Two Warehouses Inventory Model with Quadratic Demand and Maximum Life Time

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Abstract— This paper deals with a two warehouses inventory model with quadratic demand. Due to some seasonal products, all time retailers not fulfill the demand of customers, so to solve this difficulty retailer storage some product for future sales in out of season. Here we consider two warehouses system, Own Warehouse (OW) and Rent Warehouse (RW). This paper considers maximum life time for the products and shortages are not allowed. Mathematical model of this paper is proposed to obtain the total cycle time and minimum inventory cost. A numerical example is give to validate this proposed model.

Keywords- *Two warehouses; Quadratic Dmand;Maximum life time.*

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I. Introduction

During the last some decade's inventory models have been broadly applied in any business organization. **Ghare and Schrader (1963)** proposed the first inventory model considered deteriorating items. **Hartely (1976**) proposed an inventory model under the two storage facilities. In his paper, the RW holding cost is greater than the OW holding cost; the products are accumulated first in the OW, and only surplus of the stock is accumulated in the RW. In addition, the products of the RW are issued first, and then the items of the OW. **Pakkala and Acharya (1992)** proposed an inventory model under the two warehouses for deteriorating items. **Goswami and Chaudhuri (1992)** developed an inventory model with linearly demand under the shortage and two warehouses. Inventory models under the two-warehouses management system for the deteriorating items were proposed by the researchers like **Benkherouf (1997), Bhunia and Maiti (1998), Lee and Ma (2000), Kharna and Chaudhary (2003), Zhou (2003), Yang (2004), Niu and Xie (2008), Malik et al. (2008), Hsieh et al. (2008), Lee and Hsu (2009), Singh and Malik (2009), Sana (2010), Sarkar et al. (2010), Singh and Malik (2010, a&b), Singh et al (2011, a&b), Sett et al (2012), Liao et al (2012), Wang et al (2013), Hsieh** and **Dye (2013), Sarkar and Sarkar (2013)** and others.

Shah et al (2014), proposed an inventory model with maximum life time under trade credit period. **Vashisth et al. (2015)** discussed an inventory model for non-instantaneous deteriorating items. **Kumar et al. (2016)** proposed a two warehouse management system with variable demand. **Vashisth et al (2016)** discussed a two warehouse model with quadratic demand and variable holding cost. Recently, **Malik et al (2017)** proposed an inventory model under two warehouses management system with quadratic demand.

In this paper we have discussed an inventory model with quadratic demand under the two warehouses management. Here consider the two warehouses system, OW and RW with variable deteriorations; the holding cost of RW is higher than OW. The total inventory cost of developed model is optimized and demonstrated by a numerical example.

II. Assumptions and Notations

For proposing this model we considered the following mathematical notation & assumptions:

D(t)	Demand rate
A	Ordering cost per order cycle
R	Maximum inventory level in RW
W	Maximum inventory level in OR
L	Total Maximum inventory level
p	Purchasing cost per unit
C_d	Deteriorating cost in RW and OW per unit
t_1	Fresh product time *i.e.*, no deterioration occurs
TIC	Total inventory cost per unit time

1. The Demand D(t)= $d_1+d_2\,t+d_3t^2$ where ($d_1, d_2, d_3 > 0$).
2. The deterioration rates in rent warehouse and own warehouse is $\theta_1(t)$ and $\theta_2(t)$; and defined by $\theta_1(t)$ =1/(1+R$_1$-t) and $\theta_2(t)$ =1/(1+R$_2$-t) respectively, where R$_1$ and R$_2$ is the maximum life time for the product in RW and OW respectively.

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3. The holding costs in rent warehouse and own warehouse is h_1 and h_2 respectively.
4. Shortages are not allowed.
5. $I_{r1}(t)$ and $I_{r2}(t)$ are the inventory levels in RW during $[0, t_1]$ and $[t_1, t_2]$ respectively; $I_{o1}(t)$, $I_{o2}(t)$ and $I_{o3}(t)$ are the inventory levels in OW during $[0, t_1]$, $[t_1, t_2]$ and $[t_2, T]$ respectively.

III. INVENTORY MODEL

According to above mention the notation and assumptions; the inventory levels in RW and OW are as follows by the governing equations:

$$\frac{dI_{r1}(t)}{dt} = -D(t) \qquad 0 \le t \le t_1 \qquad \text{.... (1)}$$

$$\frac{dI_{r2}(t)}{dt} + \theta_1(t)\, I_{R2}(t) = -D(t) \qquad t_1 \le t \le t_2 \qquad \text{.... (2)}$$

$$\frac{dI_{O1}(t)}{dt} = 0 \qquad 0 \le t \le t_1 \qquad \text{.... (3)}$$

$$\frac{dI_{O2}(t)}{dt} + \theta_2(t)\; I_{O2}(t) = 0 \qquad t_1 \le t \le t_2 \qquad \text{.....(4)}$$

$$\frac{dI_{O3}(t)}{dt} + \theta_2(t)\; I_{O3}(t) = -D(t), \qquad t_2 \le t \le T \qquad \text{.....(5)}$$

The boundary conditions are $I_{r1}(0) = R,\quad I_{r2}(t_2) = 0,\quad I_{O1}(0) = W, I_{O2}(t_1) = W,\; I_{O3}(T) = 0.$

Solution of the above equations (10 to (5) using the above mentioned boundary conditions, we get

$$I_{r1}(t) = R - d_1 t - \frac{d_2}{2}t^2 - \frac{d_3}{3}t^3 \qquad \text{.... (6)}$$

$$I_{r2}(t) = (1+R_1-t)\left\{\frac{d_3}{2}(t^2 - t_2^2) + d_4(t-t_2) + d_5 \log\left(\frac{1+R_1-t}{1+R_1-t_2}\right)\right\} \qquad \text{.... (7)}$$

$$I_{o1}(t) = W \qquad \text{....(8)}$$

$$I_{O2}(t) = W\left(\frac{1+R_2-t}{1+R_2-t_1}\right) \qquad \text{....(9)}$$

$$I_{O3}(t) = (1+R_2-t)\left\{\frac{d_3}{2}(t^2 - T^2) + d_6(t-T) + d_7 \log\left(\frac{1+R_2-t}{1+R_2-T}\right)\right\} \qquad \text{.....(10)}$$

Where $d_4 = d_2 + d_3(1+R_1), d_5 = d_1 + d_4(1+R_1),$
$d_6 = d_2 + d_3(1+R_2), d_7 = d_1 + d_6(1+R_2)$

As per continuity of Inventory levels in RW and OW, we have $I_{r1}(t_1) = I_{r2}(t_1)$

$$R = d_1 t_1 + \frac{d_2}{2}t_1^2 + \frac{d_3}{3}t_1^3 + (1+R_1-t_1)\left\{\begin{matrix}\frac{d_3}{2}(t_1^2 - t_2^2) + d_4(t_1-t_2) \\ + d_5 \log\left(\frac{1+R_1-t_1}{1+R_1-t_2}\right)\end{matrix}\right\} \qquad \text{.....(11)}$$

Also $I_{o2}(t_2) = I_{o3}(t_2)$

$$W = (1+R_2-t_1)\left\{\frac{d_3}{2}(t_2^2 - T^2) + d_6(t_2-T) + d_7 \log\left(\frac{1+R_2-t_2}{1+R_2-T}\right)\right\} \qquad \text{.....(12)}$$

The total inventory level $L = R + W$

$$= d_1 t_1 + \frac{d_2}{2}t_1^2 + \frac{d_3}{3}t_1^3 + (1+R_1-t_1)\left\{\begin{matrix}\frac{d_3}{2}(t_1^2 - t_2^2) + d_4(t_1-t_2) \\ + d_5 \log\left(\frac{1+R_1-t_1}{1+R_1-t_2}\right)\end{matrix}\right\}$$
$$+ (1+R_2-t_1)\left\{\frac{d_3}{2}(t_2^2 - T^2) + d_6(t_2-T) + d_7 \log\left(\frac{1+R_2-t_2}{1+R_2-T}\right)\right\} \qquad \text{.....(13)}$$

For proposed inventory model the following elements are used for determining the total inventory cost:

Ordering cost is $C_O = A$ ..... (14)

Inventory holding cost in RW is

$$C_{hr} = h_1\left(\int_0^{t_1} I_{r_1}(t)\,dt + \int_{t_1}^{t_2} I_{r_2}(t)\,dt\right)$$

$$= h_1\left[\begin{matrix}\left\{Rt_1 - \frac{d_1}{2}t_1^2 - \frac{d_2}{6}t_1^3 - \frac{d_3}{12}t_1^4\right\} \\ +(1+R_1)\left\{\begin{matrix}d_5(t_1-t_2) + (1+R_1-t)\log\left(\frac{1+R_1-t_1}{1+R_1-t_2}\right) \\ -d_3\left(\frac{t_1^3 + 2t_2^3 - 3t_1t_2^2}{6}\right) - d_4\left(\frac{(t_1-t_2)^2}{2}\right)\end{matrix}\right\} \\ -\left(\begin{matrix}d_5\left\{\begin{matrix}\frac{1}{2}(1+R_1)(t_1-t_2) + \frac{(t_1^2-t_2^2)}{4} \\ +\frac{1}{2}\left((1+R_1)^2 - t_1^2\right)\log\left(\frac{1+R_1-t_1}{1+R_1-t_2}\right)\end{matrix}\right\} \\ -d_3\left(\frac{(t_1^2-t_2^2)^2}{8}\right) - d_4\left(\frac{2t_1^3 + t_2^3 - 3t_1^2t_2}{6}\right)\end{matrix}\right)\end{matrix}\right] \qquad \text{.... (15)}$$

Inventory holding cost in OW is

$$C_{ho} = h_2\left(\int_0^{t_1} I_{o1}(t)\,dt + \int_{t_1}^{t_2} I_{o1}(t)\,dt + \int_{t_2}^{T} I_{o3}(t)\,dt\right)$$

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$$= h_2 \left[\begin{array}{l} Wt_1 + \dfrac{W}{1+R_2-t_1}\left\{(1+R_2)(t_2-t_1) - \dfrac{(t_2^2-t_1^2)}{2}\right\} \\ + (1+R_2)\left\{ \begin{array}{l} d_7(t_2-T) + (1+R_2-t)\log\left(\dfrac{1+R_1-t_2}{1+R_1-T}\right) \\ - d_3\left(\dfrac{t_2^3+2T^3-3t_2T^2}{6}\right) - d_6\left(\dfrac{(t_2-T)^2}{2}\right) \end{array} \right\} \\ - \left(\begin{array}{l} d_7\left\{ \begin{array}{l} \dfrac{1}{2}(1+R_2)(t_2-T) + \dfrac{(t_2^2-T^2)}{4} \\ + \dfrac{1}{2}\left((1+R_2)^2 - t_2^2\right)\log\left(\dfrac{1+R_2-t_2}{1+R_2-T}\right) \end{array} \right\} \\ - d_3\left(\dfrac{(t_2^2-T^2)^2}{8}\right) - d_6\left(\dfrac{2t_2^3+T^3-3t_2^2T}{6}\right) \end{array} \right) \end{array} \right] \quad \ldots..(16)$$

Deteriorating cost in RW is

$$DC_r = C_d\left(\int_{t_1}^{t_2} \theta_1(t) I_{r2}(t)\, dt\right)$$

$$= h_1 \left[\begin{array}{l} d_5(t_1-t_2) + (1+R_1-t)\log\left(\dfrac{1+R_1-t_1}{1+R_1-t_2}\right) \\ - d_3\left(\dfrac{t_1^3+2t_2^3-3t_1t_2^2}{6}\right) - d_4\left(\dfrac{(t_1-t_2)^2}{2}\right) \end{array} \right] \quad \ldots (17)$$

Deteriorating cost in OW is

$$DC_o = C_d\left(\int_{t_1}^{t_2} \theta_2(t) I_{O2}(t) dt + \int_{t_2}^{T} \theta_2(t) I_{O3}(t) dt\right)$$

$$= h_2 \left[\begin{array}{l} \dfrac{W}{1+R_2-t_1}(t_2-t_1) + d_7(t_2-T) \\ + (1+R_2-t)\log\left(\dfrac{1+R_1-t_2}{1+R_1-T}\right) \\ - d_3\left(\dfrac{t_2^3+2T^3-3t_2T^2}{6}\right) - d_6\left(\dfrac{(t_2-T)^2}{2}\right) \end{array} \right] \quad \ldots. (18)$$

Purchasing cost is $PC = p \times L$

$$= \left[\begin{array}{l} d_1t_1 + \dfrac{d_2}{2}t_1^2 + \dfrac{d_3}{3}t_1^3 + \\ (1+R_1-t_1)\left\{\dfrac{d_3}{2}(t_1^2-t_2^2) + d_4(t_1-t_2) + d_5\log\left(\dfrac{1+R_1-t_1}{1+R_1-t_2}\right)\right\} \\ + (1+R_2-t_1)\left\{\dfrac{d_3}{2}(t_2^2-T^2) + d_6(t_2-T) + d_7\log\left(\dfrac{1+R_2-t_2}{1+R_2-T}\right)\right\} \end{array} \right] \quad \ldots. (19)$$

Total inventory cost per unit time is

$$TIC(t_2, T) = \frac{1}{T}\left[C_O + C_{hr} + C_{ho} + DC_r + DC_o + PC\right] \quad \ldots (20)$$

TIC is minimum if $\dfrac{\partial TIC}{\partial t_2} = 0$, $\dfrac{\partial TIC}{\partial T} = 0$ and $\dfrac{\partial^2 TIC}{\partial t_2^2} > 0$.

IV. NUMERICAL EXAMPLE

For the discussed model we given the following example: When $d_1 = 1100$, $d_2 = 0.50$, $d_3 = 0.05$, $h_1 = 0.08$, $h_2 = 0.06$, $C_d = 0.05$, $p = 100$, $R_1 = 10$, $R_2 = 8$, $t_1 = 0.20$. Putting these values in the inventory model we get the optimal solution as $t_2 = 0.45$ and $T = 5.5$, $L = 5988$ and $TIC = 7680$.

CONCLUSION

In this paper, a mathematical inventory model is proposed to analyze the demand and deterioration for the maximum life time product. In general, mostly researchers proposed their research work fully pay no attention to the time varying demand or deterioration rate. To the author's best data, such type of concept for time varying quadratically increasing demand, maximum life time product under the two warehouse models, has not yet been proposed. Thus, our inventory model has a novel decision making looming that facilitate a manufacturing industry and business organization to diminish the inventory cost up to the optimum level. Further, new research track in the study of inventory system with probabilistic and stochastic, inflation, shortages, production and price dependent demand, and partial backlogging etc can be taken.

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