

## Batch Arrival Preventive Loss Priority Queues with Preventive Distance

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**Abstract:** This paper is concerned with preemptive loss priority queues in which a batch of failed machines of each priority class arrive in a Poisson process and have general service time distribution. In this queuing system, failed machines are not considered for repair again when their services are preempted by the arrival of another batch of failed machines with higher priority. They disappear immediately. A case can be modeled by such a system in which deferred service is worthless for old demands of low priority. This model is based on the situation of strict preemption with preemption distance parameter  $d$  such that failures of only class  $1$  to  $p - d$  can preempt the service of failures of class  $p$ . The closed form expressions are obtained in the mean waiting time and source time from their distributions for each class. Several numerical examples illustrate the approach.

**Keywords:** Pre-emptive, Service Time, Busy Periods, Waiting Time, Laplace – Stieltjes Transform (LST), Poisson Distribution, Exponential Distribution

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### I. INTRODUCTION

In this chapter, we analyze a machine interference problem with preemption loss priority queues in which a bulk failure of each priority class joins the system in a Poisson process and has general service time distribution. By allowing an integer parameter  $d$ , we extend the existing model of preemptive loss priority queues such that a batch of failed machines whose priority class is  $d$  or higher than the priority class of bulk failures in service, preempt the service. In the case of bulk failures of machines, preemptive loss priority queues with preemption distance are studied for the first time in this study.

In this machine repair problem, a single repair person attends batches of failed machines of multiple classes. A batch of failed machines of class  $1$  to  $P$ , are priority classes such that class  $p$  has priority over class  $q$  if  $p < q$ . Failed machines of class  $p$  occur in a Poisson process at rate  $\lambda_p$ , where  $p = 1, 2, \dots, P$ , in a group of different size  $b_k$ . The aggregate arrival rate of batches of failed machines of class  $1$  to  $P$  is defined as

$$\lambda_p^+ = \sum_{k=1}^p b_k \lambda_k \dots\dots\dots(1.1)$$

where  $b_1, b_2, b_3, \dots, b_p$  are different group sizes for  $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_p$ .

The distribution function (DF) and its Laplace - Stieltjes transform (LST) for the service time of a group of failed machines of class  $p$  are expressed by  $B(x)$  and  $B^*(s)$ , respectively. The pattern of servicing is preemptive loss with preemption distance  $d$ .

A batch of failed machines of class  $1$  to  $p - d$  preempts the service of a batch of failed machines of class  $p$  which are then lost from the repairing system. Here, the preemption distance  $d$  is assumed to be a positive integer between  $1$  and  $P$ , inclusive. The case  $d = P$  means non - preemptive priority rule while the case  $d = 1$  corresponds to the existing preemptive priority pattern.

Now, we propose to find out the distribution for the waiting time  $W_L$ , of a batch of failed machines of class  $p$  for  $p=1, 2, \dots, P$ . The time interval from the failure to the service of a batch of failed machines of class  $p$  is termed as the waiting time  $W$ . We can obtain the distribution for the source time  $T$  of a batch of failed machines of class  $p$ , the time interval from the arrival to the service termination (either by preemption or by completion) of a batch of failed machines of class  $p$ . We define a busy period as the length of the time interval from the start of service to a batch of failed machines of class  $i$  to the first moment at which no batch of failed machines of class 1 to  $p-1$  are present in the system. The relation can be expressed as

$$T_p = W_p + \bar{X}_p \quad (1.2)$$

where  $x_p$  presents the actual service time.  $W_p$  and  $\bar{x}_p$  are independent, The interval from the start of service to a batch of failed machines of class  $p$  to the first moment after terminating the service at which no batch of failed machines of class 1 to  $p-1$  are present in the system, is termed as the completion time  $C_p$ . We may write as

$$C_p \equiv \theta^p_p.$$

We consider the steady state situation. For a subsystem consisting of batches of failed machines of class 1 to  $p$ , the stability condition is therefore given by

$$\rho_p^+ = \sum_{k=1}^p \rho_k < 1,$$

where  $\rho_k = b_k \lambda_k E[\bar{x}_k]$

$$(1.3)$$

The entire system is stable if

$$\rho = \sum_{k=1}^p \rho_k < 1 \quad (1.4)$$

## II. ACTUAL SERVICE TIME

We first analyze the actual repair time  $\bar{x}_p$ . Let  $x_p$  be the original repair time of a batch of failed machines of class  $p$ .  $\bar{x}_p$  and  $x_p$  are both equal if no batch of failed machines of class 1 to  $p-d$  join the repair facility during  $x_p$ . Whenever a batch of failed machines of class 1 to  $p-d$  arrives during  $x_p$ , the service is terminated. Therefore, we have

$$E[e^{-s\bar{x}_p} | X_p] = e^{-bp-d\lambda^+p-dxp} \cdot e^{-sxp} + \int_0^{x_p} b_{p-d} \lambda_{p-d}^+ \cdot e^{-bp-d\lambda_{p-d}^+x} \cdot e^{-sx} dx$$

$$= \frac{b_{p-d} \lambda_{p-d}^+ + s e^{-(s+b_{p-d} \lambda_{p-d}^+)x_p}}{s + b_{p-d} \lambda_{p-d}^+} x_p \quad (2.1)$$

where, we suppose that

$$b_{p-d} \lambda_{p-d}^+ \equiv 0 \quad \text{if } p \leq d.$$

We get the LST (s) of  $X_p^*$  the DF for  $\bar{x}_p$  on removing the condition on  $x_p$  in (1) as

$$\bar{X}_p^*(s) = \frac{b_{p-d} \lambda_{p-d}^+ + s B_p^*(s + b_{p-d} \lambda_{p-d}^+)}{s + b_{p-d} \lambda_{p-d}^+} \dots\dots\dots(2.2)$$

$$\therefore E[\bar{x}_p] = \frac{1 - B_p^*(b_{p-d} \lambda_{p-d}^+)}{b_{p-d} \lambda_{p-d}^+} \quad (2.3)$$

$$E[\bar{x}_p^2] = \frac{2[1 - B_p^*(b_{p-d} \lambda_{p-d}^+)]}{(b_{p-d} \lambda_{p-d}^+)^2} = \frac{2 E[x_p e^{-b_{p-d} \lambda_{p-d}^+ x_p}]}{b_{p-d} \lambda_{p-d}^+} \quad (2.4)$$

$$E[x_p e^{-b_{p-d} \lambda_{p-d}^+ x_p}] = -[d B_p^*(s) / ds]_{s = b_{p-d} \lambda_{p-d}^+}$$

where

For  $p \leq d$  (non-preemptive case)

$$\bar{X}_p^*(s) = B_p^*(s)$$

The LST of the DP for  $\bar{x}_p$ , fir a batch of failed machines of class p whose service is finished is expressed by

$$\bar{X}_p^*(s \mid \text{completed}) = \frac{\int_0^\infty e^{-b_{p-d} \lambda_{p-d}^+ x} e^{-sx} dB_p(x)}{\int_0^\infty e^{-b_{p-d} \lambda_{p-d}^+ x} dB_p(x)}$$

$$\frac{B_p^*(s + b_{p-d} \lambda_{p-d}^+)}{B_p^*(b_{p-d} \lambda_{p-d}^+)} \quad (2.5)$$

Where  $B_p^*(b_{p-d} \lambda_{p-d}^+)$  is the probability that the repair time  $x_p$  is not preempted. From (2.5) we have

$$E[\bar{x}_p | Completed] = \frac{E[x_p e^{-b_{p-d} \lambda_{p-d}^+ x_p}]}{B_p^*(b_{p-d} \lambda_{p-d}^+)} \quad (2.6)$$

The LST of the DF for  $\bar{x}_p$  for a batch of failed machines whose service is preempted is given by

$$\begin{aligned} \bar{X}_p^*(s | preempted) &= \frac{\int_0^\infty d B_p(x) \int_0^{x_p} b_{p-d} \lambda_{p-d}^+ e^{-b_{p-d} \lambda_{p-d}^+ x} e^{-sx} dx}{\int_0^\infty d B_p(x) \int_0^{x_p} b_{p-d} \lambda_{p-d}^+ e^{-b_{p-d} \lambda_{p-d}^+ x} dx} \\ &= \frac{b_{p-d} \lambda_{p-d}^+ [1 - B_p^*(s + b_{p-d} \lambda_{p-d}^+)]}{(s + b_{p-d} \lambda_{p-d}^+) [1 - B_p^*(b_{p-d} \lambda_{p-d}^+)]}, \quad p > d \end{aligned} \quad (2.7)$$

From this we have

$$E[x_p^* | preempted] = \frac{1}{b_{p-d} \lambda_{p-d}^+} \frac{E[x_p e^{-b_{p-d} \lambda_{p-d}^+ x_p}]}{1 - B_p^*(b_{p-d} \lambda_{p-d}^+)}, \quad p > d \quad (2.8)$$

### III. BUSY PERIODS

We now calculate the LST  $\theta_i^p(s)$  of the DF for the length  $\theta_i^p$  of the busy period which starts with the repair time of a batch of failed machines of class  $i$  and terminates when there are no batch of failed machines of class 1 to  $p - 1$ , present in the system

where  $1 < i, p \leq P$

Now we consider two cases, the LST of the DF for  $\theta_i^p$  conditioned on the initial repair time  $x$  for a batch of failed machines of class  $i$  can be find out. A bulk arrival of failed machines of one of classes 1 to  $i - 1$  arrive during the initial repair time  $x$ , where  $i >$

d, in the first case. The probability of a batch of failed machines of class k ( $1 \leq k \leq i - d$ ) occurring first between x and x + dx among those of classes 1 to i - d after the start of  $x_i$  is given by

$$b_k \lambda_k \cdot e^{-\sum_{j=1}^{i-d} b_j \lambda_j x} dx \quad (3.1)$$

The original repair time  $x_i$  is preempted at time x in this case.

The total number of batches of failed machines of class j such that  $i - d + 1 \leq j \leq i$  that join the system during x has Poisson distribution with mean  $b_j \lambda_j x$ . Since each of them assists  $\theta_j^p$  to  $\theta_i^p$ . The LST of the DF for total contribution is given by

$$\sum_{l=0}^{\infty} \frac{(b_j \lambda_j x)^l}{l!} \cdot e^{-b_j \lambda_j x} [\theta_j^p(s)]^l = e^{-b_j \lambda_j x [1 - \theta_j^p(s)]}$$

$$\sum_{l=0}^{\infty} \frac{(b_j \lambda_j x)^l}{l!} \cdot e^{-b_j \lambda_j x} [\theta_j^p(s)]^l = e^{-b_j \lambda_j x [1 - \theta_j^p(s)]} \quad (3.2)$$

Since the arriving process of batches of each class is independent, we have

$$E \left[ e^{-s\theta_i^p} I\{x_i \text{ is preempted} \} \mid x_i \right]$$

$$\sum_{k=1}^{i-d} \int_0^{x_i} e^{-sx} b_k \lambda_k e^{\sum_{j=1}^{i-d} b_j \lambda_j x} dx \cdot \theta_k^p(s) e^{-\sum_{j=i-d+1}^{p-1} b_j \lambda_j [1 - \theta_j^p(s)] x}$$

$$\sum_{k=1}^{i-d} b_k \lambda_k \theta_k^p(s) \int_0^{x_i} e^{[s b_{p-1} \lambda_{p-1} + \sum_{j=i-d+1}^{p-1} b_j \lambda_j \theta_j^p(s)] x} dx$$

$$= \left[ \sum_{k=1}^{i-d} b_k \lambda_k \theta_k^p(s) \right] \frac{1 - \exp \left\{ - \left[ s + b_{p-1} \lambda_{p-1} + \sum_{j=i-d+1}^{p-1} b_j \lambda_j \theta_j^p(s) \right] x_i \right\}}{s + b_{p-1} \lambda_{p-1} + \sum_{j=i-d+1}^{p-1} b_j \lambda_j \theta_j^p(s)}$$

$i > d \quad (3.3)$

where  $I_A$  is indicator function of event A.

Now we consider the second case where no batch of failed machines of class 1 to i - d join the repair facility during the initial service time  $x_i$  which is thus continued to completion.

The probability is given by

$$e^{-\sum_{j=1}^{i-d} b_j \lambda_j x_i} \quad (3.4)$$

For contribution to the conditional  $\theta_i^p$  from batches of failed machines of class  $j$  such that  $i-d \leq j \leq p-1$ , that join during  $x_i$ , the LST of the DF is given by

$$\sum_{i=0}^{\infty} \frac{(b_j \lambda_j x_i)^i}{i!} e^{-b_j \lambda_j x_i} [\theta_j^p(s)]^i = e^{-b_j \lambda_j [1 - \theta_j^p(s)] x_i} \quad (3.5)$$

Thus, we have

$$E[e^{-s\theta_i^p} | \{x_i \text{ is completed}\} | x_i] = e^{-s x_i} e^{-\sum_{j=1}^{i-d} b_j \lambda_j x_i} e^{-\sum_{j=i-d+1}^{p-1} b_j \lambda_j [1 - \theta_j^p(s)] x_i}$$

$$= \exp\{-[s + b_{p-1} \lambda_{p-1} + \sum_{j=i-d+1}^{p-1} b_j \lambda_j \theta_j^p(s)] x_i\} \quad (3.6)$$

Adding (3.3) and (3.6), and removing the condition on  $x_i$ , we have

$$\theta_i^p(s) = \left[ \sum_{k=1}^{i-d} b_k \lambda_k \theta_k^p(s) \right] \frac{1 - B_i^* [s + b_{p-1} \lambda_{p-1} + \sum_{j=i-d+1}^{p-1} b_j \lambda_j \theta_j^p(s)]}{s + b_{p-1} \lambda_{p-1} + \sum_{j=i-d+1}^{p-1} b_j \lambda_j \theta_j^p(s)} + B_i^* [s + b_{p-1} \lambda_{p-1} + \sum_{j=i-d+1}^{p-1} b_j \lambda_j \theta_j^p(s)]$$

where the condition  $i \leq d$  does not exist for the first term on the right hand side.

(3.7)

We obtain the following relation by calculating the first derivative of (8s) at  $s = 1$  from (3.7)

$$E[\theta_i^p] = E[\bar{x}_i] \left( 1 + \sum_{j=1}^{p-1} b_j \lambda_j E[\theta_j^p] \right)$$

where  $E[\bar{x}_i]$  is given in (3.3)

(3.8)

The equation is satisfied by

$$E[\theta_i^p] = \frac{E[\bar{x}_i]}{1 - \rho_{p-1}^+}$$

where  $\rho_{p-1}^+$  is defined in (3.3)

(3.9)

we find out the second derivative of  $\theta_i^p(s)$  at  $s = 1$  from (3.7) and using (3.9)

$$E[(\theta_i^p)^2] = E[\bar{x}_i] \sum_{j=1}^{p-1} b_j \lambda_j E[(\theta_i^p)^2] + \frac{E[\bar{x}_i^2] \cdot (1 - \rho_{i-d}^+)}{(1 - \rho_{p-1}^+)^2} \quad (3.10)$$

Here we assume that  $\rho_{i-d}^+ = 0$  if  $i \leq d$ . This equation is satisfied by

$$E[(\theta_i^p)^2] = \frac{E[\bar{x}_i] \sum_{k=1}^{p-1} b_k \lambda_k (1 - \rho_{k-d}^+) E[\bar{x}_k^2]}{(1 - \rho_{p-1}^+)^3} + \frac{E[\bar{x}_i^2] \cdot (1 - \rho_{i-d}^+)}{(1 - \rho_{p-1}^+)^2} \quad (3.11)$$

#### IV. WAITING TIME

We evaluate the LST  $W_p^*(s)$  of the DF for the waiting time  $W_p$  of an arbitrary hatch of failed machines of class  $p$ . Here we use the method of  $\theta$  cycle, which is introduced for similar treatment by Kella and Yechiali. In association with class  $p$  a  $\theta$ -cycle is defined as a busy period which starts with an uninterruptible initial delay  $\theta$  and terminates when there are no batch of failed machines of class  $p$  that join during a  $\theta$ -cycle is given by

$$W_p^*(s) = \frac{1 - \theta^p(s)}{E[\theta^p]s} \frac{(1 - b_p \lambda_p E[C_p])s}{s - b_p \lambda_p + b_p \lambda_p C_p^*(s)} \quad (4.1)$$

where  $\theta^p(s)$  is the LST of the DF for the length  $\theta^p$  or a busy period began with initial delay  $\theta$  and ended when there are no batches of failed machines of class 1 to  $p-1$  present in the system.  $C_p^*(s)$  denotes the LST or the DF for the completion time  $C_p$ .

$$C_p^*(s) = \theta_p^p(s) \quad (4.2)$$

Let us assume four cases with respect to the state of the system at the moment when a hatch of failed machines of class  $p$  arrives. The system is empty in the first case. Probability is  $P_0 = 1 - \rho$  and waiting time is zero. Thus we have

$$P_0 = 1 - \rho, \quad W_p^*(s | 0) = 1 \quad (4.3)$$

In the second situation, with probability  $P_j = \rho_j$ , a batch of failed machines of class  $j$  such that  $p + d \leq j \leq P$  is being served. If the class of arriving batch of failed machines is  $p$ , then it preempts the on-going service. Its waiting time is zero. Thus, we have

$$P_j = \rho_j, \quad W_p^*(s | j) = 1, \quad p + d \leq j \leq P \quad (4.4)$$

In the third situation during a  $\theta$  cycle, the system begins with the service time of a batch of failed machines of class  $j$  such that  $p + 1 \leq j \leq p + d - 1$ . Probability is  $P_j = \rho_j / (1 - \rho_j^+)$ . Here the period  $\theta^p$  is equal to  $\theta_j^p$ . Thus, we have

$$P_j = \frac{\rho_j}{1 - \rho_p^+} W_p^*(s | j) = \frac{1 - \theta_j^p(s)}{E[\theta_j^p] s} \cdot \frac{(1 - b_p \lambda_p E[C_p]) s}{s - b_p \lambda_p + b_p C_p^*(s)} \quad p + 1 \leq j \leq p + d - 1 \quad (4.5)$$

In the fourth situation, during a  $\theta$  cycle, the system is started with the service time of a batch of failed machines of class  $j$  such that  $1 \leq j \leq p$ . Probability is  $P_j = \alpha \rho_j$ . The period  $\theta^p$  is equivalent to  $\theta_j^p$ . Thus, we have

$$P_j = \alpha \rho_j W_p^*(s | j) = \frac{1 - \theta_j^p(s)}{E[\theta_j^p] s} \cdot \frac{(1 - b_p \lambda_p E[C_p]) s}{s - b_p \lambda_p + b_p \lambda_p C_p^*(s)} \quad 1 \leq j \leq p \quad (4.6)$$

The value of  $\alpha$  can be calculated from the condition:

$$\sum_{j=0}^p P_j = 1 \quad (4.7)$$

$$\text{to be } \frac{1 - \rho_{p+d-1}^+}{1 - \rho_p^+} \quad (4.8)$$

Collecting all these results, we have

$$\begin{aligned} W_p^*(s) &= \sum_{j=0}^p P_j W_p^*(s | j) \\ &= 1 - \rho + \sum_{j=p-d}^p \rho_j + \sum_{j=p-1}^{p-d-1} \frac{\rho_j}{1 - \rho_p^+} \frac{1 - \theta_j^p(s)}{E[\theta_j^p] s} \frac{(1 - b_p \lambda_p E[C_p]) s}{s - b_p \lambda_p + b_p \lambda_p C_p^*(s)} \\ &\quad + \frac{1 - \rho_{p+d-1}^+}{1 - \rho_p^+} \sum_{j=1}^p \rho_j \frac{1 - \theta_j^p(s)}{E[\theta_j^p] s} \frac{(1 - b_p \lambda_p E[C_p]) s}{s - b_p \lambda_p + b_p \lambda_p C_p^*(s)} \quad (4.9) \end{aligned}$$

where



$$\frac{\rho_j}{1 - \rho_p^+} \frac{1 - b_p \lambda_p E[C_p]}{E[\theta_j^p]} = b_j \lambda_j \tag{4.10}$$

$$\begin{aligned} W_p^*(s) &= 1 - \rho_{p+d-1}^+ + \sum_{j=p-1}^{p-d-1} \frac{b_j \lambda_j [1 - \theta_j^p(s)]}{s - b_p \lambda_p + b_p \lambda_p C_p^*(s)} \\ &+ (1 - \rho_{p+d-1}^+) \sum_{j=1}^p \frac{b_j \lambda_j [1 - \theta_j^p(s)]}{s - b_p \lambda_p + b_p \lambda_p C_p^*(s)} \\ &= \frac{(1 - \rho_{p+d-1}^+) \{s + b_{p-1} \lambda_{p-1} - \sum_{j=1}^{p-1} b_j \lambda_j \theta_j^p(s)\} + \sum_{j=p-1}^{p-d-1} b_j \lambda_j [1 - \theta_j^p(s)]}{s - b_p \lambda_p + b_p \lambda_p C_p^*(s)} \end{aligned} \tag{4.11}$$

we have the mean waiting time  $E[W_p]$  from (4.11) in terms of  $E[(\theta_j^p)^2]$  as

$$\begin{aligned} E[W_p] &= \frac{1 - \rho_{p-1}^-}{2(1 - \rho_p^+)} \left\{ (1 - \rho_{p+d-1}^+) \sum_{j=1}^{p-1} b_j \lambda_j E[(\theta_j^p)^2] \right. \\ &\left. + \sum_{j=p}^{p+d-1} b_j \lambda_j E[(\theta_j^p)^2] \right\} \end{aligned} \tag{4.12}$$

Now substituting the value of  $E[(\theta_j^p)^2]$  into (4.12) from (3.11), we have

$$E[W_p] = \frac{\sum_{k=1}^{p+d-1} b_k \lambda_k (1 - \rho_{k-d}^+) E[\bar{x}_k^2]}{2(1 - \rho_{p-1}^+)(1 - \rho_p^+)}$$

where is  $E[\bar{x}_k^2]$  Defined in (2.4) (4.13)

### V. SOURCE TIME

The source time or a batch or failed machine corresponds to the waiting time and the actual repair line. Where the two are independent Thus the LST or the DF for the source time  $T_p$  of a batch of failed machines or class  $p$  whose service is completed is given by

$$T_p^* (s | completed) = W_p^* (s) \cdot X_p^* (s | completed) \quad (5.1)$$

The mean source time of this batch of failed machines is given by

$$E[T_p | Completed] = \sum_{k=1}^{p-d-1} \frac{b_k \lambda_k (1 - \rho_{k-d}^+) E[x_k^2]}{2(1 - \rho_{p-1}^+) (1 - \rho_p^+)} + \frac{E[x_p e^{-b_{p-d} \lambda_{p-d}^+ x_p}]}{B_p^* (b_{p-d} \lambda_{p-d}^+)} \quad (5.2)$$

In the same fashion, the LST of the DF for the source time  $T_p$ , of a batch of failed machines or class  $j$  whose service is preempted, is given by,

$$T_p^* (s | preempted) = W_p^* (s) \bar{X}_p^* (s | preempted) \quad (5.3)$$

where  $\bar{X}_p^* (s | preempted)$  is defined in (2.7)

The mean source time of this batch of failed machines is given by

$$E[T_p | preempted] = \frac{\sum_{k=1}^{p-d-1} b_k \lambda_k (1 - \rho_{k-d}^+) E[x_k^2]}{2(1 - \rho_{p-1}^+) (1 - \rho_p^+)} - \frac{1}{b_{p-d} \lambda_{p-d}^+} - \frac{E[x_p e^{-b_{p-d} \lambda_{p-d}^+ x_p}]}{1 - B_p^* (b_{p-d} \lambda_{p-d}^+)} \quad (5.4)$$

$p > d$

The LST  $T_p^* (s)$  or the DF for the unconditional source time  $T_p$  or a batch of failed machines of class  $p$  is given by

$$T_p^* (s) = W_p^* (s) \bar{X}_p^* (s) \quad (5.5)$$

where  $\bar{X}_p^* (s)$  is calculated in (2.2).

Now we finally obtain the unconditional mean source time as,

$$E[T_p] = \frac{\sum_{k=1}^{p-d-1} b_k \lambda_k (1 - \rho_{k-d}^+) E[x_k^2]}{2(1 - \rho_{p-1}^+) (1 - \rho_p^+)} + \frac{1 - B_p^* (b_{p-d} \lambda_{p-d}^+)}{b_{p-d} \lambda_{p-d}^+} \quad (5.6)$$

In the case of variable, batch size  $\lambda_p^-$  can be defined in the following manner

$$\lambda_p^+ = \sum_{h=1}^P \sum_{n=1}^N b_n \lambda_k$$

## VI. NUMERICAL EXAMPLE

We consider a system with  $d=1$ ,  $b=2$ , and  $p=6$  classes of batches of failed machines for a numerical example. Assuming that the parameters of the arrival and service process are equal for batches of all classes and the service times are exponentially distributed with unit mean.

Total arrival rate of batches

$$\lambda_p = \frac{b\lambda}{p}, B_p^*(s) = \frac{l}{l+s}, p = 1, 2, \dots, P$$

We plot the mean actual repair time  $E[\bar{x}_p]$  for  $p = 1, 2, \dots, P$  against  $b$  in fig. 1. As the service to the batch of failed machines of class 1 is never preempted, we see that  $E[\bar{x}_1] = 1$ , independent of  $\lambda$ . For  $p \geq 2$ , each  $E[\bar{x}_p]$  decreases. As arrival rate increases, preemption in service occurs in a frequent manner.

We have the relation

$$1 = E[\bar{X}_1] > E[\bar{X}_2] > \dots > E[\bar{X}_p] \quad (6.2)$$

because the service to batches of the failed machines of lower priority classes are more likely to be preempted on the arrival of a batch of failed machine of high priority.

We plot the mean waiting time  $E[W_p]$  for  $p = 1, 2, \dots, P$  against  $b\lambda$  in fig. 2. We observe that each  $E[W_p]$  increases monotonously as  $b$  grows because of queuing and by the effects from batches of higher priority.

We also have the relation

$$E[W_1] < E[W_2] < \dots < E[W_p] \quad (6.3)$$

Which distinguishes between batches of failed machines of different classes.

In fig. 3, we show the mean source time  $E[T_p]$  for  $p = 1, 2, 3, \dots, P$  against  $b\lambda$ . We observe again that  $E[T_p]$  increases as  $b\lambda$  grows in a monotonic manner and that

$$E[T_1] < E[T_2] < \dots < E[T_p] \quad (6.4)$$

The mean actual repair times are small for batches of failed machines of low priority (due to preemption).

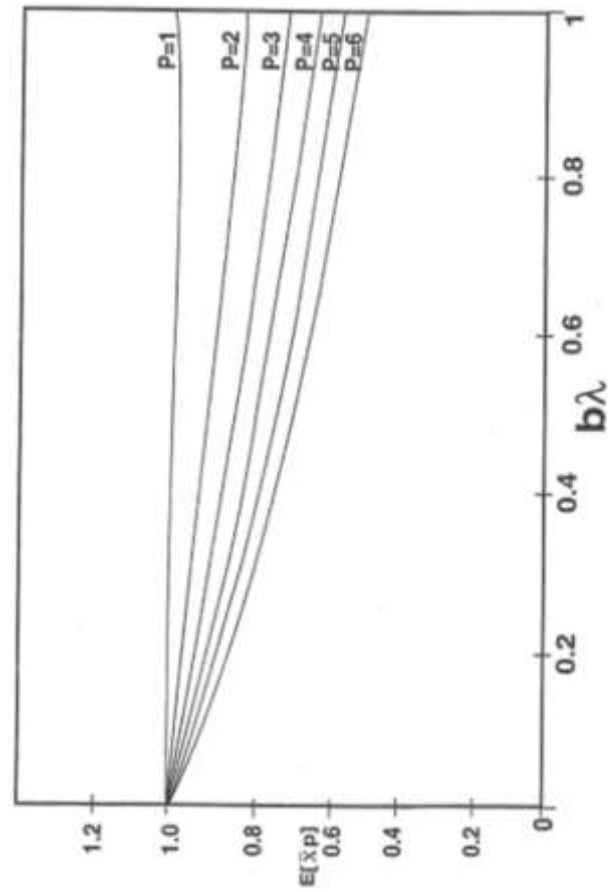


Fig. 1. Mean Actual Service Times

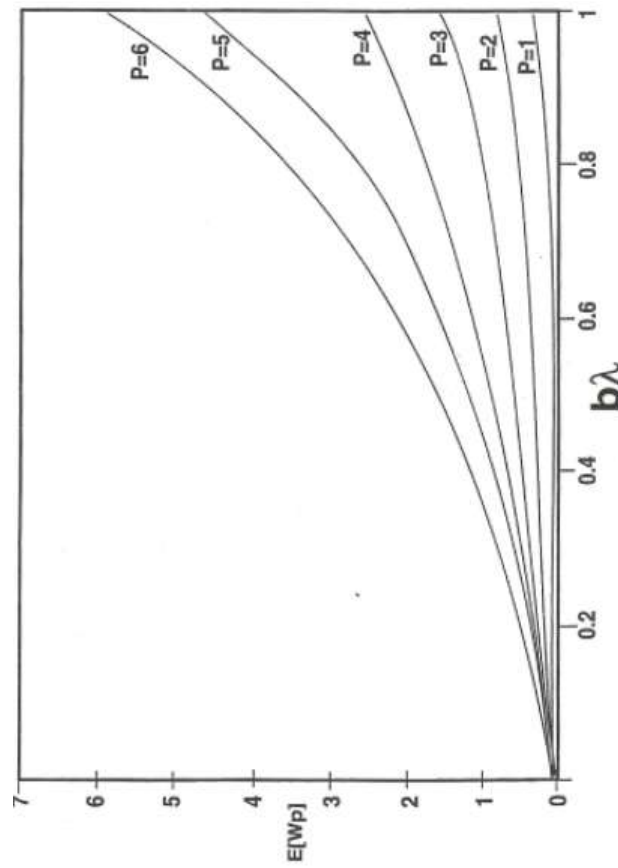
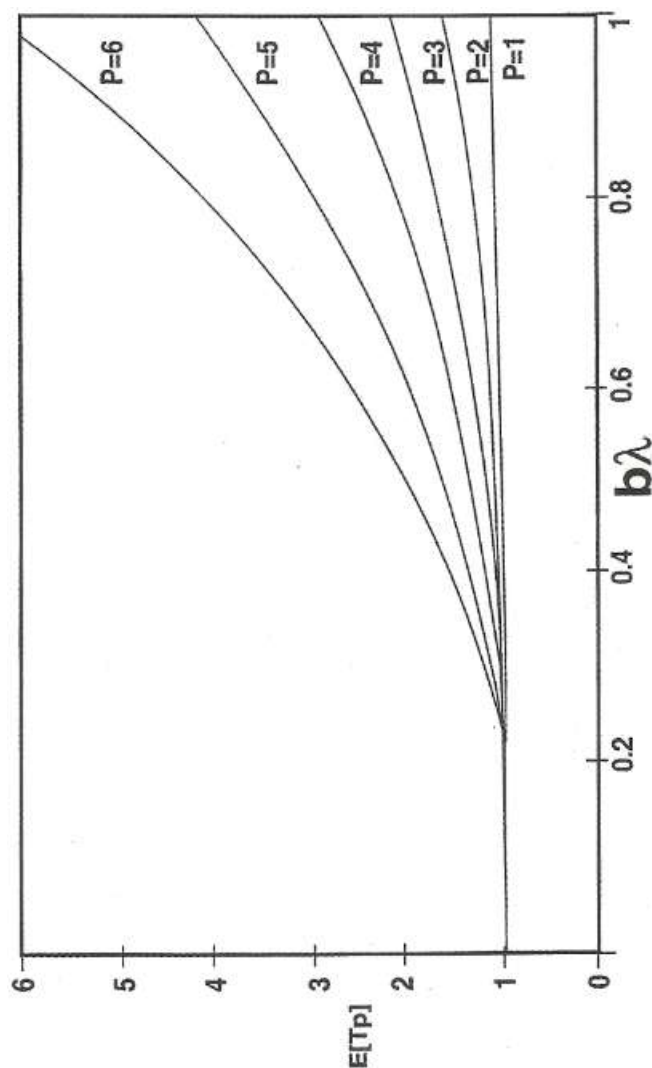


Fig. 2. Mean Waiting Times



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