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Use of Monte Carlo Simulation for Analyzing Queues in a Financial Institution

A Case Study

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Abstract—Extensive queues are big issues for financial institutions and also for the customers. Because of this, the objective of this study is to analyze queues in a financial institution in its several sectors. Statistical modeling in Monte Carlo simulation using Microsoft Excel was applied to analyze the queues in different days of the week and in different situations that include general customer service, business customer service, and drive-through customer service. During the study, some issues were found due to the number of tellers who serve these queues but simple and appropriate solutions were proposed for providing good service to the customers and minimizing waiting time of the customers in the queue.

Keywords-queues; customer service;Monte Carlo simulation; financial institution.

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I. INTRODUCTION

Queuing theory is a fascinating subject in applied probability for two contradictory reasons: it sometimes requires the most sophisticated tools of stochastic process, and it often leads to simple and explicit answers [1]. Indeed, these two aspects were easy to notice in this study, mainly the second one. Another important concept in this study is the statistical method used to analyze it. Statistical methods can help to reduce the frequency of mistakes. Program of improvement can be instituted at any institution regardless of size, for they can be tailored to meet specific needs and designed to grow in new areas of application [2]. In this particular case, several sectors of a financial institution in the United State were analyzed such as general customer service, business customer service, and drive-through customer service. In the beginning, the study did not have the goal to solve a particular problem, but to analyze the data. However, seeing the data collected, it was noticed that something was wrong and at this moment some issues were found.

Notably, the analysis was made using the Monte Carlo simulation in Microsoft Excel in order to see if the financial institution analyzed had any queuing problems. Throughout the study, it was possible to see minor problems in queues in the business and drive-through sectors. To solve all of these issues, Monte Carlo simulation was chosen for this situation because it has graphical and probabilistic results and it gives sensitivity analysis, in other words, it is easy to see which inputs had the greatest impact on the results of primary concerns [3].

The study was conducted to understand the way that the financial institution used to serve its customers, given its staffing resources. An important aspect of that understanding is to know the characteristics of the financial institution's business. Some of those characteristics are the volume of transactions, the number of workers, the customer volume, the available equipment and how each one of this information affects the queue. Based on this knowledge, several things can be analyzed; the number of lines needed, the number of tellers for each period of the day, the peak hours of that agency and others.

II. MONTE CARLO SIMULATION: BACKGROUND

To find a quick and inexpensive way to achieve results, the use of simulation is more appropriate than create an entire real environment. The basic idea of simulation is to create a "simulator" that will be used to obtain results close to reality; the way used to create this "simulator" and acquire a reliable quantitative analysis about the experimentation must be based on a mathematical model.

Generally, the model measures the nature of the proposed arrangement and additionally the amount of variability there may be in the different execution measures because of arbitrariness in the inputs. The simulation takes into account a considerable measure of experimentation and cooperation with the modeler, however it will not essentially advance the objective of hobby. The simulator is normally a much less expensive and speedier approach to explore different avenues regarding numerous interesting variables [4].

Simulation models usually are used to take a decision about something that involves risk, that is, a model in which the conduct of one or more components is not known with assurance. Bank lines or the quantities of trucks that will arrive between a particular time intervals are cases which the component that is not referred to with assurance is considered as a random variable and a probability distribution is used to simulate the behavior of a random variable. This kind of simulations is sometimes named Monte Carlo method [4, 5].

Numerous simulations in operations management can be defined as holding up line models, additionally called queuing models. The most well-known circumstances are in administration frameworks, for example, banks where the administration is given by a handling framework like a process input-processing-output [6].

In organizing holding up line models, the inputs are called "arrivals", and these arrival times are controlled by some probabilistic procedure. The time to process these arrivals is

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additionally controlled by a probabilistic procedure. The yield rate of such frameworks relies upon the interchange between the arbitrary landings and the variable administration times, and waiting line models are utilized to anticipate these qualities [7].

A common case is a bank, where the customer service at a teller window is irregular; every individual chooses how long he/she is going to be at the bank. The length of waiting line depends to some way on these waiting times. Just as essential is the time it takes for a teller to serve every person. These circumstances shift on the grounds that the time for the execution of a service is variable and, more vital; every client needs different kinds of service. For example, the first client might essentially need to make a deposit, though the second may need a service that needs more time [6].

There are four essential waiting line structures that depict the general conditions. The most straightforward structure is known as the single server case. When the quantity of service stations is expanded but at the same time draws on a single waiting line, there is multiple servers' case. The basic sequential construction system, basically, various administration offices in arrangement is a case of the single servers in arrangement case. At long last, the various servers in arrangement case can be delineated by two or more parallel sequential construction systems. On the other hand, mixes of any of the fundamental structures can likewise exist in systems in extremely complex frameworks [8]. All four basic structures of waiting line situations are illustrated in Figure 1.
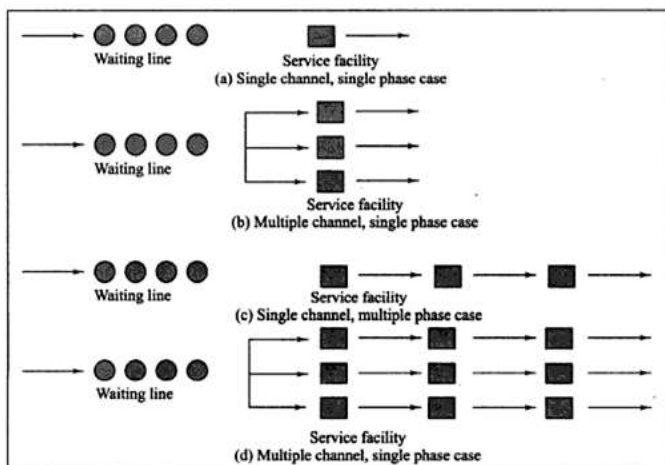

Figure 1.   Four basic structures of waiting line situations.

For the structure shown in Figure 1, there are two main classifications to divide the analytical methods for waiting lines, depending upon the measure of the source populace of the inputs: the infinite and finite waiting line models. The infinite waiting line models occur when the source is extensive and the length of the holding up line could develop without fixed limits. The finite waiting line models occur when the arrivals originate from a little, fixed-size population [8, 9].

Because of the mathematical complexity faced, Monte Carlo simulation has been the normal method of examination for frameworks including line disciplines other than first-come first-served [6,9].

The Poisson distribution function has been demonstrated to represent arrival rates in countless world circumstances. It is a discrete function that gives the likelihood of a given number of

arrivals in a unit of time. The Poisson distribution is given by [7, 9]:

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t} \qquad (1)$$

Where: t = the time interval 0 to t; n = total number of arrivals in the time interval 0 to t; and λ = the average arrival rate in arrivals/second.

Since Poisson arrivals are very common, it is expected that the mean service rate must be greater than the mean arrival rate, if this condition is not respected, the system would be unsteady and the queues become extensive. In this case one can assume the case of the single server, the first-come first-served queue order, and all arrival rates for service. Under these circumstances, the average number in the line is given by [10]:

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \qquad (2)$$

Where: L_q = the expected length of the waiting line; λ = the mean arrival rate from a Poisson distribution; μ = the mean service rate.

For the expected waiting time in the queue,

$$W_q = L_q / \lambda \qquad (3)$$

For the expected waiting time in the system,

$$W = W_q + 1/\mu \qquad (4)$$

All of these equations are used to determinate the steady state parameters for a single queue single-server system.

III.   METHODOLOGY

A.   General Customers

The general customers' data were received in spreadsheet format in Excel. These data consist of the number of customers arriving every five minutes and the service time in minutes for each one of these customers, for each working day. In order to better analyze the schedules of work, the data were collected during the periods: 9 - 11 AM, 12 - 2 PM, and 3 - 5 PM. For each period, the total number of arrivals through the sum of all customers arrived, the sum of the service time in minutes, and the transformed service time in hours were calculated. The total number of arrivals and the sum of service time for each period of general customers were calculated using Excel spreadsheet and the results are shown in Table I.

These values were used to perform calculation of the arrival rate and service rate of the day. Table II shows the results obtained for the arrival rate and service rate of the day for general customers. For the arrival rate, the total number of arrivals is divided by the total working time of each period; in this case, the period consists of two hours of work. For the service rate, the total number of arrivals is divided by the sum of service time in hours.

After calculating the arrival rate and the service rate values, using the Excel's extension queue, the probabilities and averages necessary for future data analysis were calculated. Probabilities of efficiency according to the number of employees and arrivals in the system, the probabilities of the wait time being higher than or equal to the critical wait

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time, probabilities of all servers being busy, the system be full, the critical wait time, wait time in the system, wait time in the queue, the mean time in the system and other information. Table III shows the data obtained from the Excel's extension queue for one period of general customers.

TABLE I.   CENERAL CUSTOMERS: SPREAD SHEET CALCULATIONS

A	B	C						D
09:00 - 09:05 AM	0	0						0
09:05 - 09:10 AM	2	1.3	2					3.3
09:15 - 09:15 AM	6	1.9	1.3	2.2	3.1	2.7	1.8	13
09:15 - 09:20 AM	0	0						0
09:20 - 09:25 AM	1	2.6						2.6
09:25 - 09:30 AM	3	3.3	1.1	2.6				7
09:30 - 09:35 AM	4	2.1	1.9	2.3	1.9			8.2
09:35 - 09:40 AM	1	2.8						2.8
09:40 - 09:45 AM	2	1.4	2.7					4.1
09:45 -0 9:50 AM	0	0						0
09:50 - 09:55 AM	3	2.8	1.9	1.3				6
09:55 - 10:00 AM	3	3.2	2.4	1.7				7.3
10:00 - 10:05 AM	1	1.8						1.8
10:05 - 10:10 AM	0	0						0
10:15 - 10:15 AM	0	0						0
10:15 - 10:20 AM	3	2.9	2.6	1.8				7.3
10:20 - 10:25 AM	1	2.6						2.6
10:25 - 10:30 AM	1	3.4						3.4
10:30 - 10:35 AM	2	2.6	3.4					6
10:35 - 10:40 AM	1	2						2
10:40 - 10:45 AM	2	2.5	1.7					4.2
10:45 - 10:50 AM	2	1.8	3					4.8
10:50 - 10:55 AM	0	0						0
10:55 - 11:00 AM	2	1.6	2.3					3.9
Total	40							90.3

A = Arrival time; B = Number of arrivals; C = Service time for each customer (min); D = Total service time (min).

TABLE II. ARRIVAL AND SERVICE RATES OF GENERAL CUSTOMERS

Time Interval	Arrival Rate	Service Rate
09 - 11 AM	20	26.57803709
12 - 02 PM	21	27.78390298
03 - 05 PM	29.5	26.89969605

TABLE III.   GENERAL CUSTOMERS: QUEUE EXCEL DATA FOR A PERIOD

Queue Station	Monday 9-11
Arrival Rate	20
Service Rate/Channel	26.57807309
Number of Servers	3
Type	M/M/3
Mean Number at Station	0.767398059
Mean Time at Station	0.038369905
Mean Number in Queue	0.014898087
Mean Time in Queue	0.000744904
Mean Number in Service	0.752499998
Mean Time in Service	0.037625
Throughput Rate	20
Efficiency	0.250833333
Probability All Servers Idle	0.469390124
Prob. All Servers Busy	0.044496283
Prob. System Full	0
Critical Wait Time	1
P(Wait >= Critical Wait)	5.08E-28
P(0)	0.469390124
P(1)	0.353216063
P(2)	0.132897542
P(3)	0.033335133
P(4)	0.008361562
P(5)	0.002097359
P(6)	0.000526087
P(7)	0.00013196

P(8)	3.31E-05
P(9)	8.30E-06
P(10)	2.08E-06

Finally, after the calculation of critical data probability, column charts and Box plots were created for better visualization of the results, thus facilitating the analysis. For the column charts probabilities of people in the system were used. Figure 2 illustrates the column chart obtained with the probabilities of people in the system for one period of general customers.
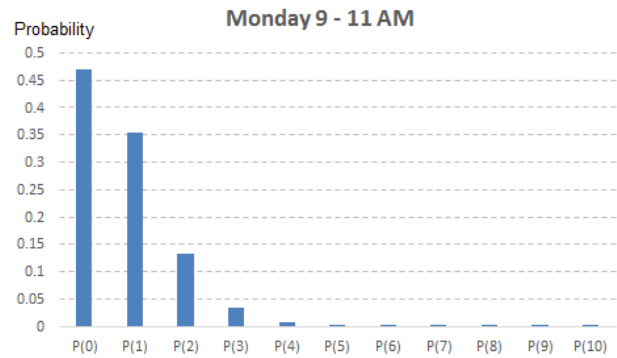

Figure 2.   General customers: probability of people in the system for a period

For the Box plot, the necessary values were calculated by using the probabilities of people in the system. The calculations necessary for the Box plot were the definition of the maximum, minimum, and median values among all probabilities, and the values from the interquartile range, the percentile of 25% and 75%. The results obtained in order to make the Box plot for a period of general customers is shown in Table IV and the Box plot obtained is shown in Figure 3.

TABLE IV.   GENERAL CUSTOMERS: BOX PLOT DATA FOR A PERIOD

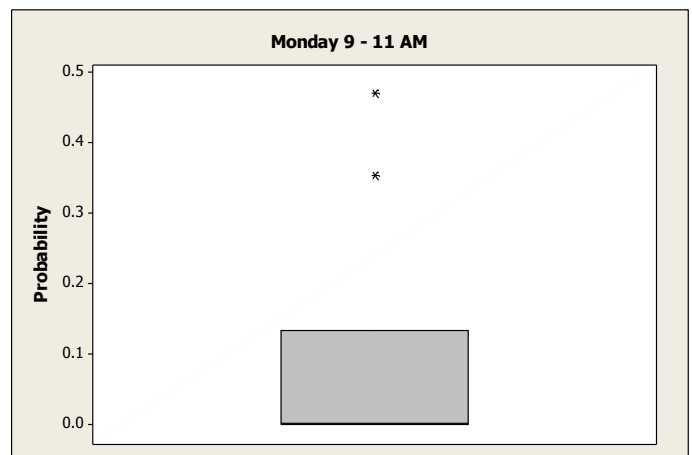| Minimum	2.08257E-06
Q_1	8.25301E-05
Median	0.002097359
Q_2	0.083116337
Maximum	0.469390124

Figure 3.   General customers: Box plot for a period

All these calculations and charts were repeated for each work period of each business day of the financial institution of study. At the end of all calculations, for verifications purposes,

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a check of the probabilities of people in the system was made. For this, the sum of these probabilities for each period was calculated. The results indicate some sort of problems related to lack of servers during certain periods (Table V).

TABLE V.   GENERAL CUSTOMERS: LACK OF SERVERS DURING ALL PERIOD OF A BUSINESS DAY (MONDAY)

Probability	Monday		
	9 - 11 AM	12 - 2 PM	3 - 5 PM
P(0)	0.46939	0.467797	0.328463
P(1)	0.353216	0.353577	0.360214
P(2)	0.132898	0.133622	0.197517
P(3)	0.033335	0.033665	0.072204
P(4)	0.008362	0.008482	0.026394
P(5)	0.002097	0.002137	0.009649
P(6)	0.000526	0.000538	0.003527
P(7)	0.000132	0.000136	0.001289
P(8)	3.31E-05	3.42E-05	0.000471
P(9)	8.30E-06	8.61E-06	0.000172
P(10)	2.08E-06	2.17E-06	6.30E-05
Verification	0.999999	0.999999	0.999964

B. Business Customers

Similar to general customers' data, the business customers' data were received in spreadsheet format in Excel. These data consist of the number of customers arriving every five minutes and the service time in minutes for each one of these customer, for each working day. In order to better analyze the schedules of work, the data were again collected during the periods: 9 - 11 AM, 12 - 2 PM and 3 - 5 PM. For each period the total number of arrivals through the sum of all customers arrived, the sum of the service time in minutes, and the transformed service time in hours were calculated. Table VI shows the results obtained after the necessary calculations, the total number of arrivals and the sum of service time for a period of business customers.

TABLE VI.   BUSINESS CUSTOMERS: SPREAD SHEET CALCULATIONS

A	B	C		D
09:00 - 09:05 AM	2	2.3	2.6	4.9
09:05 - 09:10 AM	2	2.9	7.6	10.5
09:15 - 09:15 AM	1	3.3		3.3
09:15 - 09:20 AM	0			0
09:20 - 09:25 AM	0			0
09:25 - 09:30 AM	1	11.8		11.8
09:30 - 09:35 AM	2	9.4	4.5	13.9
09:35 - 09:40 AM	1	9.8		9.8
09:40 - 09:45 AM	2	3.9	10.3	14.2
09:45 -0 9:50 AM	0			0
09:50 - 09:55 AM	1	3.2		3.2
09:55 - 10:00 AM	0			0
10:00 - 10:05 AM	2	8.3	4.3	12.6
10:05 - 10:10 AM	2	9.9	5.6	15.5
10:15 - 10:15 AM	1	7		7
10:15 - 10:20 AM	0			0
10:20 - 10:25 AM	2	4.6	10.7	15.3
10:25 - 10:30 AM	0			0
10:30 - 10:35 AM	1	6.8		6.8
10:35 - 10:40 AM	0			0
10:40 - 10:45 AM	1	5.4		5.4
10:45 - 10:50 AM	0			0
10:50 - 10:55 AM	0			0
10:55 - 11:00 AM	0			0
Total	21			134.2

A = Arrival time; B = Number of arrivals; C = Service time for each customer (min); D = Total service time (min).

These values were used to perform calculation of the arrival rate and service rate of the day. Table VII shows the results obtained for the arrival rate and service rate of the day for business customers. For the arrival rate, the total number of arrivals is divided by the total working time of each period, two hours. For the service rate, the total number of arrivals is divided by the sum of service time in hours.

TABLE VII.   ARRIVAL AND SERVICE RATES OF BUSINESS CUSTOMERS

Time Interval	Arrival Rate	Service Rate
09 - 11 AM	10.5	9.388971684
12 - 02 PM	11.5	7.931034483
03 - 05 PM	13.5	8.376421923

After calculating the arrival rate and the service rate values, using the Excel's extension queues, the probabilities and averages necessary were calculated for future data analysis. Probabilities of efficiency according to the number of employees and arrivals in the system, the probabilities of the wait time being higher than or equal to the critical wait time, probabilities of all servers being busy, the system be full, the critical wait time, wait time in the system, wait time in the queue, the mean time in the system and other information. Table VIII shows the data obtained from the Excel's extension queue for a period of business customers.

TABLE VIII.   BUSINESS CUSTOMERS: QUEUE EXCEL DATA FOR A PERIOD

Queue Station	Friday 9-11
Arrival Rate	10.5
Service Rate/Channel	9.388971684
Number of Servers	2
Type	M/M/2
Mean Number at Station	1.627062798
Mean Time at Station	0.154958367
Mean Number in Queue	0.508729402
Mean Time in Queue	0.048450419
Mean Number in Service	1.11833334
Mean Time in Service	0.106507942
Throughput Rate	10.5
Efficiency	0.55916667
Probability All Servers Idle	0.28273648
Prob. All Servers Busy	0.40106982
Prob. System Full	0
Critical Wait Time	1
P(Wait >= Critical Wait)	0.000101896
P(0)	0.28273648
P(1)	0.316193642
P(2)	0.176804952
P(3)	0.098863436
P(4)	0.055281138
P(5)	0.03091137
P(6)	0.017284608
P(7)	0.009664977
P(8)	0.005404333
P(9)	0.003021923
P(10)	0.001689758

Finally, after the calculation of critical data probability, column charts and Box plots were created for better visualization of the results, thus facilitating the analysis. For the column charts probabilities of people in the system were used. Figure 4 illustrates the column chart obtained with the probabilities of people in the system for one period of business customers.

For the Box plot, the necessary values were calculated using the probabilities of people in the system. The calculations necessary for the Box plot were the definition of

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the maximum, minimum, and median values among all probabilities, and the values from the interquartile range, the percentile of 25 and 75%. The results obtained in order to make the Box plot for each period of a day of business customers is shown in Table IX and the Box plot obtained is shown in Figure 5.
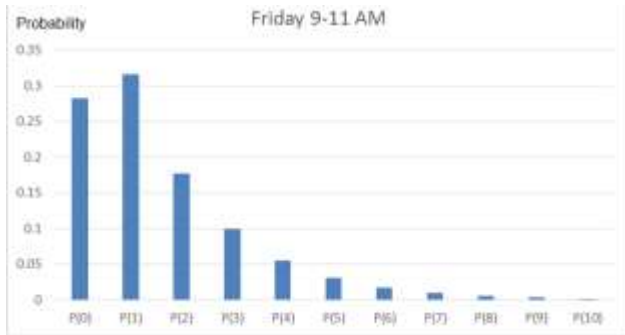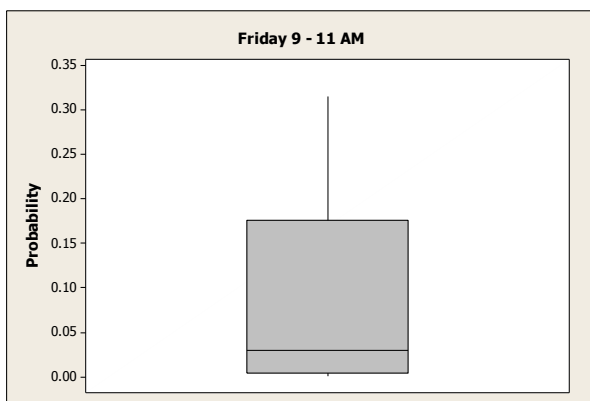

Figure 4.   Business customers: Column chart for a period

TABLE IX.   BUSINESS CUSTOMER: BOX PLOT DATA FOR A PERIOD

Friday Morning, 9 - 11 AM	
Minimum	0.0016898
Q_1	0.0075347
Median	0.0309114
Q_3	0.1378342
Maximum	0.3161936

Figure 5.   Business customers: Box plot for a period

All these calculations and charts were repeated for each work period of each business day of the financial institution of study. At the end of all calculations, for verifications purposes, a check of the probabilities of people in the system was made. For this, the sum of these probabilities for each period was calculated. The results indicate some sort of problems related to lack of servers during certain periods as shown in Table X.

TABLE X.   BUSINESS CUSTOMERS: LACK OF SERVERS DURING ALL PERIOD OF A BUSINESS DAYS (FRIDAY)

Probability	Friday		
	9 - 11 AM	12 - 2 PM	3 - 5 PM
P(0)	0.282736	0.15942	0.184568
P(1)	0.316194	0.231159	0.297462
P(2)	0.176805	0.167591	0.239705
P(3)	0.098863	0.121503	0.128775
P(4)	0.055281	0.08809	0.069181
P(5)	0.030911	0.063865	0.037165
P(6)	0.017285	0.046302	0.019966
P(7)	0.009665	0.033569	0.010726

P(8)	0.005404	0.024338	0.005762
P(9)	0.003022	0.017645	0.003096
P(10)	0.00169	0.012792	0.001663
Verification	0.997857	0.966275	0.998069

C.   Drive-Through Customers

The drive-through service consist of two machines which can form its own queue, in other words, drive-through service has two queues. Each queue was studied separately for better analysis of data.

Like other data of general and business customers, the data of drive-through customers were received in spreadsheet format in Excel. These data consist of the number of customers arriving every minute only during the peak time and the service time in minutes for each one of these customers, for each working day. In order to better analyze the schedules of work, the data were again collected during the periods: 9 - 9:30 AM, 12 - 12:30 PM and 4:30 - 5 PM. For each period the total number of arrivals through the sum of all customers arrived, the sum of the service time in minutes, and the transformed service time in hours were calculated. These values were used to perform calculation of the arrival rate and service rate of the day for each queue. Table XI shows the results obtained for the arrival rate and service rate of a day for drive-through customers. For the arrival rate, the total number of arrivals is divided by the total service time of each period; in this case the duration of each period is thirty minutes. For the service rate, the total number of arrivals is divided by the sum of service time in hours.

TABLE XI.   ARRIVAL AND SERVICE RATES OF DRIVE-THROUGH CUSTOMERS

		L1	L2
Arrival Rate	Morning	48	42
	Afternoon	40	44
	Evening	64	56
Service Rate	Morning	59.75104	64.94845
	Afternoon	51.50215	64.39024
	Evening	57.31343	61.99262

For each line of drive-through, after calculating the arrival rate and the service rate values, using the Excel's extension queues, the probabilities and averages necessary were calculated for future data analysis. Probabilities of efficiency according to the number of employees and arrivals in the system, the probabilities of the wait time being higher or equal than the critical wait time, probabilities of all servers being busy, the system be full, the critical wait time, wait time in the system, wait time in the queue, the mean time in the system, and other related information. Table XII shows the data obtained from the Excel's extension queue for one period of drive-through customers.

Finally, after the calculation of critical data probability, column charts and Box plots were created for better visualization of the results, thus facilitating the analysis. For the column charts probabilities of people in the system were used. Figure 6 illustrates the column charts obtained with the probabilities of people in the system for one period of drive-through customers in lines L1 and L2.

For the Box plot, the necessary values were calculated using the probabilities of people in the system. The calculations necessary for the Box plot were the definition of the maximum, minimum and medium values among all probabilities, and the values from the interquartile range, the percentile of 25% and 75%. The results obtained in order to make the Box plot for each period of drive-through customers

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on a Tuesday are shown in Table XIII and the Box plots obtained are shown in Figure 7.

TABLE XII. DRIVE-THROUGH CUSTOMERS: QUEUE EXCEL DATA FOR A PERIOD

Queue Station	Que_25	Que_26
Line	L1	L2
Arrival Rate	48	42
Service Rate/Channel	59.75104	64.94845
Number of Servers	2	2
Type	M/M/2	M/M/2
Mean Number at Station	0.957873	0.722165
Mean Time at Station	0.019956	0.017194
Mean Number in Queue	0.154539	0.075498
Mean Time in Queue	0.00322	0.001798
Mean Number in Service	0.803333	0.646667
Mean Time in Service	0.016736	0.015397
Throughput Rate	48	42
Efficiency	0.401667	0.323333
Probability All Servers Idle	0.426873	0.511335
Prob. All Servers Busy	0.230206	0.158002
Prob. System Full	0	0
Critical Wait Time	1	1
P(Wait >= Critical Wait)	2.04E-32	1.06E-39
P(0)	0.426873	0.511335
P(1)	0.342921	0.330663
P(2)	0.13774	0.106914
P(3)	0.055326	0.034569
P(4)	0.022222	0.011177
P(5)	0.008926	0.003614
P(6)	0.003585	0.001169
P(7)	0.00144	0.000378
P(8)	0.000578	0.000122
P(9)	0.000232	3.95E-05
P(10)	9.33E-05	1.28E-05

Figure 6. Drive-through customers: Column charts - lines L1 and L2

TABLE XIII. DRIVE-THROUGH CUSTOMERS: DATA FOR BOX PLOT

	MORNING		AFTERNOON		EVENING	
	L1	L2	L1	L2	L1	L2
MIN	0.003234	0.002636	0.005826	0.033441	0.001362	0.018426
Q1	0.011719	0.010203	0.017385	0.03976	0.006498	0.019548
MED	0.039896	0.03687	0.049571	0.047216	0.02832	0.020735
Q2	0.14458	0.142721	0.147927	0.056138	0.135146	0.021998
MAX	0.297788	0.304277	0.27487	0.066667	0.321003	0.023333
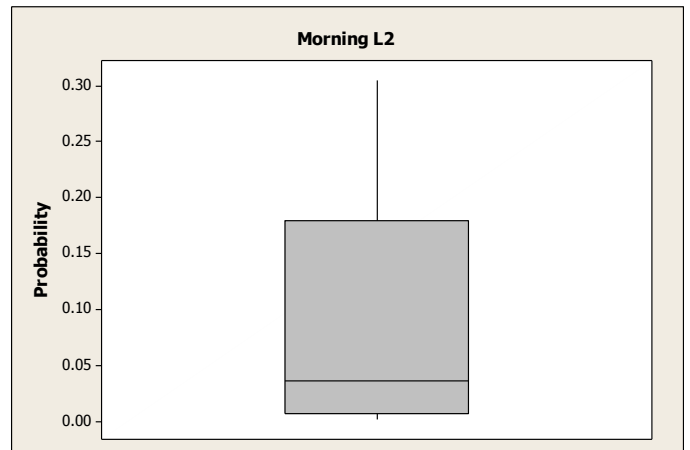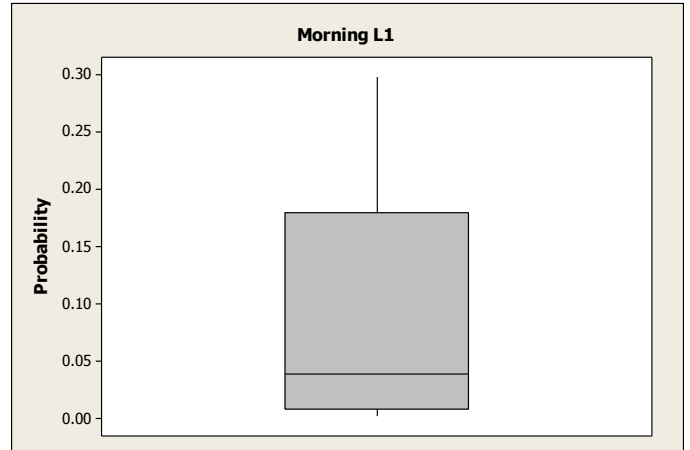
Figure 7. Drive-through customers: Box plots - lines L1 and L2

All these calculations and graphs were repeated for drive-through customers for each peak period of each business day of the financial institution of study.

IV. RESULTS AND DISCUSSIONS

A. General Customers

In the general customer situation the results were well succeeded. There were not issues related to queues in this section.

The worst result for the week analyzed was the results illustrated by the chart for Friday evening. Even though, the results were better than expected, it can be concluded that the queue was negligible. The number of tellers used to make the system work is good enough in this case. Another factor is the amount of time taken by the general customers to conclude their tasks. It is usually faster than other kinds of customers, taking only few minutes to complete their transactions. It helps the queue to be faster than the other types of queues in the financial institution. Figure 8 explains the probability of people in the queue, from zero to ten people, in the general customer service and Figure 9 shows that the data is

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asymmetric through the Box plot demonstrating that the probability of a shorter queue is higher than the probability of a longer queue [11].
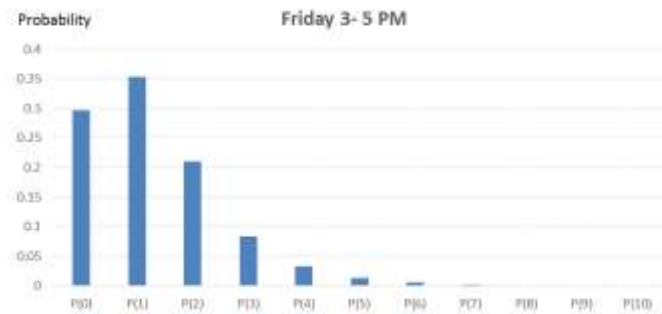

Figure 8.   General customers: Probability of people in the queue (0 -10)
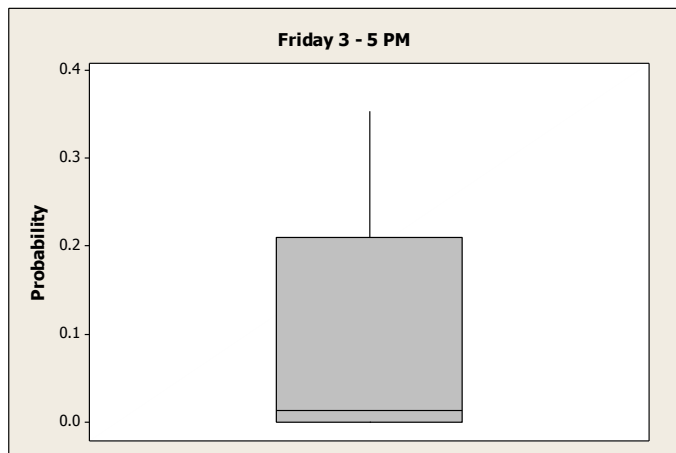

Figure 9.   General customers: Box plot for probability of people in the queue

B.   Business Customers

Considering the business customer structure, some studies revealed that in this section, there were issues detected in different days. These days were Monday, Wednesday and Friday. Monday presented issues in the last shift, then Wednesday in the second and finally Friday in the last shift. These issues were characterized as lines long enough to disrupt the functioning of the flux of people. Business customers are a different type of customers who goes to the bank to treat delicate subjects, normally business related subjects that takes a long time of service. This service varies from one client to another. This issue generates in long lines, what may represent financial loss to banking institutions because they work with time, and loss of time represents loss of money. Based on that, the issues were discussed to understand the reason why the system was presenting excessive waiting time.

The bank usually works with two tellers to attend its customers. As was observed in the collected data, the two tellers do not support the demand in certain time periods. Figures 10, 11, and 12, represent the probability of having people in the line (from zero to ten people) and show that the queues were too long during these time periods, and probably the number of tellers was not enough. Also, Figures 13, 14, and 15 reinforce the idea that the number of people in the queue is huge.
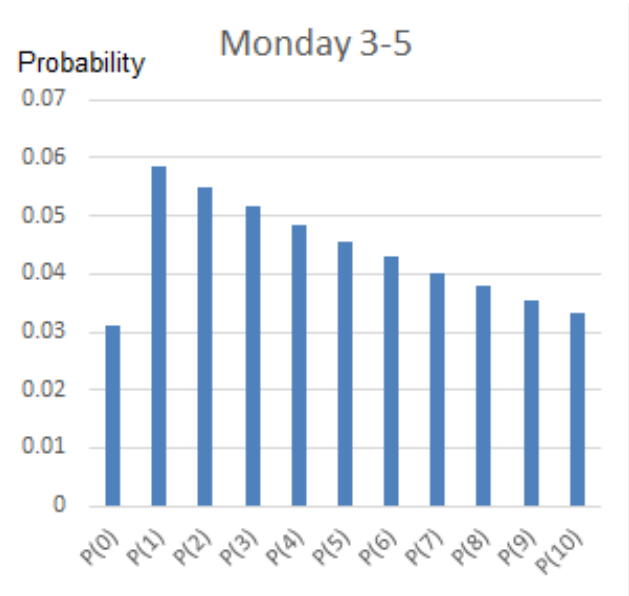
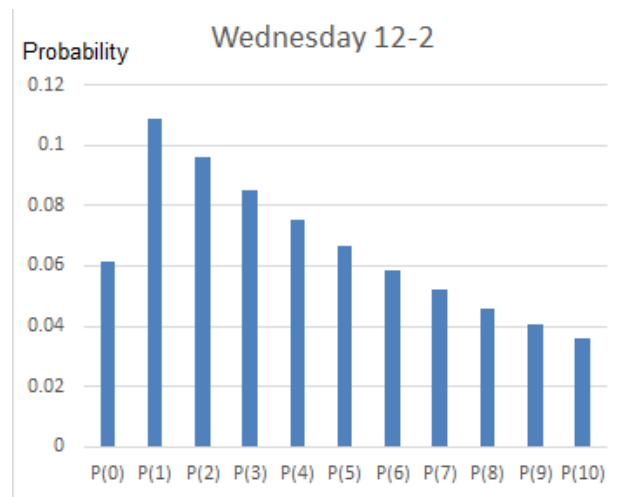Figure 10. Business customers: Monday from 3 - 5 PM

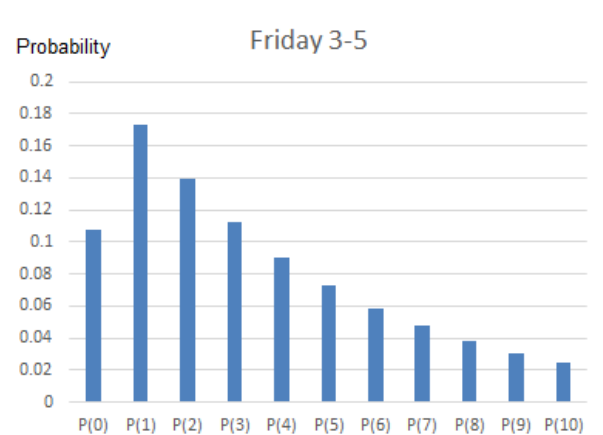Figure 11. Business customers: Wednesday from 12 - 2 PM

Figure 12. Business customers: Friday from 3 - 5 PM

Also, the simulations show that the waiting times were too long: 56.6 minutes (0.944 hour) on Monday evening, 26.2 minutes (0.437 hour) on Wednesday afternoon, and 13.3 minutes (0.221 hour) on Friday evening.
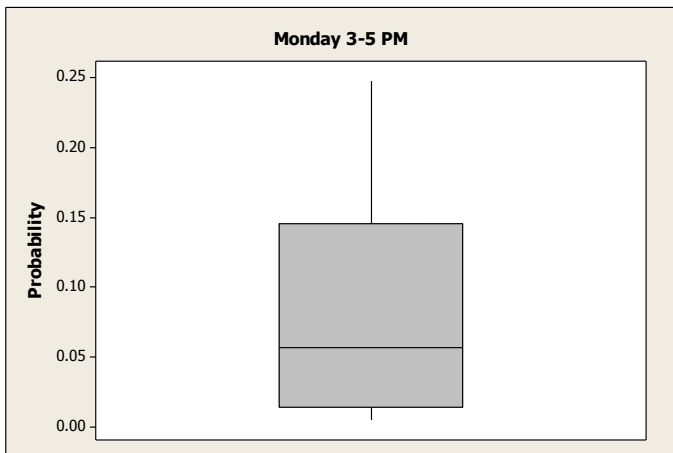
Figure 13.   Box plot of business customers: Monday from 3 - 5 PM
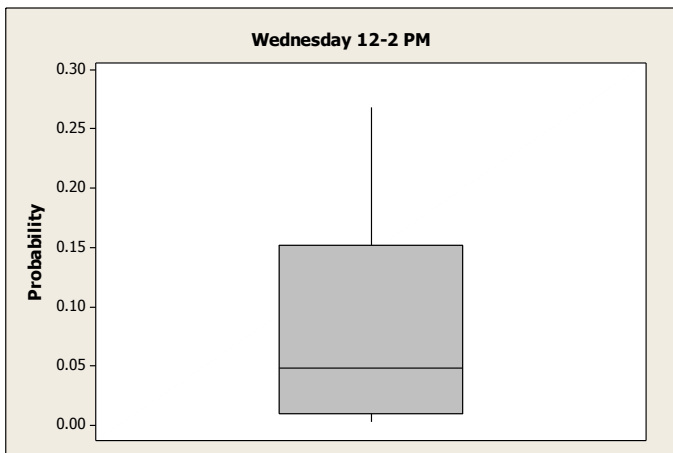

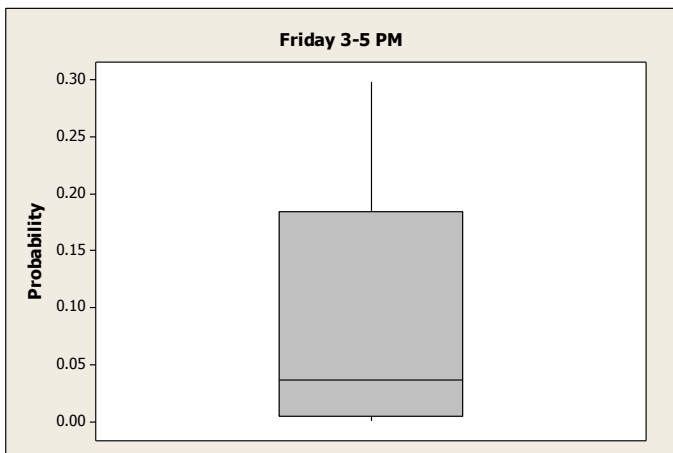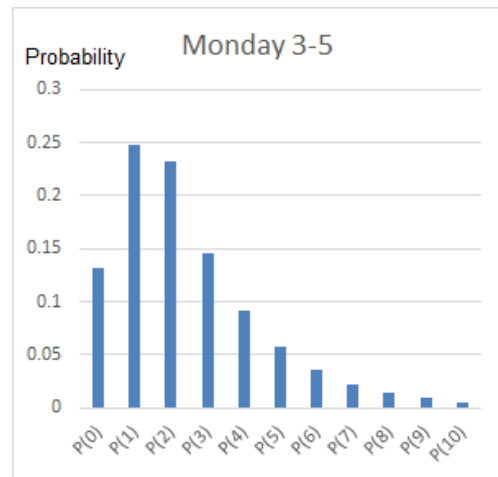Figure 14.   Box plot of business customers: Wednesday from 12 - 2 PM

Figure 15.   Box plot of business customers: Friday from 3 - 5 PM

TABLE XIV.   SIMULATIONS RESULTS ON MONDAY, WEDNESDAY, AND FRIDAY

Queue Station	Monday 3-5	Wednesday 12-2	Friday 3-5
Arrival Rate	15	14.5	13.5
Service Rate/Channel	7.982261641	8.199811499	8.376421923
Number of Servers	2	2	2
Type	M/M/2	M/M/2	M/M/2
Mean Number at Stations	16.03614044	8.102350235	4.596453667
Mean Time at Station	1.069076061	0.558782756	0.340478063
Mean Number in Queue	14.15697411	6.334016661	2.984787115
Mean Time in Queue	0.943798274	0.436828735	0.221095342

From Table XIV, it is seen that the mean time in queue: 0.944 hour on Monday, 0.437 hour on Wednesday, and 0.221 hour on Friday.

Because of this, a simulation using three tellers was made in each one of these times. The results obtained are presented in Figures 16, 17, and 18.

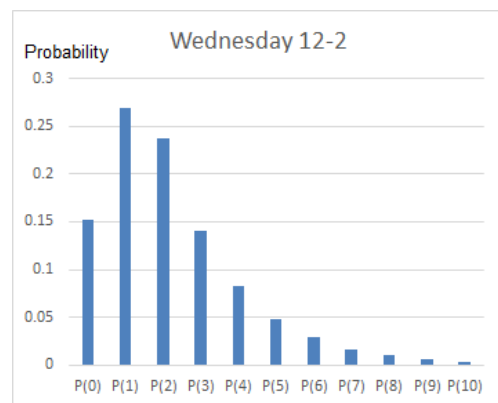Figure 16. Business customers: Monday from 3 - 5 PM (3 tellers)

Figure 17. Business customers: Wednesday from 12 - 2 PM (3 tellers)
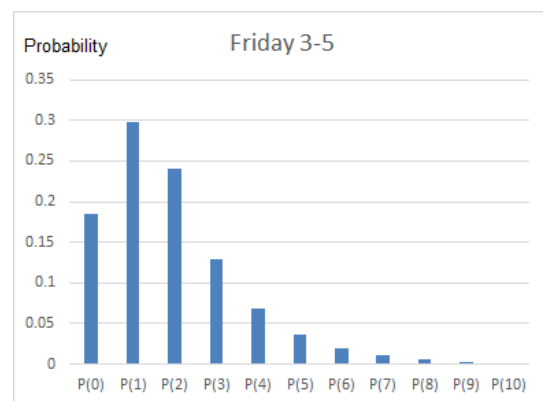

Figure 18.   Business customers: Friday from 3 - 5 PM (3 tellers)

The results found show that the number of people in the queue will decrease to a satisfactory number. Also, the waiting time will decrease to a maximum of three minutes in the three cases.

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C. Drive-Through Customers

The drive-through is the most utilized service in the bank system; In consequence of the facility of a drive-through service, people usually prefer to use it. For this reason, there were a lot of problems in the drive-through queues. In some cases, the arrival rate was higher than the service rate. This situation made the simulation impossible. These results revealed that adding just one more teller was more than enough to attend the demand. The problems were found at: Monday: L1 (morning, afternoon, and evening); Tuesday: L1 (morning, afternoon, and evening) and L2 (morning); Wednesday: L1 (morning, afternoon, and evening) and L2 (afternoon); Thursday: nothing; Friday: L1 (evening).

As shown in Figure 19, the most crowded days were those in which the longest lines appeared. On Monday, Tuesday, and Wednesday the major issues appeared, especially on Tuesday and Wednesday that had problems on both lines. As said before, the problems were easily solved by adding one more teller in each line that had long queue to attend.
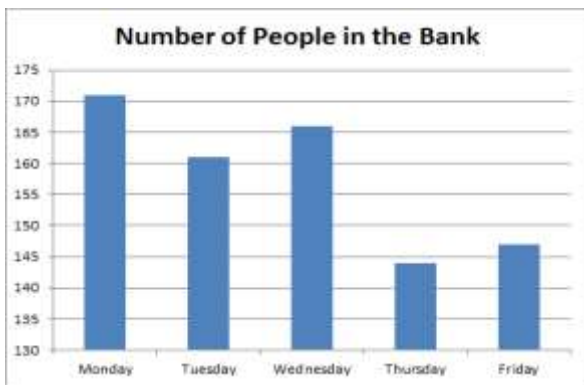

Figure 19.   Number of people coming during one and a half hour, in three different periods of thirty minutes.

From Figure 19 and the simulation results, it was clear that just one teller in each line was insufficient to attend the daily demand. As shown in the Figure 20, adding one more teller in the first line will make the number of people waiting in the line to decrease. According to that suggestion, it can be seen that Monte Carlo simulation can graphically illustrates how little changes can have an effect on the queue.
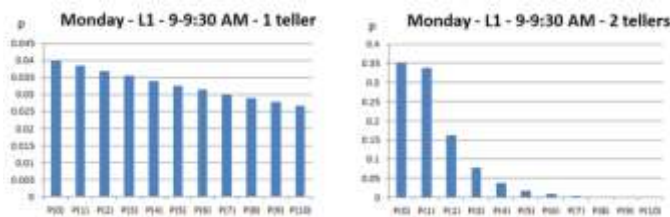

Figure 20.   Comparison between the use of one and two tellers on the drive-thru customer service on Monday

V.   CONCLUSIONS AND RECOMMENDATIONS

The observations of the queue and the simulation results clearly indicate that the institution analyzed has some problems about waiting times in queue. In order to provide solutions and assistance to the financial institution, some recommendations were studied and analyzed to state that it is not necessary to contract more people to solve the bank's problem of excessive waiting time.

Analyzing each queue, the general customers section, specially, did not presented any problem. It was observed that for the general customers, the service offered by the bank was satisfactory. However, the results presented for both business customers and drive-through customers showed excessive waiting time. The business customers had problems in few times while the drive-through customers had problems half of the time.

Based on the information collected from the charts generated by the simulation, the recommendation to solve the excessive waiting time is the addition of one more teller in the peak hours.

Figure 21 shows the probability of people in the general customer line for use of 3 and 2 tellers on Monday. Similar results for Wednesday and Friday are shown in Figures 22 and 23 respectively. Based on the tendency of the general customers queue, it was considered the reduction of one teller among the three tellers. This change could be made in order to relocate it to the area that was presenting excessive waiting time. It can be stated through the illustrations that the charts' behavior did not show significant change when reducing one teller in the general customers queue. The recommendation is the possibility of moving one of the tellers from the general customer service to the business customer service or the drive-through customer service, when needed, to improve the system. In other words, there is no need to contract more employees to solve the issues.
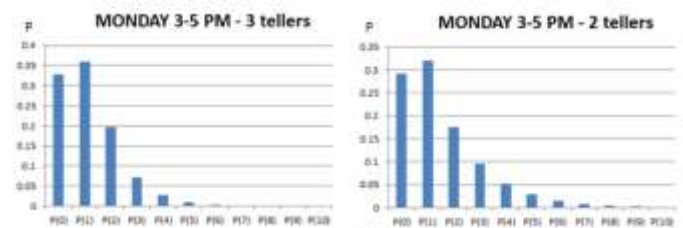

Figure 21.   Comparison between the use of three and two tellers on the general customer service on Monday.
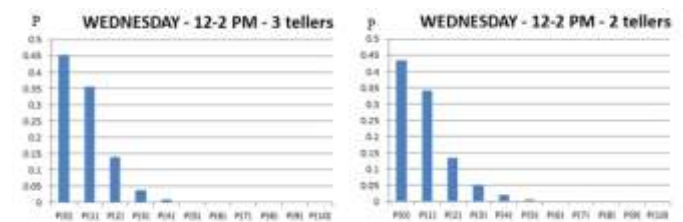

Figure 22.   Comparison between the use of three and two tellers on the general customer service on Wednesday.
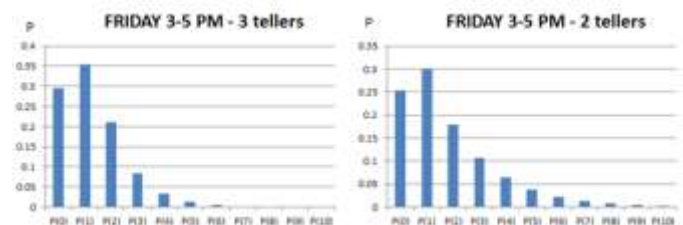

Figure 23.   Comparison between the use of three and two tellers on the general customer service on Friday.

During this study, it was grateful to experience a new way to work with queues. The Monte Carlo simulation proved to be easy and simple for some reasons. First, the results can be

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shown as graphs and charts, which make the visualization of results a lot clear. Second, it gives probability results of what and how it could happen. Moreover, the analysis of the results makes it easy to correlate the impacts to the results. Finally, Monte Carlo simulation does not demand specific knowledge.

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