Design and FPGA Implementation of Variable FIR Filters using the Spectral Parameter Approximation and Time-Domain Approach

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Abstract—This brief present a design and FPGA implementation of variable FIR filters using time domain approach of the spectral parameter approximation (SPA) technique. Farrow structure is used to implement the SPA-based filter. In the design of variable filters first design the practical filters which satisfy the given transition bandwidth, passband ripple, and stopband attenuation specifications and then approximate the coefficients of these filters by the impulse response of the Farrow structure. Least-squares technique is used to approximation problem. Various design and implementation cases with FPGA synthesis results are presented.

Keywords- *Farrow structure, FPGA, Reconfigurable FIR filter, Spectral parameter approximation technique, Variable cutoff frequency filter*.

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I. INTRODUCTION

Variable digital filters can change their frequency response characteristics based on the application requirement. Such variable finite impulse response (FIR) filters are required in many signal processing applications such as radar, sonar, and control systems, adaptive systems, vibration analysis, audio and biomedical signal processing, and wireless communications. In a special case of the variable FIR filters, the cutoff frequency of the filter needs to be varied, and the passband ripple, the stopband attenuation, and the transition bandwidth should be below predefined specified values.

For the variable coefficient filter, cutoff frequency is controlled by changing all the coefficients of the filter. However, when the cutoff frequency needs to be changed frequently, this approach becomes unsuitable for higher order filters due to the tedious updating routine (large number of memory access operations) and huge memory requirement (to store all the filter coefficients corresponding to all the desired responses). In practice, it is desirable to control the cutoff frequency using fewer number of parameters to keep the updating routine simple. A number of such reconfigurable filter design techniques have been developed to control the cutoff frequency by modifying the impulse response of the fixed-coefficient prototype filter. Details and reviews of such reconfigurable filter design techniques can be found in [3,14] and the references therein.

In the spectral parameter approximation (SPA) technique [2,4–12], the variable filter is implemented using Farrow structure. The SPA-based filter has smaller group delay, fixed transition bandwidth, requires very small number of variable multipliers (to be used for the variable weights), and its cutoff frequency can be controlled through only one variable parameter. The frequency response of the SPA-based filter is modeled as a polynomial function of the cutoff frequency parameter (α). The frequency responses of the fixed sub-filters in the Farrow structure are equivalent to the coefficients of the polynomial, and the frequency response (i.e., the cutoff frequency) of the SPA-based filter is controlled by the variable α. The cutoff frequency (or equivalently the passband frequency, as the transition bandwidth is fixed) of the SPA-based filter varies in the desired tuning range as α varies from 0 to 1.

The Farrow structure implementation of the SPA-based filter consists of $L + 1$ sub-filters, each of order N, as shown in Fig. 1. For the FIR filter design case, the frequency response of the SPA-based filter, $H(\omega, \alpha)$ is given by,

$$H(\omega,\alpha) = \sum_{k=1}^{L+1}\sum_{n=0}^{N} \alpha^{k-1} hs_k(n) e^{-jwn} \qquad (1)$$

where $hs_k(n)$ is the impulse response of the kth sub-filter.

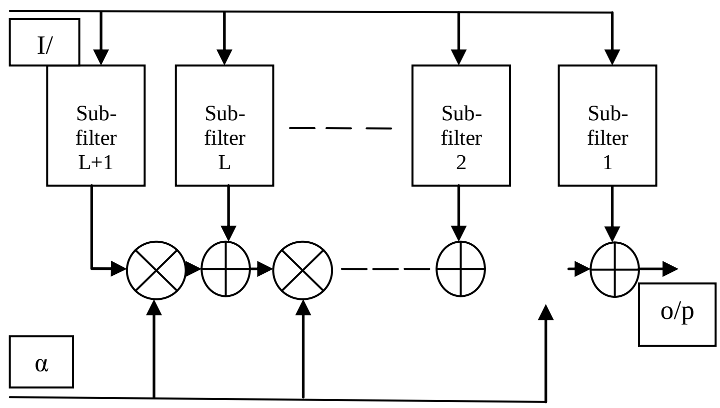

Fig.1 Farrow Structure Implementation of SPA-based filter

The objective here is to find the optimal coefficients $hs_k(n)$ such that the frequency response of the SPA-based filter will approximate the frequency response of the ideal filter for each cutoff frequency (in the desired tuning range), i.e., for each value of α (in 0–1). The ideal filter is defined as the filter with magnitude equal to 1 in its passband and 0 in its stopband. The approximation problem can be solved using the least-squares (LS) [4,7,8,12] or minimax techniques [6,9–11], by incorporating the desired peak to peak passband ripple (δ_p) and stopband attenuation (δ_s) constraints in the problem formulation. The optimal solutions can be obtained from the closed-form formulae [6–8,12], or by solving the system of linear equations obtained by the discretization method [2,4,5,9–11].

In this paper, we used a new approach for the design of SPA-based filter in the time domain, and model the approximation problem such that the optimal coefficients for the sub-filters are found so as to approximate the impulse responses of the *practical filters*, i.e., the impulse responses of the filters which satisfy the desired specifications on δ_p, δ_s, and transition bandwidth. In Section II, we present the time-domain approach in detail. Various design cases are presented in Section III, along with the comparisons and some observations. The concluding remarks are presented in the Section IV.

II. TIME DOMAIN APPROACH FOR THE DESIGN OF SPA-BASED FILTER

Let the cutoff frequency tuning range of the SPA-based filter be f_{cl} to f_{cm}, i.e., the cutoff frequency varies linearly as $f_{cl} \le f_c \le f_{cm}$ for $0 \le \alpha \le 1$. Let the corresponding passband and stopband ranges be f_{pl} to f_{pm} and f_{sl} to f_{sm}, respectively. In this case, we will formulate the SPA-based filter design problem so as to approximate the impulse response of the *practical filter* by the impulse response of the SPA-based filter for the desired tuning range. We use the discretization approach to formulate the approximation problem as follows.

First, design M practical filters, each of order N , with cutoff frequencies f_{cl}, $f_{cl} + \Delta f$, $f_{cl} + 2\Delta f, \ldots, f_{cm}$, and the transition bandwidth *tbw*, where $\Delta f = (f_{cm} - f_{cl})/(M - 1)$. These filters are designed using the standard filter design algorithm such as the Remez exchange algorithm or the Parks–McClellan algorithm [13].

Let the matrix of the impulse responses of the practical filters be denoted as

$$G_p = \begin{bmatrix} h_{0,1} & h_{1,1} & h_{2,1} & \cdot & h_{N,1} \\ h_{0,2} & h_{1,2} & h_{2,2} & \cdot & h_{N,2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{0,M} & h_{1,M} & h_{2.M} & \cdot & h_{N,M} \end{bmatrix} \qquad (2)$$

where $h_{i,j}$ denotes the ith sample of the impulse response of the jth filter, i.e. ith coefficient of the jth filter.

Discretize the cutoff frequency parameter, α, in M equidistant points in the range 0-1. The matrix of the impulse responses of the SPA-based filter is given by,

$$G = WG_s \qquad (3)$$

where

$$W = \begin{bmatrix} 1 & \alpha_1 & \alpha_1^2 & \cdot & \alpha_1^L \\ 1 & \alpha_2 & \alpha_2^2 & \cdot & \alpha_2^L \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \alpha_M & \alpha_M^2 & \cdot & \alpha_M^L \end{bmatrix} \qquad (4)$$

where α_j^i denotes the ith power of α_j, with $\alpha_1=0$, and

$$G_s = \begin{bmatrix} hs_{0,1} & hs_{1,1} & hs_{2,1} & \cdot & hs_{N,1} \\ hs_{0,2} & hs_{1,2} & hs_{2,2} & \cdot & hs_{N,2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ hs_{0,L+1} & hs_{1,L+1} & hs_{2.L+1} & \cdot & hs_{N,L+1} \end{bmatrix} \qquad (5)$$

is the matrix of the coefficients of the sub-filters of the Farrow structure and $hs_{i,j}$ denotes the ith coefficient of the jth subfilter. The system of linear equations $G_p=WG_s$, can be solved using the LS technique to find the optimal coefficients $hs_{i,j}$.

As, for $\alpha = 0$ the impulse response of the SPA-based filter is equal to the impulse response of the first sub-filter, in the proposed method, the first sub-filter in the Farrow structure is designed to have cutoff frequency f_{cl} and transition bandwidth *tbw*, i.e.,

$$\{hs_{0,1} \ldots hs_{N,1}\}=\{h_{0,1} \ldots h_{N,1}\} \qquad (6)$$

Therefore, in the proposed method, the design problem further reduces to finding the optimal coefficients for only the remaining L sub-filters. Therefore, it is sufficient to model the differences between the $M - 1$ practical filters and the first practical filter (i.e., the filter with cutoff frequency f_{cl}) using the remaining L sub-filters of the Farrow structure. Therefore, the matrices G_p, W , and G_s are modified as follows.

Matrix Gp:

$$G_{p1}(i,:)=G_p(1,:)-G_p(i,:) \text{ for } i>1 \qquad (7)$$

$$G_{p2}(i,:)=G_{p1}(i+1 , :) \text{ for } i=1 \text{ to } M-1 \qquad (8)$$

Matrix W:

$$W_1(i,:)=W(i+1, :) \text{ for } i=1 \text{ to } M-1 \qquad (9)$$

$$W_2(: , K)=W_1(: , K+1) \text{ for } k=1 \text{ to } L \qquad (10)$$

Matrix G_s:

$$G_{s1}(i , :) = G_s(i+1 , :) \text{ for } i=1 \text{ to } L \qquad (11)$$

The optimal coefficients $hs_{i,j}$ (for the remaining L sub-filters) are then obtained by solving the system of linear equations, given by $G_{p2} = W_2 G_{s1}$. Further, in the case of linear-phase filters, only half the matrices can be used due to the coefficient symmetry.

The magnitude response of the SPA-based filter designed following above procedure is checked for the M values used for the filter design, as well as for arbitrary values of α, to ensure that the desired specifications on δ_p and δ_s are satisfied. If these specifications are not satisfied, the above mentioned design procedure can be repeated by varying N and/or L such that the magnitude response of the SPA-based filter satisfies the desired specifications on δ_p and δ_s.

III. DESIGN CASES

We present five design cases of the SPA –based filter design for different tuning ranges and different transition bandwidths. For every case, the SPA-based filters are designed using the time-domain approach utilizing Least Squares technique. For every SPA-based filter, the weights are the powers of α and all the subfilters of its Farrow structure are of equal order N. All the filter designs are based on the discretization method.

Design Case.1 Let the desired cutoff frequency range be $f_{c1}=0.3$ to $f_{cm}=0.5$ with passband attenuation $\delta_p=0.1$dB, stopband attenuation $\delta_s= -45$dB and transition bandwith of 0.2. (All the frequency values mentioned in this paper are normalized with respect to π.)

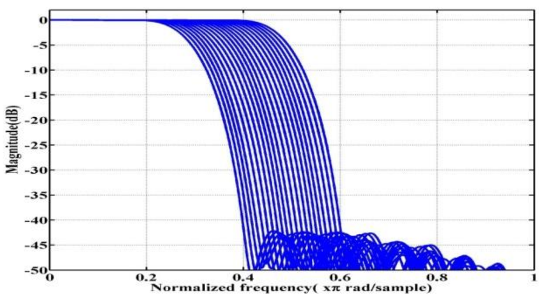

Fig.2 Magnitude Response of the Practical filters for various α values with M=21

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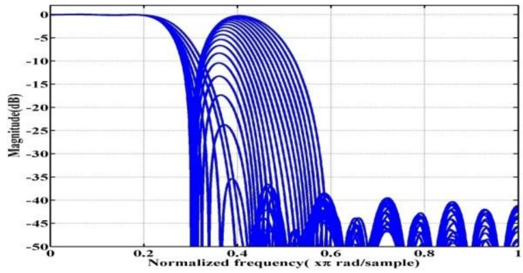

Fig.3 Magnitude Response of the SPA-based filter for various values of α with M=21

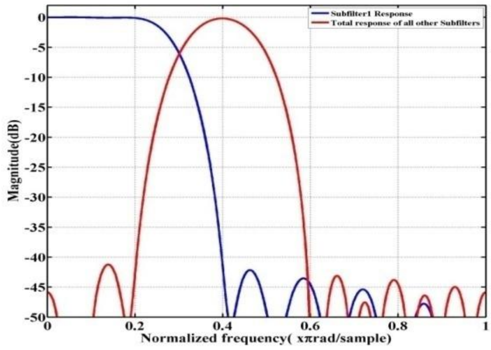

Fig.4 Magnitude Response of the subfilter1 and total response of all other subfilters with M=21

Design Case.2

Let the desired cutoff frequency range be f_{c1}=0.3 to f_{cm}=0.5 with passband attenuation δ_p=0.1dB, stopband attenuation δ_s= -45dB and transition bandwith of 0.1.

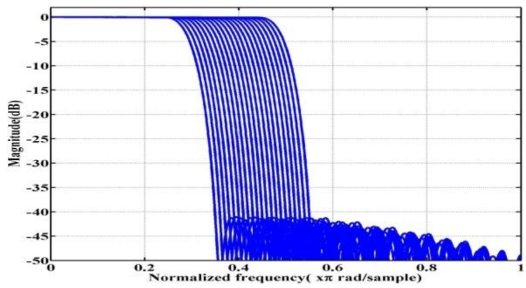

Fig.5 Magnitude Response of the Practical filters for various α values with M=21

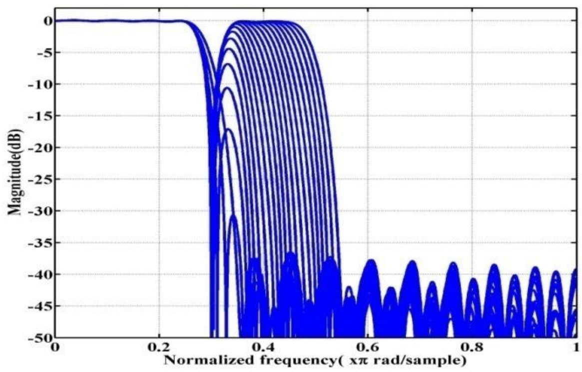

Fig.6 Magnitude Response of the SPA-based filter for various values of α with M=21

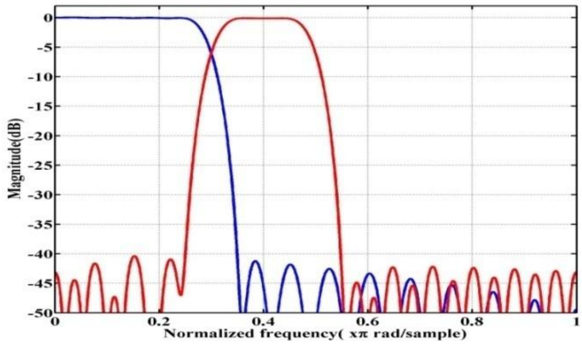

Fig.7 Magnitude Response of the subfilter1 and total response of all other subfilters with M=21

Design Case.3

Let the desired cutoff frequency range be f_{p1}=0.3 to f_{cm}=0.5 with passband attenuation δ_p=0.1dB, stopband attenuation δ_s= -45dB and transition bandwith of 0.05.

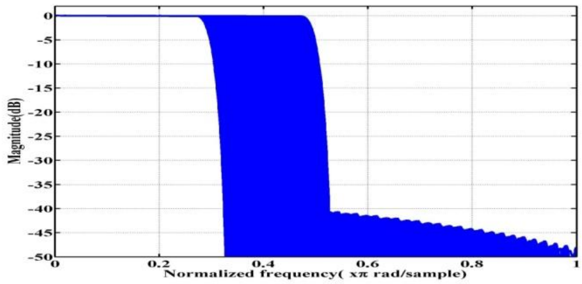

Fig.8 Magnitude Response of the Practical filters for various α values with M=21

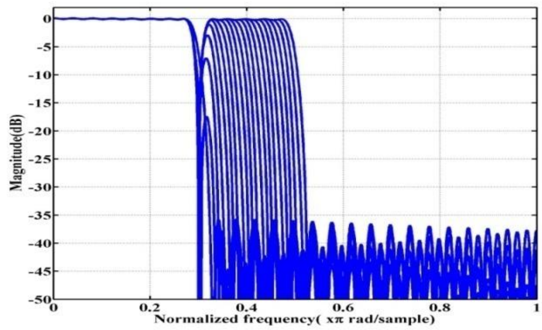

Fig.9 Magnitude Response of the SPA-based filter for various values of α with M=21

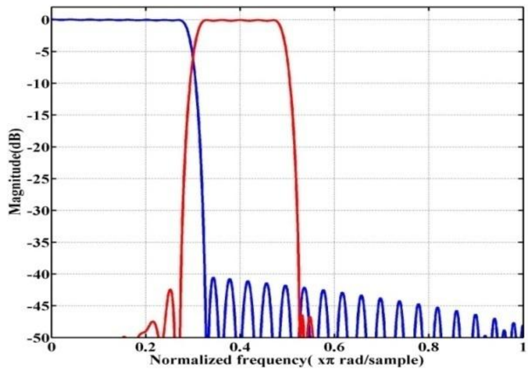

Fig.10 Magnitude Response of the subfilter1 and total response of all other subfilters with M=21

Design Case.4

Let the desired passband frequency range be f_{c1}=0.05 to f_{cm}=0.5 with passband attenuation δ_p=0.1dB, stopband attenuation δ_s= -45dB and transition bandwith of 0.2.

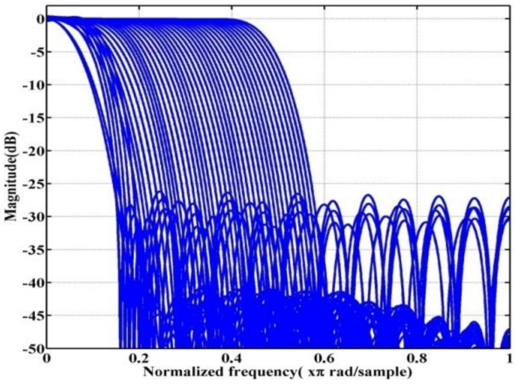

Fig. 11 Magnitude Response of the Practical filters for various α values with M=46

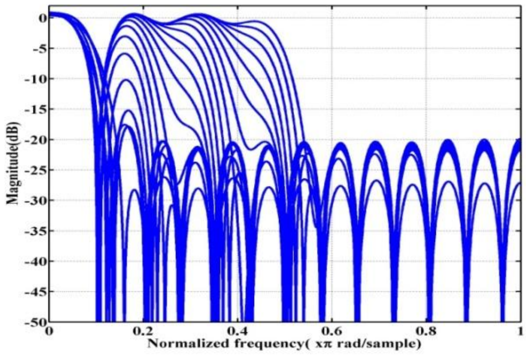

Fig.12 Magnitude Response of the SPA-based filter for various values of α with M=46

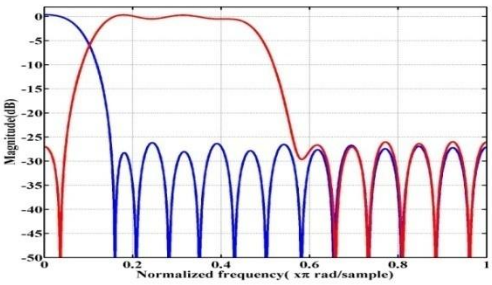

Fig.13 Magnitude Response of the subfilter1 and total response of all other subfilters with M=46

Design Case.5

Let the desired passband frequency range be f_{c1}=0.05 to f_{cm}=0.5 with passband attenuation δ_p=0.1dB, stopband attenuation δ_s= -45dB and transition bandwith of 0.1.

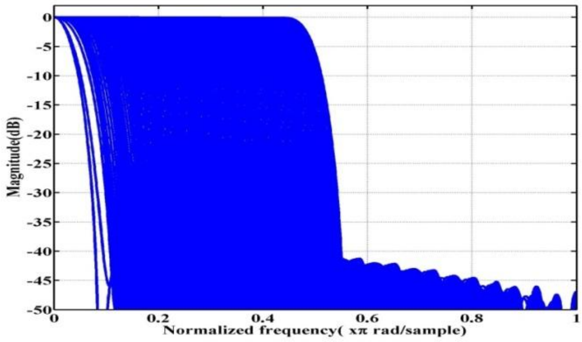

Fig.14 Magnitude Response of the Practical filters for various α values with M=46

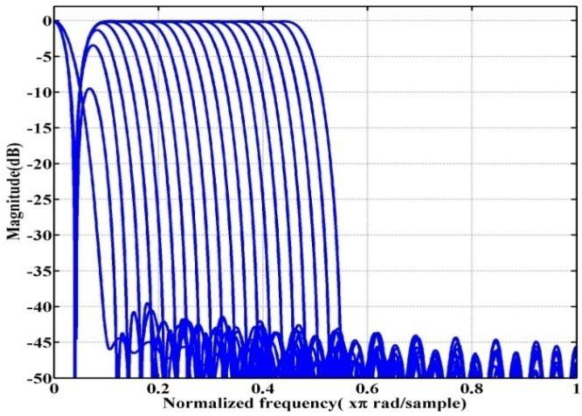

Fig.15 Magnitude Response of the SPA-based filter for various values of α with M=46

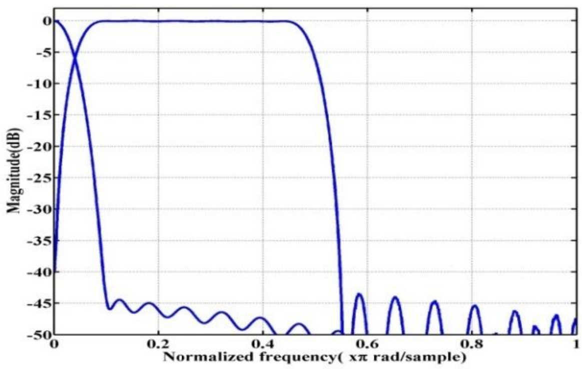

Fig.16 Magnitude Response of the subfilter1 and total response of all other subfilters with M=46

IV. FPGA IMPLEMENTATION

The design of proposed SPA-based filters were developed using Matlab, Simulink and HDL coder. The HDL code generated was tested on FPGA. The device used is Virtex-6, 6vlx240tff1156-1. Synthesis results for all the five cases of designed filters were given in the Table I and Table II. For the comparison purpose, we consider the total number of multipliers required for the SPA-based filter as the measure of complexity. For a wider or moderately wide transition bandwidths and small tuning range (Design cases 1,2, and 4) requires less number of multipliers compared to narrower transition bandwidth and wider tuning range filters (Design cases 3 and 4).

Compared to other technique, SPA-based filter design using frequency-domain [15] have less complex in Design cases 1, 2 and 4, but in design cases 3 and 5 SPA-based filter design using time domain have less complex.

Table I: FPGA Implementation of SPA filters- Comparison of Timing parameters

Parameter	Design Case1	Design Case2	Design Case3	Design Case4	Design Case5
Timing Summary					
Minimum Period(ns)	0.825	0.815	0.94	0.857	0.844
Maximum Frequency (MHz)	1212.12	1175.088	1063.401	1166.861	1184.834
Minimum input arrival time before clock (ns)	1.086	1.108	1.144	1.078	1.108
Maximum output required time after clock (ns)	66.95	106.135	209.376	93.456	184.757
Maximum combinational path delay (ns)	47.622	68.416	136.118	78.421	147.038

Table II: FPGA Implementation of SPA filters- Comparison of Device Utilization

Parameter	Design Case1	Design Case2	Design Case3	Design Case4	Design Case5
Slice Logic Utilization:					
Number of Slice Registers (301440)	475	883	1707	441	883
Number of Slice LUTs (150720)	252	855	7682	301	589
Number used as Logic (150720)	252	855	7682	301	589
Specific Feature Utilization:					
Number of DSP48E1s (768)	121	299	778	189	333

V. CONCLUSIONS

This paper presented a FPGA implementation of variable filters by using spectral parameter approximation and time-domain approach. In the implemented method, the optimal coefficients for the sub-filters in the Farrow structure are found so as to approximate the impulse responses of the practical filters by the impulse response of the Farrow structure. Least Squares approximation problem is used in the optimization. The results show that the proposed time domain method is effective compared to conventional frequency domain approach when the desired specifications are stringent.

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