/L_1$ into $\tilde{E}_U/L$ induced by ϕ_{U,U_1} under the identification just made.

Since ϕ_{U,U_1} is nuclear, it of the form $\sum_{i=1}^{\infty} \lambda_i f_i \otimes y_i$. We decompose $\tilde{E}_{U_1} = L_1 \oplus L_1^{\perp}$, $\tilde{E}_U = L \oplus L^{\perp}$ (orthogonal complements). Let $f_i = f_i + f_i^{"}$ and $y_i^{'} + y_i^{"}(i\in N)$ be the corresponding decompositions so that, for f_i, we

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have f_i, we have $f_i\left(L_1^{\perp}\right)=\left(L_1\right)=\{0\}$). Since ϕ_{U,U_1}, maps L_1 into L, it follows that $\sum_i \lambda_i f_i^{'} \otimes y_i^{''}$ vanishes whence

$$\phi_{U,U_1}=\sum_{i=1}^{\infty}\lambda_i f_i \otimes y_i{'}+\sum_{i=1}\lambda_i f_i^{''}\otimes y_i^{''}.$$

If now g_i denotes the linear fiorm on $\tilde{E}_{U1}/L_1$ determined by $f_i^{''}$ and w_i denoted the equivalence class of $y_i^{''}$ mod $L(i \in N)$, then ϕ_{V,V_1} (being the map induced by ϕ_{U,U_1} is of the form $\sum \lambda_i g_i \otimes w_i$ and hence is nuclear.

The proof of the theorem is complete.

We demonstrating that the projective tensor result of two atomic spaces is atomic. To this end, we need the idea of the tensor result of two direct mappings: Let E, F, G, H be vector spaces over K and $u \in L(E,G), v \in L(F,H)$.

The mapping $(x,y)\to u(x)\otimes v(y)$ is bilinear on E × F into G ⊗ H; the direct mapping of $E\times F$ into $G\otimes H$, which relates to the previous, is signified by $u\otimes v$, and is known as the tensor result of u and v.

Clearly $(u,v)\to u\otimes v$ is bilinear on. L(E, G) × L(F, H) into $L(E\otimes F, G\otimes H)$. Therefore agian to his guide there compare a direct guide of L(E,G)×L(F,H) into $L(E\otimes F, G\otimes H)$ (called the accepted imbedding), which, is an isomorphism. In the event that $G=H=K_0$ that is, if $u=f, v=g$ are linear forms, then tensor multiplication in $K_O \otimes K_O$ can be identified with ordinary multiplication in K and we have $f\otimes g$ so that the tensor products $E^*\otimes f^*$ considered earlier are special cases of the present definition.

On the another hand if E and F are atomic spaces, the projective tensor result of E and F, just as its fruition $E\tilde{\otimes}F$ are nuclear.

Proof. Let U,V be curved, surrounded 0-neighbourhoods in E,F separately; set $G=E\otimes F$ and $W=\Gamma U\otimes V$ in G. It is obvious that G_w is identical with the normed space $(E_U\otimes F_V, r)$, where r is the tensor product of the respective norms of E_U and F_V.

Hence ϕ_U, ϕ_V, ϕ_w denote the respective canonical maps $E\to\tilde{E}_U, F\to\tilde{F}_V, G\to\tilde{G}_w$, we have $\phi_w=\phi_U\otimes\phi_V$. Since E,F are nuclear, implies that $\phi_U=\sum\lambda_i f_i\otimes x_i, \phi_V=\sum\mu_j$ etc, have the properties enumerated. For $x\in E, y\in F$ we have by definition

$$\phi_U\otimes\phi_V(xy)=\left(\sum_{i=1}^{i=\infty}\lambda_i f_i\right)\otimes\left(\sum_{i=1}^{\infty}\mu_j g_j(y)\hat{y}_j\right),$$

Which , as an element of $\tilde{G}_W E_U\tilde{\otimes}F_V$, can be written

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$$\phi_w(x \otimes y) = \sum_{i\,j=1}^{\infty} \lambda_i \mu_i f_i(x) g_j(y)(\hat{x}_i x \hat{j}_j)$$

so that is represented by $\sum_{i,j} \lambda_i \mu_i \left(f_i \otimes_j \right) \otimes \left(\hat{x}_i \hat{y}_j \right)$. Now $\{\lambda_i \mu_i : (i, j) \in N \times N\}$ is a summable family, $\left(f_i \otimes g_i \right)$ is an equicontinuous family, namely, uniformly bounded on $\Gamma U_1 \otimes V_1$

for siutable 0-neighbourhoods U_1, V_1 in E,F, respectively and clearly, the family $\left(\hat{x}_i \otimes \hat{y}_j \right)$ is bounded in $\tilde{G}_w$

because of

$\left\| \hat{x}_j \otimes \hat{y}_j \right\| = \left\| \hat{x}_i \right\| \left\| \hat{y}_j \right\|$. Hence ϕ_w is nuclear for every component W of a 0-neighbourhood.

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