Harmonic Mean Labelling of Subdivision and Related Graphs

A. Pannerselvam
M.Sc., M.Phil.,[1]
Associate professor, Department of Mathematics,
Prist University, Thanjavur

N.Anusuya[2]
M.Phil Scholar, Department of Mathematics,
Prist University, Thanjavur

ABSTRACT: A graph G=(V,E) with p vertices and q edges is said to be a mean graph if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 0,1,2,…..q in such a way that when each edge $e = uv$ is labelled with $\left[\frac{f(u)+f(v)}{2}\right]$ if $f(u)+f(v)$ is even and $\left[\frac{f(u)+f(v)+1}{2}\right]$ if $f(u)+f(v)$ is odd, then the resulting edge labels are distinct. f is called a mean labelling of G.

In this paper, we investigate the mean labelling of caterpillar, $C_n^{(2)}$ dragon, arbitrary super subdivision of a path and some graphs which are obtained from cycles and stars.

Keywords and phrases: *Mean graph, dragon, super subdivision of a graph, caterpillar*

*****

INTRODUCTION

By a graph we mean a finite undirected graph without loops or parallel edges. The vertex set is denoted by V(G) and the edge set is denoted by E(G). A cycle of length n is C_n and a path of length n is denoted by P_n. For all detailed survey of graph labelling, we refer to J.A.Gallian. For all other standard terminology and notations we follow Harary.

The concept of Mean labelling has been introduced by S.Somasundaram and R.Ponraj and also S.Somasundaram and S.S.Sandhya introduced Harmonic mean labelling. Motivated by the above works we introduced a new type of labelling called Harmonic mean labelling.

In this paper we investigate the subdivision of Harmonic mean labelling of graphs. We will provide brief summary of definitions and other information which are necessary for our present investigation.

In this paper, we consider only finite, simple and undirected graphs. Let G(V,E) be a graph with p vertices and q edges. For notations and terminology we follow. In a graph G, the subdivision of an edge uv is the process of deleting the edge of G is subdivided exactly once, then the resultant graph is denoted by S(G) and is called the subdivision graph of G.

Somasundaram and Ponraj have introduced the concept of mean labelling of graphs. An assignment $f : V(G) \to \{0,1,2,....,q\}$ is called a mean labelling if whenever each edge e=uv is labelled with $\left[\frac{f(u)+f(v)}{2}\right]$ if $f(u)+f(v)$ is even and $\left[\frac{f(u)+f(v)+1}{2}\right]$ if $f(u)+f(v)$ is odd, then the resulting edge labels are all distinct. Any graph that admits a mean labelling is called a mean graph.

Many results on mean labelling have been proved. In a similar way, Somasundaram, Ponraj and Sandhya have introduced the concept of harmonic mean labelling of a graph. An assignment $f : V(G) \to \{1,2,.....,q+1\}$ is called a harmonic mean labelling if whenever each edge $e = uv$

ISSN: 2454-4248
19 – 27

Is labelled with $\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ then the edge labels are distinct. Any graph that admits a harmonic mean labelling is called a harmonic mean graph.

More results on harmonic mean labelling have been proved. A well collection of results on graph labelling has been done in the survey.

In this paper, we establish the harmonic mean labelling of some standard graphs like subdivision of star $S(K_{1,n})$, subdivision of bistar $S(B_{n,m})$, the disconnected graphs $S(K_{1,n}) \cup kC_m$ etc.

This dissertation entitled**"HARMONIC MEAN LABELLING OF SUBDIVISIONAND RELATED GRAPHS"** consists of four chapters.

PRELIMINARY DEFINITION

Definition 1.1:

The **corona** of two graph $G_1$and G_2 is the graph $G=G_1 \circ G_2$ formed by one copy of $G_1$and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition 1.2:

The graph $P_m AK_{1,n}$ is obtained by attaching $K_{1,n}$ to each vertex of P_m

Definition 1.3:

An assignment $f:(G)\rightarrow\{1,2,\ldots,p+q\}$ is called a **super harmonic labelling** mean if whenever each edge $e=uv$ is labeled with $\left\lceil\frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil or \left\lfloor\frac{2f(u)f(v)}{f(u)+f(v)}\right\rfloor$ then the edge labels are distinct.Any graph that admits a super harmonic mean labelling is called a **super harmonic meangraph**.

Definition 1.4:

An **Alternate Triangular snake**$A(T_n)$ is obtained from a path $u_1u_2\ldots u_n$ by joining $u_i$and u_{i+1}(alternatively) to new vertex v_i That is every alternate edge if a path is replaced by C_3

Definition 1.5:

An **Alternate Quadrilateral snake** $A(Q_n)$ is obtained from a path $u_1u_2\ldots u_n$ by joining u_i, $_{i+1}$ (alternatively) to new vertices v_iw_i respectively and then joining v_i and w_i. That is every alternate edge of a path is replaced by a cycle C_4

Definition 1.6:

An Alternate Quadrilateral Triangular snake $A(Q_n$ **)** is obtained from a path $u_1,u_2,u_3,\ldots\ldots.u_n$ by joining u_i,u_{i+1} (alternatively) to new vertices v_iw_i respectively and then joining v_i and w_i.
That is every alternate edge of a path is replaced by a cycle C_4.

HARMONIC MEAN LABELLING OF SOME

Theorem 2.1:

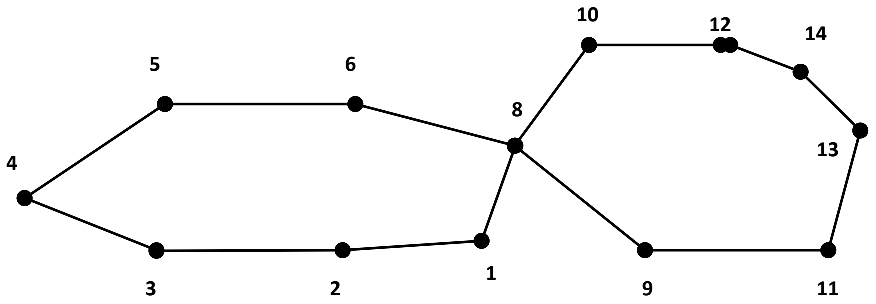
The graph $C_n(2)$ is a Harmonic mean graph

Proof:

Let u be the central vertex of $C_n^{(2)}$.
Let $u_1u_2\ldots\ldots u_nu_1$ and $v_1v_2\ldots v_nv_1$ be the vertices of first and second cycle of $C_n^{(2)}$ respectively
Take
$u = u_n= v_1$
Define $f:V(C_n^{(2)})\rightarrow\{1,2\ldots.q+1\}$ by
$f(u) = n+1$
$f(u_i) = 2,\ 1 \le i \le n$
$f(v_i) = n+1+i,\ \ 2 \le i \le n-1$
$f(v_m) = 2m$
Obviously f is a Harmonic mean labelling of $C_n^{(2)}$

Example 2.2:

A harmonic mean labelling of $C_7^{(2)}$



Theorem 2.3:
Alternate Triangular snakes $A(T_n)$ are Harmonic mean graphs

Proof:
Here we consider the following two cases

Case (i):
If the triangle starts from u_2.
Define a function $f: V\ A(T_n) \to \{1,2\ldots.q+1\}$ by

$f(u_1) = 1$

$f(u_2) = 2$

$f(u_i) = 2i-2$ for all $i=3,4\ldots n$

$f(v_i) = 2i-1$ for all $i=2\ ,4\ldots n-2$

The edges are labelled with

$f(u_iu_{i+1}) = 2i-1$ for all $i=1,\ 2\ldots\ldots n-1$

$f(u_iv_i) = 2i-2$ for all $i=2,4\ldots n-2$

$f(v_iu_{i+1}) = 2i$ for all $i= 2,4,6\ldots n-2$

In this case f is a harmonic mean labelling of $A(T_n)$

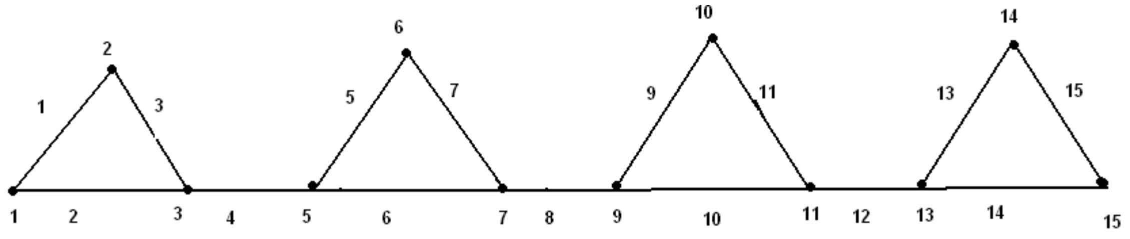


Figure 1

Case (ii):
If the triangle starts for u_1.
Define a function $f : V\ A(T_n) \to \{1,2\ldots.q+1\}$ by
$f(u_i) = 2i-1$ for all $i=1,2\ldots n$
$f(v_i) = 2i$for all $i=1,3\ldots.n-1$
The edges are labeled with
$f(u_iu_{i+1}) = 2i$for all $i=1,2\ldots.n$

 ISSN: 2454-4248
19 – 27

$f(u_iv_i) = 2i-1$ for all i=1,3,5…n-1
$f(v_iv_{i+1}) = 2i+1$ for all i=1,3,5….n-1
The f is a harmonic mean labelling of $A(T_n)$

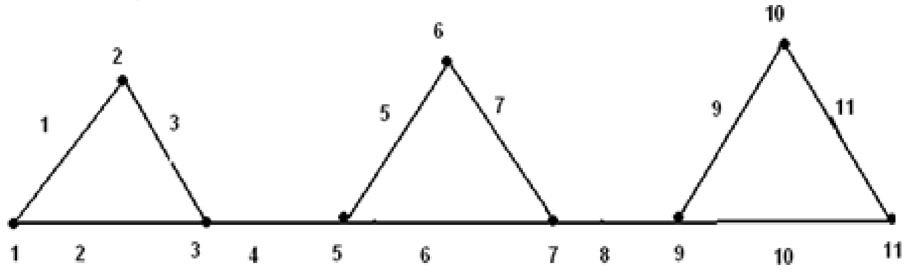


Figure 2

From case (i) and case (ii) we conclude that Alternate Triangular snake is a Harmonic mean graph.

HARMONIC MEAN LABELLING OF SUBDIVISION AND RELATED GRAPHS

Theorem 3.1

The disconnected graph $(K_{1,n})\cup kC_m$ is a harmonic mean graph for

$1\leq n\leq 5, m\geq 3$ andk ≥ 0

Proof

Let$V(S(K_{1,n})\cup kC_m)=\{v;u_1,u_2,\ldots,u_n;v_1,v_2,\ldots,v_n;w_{11},w_{12},\ldots,w_{1m};\ w_{21},w_{22},\ldots,w_{2m};\ldots w_{k1},w_{k2},\ldots,w_{km}\}$ and

$E(S(K_{1,n})\cup kC_m)=\{vu_i,u_iv_i|1\leq i\leq n\}\cup[U_{i=1}^{k}((U_{j=1}^{M-1}\{W_{ij}\ W_{ij+1}\})U\{W_{im}W_{i1}\}]$.

Here $p=2n+km+1$ and $q=2n+km$.

Define a function $f: V(S(K_{1,n})\cup kC_m)\rightarrow\{1,2,\ldots,q+1\}$ by

$f(v)=2n+1$;

$f(u_i) = n+i$,$1\leq i\leq n$;

$f(v_i)=n+1-i$,$1\leq i\leq n$ and

$f(w_{ij})=(2n+1)+(i-1)m+j, 1\leq i\leq k, 1\leq j\leq m.$

Then the induced edge labels of $(K_{1,})$ are given below:

$f^*(u_nv_n)=1$;

$f^*(u_iv_i)=n+1+i, 1\leq i\leq n$;

$f^*(u_iv_i)=n+2-i, 1\leq i\leq n-1$;

and the set of all edge labels of kC_m is $\{2(n+1),2n+3,\ldots,2n+km+1\}$.

Therefore the set of all edge labels of $(K_{1,})\cup kC_m$ is

ISSN: 2454-4248
19 – 27

$\{1, 3, 4 \ldots .2n+k\text{m}+1\}$.

Hence $(K_{1,})\cup kC_m$ is a harmonic mean graph for $1\leq n\leq 5,\geq 0$ and m≥3.

Hence the theorem.

Theorem 3.2

The disconnected graph $(B_{3,4})\cup kC_m$ is a harmonic mean graph for $k\geq 0$ and m≥3.

Proof:

Let $V(S(B_{3,4})\cup kC_m)=\{u;u_1,u_2,\ldots,u_3;v_1,v_2,\ldots,v_3;y;x;x_1,x_2,\ldots,x_4;$

$y_1,y_2,\ldots y_4;w_{11},w_{12},\ldots,w_{1m};w_{21},w_{22},\ldots,w_{2m};\ldots w_{k1},w_{k2},\ldots w_{km}\}$ .and

$E(S(B_{n1,n2})\cup kC_m)=\{\ u_iv_i,uv_i,uy,yx,xx,x_jy_j|\ 1\leq i\leq 3\ ,1\leq j\leq 4\}\cup$

$[U_{i=1}^{K}\ ((U_{j=1}^{m-1}\{w_{ij}w_{ij+1}) \cup \{w_{im}w_{i1}\})]$.

Here p =17+km and q =16+km.

Define a function f :$((_{3,4})\cup kC_m) \rightarrow\{1,2,\ldots,q+1\}$ by

f(u)=7;

f(u_i)=3+ ,$1\leq i\leq 3$;

f(v_i)=$4-i$,$1\leq i\leq 3$;

f(y)=8;

f(x)=17;

f(x_j)=12+j,$1\leq j\leq 4$;

f(y_j)=8+j,$1\leq j\leq 4$ and

f(w_{ij})=17+(i−1)m+j,$1\leq i\leq k$,$1\leq j\leq$m.

Then the induced edge labels of $(B_{3,4})$ are given below:

$f^*(u_nv_n)$=1;

$f^*(u_i\text{v}_i)$=4+i,$1\leq i\leq 3$;

$f^*(u_iv_i)$=5−i,$1\leq i\leq 2$;

$f^*(u\text{v})$=8;

$f^*(uy)$=9;

$f^*(\text{x}_j)$ =13+j,$1\leq j\leq 4$;

$f^*(\text{x}_j\text{y}_j)$=9+$j$,$1\leq j\leq 4$;

and the set of all edge labels of kC_m is $\{18,19,\ldots,17+k\text{m}\}$.

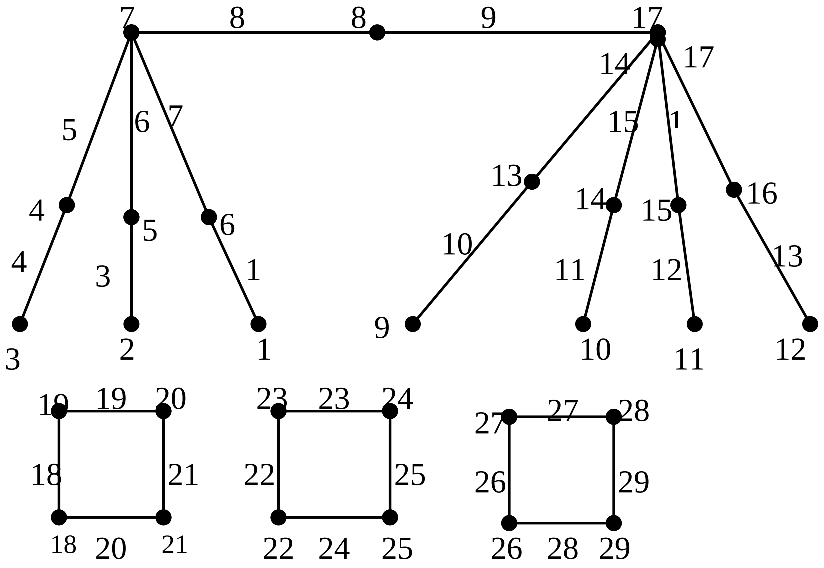
Therefore the set of all edge labels of $S(B_{3,4})\cup kC_m$ is $\{\ 1,3,4,\ldots .17+k\text{m}\ \}$.

Hence $(B_{3,4})\cup kC_m$ is a harmonic mean graph for $k\geq 0$ and m≥3.

Hence the theorem.

As an example harmonic mean labelling of $(B_{3,4})\cup 3C_4$

ISSN: 2454-4248
19 – 27



Theorem 3.3

The graph $P_n^* \cup kC_m$ is a harmonic mean graph for $k \geq 0$, m≥3 and $n \geq 2$.

Proof:

Let $V(P_n^* \cup kC_m) = \{v_1, v_2, \ldots, v_n; u_1, u_2, \ldots, u_{n-1}; w_{11}, w_{12}, \ldots, w_{1m}; w_{21}, w_{22}, \ldots, w_{2m}; w_{k1}, w_{k2}, \ldots, w_{km}\}$ and

E $(P_n^*) = \{v_i v_{i+1};\ v_i u_i;\ u_i v_{i+1} \mid 1 \leq i \leq n-1;\ u_i u_{i+1} \mid 1 \leq i \leq n-2\} \cup [U_{i=1}^{k}((U_{j=1}^{m-1}\ \{w_{ij} w_{ij+1}\}) \cup \{w_{im} w_{i1}\})]$.

Here $p = 2n + km - 1$ and $q = 4n - 5 + km$.

Define a function f $:(^*{}_n \cup kC_m) \rightarrow \{1, 2, \ldots, q+1\}$ by

$f(v_n) = 4(n-1)$;

$f(v_i) = 4i - 3, 1 \leq i \leq n-1$;

$f(u_i) = 4i - 1, 1 \leq i \leq n-1$ and

$f(w_{ij}) = 4(n-1) + (i-1)m + j, 1 \leq i \leq k, 1 \leq j \leq m$.

Then the induced edge labels of P_n^* are given below:

$f^*(v_i v_{i+1}) = 4i - 2, 1 \leq i \leq n-1$;

$f^*(u_i u_{i+1}) = 4i, 1 \leq i \leq n-2$;

$f^*(v_i u_i) = 4i - 3, 1 \leq i \leq n-1$;

$f^*(u_i v_{i+1}) = 4i - 1, 1 \leq i \leq n-1$;

and the set of all edge labels of kC_m is $\{4n-3, 4n-4, \ldots, 4(n-1) + km\}$.

Therefore the set of all edge labels of $P_n^* \cup kC_m$ is

$\{1, 2, 3 \ldots, 4(n-1) + km\}$.

Hence $P_n^* \cup kC_m$ is a harmonic mean graph for $k \geq 0$, m≥3 and $n \geq 2$.

Hence the theorem.

K-HARMONIC MEAN LABELLING OF SOME GRAPHS

Theorem: 5.1

The path P_n a k-harmonic mean graph for all k and $n \geq 2$.

ISSN: 2454-4248
19 – 27

Proof

Let $V(P_n)=\{v_i;1\le i\le n\}$ and

$$E(P_n)=\{e_i=v_i,v_{i+1};1\le i\le n-1\}$$

Define a function $f:V(P_n)\to\{k,k+1,k+2,.......,k+q\}$ by

$$f(v_i)=k+i-1,\forall 1\le i\le n$$

Then the induced edge labels are

$$f^*(e_i)=k+i-1,\forall 1\le i\le n-1$$

The above defined function f provides k-harmonic mean labelling of thegraph.

Hence P_n is a k-harmonic meangraph.

Theorem: 5.2

The Twig graph T_n is k-harmonic mean graph for all $n\ge 3$.

Proof:

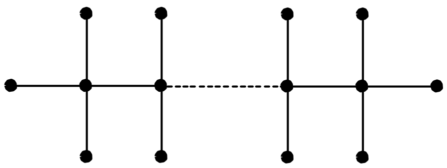
Let $V(T_n)=\{v_i;0\le i\le n-1,u_i,w_i;1\le i\le n-2$ and

$$E(T_n)=\{v_iu_i,v_iw_i;1\le i\le n-2,v_iv_{i+1};0\le i\le n-2$$

The ordinary labelling is

$u1$ $u2$ $un-3$ $un-2$

$v0$ $v1$ $v2$ $vn-3$ $vn-2$



$w1$ $w2$ $wn-3$ $wn-2$

First we label the vertices as follows

Define the function $f:V(T_n)\to\{k,k+2,k+2,......,k+q\}$ by

$$f(v_0)=k$$

$$f(v_i)=k+3i-2,\text{ for }1\le i\le n-1$$

$$f(w_i)=k+3i-1,\text{ for }1\le i\le n-2$$

$$f(u_i)=k+3i,\text{ for }1\le i\le n-2$$

Then the induced edge labels are

 ISSN: 2454-4248
19 – 27

$f^*(v_i v_{i+1}) = k + 3i$, for $0 \le i \le n-2$

$f^*(v_i u_i) = k + 3i - 1$, for $1 \le i \le n-2$

$f^*(v_i w_i) = k + 3i - 2$, for $1 \le i \le n-2$

The above defined function f provides k-harmonic mean labelling of the graph.

Theorem: 5.3

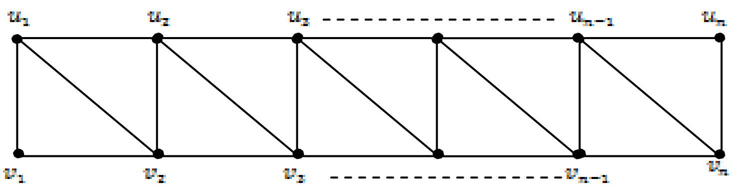
The Triangular ladder TL_n is k-harmonic mean graph for all $n \ge 2$.

Proof:

Let $V(TL_n) = \{u_i v_i ; 1 \le i \le n\}$ and

$$E(TL_n) = \{u_i u_{i+1}, v_i v_{i+1} ; 1 \le i \le n-1, u_i v_i ; 1 \le i \le n\}$$

The ordinary labelling is



First we label the vertices as follows

Define the function $f : V(TL_n) \to \{k, k+2, k+2, \ldots\ldots, k+q\}$ by

$f(u_i) = k + 4i - 3$, for $1 \le i \le n$

$f(v_1) = k$

$f(v_i) = k + 4i - 5$, for $2 \le i \le n$

Then the induced edge labels are

$f^*(u_i u_{i+1}) = k + 4i - 1$, for $1 \le i \le n-1$

$f^*(v_i v_{i+1}) = k + 4i - 3$, for $1 \le i \le n-1$

$f^*(u_i v_i) = k + 4i - 4$, for $1 \le i \le n$

$f^*(u_i v_{i+1}) = k + 4i - 2$, for $1 \le i \le n-1$

The above defined function f provides k-harmonic mean labelling of the graph.

Hence TL_n is a k-harmonic graph.

CONCLUSION

In this paper we discuss Harmonic mean labelling behaviour of some cycle related graphs such as duplication, joint sum of the cycle and identification of cycle. Also we investigate Harmonic mean labelling behaviour of Alternate Triangular Snake A(Tn), Alternate Quadrilateral Snake A(Qn).

In this paper, we establish harmonic mean labels of some well known subdivision graphs and some disconnected graphs.

In this paper we prove the Harmonic mean labelling behaviour for some special graphs.

In this paper, we establish the harmonic mean labelling of some standard graphs like subdivision of star ($K_{1,}$), subdivision of bistar ($B_{n,}$),the disconnected graph $S(K_{1,n}) \cup kC_m$.

BIBLOGRAPHY

[1] Harary.F., 1988, Graph theory, Narosa Publishing House, New Delhi.

[2] Somasundaram.S., and Ponraj R., 2003, Mean labelling of graphs National Academy of Science Letters vol.26, p.210-213

[3] Somasundaram S., Ponraj R., and Sandhya S.S., Harmonic mean labelling of graphs communicated.

[4] Sandhya S.S., Somasundaram S., and Ponraj R., Some Results on Harmonic Mean Graphs, International journal of Contemporary Mathematical Sciences 7(4) (2012), 197-208.

[5] Sandhya S.S., Somasundaram S., and Ponraj R., Some More Results on Harmonic Mean Graphs Journal of Mathematics Research 4(1) (2012) 21-29.

[6] Sandhya S.S., Somasundaram S., and Ponraj R. , Harmonic Mean Labelling of Some Cycle Related Graphs, International Journal of Mathematics Analysis vol.6, 2012. No.40 1997-2005.