

On Soft β_1 -Paracompactness in Soft Topological Spaces

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Abstract— Recently, the author [19] introduced a new class spaces namely, soft nearly paracompact spaces and established some characterizations of these spaces. In the present paper a new class of spaces, namely soft β_1 -paracompact spaces in soft topological spaces are introduced and several characterizations of such spaces are also investigated. Relationships among soft paracompact, soft β_1 -paracompact, soft S_1 -paracompact, soft α -paracompact and soft P_1 -paracompact are provided with counter examples. **Keywords**- Soft β -open sets, soft β -regular open sets, soft β -regular space, soft paracompact spaces, soft β_1 -paracompact spaces, soft locally finite open refinement etc.

I. INTRODUCTION

The real world is too complex for our immediate and direct understanding, for example, many disciplines, including medicine, economics, engineering and sociology, are highly dependent on the task of modeling uncertain data. Since the uncertainty is highly complicated and difficult to characterize, classical mathematical approaches are often insufficient to useful models or derive effective. There are some theories: the theory of rough sets[21], the theory of vague sets[22], and the theory of fuzzy sets[23], which can be regarded as mathematical tools for dealing with uncertainties. However, all these theories have their own difficulties. The main reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theory as it was mentioned by Molodtsov in [1]. Molodtsov [1] initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty. He successfully applied the soft set theory into several directions such as smoothness of functions, game theory, Riemann Integration, theory of measurement, and so on. Soft set theory and its applications have shown great development in recent years. This is because of the general nature of parametrization expressed by a soft set. Shabir and Naz [2] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Later, Zorlutuna et al.[3], Aygunoglu and Aygun [4] and Hussain et al are continued to study the properties of soft topological space. They got many important results in soft topological spaces. Weak forms of soft open sets were first studied by Chen [5]. He investigated soft semi-open sets in soft topological spaces and studied some properties of it. Yumak and Kaymakci [14] are defined soft β -open sets and continued to study weak forms of soft open sets in soft topological space. Later, Yumak and Kaymakci [9] defined soft b-open (soft b-closed) sets and Akdag and Ozkan [6] soft α -open (soft α -closed) sets respectively. Recently, the author [19] introduced a new class spaces namely, soft nearly paracompact spaces and established some characterizations

of these spaces. In the present paper a new class of spaces, namely soft β_1 -paracompact spaces by utilizing soft β -open cover are introduced and several characterizations of such spaces are also investigated. Relationships among soft paracompact, soft β_1 -paracompact, soft S_1 -paracompact, soft α -paracompact and soft P_1 -paracompact are provided with counter examples.

II. PRELIMINERIES

Throughout this work, the space X and Y will always mean soft topological spaces (X, τ, E) and (Y, ν, K) with no separation axioms assumed, unless otherwise stated. Moreover, throughout this paper, a soft mapping $f : X \rightarrow Y$ stands for a mapping, where $f : (X, \tau, E) \rightarrow (Y, \nu, K)$, $u : X \rightarrow Y$ and $p : E \rightarrow K$ are assumed mappings unless otherwise stated. If (X, τ, E) is a given space, then $SInt(U, A)$ and $SCl(U, A)$ denotes the soft interior and soft closure of (U, A) respectively in (X, τ, E) . We recall some known definitions, lemmas and theorems, which will be used throughout the work.

Definition 2.1[1]: Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a non-empty subset of E . A pair (F, A) is called a soft set over X , where F is a mapping given by $F : A \rightarrow P(X)$ defined by $F(e) \in P(X) \forall e \in A$. In other words, a soft set over X is a parameterized family of subsets of the universe X . For $e \in A$, $F(e)$ may be considered as the set of e -approximate elements of the soft set (F, A) .

Definition 2.2[10]: A soft set (F, A) over X is called a null soft set, denoted by $\tilde{\varphi}$, if $e \in A, F(e) = \varphi$.

Definition 2.3[10]: A soft set (F, A) over X is called an absolute soft set, denoted by \tilde{A} , if $e \in A, F(e) = X$. If $A = E$, then the A -universal soft set is called a universal soft set, denoted by \tilde{X} .

Theorem 2.4[2]: Let Y be a non-empty subset of X , then \tilde{Y} denotes the soft set (Y, E) over X for which $Y(e) = Y$, for all $e \in E$.

Definition 2.5[2]: Let X be an initial universal set and A be the non-empty set of parameters. Then A soft set (P,A) over X is said to be soft point if there is exactly one $\lambda \in A$, such that $P(\lambda) = \{x\}$ for some $x \in X$ and $P(\mu) = \emptyset, \forall \mu \in A \setminus \{\lambda\}$. It will be denoted by P_λ^x .

Definition 2.6[10]: The union of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C) , where $C=A \tilde{\cup} B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

We write $(F,A) \tilde{\cup} (G,B) = (H,C)$

Definition 2.7 [10]: The intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe X , denoted by $(F,A) \tilde{\cap} (G,B)$, is defined as $C=A \cap B$ and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.8 [10]: Let (F,A) and (G,B) be two soft sets over a common universe X . Then, $(F,A) \tilde{\subseteq} (G,B)$ if $A \subseteq B$, and $F(e) \subseteq G(e)$ for all $e \in A$.

Definition 2.9 [2]: Let τ be the collection of soft sets over X , then τ is said to be a soft topology on X if satisfies the following axioms.

- (1) $\tilde{\emptyset}, \tilde{X}$ belong to τ ,
- (2) the union of any number of soft sets in τ belongs to τ ,
- (3) the intersection of any two soft sets in τ belongs to τ .

The triplet (X,τ,E) is called a soft topological space over X . Let (X,τ,E) be a soft topological space over X , then the members of τ are said to be soft open sets in X . A soft set (F,A) over X is said to be a soft closed set in X , if its relative complement $(F,A)^c$ belongs to τ .

Definition 2.10 [11]: For a soft set (F,A) over X , the relative complement of (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c, A)$, where $F^c : A \rightarrow P(X)$ is a mapping given by $F^c(e) = X - F(e)$ for all $e \in A$.

Definition 2.11: A soft set (F,A) in a soft topological space X is called

- (i) soft regular open (resp.soft regular closed) set [16] if $(F,A) = \text{Int}(\text{Cl}(F,A))$ [resp. $(F,A) = \text{Cl}(\text{Int}(F,A))$].
- (ii) soft semi-open (resp.soft semi-closed) set [8] if $(F,A) \tilde{\subseteq} \text{Cl}(\text{Int}(F,A))$ [resp. $\text{Int}(\text{Cl}(F,A)) \tilde{\subseteq} (F,A)$].
- (iii) soft pre-open (resp.soft pre-closed) [12] if $(F,E) \tilde{\subseteq} \text{int}(\text{cl}(F,E))$ [resp. $\text{Cl}(\text{Int}(F,A)) \tilde{\subseteq} (F,A)$].
- (iv) soft α -open (resp.soft α -closed) [6] if $(F,E) \tilde{\subseteq} \text{Int}(\text{Cl}(\text{Int}(F,E)))$ [resp. $\text{Cl}(\text{Int}(\text{Cl}(F,A))) \tilde{\subseteq} (F,A)$].

The family of all soft α -sets of (X,τ, E) denoted by τ_E^α , forms a soft topology on X , finer than τ .

(v) soft β -open (resp.soft β -closed) set [14] if $(F,A) \tilde{\subseteq} \text{Cl}(\text{Int}(\text{Cl}(F,A)))$ [resp. $\text{Int}(\text{Cl}(\text{Int}(F,A))) \tilde{\subseteq} (F,A)$].

The family of all soft α -sets of (X,τ, E) denoted by τ_E^α , forms a soft topology on X , finer than τ . For a soft topological space (X,τ, E) , if (X, τ_E^α, E) is normal, then $\tau = \tau_E^\alpha$.

The union of all soft β -open sets of X contained in the soft set (U,A) is called soft β -interior of (U,A) and is denoted by $S\beta\text{Int}(U,A)$ and intersection of all soft β -closed sets of X containing the soft set (U,A) is called soft β -closure of (U,A) and is denoted by $S\beta\text{Cl}(U,A)$.

Definition 2.12[20]: A soft topological space (X,τ,A) is called soft paracompact if every soft β -open covering admits a locally finite soft open refinement. A subset (U,A) is called soft paracompact if the relative topology τ_U defined on it is soft paracompact.

Definition 2.13[19]: A soft topological space X is called soft nearly paracompact (briefly, SNP) if every soft regularly open covering admits a locally finite soft open refinement. A subset (U,A) is called soft nearly paracompact if the relative topology defined on it is soft nearly paracompact.

Theorem 2.14[14]: Let (X,τ,E) be soft space, $(U,A) \tilde{\subseteq} (Y,\tau_Y,E) \tilde{\subseteq} (X,\tau,E)$ and (Y,τ_Y,E) is soft β -open in (X,τ,E) . Then (U,A) is soft β -open in (X,τ,E) if and only if (U,A) is soft β -open in (Y,τ_Y,E) .

Definition 2.15[20]: A family $\mathcal{U} = \{(U,A)_\alpha : \alpha \in \Lambda, \text{infinite set}\}$ of soft subsets of a soft space (X,τ,E) is said to be locally finite if for each soft point P_λ^x of X , \exists a soft open set (V,A) containing P_λ^x and (V,A) intersects at most finitely many members of \mathcal{U} .

Lemma 2.16[20]: The union of finite family of locally finite collection of soft sets in a soft space (X,τ,E) is locally finite family of soft sets.

Theorem 2.17[20]: If $\mathcal{U} = \{(U,A)_\alpha : \alpha \in \Lambda, \text{infinite set}\}$ is a locally finite family of soft subsets in a soft space (X,τ,E) and if $(V,A)_\alpha \tilde{\subseteq} (U,A)_\alpha$, for each $\alpha \in \Lambda$, then the family $\mathcal{V} = \{(V,A)_\alpha : \alpha \in \Lambda\}$ is a locally finite in X .

Lemma 2.18[20]: If $f : (X,\tau, E) \rightarrow (Y,\sigma, K)$ is a continuous surjection mapping and $\mathcal{U} = \{(U,A)_\alpha : \alpha \in \Lambda, \text{infinite set}\}$ is locally finite in (Y,σ, K) , then $f^{-1}(\mathcal{U}) = \{f^{-1}(U,A)_\alpha : \alpha \in \Lambda\}$ is locally finite in (X,τ,E) .

Lemma 2.18[20]: Let $f : (X,\tau, E) \rightarrow (Y,\sigma, K)$ be soft almost closed surjection with soft N -closed point inverse. If $\mathcal{U} =$

$\{(U, A)_\alpha : \alpha \in \Lambda, \text{ infinite set}\}$ is a locally finite open cover of soft space (X, τ, E) , Then $\{f(U, A)_\alpha : \alpha \in \Lambda\}$ is locally finite open cover of (Y, σ, K) .

Lemma 3.7[13]: Every soft compact soft T_2 -space is Soft regular.

III. SOFT β_1 -PARACOMPACT SPACES

In this section a new class of spaces, namely soft β_1 -paracompact spaces in soft topological spaces are introduced and several characterizations of such spaces are also provided.

Definition 3.1: A soft set (F, A) in a soft topological space X is called soft β -regular open set if it is both soft β -open and soft β -closed.

Definition 3.2: A space (X, τ, E) is said to be soft β -regular if for each soft β -open set (U, A) and for each soft point P_λ^F of (U, A) , there exists a soft β -open set (V, A) such that $P_\lambda^F \in (V, A) \subseteq S\beta Cl(V, A) \subseteq (U, A)$.

For any soft topological space (X, τ, E) , it is always true that $S\beta O(X, \tau_E^\alpha, E) = S\beta O(X, \tau, E)$

Definition 3.3: A soft topological space (X, τ, A) is called soft β_1 -paracompact (briefly, $S\beta P$) if every soft β -open covering admits a locally finite soft open refinement.

A subset (U, A) is called soft β -paracompact if the relative topology τ_U defined on it is soft β -paracompact.

Definition 3.4: A soft topological space (X, τ, A) is called soft α -paracompact (briefly, $S\alpha P$) if every soft α -open covering admits a locally finite soft open refinement.

A subset (U, A) is called soft α -paracompact if the relative topology τ_U defined on it is soft α -paracompact.

Definition 3.5: A soft topological space (X, τ, A) is called soft P_1 -paracompact (briefly, SPP) if every soft pre-open covering admits a locally finite soft open refinement.

A subset (U, A) is called soft P_1 -paracompact if the relative topology τ_U defined on it is soft P_1 -paracompact.

Definition 3.6: A soft subset (U, A) of a soft space X is said to be soft N -closed relative to X if for every soft open cover $\mathcal{U} = \{(U, A)_\alpha : \alpha \in \Lambda, \text{ infinite set}\}$ of (U, A) by soft open sets of X , \exists a finite subfamily Λ_0 of Λ such that $(U, A) \subseteq \bigcup_{\alpha \in \Lambda_0} \{SInt(SCl(U, A)_\alpha)\}$.

Remark 3.7: The following diagram shows relations among soft paracompact, soft β_1 -paracompact, soft S_1 -paracompact, soft α -paracompact and soft P_1 -paracompact.

Soft β_1 -paracompact \rightarrow soft P_1 -paracompact \rightarrow soft α -paracompact \rightarrow soft paracompact

$\square \leftarrow \text{soft } S_1\text{-paracompact} \rightarrow \square$

The converses of above implications need not be true as shown by the following examples:

Example 3.8 : Let $X = \mathbb{R}$, set of real numbers; $E = \{e_1, e_2\}$ and $\tau = \{\tilde{\varphi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, where, $(F_1, E), (F_2, E)$ and (F_3, E) are soft sets over X , defined as follows: $(F_1, E) = \{(e_1, \{1\}), (e_2, \varnothing)\}$, $(F_2, E) = \{(e_1, \varnothing), (e_2, \{2\})\}$, $(F_3, E) = \{(e_1, \{1\}), (e_2, \{2\})\}$

Then, τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X . We observe that (X, τ, E) is soft paracompact space but it is not soft β_1 -paracompact, since $\mathcal{C} = \{(e_1, \{1, x\}), (e_2, \varnothing) : x \in \mathbb{R}\} \cup \{(e_1, \varnothing), (e_2, \{2, y\}) : y \in \mathbb{R}\}$ is a soft β -open cover of (X, τ, E) which has no locally finite soft open refinement.

Example 3.9 : Let $X = \mathbb{N}$, set of natural numbers ; $E = \{e_1, e_2\}$ and $\tau = \{\tilde{\varphi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, where, $(F_1, E), (F_2, E)$ and (F_3, E) are soft sets over X , defined as follows: $(F_1, E) = \{(e_1, \{n, \text{ for some fixed } n \in \mathbb{N}\}), (e_2, \varnothing)\}$ $(F_2, E) = \{(e_1, \varnothing), (e_2, \{m, \text{ for some fixed } m \in \mathbb{N}\})\}$ $(F_3, E) = \{(e_1, \{n\}), (e_2, \{m\}), \text{ for some fixed } n, m \in \mathbb{N}\}$

Then, τ defines a soft topology on X , and thus (X, τ, E) is a soft topological space over X . We observe that (X, τ, E) is both soft P_1 -paracompact and soft S_1 -paracompact space since $SSO(X, \tau, E) = SPO(X, \tau, E) = \tau$ but it is not soft β_1 -paracompact, since $\mathcal{U} = \{(e_1, \{n, p\}), (e_2, \varnothing) : n, p \in \mathbb{N}\} \cup \{(e_1, \varnothing), (e_2, \{m, q\}) : m, q \in \mathbb{N}\}$ is a soft β -open cover of (X, τ, E) which has no locally finite soft open refinement.

Example 3.10 : Let $X = \{1/n : n \in \mathbb{N}\}$, set of natural numbers; $E = \{e_1, e_2\}$ and $\tau = \{\tilde{\varphi}, \tilde{X}, (F, E)\}$ where, (F, E) is soft set over X , defined as follows:

$(F, E) = \{(e_1, \{1/n\}), (e_2, \{1/m\}), \text{ for some fixed } n, m \in \mathbb{N}\}$
 Then, τ defines a soft topology on X , and thus (X, τ, E) is a soft topological space over X . We observe that (X, τ, E) is both soft α -paracompact space since, $S\alpha O(X, \tau, E) = \tau$ but it is not soft β_1 -paracompact, since $\mathcal{U} = \{(e_1, \{1/n, 1/p\}), (e_2, \varnothing) : n, p \in \mathbb{N}\} \cup \{(e_1, \varnothing), (e_2, \{1/m, 1/q\}) : m, q \in \mathbb{N}\}$ is a soft β -open cover of (X, τ, E) which has no locally finite soft open refinement.

Theorem 3.11. If (X, τ, E) is a soft β_1 -paracompact soft T_1 -space, then $\tau = S\beta O(X, \tau, E) = \tau_E^\alpha$.

Proof: Let (U, A) be a soft β -open set in (X, τ, E) . For each soft point P_λ^x of (U, A) , we have $\mathcal{U} = \{(U, A)\} \cup \{P_\lambda^x\}$ is a soft β -open cover for (X, τ, E) and so it has a locally finite soft open refinement $\mathcal{V} = \{(V, A)_\alpha : \alpha \in \Lambda\}$. Since \mathcal{V} is a refinement of \mathcal{U} and $P_\lambda^x \in (U, A)$, there exists an $\alpha_0 \in \Lambda$

such that $P_{\lambda}^x \in (V,A)_{\alpha_0} \tilde{\subset} (U,A)$ where, $(V,A)_{\alpha_0}$ is soft open and so (U,A) is soft open. Now, we know that $\tau \tilde{\subset} \tau_E^{\alpha} \tilde{\subset} S\beta O(X,\tau,E)$ and we show $\tau = S\beta O(X,\tau,E)$. So, $\tau = S\beta O(X,\tau,E) = \tau_E^{\alpha}$.

Corollary 3.12: Let (X,τ,E) be a soft T_1 -space, Then (X,τ,E) is soft β_1 -paracompact if and only if (X,τ,E) is soft paracompact and $\tau = S\beta O(X,\tau,E)$.

Proof: The proof follows immediately from Definition 3.3 and Theorem 3.11.

Definition 3.13: A soft space (X,τ,E) is said to be extremally soft disconnected if the closure of every soft open set in (X,τ,E) is soft open.

Theorem 3.14: Let (X,τ,E) be a soft β_1 -paracompact space. Then followings are true:

- (a) If (X,τ,E) is a soft T_1 -space, then it is extremally soft disconnected.
- (b) If (X,τ,E) is a soft T_2 -space, then it is soft β -regular.

Proof: (a) Let (U,A) be an soft open set in (X,τ,E) . Then $SCI(U,A)$ is soft β -open and by Theorem 3.11, $SCI(U,A)$ is soft open set in (X,τ,E) .

(b) Let (U,A) be an soft β -open set in (X,τ,E) and $P_{\lambda}^x \in (U,A)$. By Theorem 3.11, (U,A) is a soft open set. Since (X,τ,E) is soft regular, there exists an open set (V,A) such that $P_{\lambda}^x \in (V,A) \tilde{\subset} SCI(V,A) \tilde{\subset} (U,A)$. Since $SCI(V,A) \tilde{\subset} S\beta Cl(V,A)$, thus $P_{\lambda}^x \in (V,A) \tilde{\subset} SCI(V,A) \tilde{\subset} (U,A)$. It follows that (X,τ,E) is soft β -regular.

Theorem 3.15: Let (X,τ,E) be a soft space. Then followings are true:

- (a) If (X,τ_E^{α},E) is soft β_1 -paracompact, then (X,τ,E) is soft paracompact.
- (b) If (X,τ,E) is soft β_1 -paracompact, then (X,τ_E^{α},E) is soft β_1 -paracompact. The converse is true if the space is soft T_2 .

Proof: (a) Let \mathcal{U} be an soft open cover of (X,τ,E) . Then $\mathcal{U} = \{(U,A)_{\alpha} : \alpha \in \Lambda\}$ is an open cover of the soft β_1 -paracompact space (X,τ,E) and so it has a locally finite soft open refinement $\mathcal{V} = \{(V,A)_{\gamma} : \gamma \in \Lambda\}$ in (X,τ,E) . Now for every $(V,A)_{\gamma} \in \mathcal{V}$, choose $(U,A)_{\alpha} \in \mathcal{U}$ such that $(V,A)_{\gamma} \tilde{\subset} (U,A)_{\alpha}$. One can easily show that the collection $\mathcal{C} = \{(U,A)_{\alpha} \cap SInt(SCI(SInt(V,A)_{\gamma})) : (V,A)_{\gamma} \in \mathcal{V}\}$ is a locally finite open refinement of \mathcal{U} in (X,τ,E) . (b) Let $\mathcal{U} = \{(U,A)_{\alpha} : \alpha \in \Lambda\}$ be a soft β -open cover of (X,τ_E^{α},E) . Then \mathcal{U} is a soft

β_1 -open cover of the soft β_1 -paracompact space (X,τ,E) and so it has a locally finite soft open refinement $\mathcal{V} = \{(V,A)_{\gamma} : \gamma \in \Lambda\}$ in (X,τ,E) . Since, $\tau \tilde{\subset} \tau_E^{\alpha}$, then \mathcal{V} is a locally finite soft open refinement of \mathcal{U} in (X,τ_E^{α},E) and so (X,τ_E^{α},E) is soft β_1 -paracompact.

Conversely, let (X,τ_E^{α},E) be soft β_1 -paracompact. Then (X,τ_E^{α},E) is a soft paracompact T_2 -space and so it is normal. Therefore, $\tau = \tau_E^{\alpha}$.

The following examples show that the converse of (a) in the above theorem need not be true in general and the condition T_2 on the space (X,τ,E) in (b) is essential.

Example 3.16: Let (X,τ,E) be a soft topological space as in Example 3.8. Then $\tau_E^{\alpha} = \{\tilde{\varphi}, \tilde{X}, (F_1,E), (F_2,E), (F_3,E), (F,E)_x, (F,E)_y, (F,E)_z : x,y,z \in \mathbb{R}\} = S\beta O(X,\tau_E^{\alpha},E)$ where, $\{(F,E)_x : x \in \mathbb{R}\} = \{(e_1, \{1\}) \tilde{\cup} \{(F-\{1\}):F \subseteq \mathbb{R}\}), (e_2, \varnothing)\}$, $\{(F,E)_y : y \in \mathbb{R}\} = \{(e_1, \varnothing), (e_2, \{1\}) \tilde{\cup} \{(F-\{1\}):F \subseteq \mathbb{R}\})\}$, $\{(F,E)_z : z \in \mathbb{R}\} = \{(e_1, \{1\}) \tilde{\cup} \{(F-\{1\}):F \subseteq \mathbb{R}\}), (e_2, \{1\}) \tilde{\cup} \{(F-\{1\}) : F \subseteq \mathbb{R}\})\}$. Then (X,τ,E) is soft paracompact, but (X,τ_E^{α},E) is not soft β_1 -paracompact, since $\{(F,E)_x = \{(e_1, \{1,x\}), (e_2, \varnothing) : x \in \mathbb{R}\} \tilde{\cup} \{(e_1, \varnothing), (e_2, \{2,y\}) : y \in \mathbb{R}\}$ is a soft β -open cover of (X,τ,E) which has no locally finite soft open refinement.

Example 3.17: Let $X = \{a,b\}$ and $E = \{e_1, e_2\}$. Define the soft sets on X under the parameter set E by $(F_1,E) = \{(e_1, \varnothing), (e_2, \varnothing)\}$, $(F_2,E) = \{(e_1, \varnothing), (e_2, \{a\})\}$, $(F_3,E) = \{(e_1, \varnothing), (e_2, \{b\})\}$, $(F_4,E) = \{(e_1, \varnothing), (e_2, \{a,b\})\}$, $(F_5,E) = \{(e_1, \{a\}), (e_2, \varnothing)\}$, $(F_6,E) = \{(e_1, \{a\}), (e_2, \{a\})\}$, $(F_7,E) = \{(e_1, \{a\}), (e_2, \{b\})\}$, $(F_8,E) = \{(e_1, \{a\}), (e_2, \{a,b\})\}$, $(F_9,E) = \{(e_1, \{b\}), (e_2, \varnothing)\}$, $(F_{10},E) = \{(e_1, \{b\}), (e_2, \{a\})\}$, $(F_{11},E) = \{(e_1, \{b\}), (e_2, \{b\})\}$, $(F_{12},E) = \{(e_1, \{b\}), (e_2, \{a,b\})\}$, $(F_{13},E) = \{(e_1, \{a,b\}), (e_2, \varnothing)\}$, $(F_{14},E) = \{(e_1, \{a,b\}), (e_2, \{a\})\}$, $(F_{15},E) = \{(e_1, \{a,b\}), (e_2, \{b\})\}$, $(F_{16},E) = \{(e_1, \{a,b\}), (e_2, \{a,b\})\}$

Let $\tau = \{(F_1,E), (F_5,E), (F_7,E), (F_8,E), (F_{16},E)\}$. Then, τ is a soft topology on X , and (X,τ,E) is a soft topological space over X . The collection of soft β -open sets are $\{(F_1,E), (F_5,E), (F_6,E), (F_7,E), (F_8,E), (F_{13},E), (F_{14},E), (F_{15},E), (F_{16},E)\}$ and $\tau_E^{\alpha} = \{(F_1,E), (F_5,E), (F_6,E), (F_7,E), (F_8,E), (F_{13},E), (F_{14},E), (F_{15},E), (F_{16},E)\} = S\beta O(X,\tau_E^{\alpha},E)$. Then (X,τ_E^{α},E) is a soft β_1 -paracompact space but (X,τ,E) is not

soft paracompact since $\{(F_8, E), (F_{13}, E)\}$ is soft β -open cover of (X, τ, E) which has no locally finite soft open refinement.

Definition 3.18: A collection of soft subsets $\mathcal{U} = \{(U, A)_\alpha : \alpha \in \Lambda\}$ of a soft space (X, τ, E) is soft σ -locally finite if $\mathcal{U} = \bigcup_{n \in \mathbb{N}} \mathcal{U}_n$, where, each \mathcal{U}_n is discrete collection of soft subsets of X .

Theorem 3.19: If each soft β -open cover of a space (X, τ, E) has an soft open σ -locally finite refinement, then each β -open cover of X has a locally finite refinement.

Proof: Let $\mathcal{U} = \{(U, A)_\alpha : \alpha \in \Lambda\}$ be a soft β -open cover of X . Let $\mathcal{V} = \bigcup_{n \in \mathbb{N}} \mathcal{V}_n$ be an soft open σ -locally finite refinement of \mathcal{U} where, \mathcal{V}_n is locally finite. For each $n \in \mathbb{N}$ and each $(V, A) \in \mathcal{V}_n$, let $\mathcal{V}'_n = \mathcal{V}_n - \bigcup_{k < n} \mathcal{V}^*_k$, where, $\mathcal{V}^*_k = \bigcup \{(V, A) : (V, A) \in \mathcal{V}_k\}$ and put $\mathcal{V}'_n = \{(V, A)'_n : (V, A) \in \mathcal{V}_n\}$. Now put $\mathcal{W} = \{(V, A)'_n : n \in \mathbb{N}, (V, A) \in \mathcal{V}_n\} = \bigcup \{(V, A)'_n : n \in \mathbb{N}\}$. We show that \mathcal{W} is a locally finite refinement of \mathcal{U} . Let $P_\lambda^x \in X$ and let n be the first positive integer such that $P_\lambda^x \in \mathcal{V}'_n$. Therefore, $P_\lambda^x \in (V, A)'$ for some $(V, A)' \in \mathcal{V}'_n$. Thus \mathcal{W} is a cover of X . To show that \mathcal{W} is locally finite, let $P_\lambda^x \in X$ and let n be the first positive integer such that $P_\lambda^x \in \mathcal{V}'_n$.

Then, $P_\lambda^x \in (V, A)$ for some $(V, A) \in \mathcal{V}_n$. Now, $(V, A) \tilde{\cap} (V, A)' = \emptyset$, for each $(V, A)' \in \mathcal{V}'_k$ and for each $k > n$. Therefore, (V, A) can intersect at most the elements of \mathcal{V}'_k for $k \leq n$. Since \mathcal{V}'_k is locally finite for each $k \leq n$, so we choose an open set $(G, A)_{x(k)}$ containing P_λ^x such that $(G, A)_{x(k)}$ meets at most finitely many members of \mathcal{V}'_k . Finally, we put $(G, A)_x = (V, A) \tilde{\cap} (\bigcup_{k=1}^n (G, A)_{x(k)})$. Then, $(G, A)_x$ is an soft open set containing P_λ^x such that $(G, A)_x$ meets at most finitely many members of \mathcal{W} .

Theorem 3.20: Let (X, τ, E) be a soft β -regular space. If each soft β -open cover of the space X has a locally finite soft refinement, then each soft β -open cover of X has a locally finite soft β -closed refinement.

Proof: Let $\mathcal{U} = \{(U, A)_\alpha : \alpha \in \Lambda\}$ be a soft β -open cover of X . For each $P_\lambda^x \in X$, Pick a $(U, A)_x \in \mathcal{U}$ such that $P_\lambda^x \in (U, A)_x$. Since (X, τ, E) is soft β -regular, \exists a soft β -open set $(V, A)_x$ such that $P_\lambda^x \in (V, A)_x \tilde{\subset} S\beta Cl((V, A)_x) \tilde{\subset} (U, A)_x$. The family $\mathcal{V} =$

$\{(V, A)_x : x \in X\}$ is soft β -open cover of X and so has a locally finite refinement $\mathcal{W} = \{(W, A)_\alpha : \alpha \in \Lambda\}$. The collection $S\beta Cl(W, A) = \{S\beta Cl(W, A)_\alpha : \alpha \in \Lambda\}$ is locally finite for each $\alpha \in \Lambda$; if $(W, A)_\alpha \tilde{\subset} (V, A)_\alpha$, then $S\beta Cl(W, A)_\alpha \tilde{\subset} (U, A)^*$ for some $(U, A)^* \in \mathcal{U}$. Thus $S\beta Cl(W, A)$ is a soft β -closed locally finite refinement of \mathcal{U} .

Theorem 3.20: If (X, τ, E) is soft β -regular space, then the following are equivalent:

- (a) (X, τ, E) is soft β_1 -paracompact.
- (b) Each soft β -open cover of X has a σ -locally finite soft open refinement.
- (c) Each soft β -open cover of X has a locally finite soft refinement.
- (d) Each soft β -open cover of X has a locally finite soft β -closed refinement, provided that the space (X, τ, E) is extremely soft disconnected.

Proof: The proof follows from *Theorems 3.19 and 3.20*.

IV. CONCLUSION

The soft set theory proposed by Molodtsov offers a general mathematical tool for dealing with uncertain or vague objects. It is shown that soft sets are special type of information system known as single valued information system. Recently, the author [19] introduced a new class spaces namely, soft nearly paracompact spaces and established some characterizations of these spaces. In the present paper a new class of spaces, namely soft β_1 -paracompact spaces by utilizing soft β -open cover are introduced and several characterizations of such spaces are also investigated. Relationships among soft paracompact, soft β_1 -paracompact, soft S_1 -paracompact, soft α -paracompact and soft P_1 -paracompact are provided with counter examples.. I hope that the findings in this paper will help researcher enhance and promote the further study on soft topological spaces to carry out a general framework for their applications in generalized space and also in practical life.

ACKNOWLEDGMENT

The author wishes to thank the learned referee for his valuable suggestions which improved the paper to a great extent.

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