Optimizing Triangular Parabolic Fuzzy EOQ Model with Shortage Using Nearest Interval Approximation

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Abstract: Many Re-searchers have introduced different topics using fuzzy numbers. Triangular parabolic, Trapezoidal parabolic, Hexagonal and octagonal fuzzy numbers are developed in such a way that their membership function attains the highest value only between an interval. If that fuzzy numbers are parabolic in shape when they attains the highest value at midpoint of an interval and called as Triangular parabolic fuzzy number. This paper deals with developing α-cut from Triangular parabolic membership function and using Triangle shaped values with an Economic Order Quantity(EOQ)model with shortage,here the setup cost ,holding cost, shortage cost are defined as fuzzy numbers. The purpose of this research is to analyse in which point attains it's maximum value also using midpoint of an interval. Finally numerical examples along with graphical representation of the results are presented.

Keywords: *Fuzzy number, EOQ, Triangular fuzzy number, α –cut, optimization, Interval number, fuzzy inventory, parabolic, membership function.*

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I. INTRODUCTION

Many researchers have worked with different types of fuzzy numbers. Triangular and Trapezoidal fuzzy numbers are the most commonly used fuzzy numbers. But these fuzzy numbers are not sufficient where there are more parameters. Here Triangular fuzzy numbers are used in three parameters. In Inventory practical problems, uncertainty always exists, also we deal with such uncertain situations fuzzy model is used [1],[5] in such cases, fuzzy set theory, introduced by Zadeh [7] is acceptable. There are several studies on fuzzy EOQ model.

This paper discusses a fuzzy EOQ model with shortage. Demand, holding cost, ordering cost, shortage cost are taken as triangular fuzzy numbers and expression for fuzzy cost is established for minimizing the cost function we transformed the fuzzy objective function into interval object function. Then, this single objective function is converted to multi-objective problem by defining left limit, right limit and center of the objective function.

II. RELATED WORK

Many Researchers doing fuzzy related work by using fuzzy numbers with EOQ model . Some of them using defuzzification with optimization twchnique, Fuzzy matrix games Defuzzificated by Trapezoidal parabolic fuzzy numbers etc…Many of them were done these type of works had been published. Here we took a new assumption of fuzzy numbers with Triangular parabolic using EOQ model to determine the optimum solution to reduce the average cost.

PRILIMINARIES

Definition 1:

A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0,1]. (i.e) $A = \{(x, \mu_A(X)); x \in X\}$,here $\mu_A: X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and μ_A is called the membership value of x $\in X$ in the fuzzy set A.

Definition 2:

Let $\Re$ be the set of all real numbers. An interval, may be expressed as

$$\bar{a} = [a_L, a_R] = \{x: a_L \leq x \leq a_R, a_L \in \Re, a_R \in \Re\} \qquad \textbf{(1)}$$

where $a_L\ and\ a_R$ are called the lower and upper limits of the interval $\bar{a}$, respectively.

If $a_L = a_R$ then $\bar{a} = [a_L, a_R]$ is reduced to a real number a, where $a = a_L = a_R$. Alternatively an interval $\bar{a}$ can be expressed in mean-width or center-radius form as $\bar{a} = \langle m(\bar{a}), w(\bar{a})\rangle$,

where $m(\bar{a}) = \frac{1}{2}(a_L + a_R)$ and $w(\bar{a}) = \frac{1}{2}(a_R - a_L)$ are respectively the mid-point and half-width of the interval $\bar{a}$. The set of all interval numbers in $\Re$ is denoted by I($\Re$).

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Optimization in interval environment

Now we define a general non-linear objective function with coefficients of the decision variables as interval numbers as

$$\text{Minimize } \bar{Z}(x) = \frac{\sum_{i=1}^{n}[a_{L_i}, a_{R_i}] \prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l}[b_{L_i}, b_{R_i}] \prod_{j=1}^{n} x_j^{q_j}} \quad \textbf{(2)}$$

subject to $x_j > 0, j = 1,2, \ldots, n$ and $x \in S \subset \Re$.

where S is a feasible region of x, $0 < a_{L_i} < a_{R_i}, 0 < b_{L_i} < b_{R_i}$ and r_i, q_j are positive numbers.

Now we exhibit the formulation of the original problem (2) as a multi-objective non-linear problem.

Now $\bar{Z}(x)$ can be written in the form $\bar{Z}(x) = [Z_L(x), Z_R(x)]$

where

$$Z_L(x) = \frac{\sum_{i=1}^{n} a_{L_i} \prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} b_{R_i} \prod_{j=1}^{n} x_j^{q_j}} \quad \textbf{(3)}$$

$$Z_R(x) = \frac{\sum_{i=1}^{n} a_{R_i} \prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} b_{L_i} \prod_{j=1}^{n} x_j^{q_j}} \quad \textbf{(4)}$$

The center of the objective function

$$Z_c(x) = \frac{1}{2}[Z_L(x), Z_R(x)] \quad \textbf{(5)}$$

Thus the problem (2) is transformed in to minimize

$$\{Z_c(x), Z_R(x); x \in S\} \quad \textbf{(6)}$$

Subject to the non-negativity constraints of the problem, where Z_C, Z_R are defined by (4) and (5).

III. NEAREST INTERVAL APPROXIMATION METHOD

According to Gregorzewski [3] we determine the interval approximation of a fuzzy number as: Let $\tilde{A} = (a_1, a_2, a_3)$ be an arbitrary triangular parabolic fuzzy number with a $\propto$-cuts $[A_L(\propto), A_R(\propto)]$ and with the following membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \left(\frac{(b-x)}{(b-a)}\right)^2 & ; \quad a \le x \le b \\ 1 & ; \quad x = b \\ 1 - \left(\frac{(x-b)}{(c-b)}\right)^2 & ; \quad b \le x \le c \end{cases} \quad \textbf{(7)}$$

Then by nearest interval approximation method, the lower limit C_L and upper limit C_R of the interval are

$$C_L = \int_0^1 A_L(\propto) d \propto \quad = \int_0^1 [b - (b-a)\sqrt{1-\alpha}] d \propto$$
$$= \frac{2a+b}{3}$$
$$C_R = \int_0^1 A_R(\propto) d \propto \quad = \int_0^1 [lp.o\ b + (c-b)\sqrt{1-\alpha}] d \propto$$
$$= \frac{b+2c}{3}$$

Therefore,

The interval number considering $\tilde{\boldsymbol{A}}$ as triangular fuzzy number is $\left[\frac{a_1+a_2}{2}, \frac{a_2+a_3}{2}\right]$ **(8)**

MODEL FORMULATION:

In this model, an inventory with shortage is taken into account. The purpose of this EOQ model is to find out the optimum order quantity of inventory item by minimizing the total average cost. We discuss the model using the following notations and assumptions throughout the paper.

Notations:

C_1: Holding cost per unit time per unit quantity.

C_2: Shortage cost per unit time per unit quantity.

C_3: Setup cost per period.

D: The total number of units produced per time period.

Q_1: The amount which goes into inventory.

Q_2: The unfilled demand.

Q: The lot size in each production run.

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Assumptions: (i)
Demand is known and uniform. (ii)
Production or supply of commodity is instantaneous. (iii)
Shortages are allowed. (iv)
Lead time is Zero.

Let the amount of stock for the item be Q_1 at time t=0 in the interval (0, t (=$t_1 + t_2$)), the inventory level gradually decrease to meet the demands. By this process the inventory level reaches zero level at time t_1 and then shortages are allowed to occur in the interval(t_1, t). The cycle repeats itself. (Fig.1)

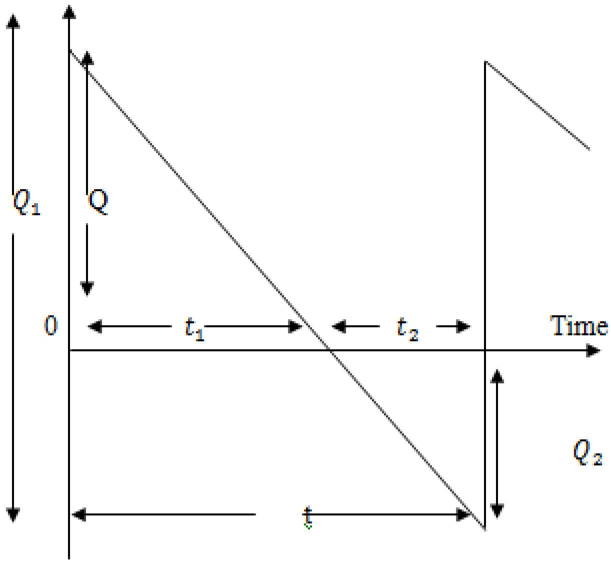

Fig .1 Inventory level

The order level Q>0 which minimizes the average total cost (Q) per unit time is given by

$$\min C(Q) = \frac{1}{2} C_1 \left(\frac{Q_1^2}{Q}\right) + \frac{1}{2} C_2 \left(\frac{Q_2^2}{Q}\right) + C_3 \left(\frac{D}{Q}\right) \quad \textbf{(9)}$$

Up to this stage, we are assuming that the demand, ordering cost, holding cost etc. as real numbers i.e. of fixed value. But in real life business situations all these components are not always fixed, rather these are different in different situations. To overcome these ambiguities we approach with fuzzy variables, where demand and other cost components are considered as triangular fuzzy numbers.

Let us assume the fuzzy demand $\widetilde{D} = (D{-}\propto, D, D + \beta)$ fuzzy holding cost $\tilde{C_1} = (C_1{-}\propto, C_1, C_1 + \beta)$, fuzzy shortage cost $\tilde{C_2}$= $(C_2{-}\propto, C_2, C_2 + \beta)$, fuzzy ordering cost $\tilde{C_3} = (C_3{-}\propto, C_3, C_3 + \beta)$ Replacing the real valued variables $D, C_1, C_2 \& C_3$ by the triangular fuzzy variables $\widetilde{D}, \widetilde{C_1}, \widetilde{C_2}, \widetilde{C_3}$ We get,

$$\tilde{C}(Q) = \frac{1}{2} \tilde{C_1} \left(\frac{Q_1^2}{Q}\right) + \frac{1}{2} \tilde{C_2} \left(\frac{Q_2^2}{Q}\right) + \tilde{C_3} \left(\frac{\widetilde{D}}{Q}\right) \quad \textbf{(10)}$$

Now we represent the fuzzy EOQ model to a deterministic form so that it can be easily tackled. Following Grzegorzewski [3] , the fuzzy numbers are transformed into interval numbers as

$\widetilde{D} = (D{-}\propto, D, D + \beta) = [D_1, D_2]$
$\tilde{C_1} = (C_1{-}\propto, C_1, C_1 + \beta) = [C_{1L}, C_{1R}]$
$\tilde{C_2} = (C_2{-}\propto, C_2, C_2 + \beta) = [C_{2L}, C_{2R}]$
$\tilde{C_3} = (C_3{-}\propto, C_3, C_3 + \beta) = [C_{3L}, C_{3R}]$

Using the above expression (10) becomes

$$\tilde{C}(Q) = [f_L, f_R] \quad \textbf{(11)}$$

Where ,

$$f_L = \frac{1}{2} C_{1L} \left(\frac{Q_1^2}{Q}\right) + \frac{1}{2} C_{2L} \left(\frac{Q_2^2}{Q}\right) + C_{3L} \left(\frac{D_L}{Q}\right) \quad \textbf{(12)}$$

$$f_R = \frac{1}{2} C_{1R} \left(\frac{Q_1^2}{Q}\right) + \frac{1}{2} C_{2R} \left(\frac{Q_2^2}{Q}\right) + C_{3R} \left(\frac{D_L}{Q}\right) \quad \textbf{(13)}$$

The composition rules of intervals are used in these equations.

Hence the proposed model can be stated as

Minimize $\{ f_L(Q),\ f_R(Q) \}$, **(14)**

Generally , the multi-objective optimization problem(14),in case of minimization problem, can be formulated in a conservative sense from(3)as

Minimize $\{ f_L(Q),\ f_R(Q) \}$, **(15)**

Subject to Q≥0. Where $f_C = \frac{f_L + f_R}{2}$.

Here the interval valued problem(14) is represented as

Minimize $\{ \boldsymbol{f_L(Q)},\ \boldsymbol{f_C(Q)},\ \boldsymbol{f_R(Q)} \}$, **(16)**

Subject to Q≥0

The expression (16) gives a better approximation than obtained from(14).`

Fuzzy programming technique for solution:

To solve multi-objective minimization problem given by (16), we have used the following fuzzy programming technique. For each of the objective functions $f_L(Q), f_C(Q), f_R(Q)$, we first find the lower bounds L_L, L_C, L_R (best values) and the upper bounds U_L, U_C, U_R (worst values), where L_L, L_C, L_R are the aspired level achievement and U_L, U_C, U_R are the highest acceptable level achievement for the objectives $f_L(Q), f_C(Q), f_R(Q)$ respectively and $d_k = U_k - L_k$ is the degradation allowance for objective $f_k(Q), k = L, C, R$. Once the aspiration levels and degradation allowance for each of the objective function has been specified, we formed a fuzzy model and then transform the fuzzy model into a crisp model. The steps of fuzzy programming technique is given below.

Step 1: Solve the multi-objective cost function as a single objective cost function using one objective at a time and ignoring all others.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived.

Step 3: From step 2, we find for each objective, the best L_k and worst U_k value corresponding to the set of solutions. The initial fuzzy model of (10) can then be stated as, in terms of the aspiration levels for each objective, as follows:

Find Q satisfying $f_k \lessapprox L_{k,} k = L, C, R$ subject to the non negativity conditions.

Step 4: Define fuzzy linear membership function $\left(\mu_{f_k}; k = L, C, R\right)$ for each objective function is defined by

$$\mu_{f_k} = \begin{cases} 0 & ; \quad f_k \geq U_k \\ 1 - \left(\frac{f_k - L_k}{d_k}\right)^2 & ; \quad L_k \leq f_k \leq U_k \\ 1 & ; \quad f_k \leq L_k \end{cases} \quad \textbf{(17)}$$

Step 5: After determining the linear membership function defined in (17) for each objective functions following the problem (16) can be formulated an equivalent crisp model $Max\ \alpha \leq \mu_{f_x}(x); k = L, C, R.\ \ \alpha \geq 0, Q \geq 0.$

IV.NUMERICAL EXAMPLE

In this section, the above mentioned algorithm is illustrated by a numerical example. Here the parameters demand, ordering cost, holding cost and shortage cost are considered as triangular parabolic fuzzy numbers (TPFN). After that, the fuzzy numbers are transformed into interval numbers using nearest interval approximation following [3].

Let $C_1 = 7, C_2 = 27, C_3 = 86, D = 4800$ units.

Taking these as triangular fuzzy numbers we have,

$$\tilde{C}_1 = (5,8,9), \tilde{C}_2 = (26,27,29), \tilde{C}_3 = (85,86,87), \quad , \tilde{D} = (4400,4800,5100).$$

The fuzzy numbers $\tilde{D}$, $\tilde{C}_1$, $\tilde{C}_2$, $\tilde{C}_3$ are transformed into interval numbers as,

$\tilde{D} = [D_L, D_R]$ = [4600, 4950]

$\tilde{C}_1 = [C_{1L}, C_{1R}]$ = [6.5, 8.5]

$\tilde{C}_2 = [C_{2L}, C_{2R}]$ = [26.5, 28]

$\tilde{C}_3 = [C_{3L}, C_{3R}]$ = [85.5, 86.5]

Individual minimum and maximum of objective functions $\boldsymbol{f_L}, \boldsymbol{f_C}, \boldsymbol{f_R}$ are given in Table 1.

Objective functions	Optimize f_L	Optimize f_C	Optimize f_R
$\boldsymbol{f_L}$	f_L'=1915.60	f_L''=3465.90	f_L'''=1936.03
$\boldsymbol{f_C}$	f_C'=1241.96	f_C''=684.11374	f_C'''=1160.15
$\boldsymbol{f_R}$	f_R'=2425.26	f_R''=4059.36	f_R'''=2399.81

Now we calculate

L_L=min (f_L',f_L'',f_L''') =1915.60 U_L=max (f_L',f_L'',f_L''') =3465.90

L_C=min $(f_L{}', f_L{}'', f_L{}''')$ =684.11374 U_C=max $(f_L{}', f_L{}'', f_L{}''')$ =1241.96
L_R=min $(f_L{}', f_L{}'', f_L{}''')$ =2399.8051 U_R=max $(f_L{}', f_L{}'', f_L{}''')$ =4059.36

Using the equation (18) , we formulate the following problem as ,

$$3\left(\frac{Q_1^2}{Q}\right) + 13.17\left(\frac{Q_2^2}{Q}\right) + \left(\frac{375452}{Q}\right) + (1550.3)\alpha \le 3465.90$$
$$3.67\left(\frac{Q_1^2}{Q}\right) + 13.67\left(\frac{Q_2^2}{Q}\right) + \left(\frac{40440}{Q}\right) + (557.85)\alpha \le 1241.96$$
$$4.34\left(\frac{Q_1^2}{Q}\right) + 14.17\left(\frac{Q_2^2}{Q}\right) + \left(\frac{433350}{Q}\right) + (1659.55)\alpha \le 4059.36 \quad \textbf{(18)}$$

Result and Discussion:

The solution obtained from (18) are given in table 2 and 3. Using non-linear programming technique equation (18) solved and the optimum value of α is found. It is given in table 2

Table 2: Optimum value of α

Maximum
0.947857

The optimum results for the total average cost, the economic order quantity average cost, the economic order quantity and level of inventory are found and given in table 3.

Table 3: The optimum results.

f_L^*	f_C^*	f_R^*	Q^*	Q_1^*
2269.63	811.53	2729.81	215.59182	171.37616

V .CONCLUSION

In this paper, a method of solving triangular parabolic membership function using to non-linear equations of fuzzy numbers has been considered, This impreciseness may be represented by various ways. In this paper we have represented by fuzzy numbers. Here we have present an inventory model with shortage cost, carrying or holding cost, setup or ordering cost and Demand are assumed as fuzzy numbers. In starting expression for the total cost is developed containing fuzzy parameters. Then each fuzzy quantities are approximated by interval number. After that the problem of minimizing the cost function is transformed into a multi-objective inventory problem, where the objective functions are left or right limit of the interval function. Fuzzy optimization technique is used to get the optimal results. At last numerical example is presented in it.

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