Performance of MMSE Based MIMO Radar Waveform Design in White and Colored Noise

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Abstract - In this paper we consider multiple input multiple output (MIMO) radar waveform design based on white noise and colored noise. The estimation oriented measure are used as criterions for optimal waveform design under transmitted power constraint. One of the estimation measure named the mean square error is minimized between target impulse response and target echoes, the optimal solutions are derived and the optimality of matching of the singular vectors are proved. This paper uses Sum power constraint power allocation strategy for allocation of power to the antennas which shares the total power among the antennas. The MMSE performance under white noise is better compared to colored noise. Earlier works on multiple input multiple output radar is based on MMSE considered the optimization of singular value of the waveform matrix. Here the optimization of the singular vectors was also considered which forms the basis of singular value optimization. To obtain minimum MMSE the pairing of the eigenvectors of the target and noise should be carefully designed. The optimal pairing of the eigenvectors based on MMSE is fixed.

Keywords- MIMO radar, MMSE, Sum Power Constraint, Waveform design.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) radar, which refers to radar systems with multiple antennas for transmitting independent waveforms and multiple receivers for receiving the target echoes. In [1] and [2], MIMO radar systems are classified in to two types which are colocated MIMO radar and statistical MIMO radar. In colocated MIMO radar, the transmitters and the receivers are close enough so that all the transmitters observe the same target RCS where we can have improved parameter identifiability and estimation accuracy. MIMO radar

with widely separated antennas are also called statistical MIMO radar which has the ability to improve the detection performance through spatial diversity and can obtain high resolution target localization.

For both types of MIMO radar, the problem to be considered is how to design the transmitted waveform [3]–[6]. In [7], waveform design for MIMO radar with widely separated antennas based on mutual information and Chernoff bound respectively was derived. In [8] a procedure is developed to design the optimal waveform which maximizes the signal-tointerference plus-noise ratio (SINR) at the output of the detector. In [9] the use of multiple signals with arbitrary crosscorrelation matrix has been proposed and that the crosscorrelation matrix can be chosen to achieve a desired spatial transmit beam pattern. In [10] considers the waveform design for MIMO radar by optimizing two criteria: maximization of the MI and minimization of the minimum mean-square error (MMSE). It was demonstrated that these two different criteria yield essentially the same optimum solution. Therefore, waveform design for MIMO radar becomes of great interest and there is much work pertaining to it [7]-[20],[24],[25].It should be noted that the optimal waveform design of MIMO radar in noise is not only related to the power allocation strategy, but also the optimization of the singular vectors of waveform matrix. The power allocation strategy usually depends on the optimization result of the singular vectors. However, the optimization of the singular vectors is not sufficiently considered in [18], in which the left singular vectors are constrained without proof of optimality to be the eigenvectors of the colored noise, while the right singular vectors are constrained to be the target eigenvectors by using the result derived for the white noise case in [10]. Therefore, by using these constraints, the waveform design problems in [18] are simplified to power allocation problems. Moreover, the ordering of the eigen values in the eigen decomposition of the covariance matrix of the target and colored noise, which has significant impact on the waveform design result as shown in [19], is also ignored in [18]. Thus, further results about the optimal waveform design based on MMSE should be derived. The outline is given as follows. We present the signal model in Section II. In Section III, the waveform optimization problems based on MMSE is explained. We prove that to attain minimum MMSE, the left singular vectors of the waveform matrix should be the eigenvectors of the colored noise, while the right singular vectors should be the target eigenvectors. Moreover, for the optimal waveform based on MMSE, the eigenvector of the k th smallest noise eigen value and that of the k th largest target eigen value, should correspond to the same singular value. In Section IV, simulation results are given. Finally, we draw the conclusion in Section V.

Notation: Throughout this paper, superscript $(.)^T$ and $(.)^H$ denote transpose and conjugate transpose, respectively. $\|.\|_F$ and tr(.) represent the Frobenius norm and trace of a matrix, respectively. Complex Gaussian distribution is denoted by $\mathcal{CN}(...)$. Finally, $(x)^+$ means $\max(x, 0)$.

II. SIGNAL MODEL

The MIMO radar system has M transmitters and N receivers, in which the receivers are assumed to be colocated. After coherently combining the received signal, the signal model is given by

$$\boldsymbol{r_1} = \boldsymbol{X}\boldsymbol{h_1} + \boldsymbol{w_1} \tag{1}$$

where $r_1 \in \mathbb{C}^{L \times 1}$ is the signal received at first receiver, $\mathbf{X} = [X_1 \dots X_M] \in \mathbb{C}^{L \times RM}$ is the waveform matrix of the signal, $X_m \in \mathbb{C}^{L \times R}$ is the m^{th} transmitter waveform, $h_1 = [h_{1,1}^T \dots h_{1,M}^T]^T \in \mathbb{C}^{RM \times 1}$, $h_{1,m} \in \mathbb{C}^{R \times 1}$ is the target viewing aspect from the m^{th} transmitter to the first receiver, $w_1 \in \mathbb{C}^{L \times 1}$ is the first receiver colored noise, L is the number of samples, K is the channel delay.

The assumptions considered in the design of transmit waveform.

1.
$$h_1 \sim (\mathcal{N}(0, R_H), w_1 \sim (\mathcal{N}(0, R_u)),$$

2. w_1 does not depend on X and is independent of h_1

III. OPTIMAL WAVEFORM DESIGN BASED ON MINIMIZING MMSE

The MMSE estimator of h_1 which is given by

$$\widehat{h}^{MMSE} = R_H X^H (R_u + X R_H X^H)^{-1} r_1$$
(2)

The MMSE of arbitrary **X**, is denoted by $Err_{MMSE} = [\|h_1 - \hat{h}^{MMSE}\|_F^2]$ is given by

Err_{MMSE}

$$= tr(\mathbf{R}_{H}) - tr(\mathbf{R}_{H}\mathbf{X}^{H}(\mathbf{R}_{u} + \mathbf{X}\mathbf{R}_{H}\mathbf{X}^{H})^{-1}\mathbf{X}\mathbf{R}_{H})$$
$$= tr[(\mathbf{R}_{H}^{-1} + \mathbf{X}^{H}\mathbf{R}_{u}^{-1}\mathbf{X})^{-1}]$$
(3)

The waveform optimization problem based on minimizing MMSE can be formulated as

 $\frac{\min}{X} tr[(R_{H}^{-1} + X^{H} R_{u}^{-1} X)^{-1}]$

s.t.
$$tr(XX^H) \le P_0$$
 (4)

where P_0 is the total power transmitted.

To solve (4), two lemmas are given.

Lemma 1:

Let **A** and **B** be $n \times n$ positive-definite Hermitian matrices with eigen decomposition $A = U_A \sum_A U_A^H$ and $B = U_B \sum_B U_B^H$

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respectively, the eigen values of **A** and **B** satisfy that $\alpha_1 \ge \alpha_2 \ge \cdots \ge \alpha_n$ and $\beta_1 \ge \beta_2 \ge \cdots \ge \beta_n$, then

$$tr[(\boldsymbol{A} + \boldsymbol{B})^{-1}] \ge \sum_{i=1}^{n} \frac{1}{\alpha_i + \beta_{n-i} + 1}$$
(5)

 $U_A = U_B P$ lower bound is achieved, where

$$P = \begin{bmatrix} 0 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 0 \end{bmatrix}$$

Proof:

Denote the eigen values of A + B by $\lambda_1(A + B) \ge \cdots \ge \lambda_n(A + B)$ and $\lambda(A + B) = [\lambda_1(A + B), \dots, \lambda_n(A + B)]^T$.

$$\lambda^{\downarrow}(\mathbf{A}) + \lambda^{\uparrow}(\mathbf{B}) < \lambda(\mathbf{A} + \mathbf{B})$$

where $\lambda^{\downarrow}(\mathbf{A}) = [\alpha_1, ..., \alpha_n]^T, \lambda^{\uparrow}(\mathbf{B}) = [\beta_1, ..., \beta_n]^T$,

x < y denotes x is majorized by y.

Since $f(\mathbf{x}) = \frac{1}{x}$ is convex for $\mathbf{x} > 0$, then $f(\mathbf{y}) = \sum_{i=1}^{n} \frac{1}{y_i}$ is schur-convex for $\mathbf{y} = [\mathbf{y}_1 \dots \mathbf{y}_n]^T \in \mathbb{R}^n_+$. Nothing that $tr[(\mathbf{A} + \mathbf{B})^{-1}] \ge \sum_{i=1}^{n} \frac{1}{\lambda_i(\mathbf{A} + \mathbf{B})}$

$$tr[(A+B)^{-1}] \ge \sum_{i=1}^{n} \frac{1}{\alpha_i + \beta_{n-i} + 1}$$

Lemma 2:

Let $\mathbf{R} \in \mathbb{C}^{n \times n}$ be a positive –semi definite matrix and its eigen decomposition is given by $\mathbf{R} = \mathbf{V} \sum \mathbf{V}^H$ where $\sum =$ diag[$\sigma_1, ..., \sigma_n$] and $\sigma_1 \ge \cdots \ge \sigma_n$. Assume that $\mathbf{\tilde{X}} \in \mathbb{C}^{n \times m}$ satisfies $\mathbf{\tilde{X}}^H \mathbf{R} \mathbf{\tilde{X}} = \mathbf{D}$, where $\mathbf{D} =$ diag[$d_1, ..., d_m$]), $d_1 \ge \cdots \ge d_m$ and m < n, then it is always possible to find $\mathbf{\bar{X}} \in \mathbb{C}^{n \times m}$, which satisfies $\mathbf{\bar{X}}^H \mathbf{R} \mathbf{\bar{X}} = \alpha \mathbf{\tilde{X}}^H \mathbf{R} \mathbf{\tilde{X}}$ with $tr(\mathbf{\tilde{X}} \mathbf{\tilde{X}}^H) = tr(\mathbf{\bar{X}} \mathbf{\bar{X}}^H)$ and $\alpha \ge 1$, where $\mathbf{\bar{X}}$ can be expressed as

$$\overline{X} = V[\sum_{x}^{\frac{1}{2}} \mathbf{0}_{m \times (n-m)}]^{T}$$
(6)

$$\begin{split} & \sum_{X} = \alpha \ diag\left(\left[d_{1}\sigma_{1}^{-1}, \ldots, d_{m}\sigma_{m}^{-1}\right]\right), = \frac{\left(\sum_{i=1}^{m} d_{i}d_{Q,i}\right)}{\left(\sum_{i=1}^{m} d_{i}\sigma_{i}^{-1}\right)}, \ d_{Q,i} \text{ is the} \\ & i^{th} \text{ diagonal element of } QR^{-1}Q^{H}, \text{ and } Q \in \mathbb{C}^{n \times n} \text{ is a unitary} \\ & \text{matrix which satisfies that } R^{\frac{1}{2}} \widetilde{X} = Q[D^{\frac{1}{2}} \mathbf{0}_{m \times (n-m)}]^{T}. \end{split}$$

Proof:

Let the eigen decomposition of $X^H R_u^{-1} X$ and R_H be $X^H R_u^{-1} X = V_1 \sum_{1}^{\downarrow} V_1^H$ and $R_H = V_H \sum_{H}^{\downarrow} V_H^H$ respectively, where $\sum_{1}^{\downarrow} = diag([\sigma_{1,1} \dots \sigma_{1,KM}]), \sigma_{1,1} \ge \dots \ge \sigma_{1,KM}, \qquad \sum_{H}^{\downarrow} = diag([\sigma_{1,1} \dots \sigma_{1,KM}])$ and $\sigma_{h,1} \ge \dots \ge \sigma_{h,KM}$. Then according to Lemma 1. We have

$$tr[(\mathbf{R}_{H}^{-1} + \mathbf{X}^{H}\mathbf{R}_{u}^{-1}\mathbf{X})^{-1}] \ge \sum_{i=1}^{KM} \frac{1}{\sigma_{h,i}^{-1} + \sigma_{1,i}}$$
(7)

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where the equality holds if $V_{H} = V_{1}$.

The optimal solution for (4) should satisfy $V_H^H X^H R_u^{-1} X V_H = \sum_{1}^{\downarrow}$. Let $X^H = X V_H$ and since that s.t. $tr(X_H X_H^H) = tr(X X^H)$, then the optimal solution of (4) can be obtained by solving

$$\min_{X_{H}} tr[((\sum_{H}^{\downarrow})^{-1} + X_{H}^{H} R_{u}^{-1} X_{H})^{-1}]$$
s.t. $tr(X_{H} X_{H}^{H}) \leq P_{0}, X_{H}^{H} R_{u}^{-1} X_{H} = \sum_{1}^{\downarrow}$ (8)

If the solution of (8) is denoted by X_{H}^{opt} , then the optimal solution of (4) satisfies $X^{opt} = X_{H}^{opt} V_{H}^{H}$.

Next we decompose $R_u = V_u \sum_u^{\dagger} V_u^{H}$, where $\sum_u^{\dagger} = diag([\sigma_{u,1} \dots \sigma_{u,L}])$ and $\sigma_{u,1} \leq \dots \leq \sigma_{u,L}$, then we have $R_u^{-1} = V_u (\sum_u^{\dagger})^{-1} V_u^{H}$ and the diagonal elements of $(\sum_{1}^{\downarrow})^{-1}$ are in decreasing order. Note that the diagonal elements of \sum_u^{\dagger} are in decreasing order, then by Lemma 2 and assume that L > KM without loss of generality, it is always possible for us to find \bar{X} which satisfies that $tr(X_H X_H^H) = tr(\bar{X} \bar{X}^H)$ and $\bar{X}^H R_u^{-1} X^H = \alpha X_H^H R_u^{-1} X_H$, where $\alpha \geq 1$. Moreover, \tilde{X} can be written

$$\overline{\boldsymbol{X}} = \boldsymbol{V}_{u} \left[\sum_{x1}^{\frac{1}{2}} \boldsymbol{0}_{KM \times (L-KM)} \right]^{T}$$
(9)

where $\sum_{x_1} \in \mathbb{C}^{KM \times KM}, \sum_{x_1} = diag([\sigma_{x,1}, \dots, \sigma_{x,KM}]).$

Since $tr(A)^{-1}$ is a monotonic decreasing function of the positive definite matrix **A**, then $tr[((\sum_{H}^{\downarrow})^{-1} + X_{H}^{H}R_{u}^{-1}X_{H})^{-1}] \ge tr[((\sum_{H}^{\downarrow})^{-1} + \overline{X}_{u}^{H}R_{u}^{-1}\overline{X})^{-1}]$

. Therefore X_{H}^{opt} should have a structure like \bar{X} and by solving the following power allocation problem the optimal solution of (8) can be obtained.

$$\min_{\sigma_{x,i}} \sum_{i=1}^{KM} \frac{1}{\sigma_{h,i}^{-1} + \sigma_{x,i} \sigma_{u,i}^{-1}}$$

s.t. $\sum_{i=1}^{KM} \sigma_{x,i} \le P_0.$ (10)

The optimal solution of (10) is given by the Lagrange multipliers,

$$\sigma_{x,i}^{opt} = \left(\sqrt{\frac{\sigma_{u,i}}{\lambda} - \frac{\sigma_{u,i}}{\sigma_{h,i}}}\right)^+$$
(11)

where λ can be found by solving

$$\sum_{i=1}^{KM} \left(\sqrt{\frac{\sigma_{u,i}}{\lambda}} - \frac{\sigma_{u,i}}{\sigma_{h,i}} \right)^+ = P_0.$$

The optimal solution of (4) X^{opt} can be written by summing up the above results,

$$\boldsymbol{X}^{opt} = \boldsymbol{V}_{u} [(\boldsymbol{\Sigma}_{x1}^{opt})^{\frac{1}{2}} \boldsymbol{0}_{KM \times (L-KM)}]^{T} \boldsymbol{V}_{H}^{H}$$
(12)

where $\sum_{x1}^{opt} = diag([\sigma_{x,1}^{opt}, \sigma_{x,2}^{opt} \dots \sigma_{x,KM}^{opt}]).$

In [18], how to pair the eigenvectors is not considered. The authors claimed without proof that the singular vectors should be the target and noise eigenvectors, respectively. Therefore, if we eigen decompose \mathbf{R}_u as $\mathbf{R}_u = \mathbf{V}_{u,P} \sum_{u}^{\downarrow} \mathbf{V}_{u,P}^{H}$, where $\mathbf{V}_{u,P} = \mathbf{V}_u \mathbf{P}$ and $\sum_{u}^{\downarrow} = diag([\sigma_{u,L} \dots \sigma_{u,1}])$, and assume the waveform matrix as $\mathbf{X} = \mathbf{V}_{u,P} [\sum_{x=1}^{\frac{1}{2}} \mathbf{0}_{KM \times (L-KM)}]^T \mathbf{V}_H^H$ like [18], then

$$tr[(\mathbf{R}_{H}^{-1} + \mathbf{X}^{H}\mathbf{R}_{u}^{-1}\mathbf{X})^{-1}] = \sum_{i=1}^{KM} \frac{1}{\sigma_{h,i}^{-1} + \sigma_{x,i}\sigma_{L+1-i}^{-1}}$$
(13)

In (12), we can observe that for the k^{th} diagonal element of the corresponding left singular vector is the eigenvector of the k^{th} smallest noise eigenvalue while its right singular vector is the eigenvector of the k^{th} largest target eigenvalue. Therefore, the pairing strategy of the singular vectors of the optimal waveform matrix based on MMSE is fixed.

IV.SIMULATION RESULTS

In this section, simulation results are shown to explain the performance of MIMO radar with white and colored noise. As we know that the white noise affects all the frequency components it has the eigen values as 1. Colored noise performance is shown with optimal pairing of eigen values. The parameter used here are as follows: M = 5, L = 5 and K = 1. And for clarity, if we say $(\sigma_{u,i})$, $\sigma_{h,i}$) is paired together, it means the eigenvector of $\sigma_{u,i}$ and the eigenvector of $\sigma_{h,i}$ correspond to the same singular value of the transmitted waveform matrix.

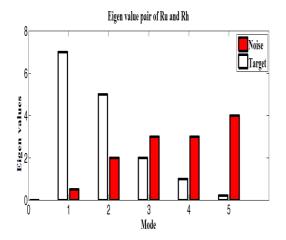


Fig.1 Power allocation based on MMSE, Po = 10. The eigen values for colored noise are paired as {((0.5,7),(2,5),(3,2),(3,1),(4,0.2)}.

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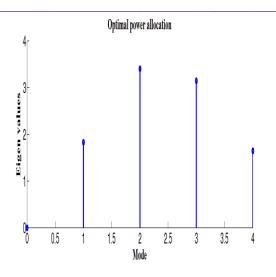


Fig.2 Power allocation for colored noise, the eigen values are $\{(0.5,7),(2,5),(3,2),(3,1),(4,0.2)\}.$

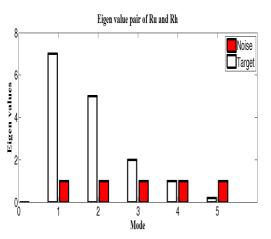


Fig.3 Power allocation based on MMSE, Po = 10. The eigen values of white noise are paired as $\{(1,7),(1,5),(1,2),(1,1),(1,0.2)\}.$

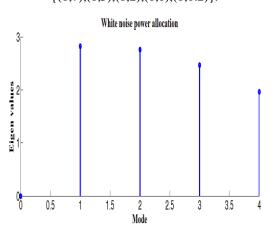


Fig.4 Power allocation for the eigen values of white noise $\{(1,7),(1,5),(1,2),(1,1),(1,0.2)\}.$

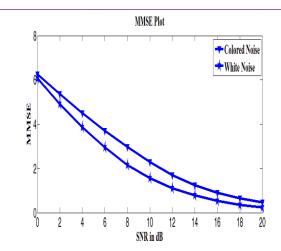


Fig.5 MMSE plot for colored Noise and White Noise .

In Fig.1 and Fig.2 the eigen values plot of colored noise and their corresponding power allocation are shown. In Fig.3 and Fig.4 the eigen values plot of white noise and their corresponding power allocation are shown. The MMSE plot for colored noise and white noise are shown in Fig.5, in which the white noise has the less minimum mean square error compared to colored noise.

V. CONCLUSION

This paper clearly explains the optimal waveform design for MIMO radar in colored noise and white noise based on minimizing MMSE. Here the eigen vector of the k^{th} largest eigen value of R_{H} and eigen vector of the k^{th} smallest eigen value of R_{u} are paired together. The pairing of singular vectors of the waveform matrix is fixed for MMSE. The Sum Power Constraint is good mode will be allotted more power so that more capacity of information can be sent over the mode and poor mode will be allotted less power or zero power. REFERENCES

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