Square Sum Difference Product Prime Labeling of Some Tree Graphs

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Abstract— Square sum difference product prime labeling of a graph is the labeling of the vertices with {0,1,2---------,p-1} and the edges with absolute difference of the sum of the squares of the labels of the incident vertices and product of the labels of the incident vertices. The greatest common incidence number of a vertex (***gcin***) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the ***gcin*** of each vertex of degree greater than one is one, then the graph admits square sum difference product prime labeling. In this paper we investigate some tree graphs for square sum difference product prime labeling.

Keywords- *Graph labeling, prime labeling, greatest common incidence number, square sum, trees.*

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I. Introduction

All graphs in this paper are simple, finite, connected and undirected. The symbol V(G) and E(G) denote the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1] and [2]. In [5], we introduced the concept of square sum difference product prime labeling and proved that some path related graphs admit this kind of labeling In this paper we investigated square sum difference product prime labeling of some tree graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor(gcd) of the labels of the incident edges.

MAIN RESULTS

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection f : V(G) → {0,1,2,3,------,p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping $f^*_{sqsdppl}$: E(G) → set of natural numbers N by $f^*_{sqsdppl}(uv) = |\{f(u)\}^2 + \{f(v)\}^2 - f(u)f(v)|$.The induced function $f^*_{sqsdppl}$ is said to be square sum difference product prime labeling, if for each vertex of degree at least 2, the greatest common incidence number is 1.

Definition 2.2 A graph which admits square sum difference product prime labeling is called a square sum difference product prime graph.

Definition 2.3 Let G be the graph obtained by joining one edge to each vertex of path P_n. G is called corona of path P_n or comb graph and is denoted by $P_n \odot K_1$.

Theorem 2.1 Corona of path P_n admits square sum difference product prime labeling.

Proof: Let G = $P_n \odot K_1$ and let v_1,v_2,----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 2n-1

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

$f(v_i) = i-1$, i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i\, v_{i+1}) = i^2-i+1$, i = 1,2,-------,n+1

$f^*_{sqsdppl}(v_{n-i+1}\, v_{n+i+2}) = (2i+1)^2 + (n+i+1)(n-i)$, i = 1,2,-------,n-2

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_{i+1}) = gcd of { $f^*_{sqsdppl}(v_i\, v_{i+1})$, $f^*_{sqsdppl}(v_{i+1}\, v_{i+2})$ }
= gcd of { i^2-i+1, i^2+i+1}
= gcd of {2i , i^2-i+1}
= gcd of {i , i^2-i+1}
= 1, i = 1,2,--------,n

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $P_n \odot K_1$, admits square sum difference product prime labeling. ■

Definition 2.4 Let G be the graph obtained by joining two edges to each vertex of path P_n. G is denoted by $P_n \odot 2K_1$.

Theorem 2.2 $P_n \odot 2K_1$, admits square sum difference product prime labeling.

Proof: Let G = $P_n \odot 2K_1$ and let v_1,v_2,----------------,v_{3n} are the vertices of G

Here |V(G)| = 3n and |E(G)| = 3n-1

Define a function f : V → {0,1,2,3,----------------,3n-1 } by

$f(v_i) = i-1$, i = 1,2,------,3n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i\, v_{i+1}) = i^2-i+1$, i = 1,2,------------,n+1

$f^*_{sqsdppl}(v_{n-i+1}\, v_{n+i+2}) = (2i+1)^2 + (n+i+1)(n-i)$, i = 1,2,------------,n-2

$f^*_{sqsdppl}(v_{i+1}\, v_{2n+i}) = (2n-1)^2 + (2n+i-1)(i)$, i = 1,2,------------,n

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,n

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $P_n \Theta 2K_1$, admits square sum difference product prime labeling. ■

Definition 2.5 Let G be the graph obtained by joining two edges to each internal vertex of path P_n. G is called twig graph and is denoted by $T_w(n)$.

Theorem 2.3 Twig graph $T_w(n)$, admits square sum difference product prime labeling.

Proof: Let G = $T_w(n)$, and let v_1,v_2,----------------,v_{3n-4} are the vertices of G

Here |V(G)| = 3n-4 and |E(G)| = 3n-5

Define a function f : V → {0,1,2,3,----------------,3n-5 } by

$f(v_i)$ = i-1 , i = 1,2,------,3n-4

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i\, v_{i+1})$ = i^2-i+1, i = 1,2,------------,n-1

$f^*_{sqsdppl}(v_{i+1}\, v_{2n-2+i})$ = $(2n-3)^2$ +(2n-3+i)(i), i = 1,2,------------,n-2

$f^*_{sqsdppl}(v_{i+1}\, v_{n+i})$ = $(n-1)^2$ +(n+i-1)(i), i = 1,2,------------,n-2

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,n-2

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $T_w(n)$, admits square sum difference product prime labeling. ■

Definition 2.6 Let G be the graph obtained by joining n edges to one of the end vertex of path P_m. G is called coconut tree graph and is denoted by CT(m,n)

Theorem 2.4 Coconut tree graph, admits square sum difference product prime labeling.

Proof: Let G = CT(m,n), and let v_1,v_2,----------------,v_{m+n} are the vertices of G

Here |V(G)| = m+n and |E(G)| = m+n-1

Define a function f : V → {0,1,2,3,----------------,m+n-1 } by

$f(v_i)$ = i-1 , i = 1,2,------,m+n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i\, v_{i+1})$ = i^2-i+1, i = 1,2,------------,m

$f^*_{sqsdppl}(v_m\, v_{m+i+1})$ = $(i+1)^2$ +(m+i)(m-1), i = 1,2,------------,n-1

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,m-1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence CT(m,n), admits square sum difference product prime labeling. ■

Definition 2.7 Let G be the graph obtained by joining m edges to one end and n edges to the other end of path P_2. G is called bistar and is denoted by Bistar B(m,n).

Theorem 2.5 Bistar B(m,n), admits square sum difference product prime labeling.

Proof: Let G = B(m,n), and let a,b,x_1,x_2,----------------,x_m ,y_1,y_2,----------------,y_n are the vertices of G

Here |V(G)| = m+n+2 and |E(G)| = m+n+1

Define a function f : V → {0,1,2,3,----------------,m+n-1 } by

$f(x_i)$ = i+1 , i = 1,2,------,m

$f(y_i)$ = m+i+1 , i = 1,2,------,n

f(a) = 0, f(b) = 1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(ab)$ = 1,

$f^*_{sqsdppl}(a\, x_i)$ = $(i+1)^2$, i = 1,2,-----------,m

$f^*_{sqsdppl}(b\, y_i)$ = $(m+i)^2$ +(m+i+1),i = 1,2,------------,n

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (a) = 1

gcin of (b) = 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence B(m,n), admits square sum difference product prime labeling. ■

Theorem 2.6 Star $K_{1,n}$, admits square sum difference product prime labeling.

Proof: Let G = $K_{1,n}$ and let a,x_1,x_2,----------------,x_n are the vertices of G

Here |V(G)| = n+1 and |E(G)| = n

Define a function f : V → {0,1,2,3,----------------,n } by

$f(x_i)$ = i , i = 1,2,------,n

f(a) = 0, .

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(a\, x_i)$ = i^2 , i = 1,2,------------,n

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (a) = 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $K_{1,n}$ admits square sum difference product prime labeling. ■

Theorem 2.7 Sub division graph of Star $K_{1,n}$, admits square sum difference product prime labeling.

Proof: Let G = Sd($K_{1,n}$) and let a,x_1,x_2,----------------,x_{2n} are the vertices of G

Here |V(G)| = 2n+1 and |E(G)| = 2n

Define a function f : V → {0,1,2,3,----------------,2n } by

$f(x_i)$ = i , i = 1,2,------,2n

f(a) = 0, .

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdp\,pl}(a\, x_{2i-1})$ = $(2i-1)^2$, i = 1,2,------------,n

$f^*_{sqsdppl}(x_{2i-1}\, x_{2i})$= $4i^2$-2i+1, i = 1,2,------------,n

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (a) = 1

gcin of (v_{2i-1}) =gcd of {$f^*_{sqsdppl}(a\, x_{2i-1})$, $f^*_{sqsdppl}(x_{2i-1}\, x_{2i})$ }

= gcd of { $(2i-1)^2$, 2i(2i-1)+1}

= 1, i = 1,2,------------,n

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence Sd($K_{1,n}$) ,admits square sum difference product prime labeling. ■

Definition 2.8 Let G be the graph obtained by joining m copies of path P_n to a single vertex by edges . G is denoted by $S_{m,n}$.

Theorem 2.8 $S_{m,n}$ admits square sum difference product prime labeling.

Proof: Let G = $S_{m,n}$ and letu, u,v_1,v_2,----------------,v_{mn} are the vertices of G

Here |V(G)| = mn+1 and |E(G)| =mn

Define a function f : V → {0,1,2,3,----------------,mn } by

$f(v_i)$ = i , i = 1,2,------,mn

f(u) = 0.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(u\, v_{(i-1)n+1})$ = $\{(i-1)n+1\}^2$, i = 1,2,------------,m

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$f^*_{sqsdppl}\ (v_{(j-1)n+i}\ v_{(j-1)n+i+1})$= 1+{(j-1)n+i}{(j-1)n+i+1}
j = 1,2,------------,m; i = 1,2,------------,n-1

Clearly $f^*_{sqsdppl}$ is an injection

gcin of (u) = 1.

gcin of $(v_{(i-1)n+1})$ = gcd of { $f^*_{sqsdppl}\ (u\ v_{(i-1)n+1})$, $f^*_{sqsdppl}\ (v_{(i-1)n+1}\ v_{(i-1)n+2})$}
= 1 , i = 1,2,------------,m

gcin of $(v_{(j-1)n+i+1})$ = gcd of {$f^*_{sqsdppl}\ (v_{(j-1)n+i}\ v_{(i-1)n+i+1})$, $f^*_{sqsdppl}\ (v_{(j-1)n+i+1}\ v_{(i-1)n+i+2})$}
= 1 , j = 1,2,------------,m,
i = 1,2,------------,n-2

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $S_{m,n}$, admits square sum difference product prime labeling. ■

Theorem 2.9 H-graph of path P_n admits square sum difference product prime labeling.

Proof: Let G = H(P_n) and let v_1, v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| =2n-1

Define a function f : V → {0,1,2,3,----------------,2n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}\ (v_i\ v_{i+1})$ = $i^2 - i + 1$, i = 1,2,------------,n-1

$f^*_{sqsdppl}\ (v_{n+i}\ v_{n+i+1})$ = $(n+i)^2$ - (n+i) + 1
i = 1,2,-----------,n-1

Case(i) n is odd

$f^*_{sqsdppl}\ (v_{(\frac{n+1}{2})}\ v_{(\frac{3n+1}{2})})$ = $\frac{7n^2-4n+1}{4}$

Case(ii) n is even

$f^*_{sqsdppl}\ (v_{(\frac{n+2}{2})}\ v_{(\frac{3n}{2})})$ = $\frac{19n^2-18n+4}{4}$

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,n-2

gcin of (v_{n+i+1}) = 1, i = 1,2,------------,n-2

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence H(P_n) , admits square sum difference product prime labeling. ■

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