

Exchange Rate Synchronization for a Set of Currencies from Different Monetary Areas

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Abstract

The degree of co-movement between currencies remains an important subject for international trade and monetary integration across countries. However, the economic literature has given limited answers about the directional relationships among currencies, and whether they have a leader or a driver. Using the Hodrick-Prescott filter and the wavelet methodology, this paper analyzes exchange rate synchronization for a set of twelve currencies belonging to different monetary areas covering the period between January 1980 and July 2020. The empirical results reveal that: i) the U.S. dollar still plays an essential role as a foreign exchange anchor; ii) the euro shows an out-of-phase relationship with the vast majority of currencies, including with the other European currencies; iii) the British pound seems to have departed significantly from the European single currency; iv) the Brazilian real leads the Chinese yuan for most of the sample, and both currencies record great dissimilarities with the other currencies; v) in the absence of short-term foreign exchange market frictions, average bilateral distances between currencies are smaller, and vi) during the international financial crisis, exchange rates became more synchronized.

Keywords: exchange rate; co-movements; Hodrick-Prescott filter; wavelets; synchronization.

JEL classification: E31; F41; F42; C51.

1. INTRODUCTION

It is recognized that the volatility of both nominal bilateral and effective U.S. dollar exchange rates increased markedly after the collapse of the Bretton Woods international monetary system in the early 1970s. For this reason, exchange rates have received much attention recently from investors, traders, policy-makers and producers, partly because of the possible synchronization of their movements. An environment of high uncertainty invokes tendencies towards the use of foreign exchange reserves as safe havens to avoid foreign exchange risks. Identical strategies can also be enhanced by developing joint co-movements between different currencies, namely regarding large monetary areas, such as the Euro Zone.

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Neither the economic theory nor empirical literature has given us enough answers about the directional relationships among the major foreign exchange reserves currencies, whether they have a leader or a driver, and how they are related to other currencies belonging to different monetary areas or distant geographic spaces. Some authors (e.g., [Chitu, Eichengreen, and Mehl \(2014\)](#); [Goldberg and Tille \(2006\)](#); [Seghezza and Morelli \(2018\)](#) and [Dabrowski \(2020\)](#)) would consider the U.S. dollar as the leader of foreign exchange market. Nevertheless, the euro, the Chinese yuan or the British pound sterling have been gaining weight as a nominal anchor for other currencies and as a medium of exchange and reserve of value with high liquidity. In these circumstances, would currencies such as the Australian dollar, Brazilian real, Canadian dollar, Japanese yen, Mexican peso, Norwegian krone, Polish zloty and Swiss franc relate more to the U.S. dollar, the euro, the British pound sterling or the Chinese yuan? Which of these currencies will be more synchronized with each other? Is it possible to identify cyclical co-movements and similar behavioral patterns between them? It would be useful and interesting to answer these questions, particularly when the international monetary system started to be divided into large monetary areas that, despite competing with each other, are also closely interdependent given the increasing globalization of international financial markets. This is precisely the main aim of this study.

For the post-Bretton Woods period, we want to investigate the existence of exchange rate synchronization for a set of twelve currencies belonging to different monetary areas and different geographic spaces, such as Australia, Latin America, Canada, China, Japan, United States of America and Europe. For this purpose, we use monthly data for nominal bilateral exchange rates vis-à-vis the U.S. dollar, as well as for both the nominal effective exchange rate of the euro and the U.S. dollar, trade partners by consumer price index, between January 1980 and July 2020.

We intend to analyze the existence of co-movements and possible synchronization between the cyclical components of the exchange rates by using two different methodologies, the Hodrick-Prescott filter and the wavelet methodology. These are methodologies usually applied in the study of business cycles synchronization but can also be used in the analysis of behavioral dynamics in the context of the foreign exchange market, similarly to what has been done, for example, by [Nikkinen, Pynnönen, Ranta, and Vähämaa \(2011\)](#), [Andries, Ihnatov, and Tiwari \(2014\)](#), [Rabanal and Rubio-Ramírez \(2015\)](#), [Tiwari and Albuлесcu \(2016\)](#), and [da Silva and Fernandes \(2018\)](#). However, these two methods tend to be applied separately. This study's originality results from the fact that these two methodologies are combined in the same investigation. This research strategy has never been carried out before, at least to the best of our knowledge, but that due to their interrelationship, it allows for additional gains in terms of descriptive analysis and a better understanding of the phenomenon. In addition, this study investigates the co-movements and possible synchronizations between exchange rates currencies of countries belonging to both different monetary areas and geographic spaces for approximately forty years, when most of the work developed so far has focused exclusively on a single currency area or region and a relatively short period. The merit of such an approach is that short sample periods may mask significant trends in analyzing synchronization patterns. In the short term, much of the exchange rate behavior depends on the shock dynamics, which tends to overshadow monetary integration. Against this, we analyze and attempt to explain changes in exchange rate synchronization over 40 years, which for monthly data corresponds to the use of a sample with about 480 observations.

The work is organized into four sections. In addition to the introduction, in [Section 2](#), we present a brief review of the topic's literature. In [Section 3](#), synchronization between the

various exchange rates considered in this study is analyzed. We started this analysis by presenting the data, followed by studying the joint movements of the cyclical components of exchange rates using the Hodrick-Prescott filter methodology. This section ends with the investigation of exchange rate synchronization within the scope of the wavelet methodology. Finally, in [Section 4](#), we present the main conclusions of this study.

2. A BRIEF REVIEW OF THE LITERATURE

Exchange rate synchronization refers to the fact that foreign exchange markets display significant co-movements. The degree of co-movement or correlation between different currencies remains very important for international trade and monetary integration between countries and regions. This correlation is likely to change over time due to rising economic interdependence through foreign exchange and international financial markets. Foreign exchange market synchronization will also be affected by monetary integration, say through the removal of exchange rate volatility and the promotion of price stability, as it happened in the European integration process. It is expected that participation in large monetary areas or anchoring their own currency to international reference currencies could improve exchange rate synchronization by lowering transaction costs and inflation expectations in cross-border countries. Moreover, lower the risks of exchange rate volatility could also lead to enhanced international trade in goods, which is usually thought of as fostering business cycle synchronization, thereby leading to higher foreign exchange market co-movements. Other factors such as the similarity of economic structure across countries, informational asymmetries, geographical proximity and a common language could also contribute to exchange rate synchronization. However, trade integration may also lead countries to specialize in producing goods for which they have a comparative advantage, hence reducing co-movements. The net impact of these determinants of exchange rate co-movements is therefore ambiguous in explaining their possible synchronization. Using daily data for a set of thirteen East Asian countries, [Wang and Yang \(2009\)](#) examines exchange rate co-movements in the region before and after the China exchange rate reform in 2005. The authors found that the correlation between Asian currencies and the U.S. Dollar, the previous regional reference currency, has become weaker and intra-Asia interactions have increased¹. Cross-sample entropy and cross-entropy approaches are also applied to examine the synchrony behavior among the Asian currencies. The study shows that exchange rate markets in Asia are surprisingly collectively correlated. The temptation of Central Bank intervention in the foreign exchange markets plays a role in such an outcome, even though most central banks would not officially admit what they have done. The study also finds that the interaction among Asian currencies has been intensified. The dependence upon the U.S. Dollar has been weakened, which has been proved by evaluating the correlation coefficient matrix, the cross-sample entropy and the cross-entropy. This evolving trend became much more significant after the Chinese currency reform in 2005. As a result, the synchronization of currency movements in the region has increased.

An extensive literature also focuses on analyzing the co-movements and volatility spillovers among exchange rate series since the introduction of the European single currency on January 1, 1999. Employing the dynamic conditional correlation model developed by [Engle \(2002\)](#), e.g., [Pérez-Rodríguez \(2006\)](#) finds that the correlations among the euro, British pound sterling, and Swiss franc fluctuate significantly between 1999 and 2004 and that the dynamic correlation between the euro and the British pound was particularly high. Using a

VAR model on currency options data for the same three currencies over the period 2001-2003, [Nikkinen, Sahlström, and Vähämaa \(2006\)](#) conclude that the highest correlation is between the euro and the Swiss franc and that the euro is the dominant currency in volatility transmission. Using residual cross-correlation functions, [Inagaki \(2007\)](#) analyzes the volatility spillovers between the British pound sterling and the euro vis-à-vis the U.S. dollar nominal exchange rates between 1999 and 2004 similarly, finding a unidirectional volatility spillover from the euro to the British pound sterling. Following a similar investigation line, [McMillan and Speight \(2010\)](#) also finds that the U.S. dollar dominates the Japanese yen and the British pound sterling vis-à-vis the euro between 2002 and 2006 in terms of both return and volatility spillovers. More recently, [Antonakakis \(2012\)](#) investigates return co-movements and volatility spillovers between major exchange rates before and after the introduction of the euro. Dynamic correlations and VAR-based spillover index results suggest significant return co-movements and volatility spillovers, on average, lower in the post-euro period. Co-movements and spillovers are positively associated with extreme episodes and U.S. dollar appreciations. The author also concludes that the euro is the dominant net transmitter of volatility, while the British pound sterling is the dominant net receiver of volatility in both periods. Nevertheless, cross-market volatility spillovers are bidirectional, and the highest spillovers happen between European foreign exchange markets.

Some research also focuses on analyzing the asymmetric responses of higher moments (depreciation v.s. appreciation) of exchange rates. Based on exchange rate data between 1996 and 2004, [Wang and Yang \(2009\)](#) find evidence of asymmetric volatility in the Australian dollar, British pound sterling and the Japanese yen against the U.S. dollar exchange rates. Specifically, a depreciation against the U.S. dollar tends to lead to significantly greater volatility than an appreciation for the Australian dollar and the British pound sterling, while the opposite is not true for the Japanese yen. [Boero, Silvapulle, and Tursunalieva \(2011\)](#) also find varying degrees of pairwise co-movements of the euro, the British pound sterling and the Japanese yen during episodes of appreciations and depreciations against the U.S. dollar for the period between 1994 and 2007. Starting from a multivariate asymmetric dynamic conditional correlation GARCH model, [Tamakoshi and Hamori \(2014\)](#) examine for the period between January 1, 1999, and December 31, 2010, the interdependence of daily U.S. dollar exchange rates expressed in terms of euro, British pound sterling, and Swiss franc. The effect of Europe's recent financial crisis on these dynamic correlations is particularly inspected. The authors suggest asymmetric responses in correlations among the three exchange rates, namely, higher dependency during periods of joint appreciation than during periods of joint depreciation. Moreover, the results indicate that the European turmoil may have triggered the shift of fund flows to the Swiss franc in particular, which is widely believed to be a safe-haven currency.

To conclude this brief review of the literature, it is also important to highlight the recent study developed by [Orlowski \(2016\)](#), which investigates for the period between January 4, 2000, and March 11, 2015, the existence of co-movements of non-euro European Union (EU) members' currencies and the euro. The author proposes a cross-elasticity model of exchange rates and performs the Bai-Perron multiple breakpoints, GARCH and BVAR estimations on the daily data series. The main question is whether local currencies move together, or against, the euro in foreign exchange markets and if and how volatility shocks are transmitted from one currency to another. Positive co-movements imply substitution, while negative indicate complementarity between the local currencies and the euro. Since the analyzed countries are members of the EU, some inherent substitution between their currencies and the euro can be reasonable. The results show high

positive cross-elasticity between the euro and the currencies of Denmark, Sweden, Poland, the Czech Republic and Hungary. For the Romanian Leu, cross-elasticity (co-movements) with the euro is initially nonexistent but steadily increasing over the sample period. This implies a strong substitution between these currencies and the euro in foreign exchange markets. In contrast, cross-elasticity between the British pound sterling and the euro is considerably lower. Orłowski (2016) also find that for all examined non-euro currencies substitution with the euro increased substantially during the recent international financial crisis, which could indicate that at times of financial distress, a flexible exchange rate can serve as an important tool for mitigating financial risk. Monetary authorities of the countries with flexible exchange rates can always resort to currency depreciation in order to sustain exports and economic growth, a behavior that may explain the lack of synchronization between the different currencies.

3. EXCHANGE RATE SYNCHRONIZATION

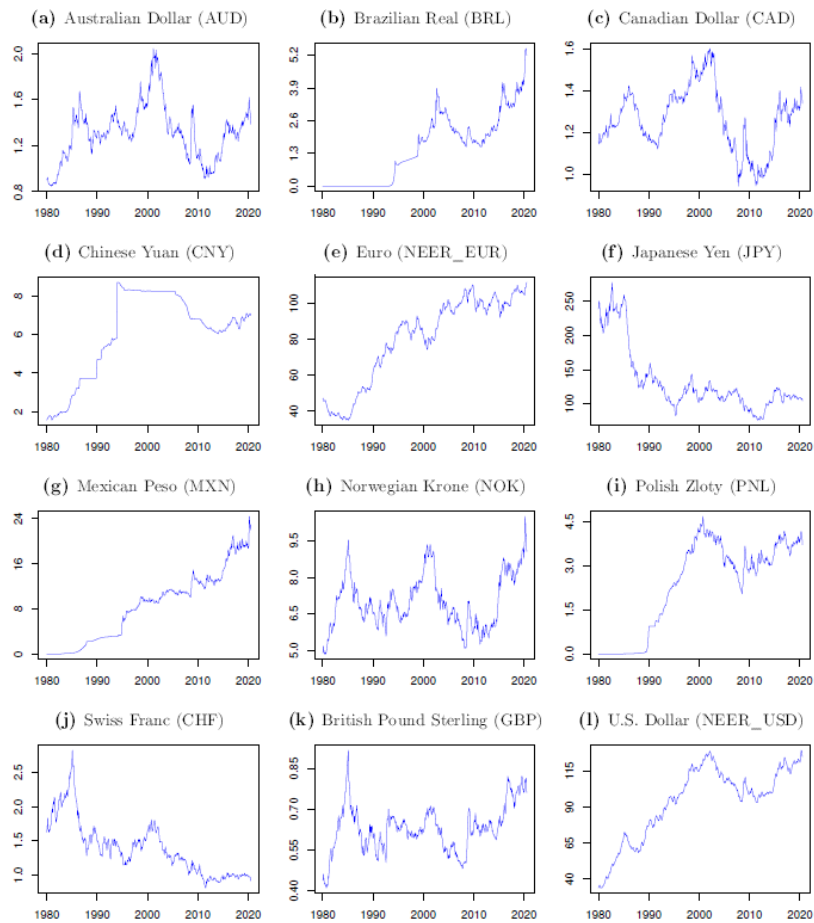
To study the synchronization between various currencies belonging to different monetary areas and different geographical spaces, we chose to develop an analysis based on two methodologies: the Hodrick-Prescott filter (Hodrick & Prescott, 1997) and the wavelet methodology (see, Grossmann and Morlet (1984) and Goupillaud, Grossmann, and Morlet (1984)). We begin by studying the joint movements of the cyclical components of exchange rates to ascertain whether they are positively or negatively correlated and whether the correlation obtained is strong or weak and statistically significant. This analysis is carried out using the Hodrick-Prescott filter. Complementing this investigation, we then proceed to analyze the exchange rate synchronization for the twelve currencies considered in this study, using the wavelet methodology for this purpose.

In the following sections, we describe the main steps developed under each of these methodologies and analyze the main empirical results. Before that, we present the data used in this study.

3.1 Data

The data used in this study consist of time series of the monthly nominal bilateral exchange rates, end of period, of the Australian dollar (AUD), Brazilian real (BRL), Canadian dollar (CAD), Chinese yuan (CNY), Japanese yen (JPY), Mexican peso (MXN), Norwegian krone (NOK), Polish zloty (PLN), Swiss franc (CHF) and British pound sterling (GBP) against the United States dollar (U.S. dollar), for the period between January 1980 and July 2020². The nominal effective exchange rates of the euro (NEER_EUR) and the U.S. dollar (NEER_USD), trade partners by consumer price index, were also considered. In the euro case, we use an effective or multilateral exchange rate due to data availability. Considering just the euro's nominal bilateral exchange rate against the U.S. dollar, we would only have data starting from January 1999, but also because it is a common currency adopted by several countries that integrate the same monetary area, the Euro Zone. In the U.S. dollar case, we used the effective nominal exchange rate as a reference currency for the ten nominal bilateral exchange rates considered in our study. The behavior of the twelve exchange rates included in this study is presented in Figure no. 1. All our empirical results and figures were obtained using Gretl2019a and R software.

The choice of currencies was thus essentially determined by three reasons. First, the International Monetary Fund (IMF) considered the five international reserve currencies as part of the Special Drawing Rights (SDR) basket included in the analysis³. Second, some of the most liquid currencies defined by the trading volume and the size of the economy in the BIS (2019) report were also selected. Thirdly, we sought to integrate into the analysis currencies representative of various monetary areas and geographically distant spaces so that it was possible to study their co-movements and synchronization in addition to these determining factors.



Notes: Australian dollar (AUD), Brazilian real (BRL), Canadian dollar (CAD), Chinese yuan (CNY), Japanese yen (JPY), Mexican peso (MXN), Norwegian krone (NOK), Polish zloty (PNL), Swiss franc (CHF) and British pound sterling (GBP) are the monthly nominal bilateral exchange rates, end of period, against the U.S. dollar (USD). U.S. dollar (NEER_USD) and Euro (NEER_EUR) are the nominal effective exchange rates, trade partners by consumer price index.

Source: authors, using the research database

Figure no. 1 – Nominal Exchange Rates

Except for the Chinese yuan, where the series of exchange rate values was taken from the Federal Reserve Economic Data of Federal Reserve Bank of St. Louis, the remaining data on nominal bilateral exchange rates and nominal effective exchange rates were obtained from IMF's International Financial Statistics, Metadata by Country. Both databases were consulted online during October and November 2020. It should be noted that the series of values for the CNY/USD exchange rate was only available from January 1981, so, for simplicity, it was assumed that the values to be considered during the twelve months of 1980 would be the same as those observed during the same months of 1981.

The choice of a period of analysis of about 40 years and after 1973 had the general objective of capturing the various behavioral dynamics of the referred currencies in different exchange rate regimes, since after the collapse of the Bretton Woods international monetary system in March 1973, there were several options followed by countries in terms of exchange rate regime choice, which is why the post-Bretton Woods period is often referred to in the economic literature as "nonsystem" (see, e.g., Williamson (1976) and Truman (2012)). More particularly, the reason the sample period starts on January 1980 and ends on July 2020 is to give similar numbers of observations in the pre-and the post-euro period (specifically, 228 and 259 monthly observations, respectively). In addition, as mentioned earlier, in order to make it possible to have a uniform analysis period for all twelve currencies, it was also for this reason that instead of using the nominal bilateral exchange rate of the euro against the U.S. dollar (after January 1999) or in relation to the Deutsche mark (until December 1998), we decided to use only a nominal effective exchange rate, i.e., the NEER_EUR.

Based on the observation of this set of graphs, we then proceed to the synchronization analysis of the cyclical components of exchange rates within the scope of the Hodrick-Prescott filter and the wavelet methodology.

3.2 Evidence from the Hodrick-Prescott filter

The Hodrick-Prescott filter decomposes a time series (in our case, the series of values for the exchange rate) into two other time series, one formed by the series of values of its trend, the so-called trend component of the variable, the other corresponding to the cyclical or random part of the original series of values, the so-called cyclical component of the variable:

$$y_t = \tau_t + C_t \quad (1)$$

where y_t is the series' original values and τ_t and C_t are, respectively, the trend component and the cyclical component extracted through the Hodrick-Prescott filter. Thus, the Hodrick-Prescott filter allows determining the cyclical component as the difference between the series' original values and its trend component. For this purpose, the extraction of the trend component is performed by minimizing the equation (2):

$$\min \left\{ \sum_{t=1}^T (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 \right\} \quad (2)$$

where $t = 1, 2, \dots, T$. The first term of equation (2) is the sum of the square of deviations between the values of the original series and the respective values of the trend series, thus representing a measure of the degree of adjustment. The second term of the equation (2) is the sum of the square of the second difference between the trend components, indicating their

degree of "smoothing" or "penalty". The smoothing parameter, λ , controls the variations in the trend component's growth rate and should therefore assume positive values, since $\lambda = 0$, the series corresponding to the trend would be equal to the series of original values. In turn, the greater the value of the smoothing parameter, the more "smoothed" will be the trend component extracted by the Hodrick-Prescott filter. At the limit, for values of λ close to infinity, the solution of the problem (3.2) will correspond to the least squares fit of a linear time trend model of the type $y_t = \alpha + \delta t + \epsilon_t$, where α is a drift, δt is the trend component and ϵ_t is a residual, i.e., the trend will approach from a straight line.

In these circumstances, the critical element associated with the use of the Hodrick-Prescott filter will be choosing the value to adopt for the smoothing parameter λ . [Hodrick and Prescott \(1997, p. 6\)](#) draw attention to the fact that any filter can change the serial correlation properties of the data, which should be interpreted with caution. The suggested values for the smoothing parameter λ for annual, quarterly and monthly data are, respectively, 400, 1600 e 6400. [Canova \(1998, p. 485\)](#) states that the value of λ is debatable, having investigated the issue for quarterly data with $\lambda = 1600$. In our study, we have used $\lambda = 14400$ as the value for the smoothing parameter, suggested by Gretl software, as appropriate for the work on monthly data⁴.

However, obtaining the cyclical components of exchange rates through the Hodrick-Prescott filter implies that the stationarity characteristics of the original series of exchange rates are previously analyzed and that the so-called "end-points problem" (initial and final) associated with the use of this filter must be corrected to avoid bias in the analysis⁵. For this purpose, the study of the stationarity characteristics of the exchange rates will be carried out at the first stage using the traditional unit root Augmented Dickey-Fuller (ADF) test ([Dickey & Fuller, 1979](#)), as well as the stationarity test of Kwiatkowski-Phillips-Schmidt-Shin, usually known as the KPSS test ([Kwiatkowski, Phillips, Schmidt, & Shin, 1992](#)). In a second step, in order to eliminate the end-points problem, the most appropriate ARIMA (autoregressive integrated moving average) forecasting model will be selected. Finally, after having already obtained the cyclical components of exchange rates through the Hodrick-Prescott filter application, we analyzed the correlation coefficients between the various cyclical components of exchange rates to identify behavioral patterns and joint movements.

3.2.1 Stationarity characteristics of exchange rates

As previously mentioned, the study of the stationarity characteristics of the exchange rate series was conducted by applying the ADF and KPSS tests. For the remainder of the paper, we work with the exchange rates in logarithms. Moreover, since we use monthly data, seasonal adjustments were included in the analysis. We understand that seasonal factors can influence the foreign exchange market, say those related to tourism. Naturally, the currency of a country may be more (less) demanded during periods of greater (less) tourist activity and, therefore, to observe a greater appreciation (depreciation) of their exchange rate. Hence, it is necessary to adjust our analysis to these types of circumstances. [Table no. 1](#) presents the results of standard ADF and KPSS tests on log-values and first differences of log-values (logarithmic rates of change of the exchange rates, i.e., the nominal exchange rate appreciation or depreciation of the currencies).

Table no. 1 – Unit root and stationarity tests

	ADF				KPSS			
	Level		First Difference		Level		First Difference	
	T	C	C	NC	T	C	T	C
AUD	-2.69	-2.72*	-6.76***	-6.74***	0.9***	0.91***	0.07	0.11
BRL	-0.772	-2.89**	-3.69***	-2.14**	1.94***	6.31***	0.42***	2.93***
Δ_BRL	—	—	-9.19***	-9.28***	—	—	0.01	0.02
CAD	-1.85	-1.81	-5.89***	-5.92***	0.65***	1.07***	0.09	0.09
CNY	-1.61	-3.25**	-19.24***	-19.18***	1.92***	4.75***	0.12	1.15***
NEER_EUR	-1.58	-1.4	-5.89***	-10.55***	1.52***	6.81***	0.12*	0.16
JPY	-2.14	-1.79	-5.38***	-5.45***	1.09***	5.07***	0.04	0.16
MXN	-3.29*	-3.96***	-3.46***	-3.04***	1.61***	6.06***	0.25***	1.92***
Δ_MXN	—	—	-16.12***	-16.34***	—	—	0.01	0.01
NOK	-2.2	-2.14	-5.67***	-5.6***	0.4***	0.52**	0.1	0.1
PNL	-1.44	-2.58*	-3.78***	-3.51***	1.82***	5.75***	0.14**	1.05***
Δ_PNL	—	—	-9.18***	-9.18***	—	—	0.01	0.01
CHF	-2.88	-1.26	-6.36***	-6.27***	0.29***	6.29***	0.05	0.06
GBP	-3.13*	-3.15**	-6.55***	-6.6***	0.42***	1.07***	0.07	0.07
NEER_USD	-2.99	-3.44***	-4.03***	-3.79***	1.61***	5.59***	0.15**	0.78***
Δ_NEER_USD	—	—	-9.09***	-9.30***	—	—	0.01	0.01

Notes: The number of lags included in the test regressions was chosen according to the AIC criterion. “T” identifies tests ran with a constant and a trend. “C” identifies tests ran with only a constant. “NC” identifies tests ran without a deterministic term. “Δ” identifies the first difference of the series. The null hypothesis of the ADF test is the existence of a unit root, while for KPSS under the null, the series is (trend-) stationarity. Significance at the 1%, 5% and 10% levels is denoted by “***”, “**” and “*”, respectively.

Source: authors, using the research database.

As can be observed, none of the twelve exchange rates analyzed is stationary in the period under analysis. Most (eight) of the exchange rates are presented as a variable I(1), thus needing only a differentiation to become stationary. The remaining four exchange rates are I(2), therefore needing a second differentiation to become I(0). As variables I(1), we find the logarithmic of the exchange rates of the following currencies: AUD, CAD, CNY, NEER_EUR, JPY, NOK, CHF and GBP. In turn, the four currencies whose behavior shows I(2) are BRL, MXN, PNL and NEER_USD. The analysis of the descriptive statistics of the logarithmic rates of change of the exchange rates presented in Table no. 2 may help us to better understand these dynamics.

Through the analysis of Table no. 2 it is possible to verify that the currency whose exchange rate presented a greater average variation in the period under analysis was the BRL, with an average depreciation against the U.S. dollar of 5.45%, followed by the MXN, with a nominal average depreciation of 1.41%, and PNL, with an average depreciation against the U.S. dollar of 1.38%. On the opposite side, we find the JPY and CHF, with a nominal average appreciation against the U.S. dollar, respectively, of 0.16% and 0.11%.

It should also be noted that the maximum depreciation against the U.S. dollar was observed in the MXN, with a depreciation of 65.7%, followed by the BRL, PNL and CNY, with a depreciation of 54.6%, 53.6% and 40.4%, respectively. Interestingly, the largest nominal appreciation against the U.S. dollar was also recorded in two of the previous currencies, namely BRL and MXN, with an appreciation of -18.1% and -16.4%, respectively. This may explain the reason why both currencies were I(2). The CHF and GBP appreciation of -12.8% and -13.1%, respectively, are also interesting.

Table no. 2 – Descriptive statistics of the logarithmic rates of change of the exchange rates

	AUD	BRL	CAD	CNY	NEER_EUR	JPY
Mean (%)	0.0881	5.4547	0.0301	0.3101	0.1779	-0.1699
Median (%)	0.0189	1.9472	0.0038	0.0000	0.0969	0.0335
Minimum (%)	-8.8525	-18.1624	-8.8735	-11.476	-4.2108	-15.0092
Maximum (%)	17.9823	54.6861	12.3841	40.437	5.4636	11.3918
Std. Dev. (%)	3.1964	10.2472	2.0766	2.3570	1.5906	3.0740
C.V.	36.2735	1.8785	68.7930	7.6010	8.9400	18.0877
Skewness	0.7494	1.5884	0.43907	10.878	0.24959	-0.3483
Excess Kurtosis	2.9459	2.8930	4.0624	176.23	0.2573	1.3548
	MXN	NOK	PNL	CHF	GBP	NEER_USD
Mean (%)	1.4136	0.1269	1.3897	-0.1198	0.1123	0.2676
Median (%)	0.5946	0.0144	0.4103	0.0376	0.0499	0.1957
Minimum (%)	-16.4224	-8.0569	-9.9499	-12.8702	-13.1350	-4.4229
Maximum (%)	65.7313	13.7597	53.6801	10.9297	12.7690	6.4646
Std. Dev. (%)	5.4413	3.1328	6.3189	3.2642	2.9004	1.4671
C.V.	3.8490	24.6688	4.5469	27.2421	25.8114	5.4821
Skewness	6.2992	0.2698	4.4509	-0.1071	0.1476	0.2128
Excess Kurtosis	61.6748	1.0963	28.3652	0.8506	2.2331	0.6713

Notes: “Std. Dev.” is the standard deviation. “C.V.” is the coefficient of variation. “AUD”; “BRL”; “CAD”; “CNY”; “EUR”; “JPY”; “MXN”; “NOK”; “PNL”; “CHF” and “GBP” are, respectively, the nominal bilateral exchange rate per U.S. dollar (end of the period) of the Australian dollar, Brazilian real, Canadian dollar, Chinese yuan, Japanese yen, Mexican peso, Norwegian krone, Polish zloty, Swiss franc and British pound sterling. “NEER_EUR” and “NEER_USD” are, respectively, the nominal effective exchange rate of the euro and U.S. dollar, trade partners by consumer price index.

Source: authors, using the research database.

Table no. 2 also shows that the nominal bilateral exchange rates of BRL, PNL and MXN against the U.S. dollar showed high volatility during the period under study, taking into account the values observed in the standard deviation of these variables, respectively, 10.2%, 6.3% and 5.4%. This fact also helps us understand why these variables presented I(2) when analyzing their stationarity characteristics. Interestingly, NEER_USD also presented I(2). However, its standard deviation shows a relatively low value, of approximately 1.4%, as well as its mean, maximum and minimum variation values. Hence, the result obtained in terms of stationarity analysis seems to be explained by the fact that a multilateral exchange rate involving one of the international monetary system’s main reference currencies is at stake. Its behavior may also have been directly influenced by a whole set of shocks that hit the international monetary and financial system over the period of analysis, disturbing, consequently, in a permanent way its evolutionary dynamic.

Finally, it is important to mention the relatively higher skewness for CNY, MXN and PNL, which indicates that extreme changes tend to occur more frequently for these currencies. The high values of excess kurtosis for these currencies also indicate the existence of fat tails in their return distribution.

3.2.2 The end-points problem: ARIMA model selection

After analyzing the stationarity characteristics of the exchange rates, the next step is to eliminate the end-points problem associated with the use of the Hodrick-Prescott filter. This

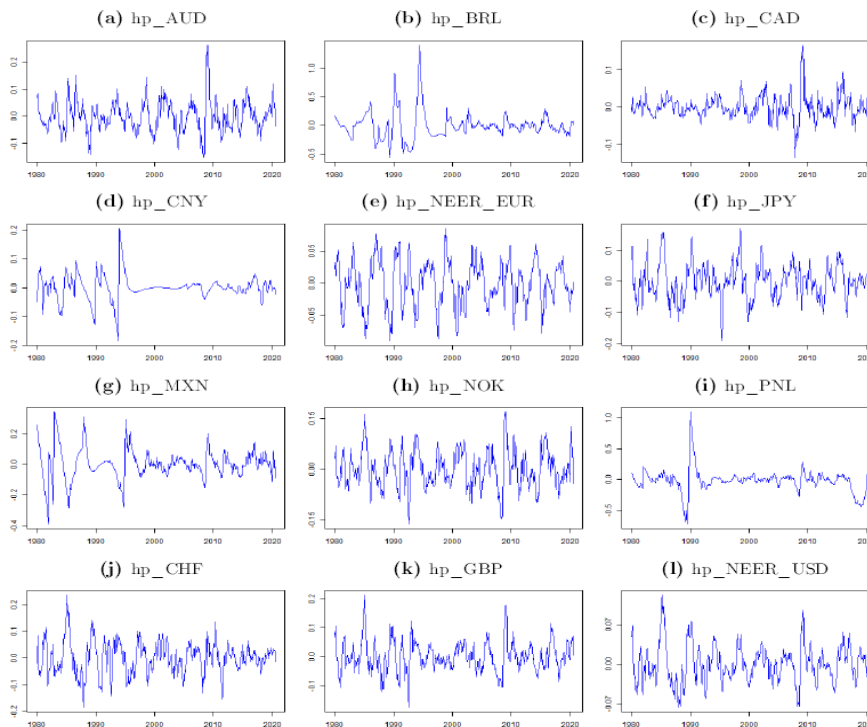
filter tends to underestimate the cyclical component of the variables, so it is necessary to correct this problem by adding observations to the original series, using forecast models for this purpose, as is the case with the ARIMA model. For simplicity, we will consider only the final-point problem, ignoring the initial-point problem consequently.

Since we are working with monthly data, for each series of exchange rates, 36 additional observations were estimated, not without first having proceeded to the selection of the most appropriate ARIMA model. For this purpose, it was considered the minimum value of the Schwarz information criterion, also known as the BIC criterion⁶. The results of this analysis are summarized in [Table no. 3](#).

Table no. 3 – ARIMA Model Selection

Variable	ARIMA Model Selection (AR, d, MA) Schwarz information criterion (BIC)							
	AUD	CAD	CNY	NEER_EUR	JPY	NOK	CHF	GBP
I(1)	(0,1,0)	(0,1,0)	(0,1,0)	(0,1,0)	(0,1,0)	(0,1,0)	(0,1,0)	(0,1,0)
I(2)	BRL	MXN	PNL	NEER_USD				
	(0,2,0)	(0,2,2)	(0,2,0)	(0,2,0)				

Source: authors, using the research database



Notes: Cyclical components of the exchange rates obtained using the Hodrick-Prescott filter.

Source: authors, using the research database.

Figure no. 2 – Cyclical components of the exchange rates

As can be seen, the analysis of the minimum value of the Schwarz information criterion pointed to the choice of an ARIMA forecasting model (0,1,0) for all exchange rates I(1). In the case of exchange rates I(2), two types of ARIMA models were selected, an ARIMA model (0,2,0) for BRL, PNL and NEER_USD, and a model (0,2,2) for the MXN exchange rate. After choosing the most appropriate ARIMA forecasting model, the cyclical components of exchange rates were determined by applying the Hodrick-Prescott filter methodology in order to carry out a first analysis of the joint movements in these variables and subsequent identification of synchronizations in its evolutionary behavior, this last analysis developed in the framework of the wavelet methodology. [Figure no. 2](#) shows the cyclical components of exchange rates for the twelve currencies considered in this study.

Although in an exploratory way, the analysis of this set of figures allows us to have an idea of the existence or not of joint movements between the cyclical components of exchange rates and, therefore, a possible synchronization between the different currencies involved. This analysis can, however, be complemented with a more particular examination of the values of the correlation coefficients of the cyclical components of exchange rates, a task developed in the following section.

3.2.3 The cyclical component of exchange rates: a correlation analysis

A first approach to the study of the exchange rate synchronization included in our study was carried out by calculating the correlation coefficients between the cyclical components of the exchange rates, shown in [Table no. 4](#).

Table no. 4 – Correlation coefficients between the cyclical components of exchange rates

	AUD	BRL	CAD	CNY	NEER_EUR	JPY
AUD	—	0.1061**	0.7091***	0.0406	-0.1244***	0.2745***
BRL		—	0.2895***	0.4622***	0.0775*	0.0377
CAD			—	0.1201***	-0.0672	0.1439***
CNY				—	0.1649***	0.0486
NEER_EUR					—	-0.2229***
MXN	0.1501***	-0.1856***	0.1045**	-0.1016**	0.3562***	-0.2775***
NOK	0.5619***	0.1539***	0.5159***	0.0359	-0.6119***	0.3199***
PNL	0.2303***	0.4630***	0.2297***	0.2994***	0.1123**	0.2151***
CHF	0.2670***	0.0518	0.2178***	-0.0721	-0.7612***	0.4720***
GBP	0.4664***	0.0703	0.3965***	0.0250	-0.4131***	0.2922***
NEER_USD	0.5445***	0.3649***	0.5229***	0.1678***	-0.4497***	0.5943***
	MXN	NOK	PNL	CHF	GBP	NEER_USD
MXN	—	-0.1006**	-0.0015	-0.3687***	-0.0364	-0.1399***
NOK		—	0.1785***	0.7495***	0.7257***	0.7113***
PNL			—	0.0123	0.1760***	0.3124***
CHF				—	0.6103***	0.6594***
GBP					—	0.6777***

Notes: As usual, “*”, “**” and “***” are the 10%, 5% and 1% significance levels of the correlation coefficients, respectively.

Source: authors, using the research database.

The analysis of the previous table allows us to draw the following conclusions:

i) In general, we find the highest and statistically significant correlation coefficients when the cyclical component of the U.S. dollar (NEER_USD) is involved in the exchange

rate pair under analysis. This may mean that the U.S. dollar has assumed an essential role as a foreign exchange anchor over the period under review, a fact that will not be alien to its status as a privileged international reserve currency. In these circumstances, as one would expect, it stands out with a relatively strong positive correlation with the Australian dollar and the Canadian dollar, with correlation coefficients of 0.5445 and 0.5229, respectively, for a 1% statistical significance level. Somewhat more surprising is the strong positive correlation observed between the cyclical components of NEER_USD and the cyclical components of JPY, NOK, CHF and GBP. This may indicate that these exchange rate pairs have been relatively synchronized, a compelling aspect if we consider that two of the currencies involved (JPY and GBP) also have a relevant role as international reserve currencies. The opposite relationship, however, seems to be evident between NEER_USD and NEER_EUR, where there is a negative correlation of -0.4497 for a 1% significance level, only seconded by the Mexican currency, which contrary to what would be expected, shows a negative correlation with the U.S. dollar, although relatively weak (-0.1399), but statistically significant. The relationship between NEER_USD and NEER_EUR suggests that if there is synchronization between the American currency and the European single currency, it will be in the opposite direction, which can be explained by the fact that the two currencies and the respective monetary areas are more and more competitors in the foreign exchange market, but also due to the existence of possible lags in the monetary transmission mechanisms between the two areas. It should also be noted that in this case, nominal effective exchange rates are involved, i.e., relations between a given currency and a basket of currencies representative of its main trading partners, with the two baskets of currencies associated with the U.S. dollar and the euro not necessarily equal.

ii) The cyclical component of the nominal effective exchange rate of the euro (NEER_EUR) has a negative correlation with the vast majority of the cyclical components of the currencies considered in the analysis, including with the other European currencies, with particular emphasis on the correlations with the GBP, NOK and CHF, with correlation coefficients, respectively, of -0.4131, -0.6119 and -0.7612, for a 1% significance level. This is a meaningful result, and at the same time worrying, for the process of European construction, for the affirmation of the euro as an international reserve currency, if we add the United Kingdom's recent exit from the European Union, but also for possible future enlargements of the Eurozone to the East, a fact that is not unrelated to the relatively weak correlation (0.1123) observed between NEER_EUR and PNL, although positive and statistically significant at 5%. In this way, we can anticipate a possible lack of synchronization or a counter-cyclical relationship between NEER_EUR and the other European currencies considered in our study. Taking into account the correlation coefficients between the cyclical components of exchange rates shown in [Table no. 4](#), it appears that the higher positive correlations observed for NEER_EUR occur with currencies that integrate other monetary areas and distant geographical spaces, specifically concerning the Chinese yuan and the Mexican peso, with correlation coefficients of 0.1649 and 0.3562, respectively. This result can be explained by China's growing presence in many European countries' economic activity, associated with strong capital outflows, and the increasing preponderance of CNY in the foreign exchange market. On October 1, 2016, the International Monetary Fund had considered the Chinese currency as part of the Special Drawing Rights (SDR) basket [?]. On the other hand, the positive correlation observed between NEER_EUR and the MXN is entirely unexpected. This co-movement may be due to the relatively increasing importance of

the European single currency in the context of the recent functioning of the international monetary and financial system.

iii) Over the period under review, the British currency seems to have departed significantly from the European single currency, taking into account the statistically significant negative correlation (-0.4131) observed between the two currencies' cyclical components. Following what we mentioned in ii), this result can be explained if we consider the historically challenging and very peculiar relationship that has always characterized the United Kingdom's participation in the European Union (Bento & Duarte, 2020) and which recently culminated in the so-called BREXIT. Therefore, it is to be expected that in the framework of the wavelet analysis, we can confirm this progressive departure between the two currencies through an absence of synchronization or the observation of counter-cyclical behavior. On the contrary, GBP has a strong positive correlation with two other European currencies, NOK and CHF, with correlation coefficients of 0.7257 and 0.6103, respectively, showing a weaker positive correlation with PNL (correlation coefficient of only 0.1760). In any case, the relationships are statistically significant, indicating synchronization between the respective cyclical components. Similarly, it is also noteworthy to highlight the positive correlation observed between GBP and a set of three currencies belonging to geographically distant areas, namely AUD, CAD and JPY, with correlation coefficients, respectively, of 0.4664, 0.3965 and 0.2922. The correlations (co-movements) obtained between GBP and those currencies are similar to those observed for NEER_USD. However, of lesser value, which can be explained by the still significant weight held by the British currency as a reference currency in the International Monetary System, despite the loss of relative importance for the U.S. dollar, namely after the conclusion of the Bretton Woods Agreement in 1944. Finally, it is important to mention the lack of statistical significance in the relationship between the cyclical components of GBP, BRL, CNY and MXN. This result may be due to the weak commercial links between the monetary areas and geographical spaces concerned, at least in the most of the period under analysis.

iv) The Chinese currency has nearly identical co-movements with the euro (correlation coefficient of 0.1649) and the U.S. dollar (correlation coefficient of 0.1678), with a 1% significance level in both cases. This may mean that the Chinese monetary authorities have not immensely privileged either of these two monetary areas, preferring a moderate synchronization with the two currencies. Interestingly, although belonging to the same geographical area as the Japanese yen, the Chinese yuan shows a weak correlation, although not statistically significant, suggesting that neither of the two currencies has monetary hegemony in Asia and may soon compete with each other as the reference currency. It should also be noted the presence of a moderate positive correlation between the cyclical component of the Chinese currency and the cyclical component of the Brazilian currency, with a correlation coefficient of 0.4622. This value can be explained by the growing narrowing trade relations between the two countries, due, in particular, to a greater presence of Chinese business activity in Latin America. In the opposite direction, the CNY shows a negative correlation with the MXN at a 5% significant level. Finally, it is important to mention the existence of a moderate positive correlation (0.2994) between the cyclical component of CNY and the cyclical component of PNL, a result that could also mean an increasing presence of China in Eastern Europe, in this case, in Poland, with the natural strengthening of trade relations between the two countries and, consequently, with an expected behavior in terms of the evolution of their currencies.

v) European countries with their own currency have their exchange rates relatively more correlated with each other and in the same direction than in relation to the euro. In particular, there is a strong co-movement, statistically significant at 1%, between NOK and CHF, with a correlation coefficient of 0.7495, while these two currencies have a resilient negative correlation vis-à-vis the European single currency. This will certainly mean that the two countries in question favor the maintenance of their own currency and, consequently, autonomy in the conduct of their foreign exchange policy, rather than sharing a large monetary area such as the Eurozone. Therefore, it is expected that there will be a high synchronization of the Norwegian krone and the Swiss franc against currencies belonging to other monetary areas, rather than in relation to the euro. Following the same line of thought, this is the only way to explain the strong positive correlation observed in NOK and CHF in relation to GBP and NEER_USD, with correlation coefficients, respectively, of 0.7257 and 0.7113, in the case of the Norwegian krone, and of 0.6103 and 0.6594, in the case of the Swiss franc. Also noteworthy is the presence of a relatively strong co-movement between NOK and AUD, with a correlation coefficient of 0.5619 and between NOK and CAD, with a correlation coefficient of 0.5159, statistically significant at 1% in both cases. Also, very interesting is the finding of a relatively strong positive correlation (0.4720) between the Swiss franc and the Japanese yen, a fact that can be mainly explained by substantial capital flows in the banking and financial sector between the two countries, particularly in the sequence of the recent global financial crisis, more than by strengthening commercial relations.

vi) Although geographic proximity between countries or regions can be considered an important determinant in monetary integration processes or cycle synchronization between two currencies, the analysis of Table no. 4 allows us to note some exceptions. It is the case, for example, of the relationship between AUD and CAD, which, although belonging to very distant geographical areas, show the presence of strong co-movements between the cyclical components of the respective exchange rates, with a correlation coefficient of 0.7091 at a 1% significance level. On the same line, although less significant, we also find the correlation between BRL and PNL, with a correlation coefficient of 0.4630, for a significance level of 5%. Conversely, it is important to mention the negative correlation of -0.1856 observed between MXN and BRL and the correlation coefficient of -0.1399 between MXN and NEER_USD, even though the countries in question are geographically close or even sharing borders.

3.3 Evidence from a wavelet analysis

The wavelets methodology is based on the Fourier spectral analysis and it is an alternative to the most used methodologies in the treatment of time-series. Being a much more flexible approach than the spectral analysis and the Fourier transforms, it allows for the estimation of a time-series' spectral characteristics as a function of time, thus revealing how its different periodic components, i.e., frequency bands, change over time. In a univariate framework, multiple wavelet analysis applications relied on discrete transformations for specific (discrete) choices of parameter values for time and frequency. However, in the last decades, a significant body of literature emerged using the Continuous Wavelet Transform applied to economic data, which also accommodates the analysis of time-frequency dependencies between two (or more) time-series. Some examples are found in [Aguiar-Conraria, Azevedo, and Soares \(2008\)](#); [Aguiar-Conraria, Martins, and Soares \(2013\)](#); [Aguiar-Conraria and Soares \(2011\)](#); [Aloui, Hkiri, and Nguyen \(2016\)](#); [Andries et al. \(2014\)](#); [Jammazi](#)

(2012); Raihan, Wen, and Zeng (2005); Rua (2012); Tiwari (2013); Tiwari and Albuлесcu (2016); Aguiar-Conraria, Martins, and Soares (2018); Chen, Chen, and Tseng (2017); Duarte and Silva (2020); Ojo, Aguiar-Conraria, and Soares (2020) to mention just a few. This section presents the most important definitions, properties, and wavelet tools used by these authors.

3.3.1 The continuous wavelet transform

Spectral analysis and Fourier transforms provide methods and tools to study a time-series' cyclical or seasonal nature in the frequency-domain. Such techniques permit quantifying the importance of various frequency components and infer information about the length of a cycle or phase. Despite its usefulness, under the Fourier transform, the time information of a time-series is very difficult to recover, being hard to distinguish transient relations or to identify structural changes. The continuous wavelet transform (CWT) overcomes this main limitation since it preserves the information about time and defines a window oscillatory function in the sense that the length of wavelets varies endogenously: it stretches into a long wavelet function to measure the low-frequency movements; and it compresses into a short wavelet function to measure the high-frequency movements (Aguiar-Conraria & Soares, 2011). In most applications, it is enough to require the square integrability of this wavelet function, ψ , called the mother wavelet, and that $\int_{-\infty}^{+\infty} \psi(t)dt = 0$. That is the mother wavelet wiggles over the t-axis, behaving like a small wave.

Given a time-series x_t , its CWT with respect to the wavelet ψ is a function of two variables, $W_x(\tau, s)$:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x_t \left[\frac{1}{\sqrt{|s|}} \bar{\psi} \left(\frac{t - \tau}{s} \right) \right] dt \quad (3)$$

where $\bar{\psi}$ is the complex conjugation of ψ , τ is a translation parameter and s is a scaling factor that controls the width of the wavelet. At each translation and scale, the signal is decomposed in sets of coefficients, each associated with a scale and each coefficient in the set associated with a location. The scale compresses (if $|s| < 1$) or stretches (if $|s| > 1$) the wavelet function and measures movements in high or low frequencies, respectively. A small scale is associated with an high frequency and allows to establish a local analysis while a large scale, associated with a low frequency, allows to establish a global analysis.

Since the coefficients extracted from equation (3) contain information about the original function, x_t , and $\psi(t)$, the choice of the wavelet is an important aspect to be taken into account. In the context of time-series analysis, and in particular, to study the co-movement between them, one should select a wavelet that is able to incorporate information about the amplitude and the phase. The natural choice involves a complex-valued wavelet. One of the most used is the Morlet wavelet, defined as

$$\psi_{\omega_0} = \pi^{-1/4} e^{-i\omega_0 t} e^{-t^2/2} \quad (4)$$

where ω_0 is the localization parameter in the frequency domain. The functions used in the Morlet wavelet are given essentially by sines and cosines like in the Fourier analysis. In this sense, these are not true wavelets since they fail to satisfy the square integrability condition⁷. However, the Morlet wavelet has optimal joint time-frequency localization, and, for the

particular choice $\omega_0 = 6$, we obtain a one-to-one (inverse) relationship between scale and frequency (see [Aguiar-Conraria and Soares \(2014\)](#) for more details on the Morlet wavelet)⁸.

In analogy with the terminology used in the Fourier analysis, we define the (local) wavelet power spectrum (WPS) as:

$$\psi_{\omega_0} = \pi^{-1/4} e^{-i\omega_0 t} e^{-t^2/2} \tag{5}$$

which gives a measure of the relative contribution, for each moment and scale, for the variance of the distribution of the time-series.

[Torrence and Gilbert \(1998\)](#), in their seminal paper, based on a large number of Monte Carlo simulations, discuss significance testing for wavelet (and cross-wavelet) power. The authors concluded that the wavelet (and cross-wavelet) power spectrum of a white noise or an AR(1) process is reasonably well approximated by a chi-squared distribution. Our significance tests were obtained by fitting an AR(1) model to each series (or each pair of series), constructing new samples by drawing errors from a Gaussian distribution with a variance equal to that of the estimated error terms⁹.

3.3.2 Wavelet Coherency and the Phase Difference

The concepts of wavelet transform and wavelet power spectrum can be generalized and enable us to deal with the time-frequency dependencies between two time-series. With this purpose, consider two time-series, x_t and y_t , and their respective wavelet transforms, $W_x(\tau, s)$ and $W_y(\tau, s)$. The cross-wavelet transform (XWT) between x_t and y_t is defined as

$$W_{xy}(\tau, s) = W_x(\tau, s) \overline{W_y(\tau, s)} \tag{6}$$

where $\overline{W_y(\tau, s)}$ is the complex conjugation of $W_y(\tau, s)$. The cross-wavelet power is the absolute value of XWT, $|W_{xy}(\tau, s)|$, and measures the local covariance between the two time-series in the time-scale/frequency plane. In addition, the wavelet coherency, given by,

$$R_{xy} = \frac{|S(W_x(\tau, s))|}{[S(|W_x(\tau, s)|^2)S(|W_y(\tau, s)|^2)]^{1/2}} \tag{7}$$

with $0 < R_{xy} < 1$, and where S denotes a smoothing parameter in both time and scale, gives us a measure of the correlation between x_t and y_t , and has the advantage of being normalized by the power spectrum of the two series ([Aguiar-Conraria et al., 2013](#)).

Since we are using a complex-valued wavelet, its corresponding wavelet transform is also complex-valued and can be written in polar form. We can then extract the phase of the wavelet transform of each series and thus obtain information about the oscillations of two series as a function of time and scale, by computing the phase difference, using the formula,

$$\phi_{xy} = \text{Arctan} \left(\frac{\Im\{SW_{xy}(\tau, s)\}}{\Re\{SW_{xy}(\tau, s)\}} \right) \tag{8}$$

where \Im and \Re correspond to the imaginary and real parts, respectively. For each frequency, a phase of zero indicates that both time-series move together; if $\phi_{xy} \in [0, \pi/2]$, the series move in phase and x_t leads y_t ; if $\phi_{xy} \in [-\pi/2, 0]$, the series also move in phase, but in this case y_t leads x_t ; if $\phi_{xy} \in [\pi/2, \pi]$, the series move out-of-phase with y_t leading; if $\phi_{xy} \in [-\pi, -\pi/2]$, we still have an anti-phase relation, but with x_t leading; finally, if $\phi_{xy} = 0$, both series move together.

3.3.3 Wavelet Spectra Distance

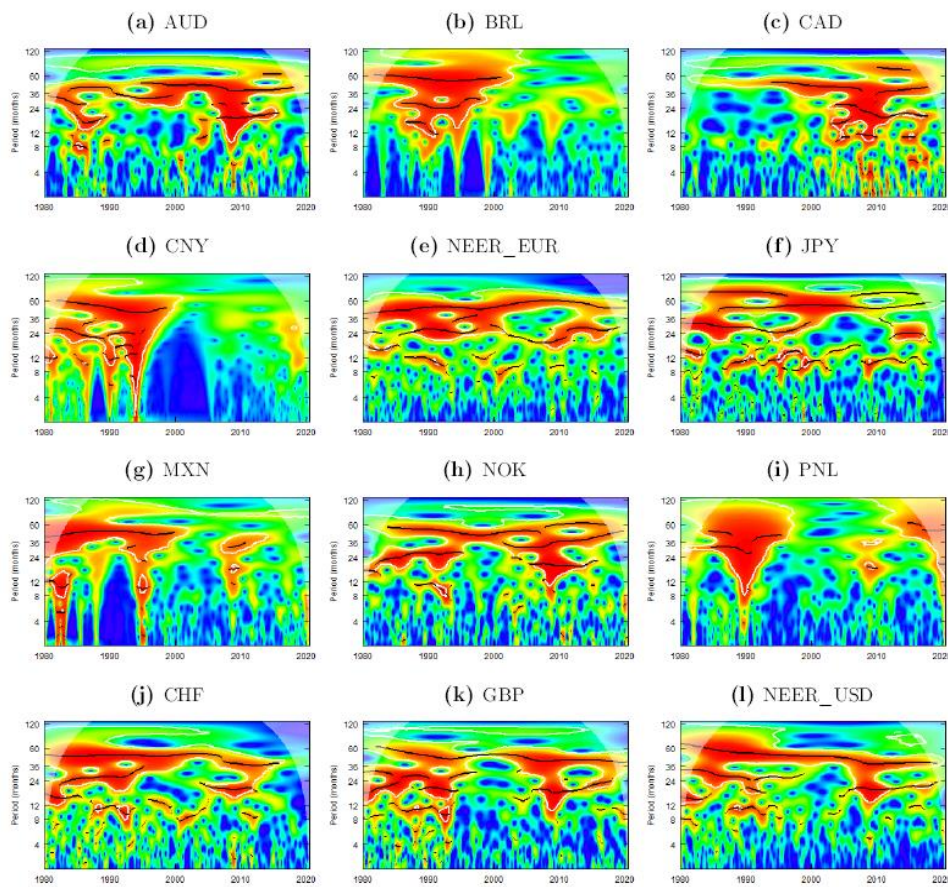
To study the synchronization between exchange rates, we will also use a metric first proposed by [Aguilar-Conraria and Soares \(2011\)](#). As argued by the authors, a direct comparison of two wavelet spectra can be clouded in low power regions. Using the Singular Value Decomposition (SDV) of the matrix $Q_{x,y} = W_x(\tau, s)W_y^H(\tau, s)$, where W_y^H is the conjugate transpose of W_y , with the first extracted components to the most important patterns between wavelet spectra, it is possible to construct the following leading patterns, l_x^k and l_y^k , and leading vectors, u_x^k and u_y^k . For K of these, we can approximately reconstruct the original spectral matrices. To compare two wavelet spectra between two exchange rates, we measure the distance:

$$\text{dist}(W_x, W_y) = \frac{\sum_{k=1}^K \sigma_k^2 [d(l_x^k, l_y^k) + d(u_x^k, u_y^k)]}{\sum_{k=1}^K \sigma_k^2} \quad (9)$$

where σ_k^2 are the singular values (of the SDV decomposition) and where the distance $d(\cdot)$ between two vectors is obtained by measuring the angle between each pair of corresponding segments and by taking the mean of these values. By computing [equation \(9\)](#) for each pair of currencies, we can then construct a distance matrix suitable for cluster analysis. A value close to zero indicates that two currencies have similar wavelet transforms, which implies high synchronization.

3.3.4 Results: the co-movement between exchange rates

[Figure no. 3](#) shows the wavelet power spectrum for each exchange rate time-series. We estimate the wavelet power spectra between 2 and 128 months frequencies. At first glance, the figure reveals a long cycle around the 60-months frequency, shared by most currencies. Regarding the Canadian dollar, we only capture that lower frequency spike after 1990 while the Brazilian real, the Chinese yuan, the Polish Zloty and the Mexican peso appear to be muted after 2000. In turn, when we look at higher frequencies (8~24-months), we find high power regions during the international financial crisis period for the AUD, CAD, NEER_EUR, JPY, NOK, CHF, GBP and NEER_USD. Again, this is not the case for the BRL, CNY and PNL. Instead, the 8~24-months frequency band is significant until mid 90s for CNY and late 90s for BRL, periods characterized by sharp fluctuations in the their cyclical components. We can also observe increased volatility at higher frequencies during the 80s for the CHF, spanning through the 90s for the JPY, NEER_EUR, NOK and GBP. In general, the results suggest a lower volatility period between the late 90s and the mid 2000s at all frequencies and across all currencies.



Notes: The white counter designates the 1% significance level against an AR(1) null. The cone of influence (COI), which indicates the region affected by edge effects, is delimited by the shaded area. The color code for power ranges from blue (low power) to red (high power). The black lines show the maxima of the undulations of the wavelet power spectrum.

Source: authors, using the research database

Figure no. 3 – Wavelet Power Spectra

The wavelet power spectra detect that exchange rates' fluctuations develop along a 60-months cycle for most of the sample with a few exceptions, as mentioned above. Regions of high energy were also detected for higher frequencies (around the 8~24-months frequency band) in the first half of the sample and during the financial crisis. As a first step in assessing the co-movement between the exchange rates present in our sample, we compute a measure of the distance between the wavelet transform of each series, based on [formula \(9\)](#). As argued by [Aguilar-Conraria et al. \(2013\)](#), smaller distances (close to zero) indicate that two wavelet transforms share their high power regions and that their phases are almost perfectly aligned. However, if there are episodes of statistically significant out-of-phase co-movements, which, as we will show, is the case for some currencies, distances may be much closer to zero than the

distance between two independent processes. This means that our results should be regarded with caution. Hence, the analysis of coherency and phase-difference complements this analysis.

Table no. 5 – Exchange rate dissimilarities matrix

	AUD	BRL	CAD	CNY	NEER_EUR	JPY	MXN	NOK	PNL	CHF	GBP	NEER_USD
AUD	—	0.65	0.55*	0.71	0.56*	0.58*	0.56**	0.54**	0.67	0.63	0.59	0.50***
BRL	0.64	—	0.75	0.63	0.60**	0.69	0.72	0.64	0.59	0.64	0.69	0.66
CAD	0.31***	0.57	—	0.86	0.67	0.73	0.68	0.59	0.72	0.69	0.62	0.57*
CNY	0.59	0.40***	0.53	—	0.76	0.69	0.63	0.73	0.69	0.82	0.75	0.71
NEER_EUR	0.53	0.58	0.50	0.62	—	0.52**	0.73	0.44***	0.69	0.39***	0.49**	0.45***
JPY	0.51*	0.62	0.48*	0.59	0.51	—	0.72	0.48***	0.76	0.41***	0.49**	0.37***
MXN	0.55	0.53	0.57	0.57	0.51	0.47*	—	0.76	0.66	0.72	0.72	0.72
NOK	0.44**	0.54	0.37***	0.46**	0.36***	0.49*	0.67	—	0.58**	0.36***	0.29***	0.39***
PNL	0.62	0.51	0.56	0.36**	0.65	0.69	0.66	0.56	—	0.61**	0.61**	0.63
CHF	0.51	0.63	0.49**	0.51	0.29***	0.44**	0.49*	0.32***	0.73	—	0.45***	0.39***
GBP	0.48*	0.67	0.47*	0.58	0.46***	0.49*	0.67	0.34***	0.55	0.38***	—	0.35***
NEER_USD	0.31***	0.48*	0.28***	0.43**	0.36***	0.43**	0.62	0.21***	0.55	0.30***	0.31***	—

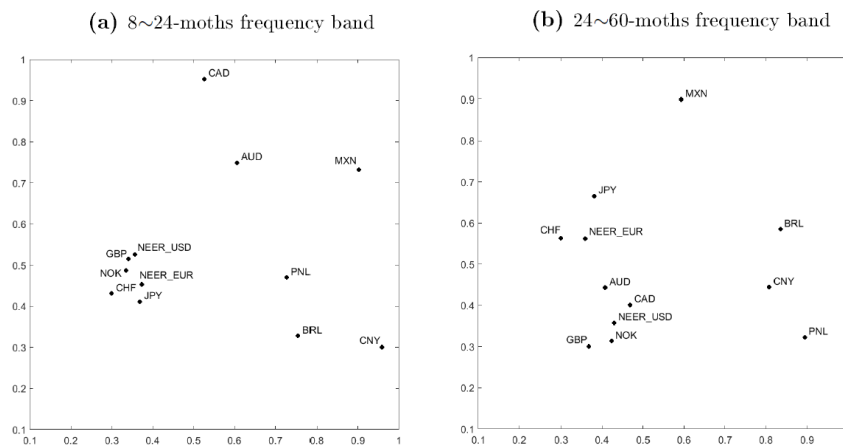
Notes: The upper triangle shows the exchange rate dissimilarities for the 8~24-months frequency band. The lower triangle shows the exchange rate dissimilarities for the 24~60-months frequency band. Significance at the 1%, 5% and 10% is denoted by "*", "**" and "***", respectively.

Source: authors, using the research database.

Table no. 5 presents the distance matrix computed for all currencies. Since we observed high power regions at different frequencies, we estimate, for each case, the wavelet spectra for a higher 8~24-months frequency band, shown in the lower triangle, and a lower 24~60-months frequency band, shown in the upper triangle. Looking at the lower frequency band, we observe that the most dissimilar pair is the Chinese yuan and the Australian dollar. By computing the average pairwise distances, we can also argue that the Chinese yuan is the most dissimilar currency, with an average distance of 0.72, and records no significant bilateral distances, followed by the Mexican peso, the Canadian dollar and the Brazilian real. A second conclusion is that many currencies, namely the AUD, CAD, NEER_EUR, JPY, NOK, CHF and GBP, share statistically significant similarities with the U.S. dollar and that on average, distances shrink, with the BRL and CNY joining the statistically significant club, at higher frequencies. The U.S. dollar is the closest currency to that of the other economies, with an average distance of 0.52 and 0.39 for the higher and lower frequency bands. Interestingly, the most dissimilar currency with the U.S. dollar at both frequency bands is the Mexican peso. Thirdly, the results suggest that the bilateral distances decrease at the 24~60-months frequency band. On average, we obtained a decrease of 0.11 for each currency vis-à-vis the remaining ones. Performing a t-test, we can reject the null that the means are equal between frequencies at 1%. These results are striking and suggest that in the absence of short-term foreign exchange market frictions, currencies are more synchronized.

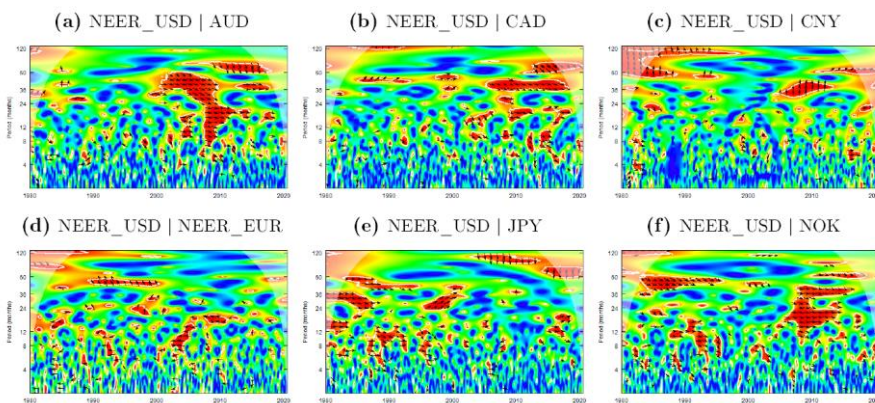
For a better insight on these findings, Figure no. 4 summarizes the distances in two multidimensional scaling maps, one for each frequency band. There is a clear cluster in higher frequencies on the left comprised of the NEER_EUR, NOK, CHF, GBP, JPY, arguably gravitating around the U.S. dollar. At lower frequencies, the Canadian dollar and the Australian dollar join this cluster. While currencies become closer to each other on average, there are a few pairwise exceptions. For example, the Japanese yen increases the bilateral distance relative to the British pound and the Swiss franc. It is surprising that the two

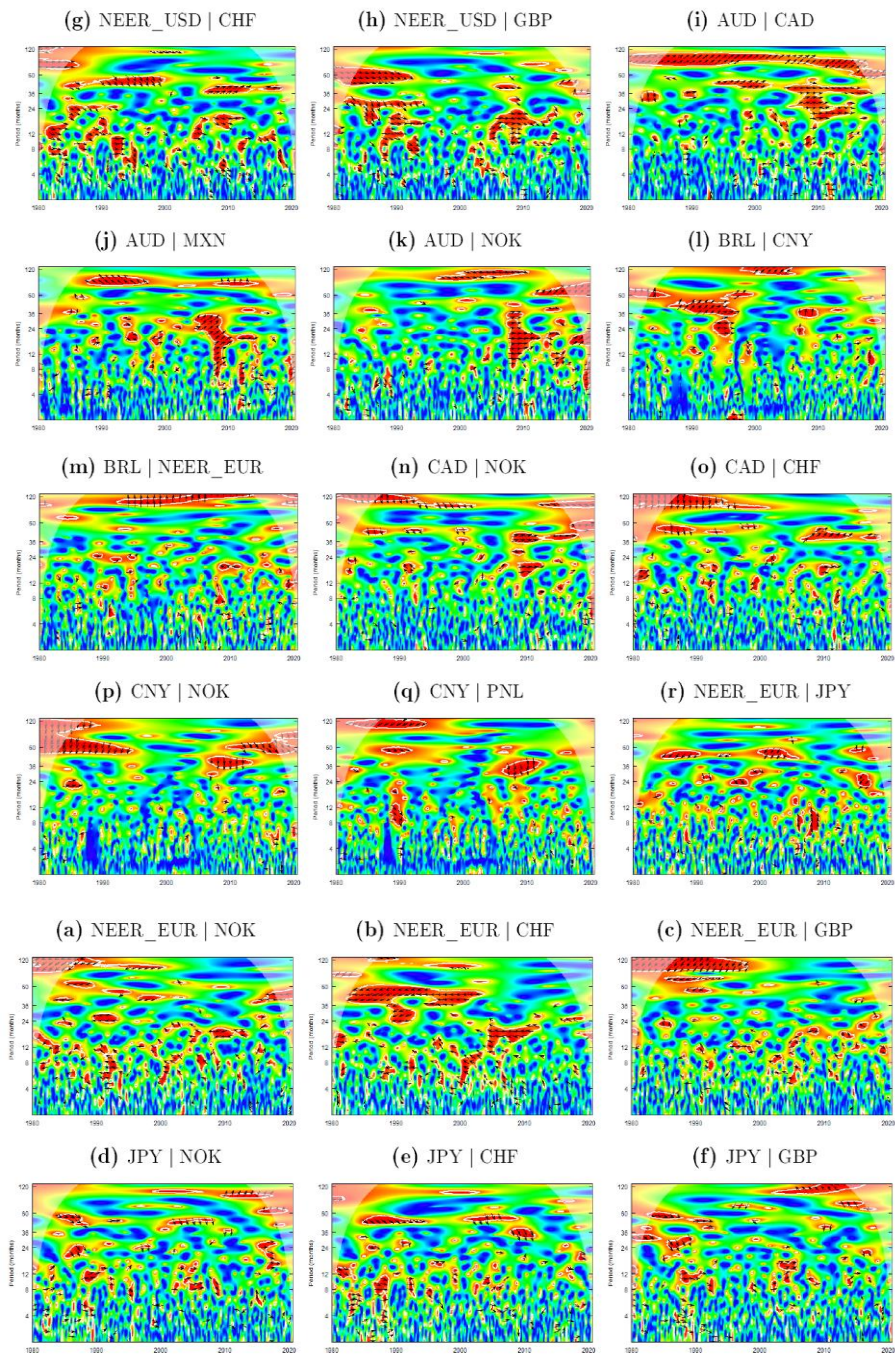
currencies that were pegged to the Deutsch mark/European currency unit between 1990 and 1992, the British pound and the Norwegian Krone, show lower dissimilarities with other currencies than the NEER_EUR. For the GBP case, the lowest bilateral distance is against the NEER_USD, at both frequencies. For the NOK case, the lowest bilateral distances were found respectively with the GBP and the NEER_USD at the lower and the higher frequency bands. We conclude the analysis of Table no. 4 and Figure no. 4 by highlighting the high dissimilarity between the Polish zloty and the European euro, reinforcing the results found in the correlation coefficients analysis.

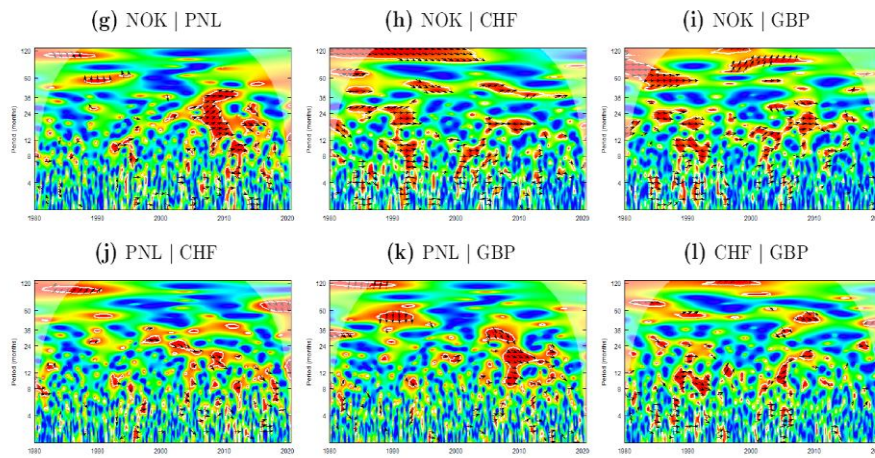


Source: authors, using the research database.
Figure no. 4 – Multidimensional Scaling Maps

We now turn to the analysis of the wavelet coherencies and phase-differences as a second step in the assessment of the co-movements between the exchange rates. The results obtained are shown in Figure no. 5.







Notes: The white counter designates the 1% significance level against an AR(1) null. The cone of influence (COI), which indicates the region affected by edge effects, is delimited by the shaded area. The color code for coherency ranges from blue (low coherency) to red (high coherency). The phase difference is shown by the black arrows. Arrows pointing to the right (\rightarrow) indicate the currencies are in-phase (and move together); (\nearrow) indicates that the currency on the left is leading; (\searrow) indicates that the currency on the right is leading. Arrows pointing to the left (\leftarrow) indicate that currencies are out-of-phase; (\nwarrow) indicates that the currency on the right is leading; (\swarrow) indicates that the currency on the left is leading.

Source: authors, using the research database.

Figure no. 5 – Cross-Wavelet Coherencies and Phase-Differences

Focusing on the U.S. dollar, we find significant regions of high coherency with most currencies and that the co-movements became contemporaneous with the Australian dollar after 2000 and with the Canadian dollar, the Norwegian krone, the Swiss Franc and the British pound somewhere around 2005. This evidence suggests that exchange rates became more synchronized during the international financial crisis. We also observe other examples, such as the AUD|CAD, AUD|MXN, AUD|NOK, CAD|NOK, NOK|PNL, NOK|GBP, NOK|CHF, PNL|GBP and CHF|GBP cases. Conversely, we barely find any in-phase periods between the U.S. dollar and the NEER_EUR or the CNY. A close inspection of the European euro shows significant high coherency areas with the BRL, JPY, NOK and GBP, but the series are mostly out-of-phase. We detect a brief period (between 2000 and 2005) in which the Swiss franc is leading. Also, until 2000, the euro was leading the Norwegian krone and the British pound at lower frequencies.

4. CONCLUSION

This paper combined the Hodrick-Prescott filter and wavelet tools to study the co-movement between exchange rates from different monetary areas using monthly data between January 1980 and July 2020. We obtain similar results between the two methodologies and reveal important caveats of the Hodrick-Prescott filter: exchange rate co-movements are highly driven by time and frequency-varying patterns. We focused on nominal bilateral exchange rates of the Australian dollar (AUD), Brazilian real (BRL), Canadian dollar (CAD), Chinese yuan (CNY), Japanese yen (JPY), Mexican peso (MXN), Norwegian krone (NOK), Polish zloty (PNL), Swiss franc (CHF) and British pound sterling (GBP) against the United

States dollar (U.S. dollar) and the nominal effective exchange rates of the euro (NEER_EUR) and the U.S. dollar (NEER_USD). Our novel approach allowed for a set of results and conclusions from which we highlight a few.

The U.S. dollar still plays an essential role as a foreign exchange anchor, showing a high degree of correlation and similarity with most currencies. In contrast, the euro shows an out-of-phase relationship with the vast majority of currencies, including the other European currencies, such as the Swiss Franc, the British pound and the Norwegian krone. These last currencies reveal low coherency with the euro after leaving the hard peg regimes in the early 90s. The Brazilian real leads the Chinese yuan for most of the sample, and both currencies record great dissimilarities with the other currencies.

From a cluster analysis point of view, the evidence suggests important differences across frequencies. In the absence of short-term foreign exchange market frictions, average bilateral distances between currencies are smaller, denoting higher synchronization in lower frequencies. Additionally, the phase-differences support that during the international financial crisis, exchange rates became more synchronized.

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Notes

¹ East Asia countries have mainly chosen to peg their currencies to the U.S. Dollar. East Asia, therefore, became an “Asia Dollar Standard” region. After the Asian Financial Crisis, further claims that East Asia has returned to the dollar standard. There is, however, no consensus on this view. After the Asian Financial Crisis, China has initiated a currency reform aiming for a much more flexible exchange rate regime by pegging the Chinese yuan to a basket of currencies (Feng, Hu and Wang, 2010).

² The exchange rate is defined as the number of the national currency (e.g., AUD) per U.S. dollar. Thus, an increase (decrease) in the exchange rate (AUD/USD) denotes a depreciation (appreciation) of the domestic currency (AUD).

³ The SDR is an international reserve asset, created by the IMF in 1969 as a supplementary international reserve asset in the context of the Bretton Woods fixed exchange rate system. The collapse of Bretton Woods’s international monetary system in 1973 and the shift of major currencies to floating exchange rate regimes lessened the SDR’s reliance as a global reserve asset. Nonetheless, SDR allocations continue to play an important role in providing liquidity and supplementing member countries’ official reserves, as was the case in the recent global financial crisis. The SDR value is based on a basket of five currencies: the U.S. dollar, the euro, the Chinese yuan, the Japanese yen, and the British pound sterling. For more details, see Duarte (2015).

⁴ Using the Hodrick-Prescott filter to analyze the exchange rate risk contribution to the performance of international investments in Romania, Horobet and Ilie (2009) also use $\lambda = 14400$ as the value for the smoothing parameter for monthly data. Maravall and del Río (2001) suggest similarly a smoothing parameter $\lambda = 14400$ for monthly data and a reference cycle of 5.7 years. For additional discussion see Ravn and Uhlig (2002).

⁵ The Hodrick-Prescott filter tends to underestimate the cyclical component of the variables in the first and last 36 monthly observations. We will return to this question in the next sections.

⁶ Similar to what was done in the analysis of stationarity, seasonal adjustments were also included in the selection of the ARIMA model using the X-12-ARIMA package, so in practical terms, we ended up working with a SARIMA model.

⁷ It should be noted nonetheless that, for numerical purposes, for sufficiently large choices of ω_0 , it can be treated as an analytical wavelet.

⁸ All our results are obtained using $\omega_0 = 6$.

⁹ For each case, we perform the exercise 5000 times and then extract the critical values.

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