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from Oberwolfach

№ 3/2019

Diophantine equations and why they are hard

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Diophantine equations are polynomial equations whose solutions are required to be integer numbers. They have captured the attention of mathematicians during millennia and are at the center of much of contemporary research. Some Diophantine equations are easy, while some others are truly difficult. After some time spent with these equations, it might seem that no matter what powerful methods we learn or develop, there will always be a Diophantine equation immune to them, which requires a new trick, a better idea, or a refined technique. In this snapshot we explain why.

1 Diophantine equations

Let's start with a simple question. Is 3 the sum of two squares? Well, yes. With no further instructions, here are some solutions:

$$0^2 + (\sqrt{3})^2 = 3, \qquad 1^2 + (\sqrt{2})^2 = 3,$$
$$2^2 + \mathrm{i}^2 = 3, \qquad 3^2 + (\mathrm{i}\sqrt{6})^2 = 3$$

where i is the usual imaginary unit which yields $\mathrm{i}^2 = -1$. Perhaps a more interesting question is the following: is 3 the sum of two *integer* squares? In

[1] Hector Pasten was supported by a Benjamin Peirce Fellowship at Harvard University.

this case the answer is negative. In other words, the equation $x^2 + y^2 = 3$ has no integer solutions, as one can quickly check.

Definition 1 (Diophantine equation) — *A* Diophantine equation *is a polynomial equation with integer coefficients, possibly in several variables, for which we require integer solutions.*

For instance, $x^2 + y^2 = 3$ is a Diophantine equation with no solutions. On the other hand, $x = 1, y = 2$ is a solution of the Diophantine equation $x^2 + y^2 = 5$.

The word "Diophantine" is derived from the name of *Diophantus of Alexandria.* He was a Greek mathematician who lived in the third century and who wrote the very influential series of books *Arithmetica.* The central topic of *Arithmetica* were algebraic equations and methods for solving them in (positive) integers or rational numbers, depending on the problem. It is worth pointing out that integer solutions to equations have been investigated long before Diophantus; the earliest known record of such an investigation is the Babylonian tablet *Plimpton 322*, dating to 1800 BC, which records a number of Pythagorean triples. (Note that this is long before *Pythagoras of Samos*, who lived in the sixth century BC.)

The most basic Diophantine problem that one can ask is the following: *given a Diophantine equation, does it have integer solutions?* There are a number of methods to approach this problem, either to show that there is a solution or to prove rigorously that there is none. Just to mention some:

- *Brute force.* Search for solutions until you find one. There is no guarantee that we will ever find a solution, but if we do, then we know for sure that the equation can be solved.
- *Congruences.* Show that an equation has no solutions by reducing modulo a suitable integer and proving that there is no solution to the resulting congruence. For instance, the equation $x^2 + 3y^5 = 2$ doesn't have integer solutions. This is because the equation seen modulo 3 gives the congruence

$$x^2 \equiv 2 \mod 3,$$

  which doesn't have solutions as we can check with finitely many calculations.
- *Infinite descent.* One proves that an equation has no solution by showing that *if* any (hypothetical) integer solution is given, then one can construct a smaller one (where smaller is meant in the sense of absolute value), and then a smaller one, and so on, which is not possible because one cannot find integers with smaller and smaller absolute value. This contradiction shows that there is no solution to start with.

Mathematicians have developed much more sophisticated methods to search for solutions of a Diophantine equation, and to prove that there is no solution in suitable cases. Unfortunately, some simple Diophantine equations remain resistant to all our techniques. For example, it is a well-known open problem to decide (with a complete and rigorous proof!) whether the Diophantine equation

$$x^3 + y^3 + z^3 = 42$$

has integer solutions or not. (Negative numbers are allowed.) Can you answer this question? If so, you would become famous![2]

Sometimes it is more convenient to focus only on Diophantine equations for which *non-negative* integer solutions are required. It turns out that this problem is of equal difficulty to the problem of existence of integer solutions.

For instance, suppose that we want to study integral solutions of a Diophantine equation such as $x^3 + y^3 + z^3 = 42$ (mentioned above). If we want to use only techniques that apply to *non-negative* integer solutions of Diophantine equations, then we could instead study non-negative integer solutions of the 6–variables equation

$$(x_1 - x_2)^3 + (y_1 - y_2)^3 + (z_1 - z_2)^3 = 42.$$

Note that for a solution of the first equation with positive x, we have a solution of the second equation with $x_1 - x_2 = x - 0$ and for a solution with negative x, we have a solution with $x_1 - x_2 = 0 - (-x)$. In the other direction a similar trick works. For this we use a theorem by the Italian mathematician Joseph-Louis Lagrange (1736–1813), called Lagrange's Four Squares Theorem:

Theorem 1 (Lagrange) — *If an integer number n is the sum of four squares of integers, then we have $n \geq 0$. And on the other hand, if $n \geq 0$ is an integer number, then it can be written as the sum of four squares of integers.*

So, solving a Diophantine equation $F(x, y) = 0$ in non-negative integers is "the same" as solving the equation

$$F(x_1^2 + x_2^2 + x_3^2 + x_4^2, y_1^2 + y_2^2 + y_3^2 + y_4^2) = 0$$

with x_i, y_j integers, negative numbers allowed. Of course the same remark applies to any number of variables, not just two.

2 Hilbert's Tenth Problem

The German mathematician David Hilbert (1862–1943) was a speaker at the International Congress of Mathematicians in 1900. In relation to his talk,

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[2] For a long time, it was not known whether 33 could be written as sum of three cubes. This was recently shown to be possible by Andrew Booker.

he proposed a list of 23 problems that proved to be very influential in the development of mathematics in the last century, see [9]. The tenth problem in the list was the following:

Hilbert's Tenth Problem — *Given a Diophantine equation with any number of unknown quantities and with rational integral numerical coefficients: To devise a process according to which it can be determined in a finite number of operations whether the equation is solvable in rational integers.*[3]

It took some time to make sense of the question. First of all, what is a "process" and what is a "finite number of operations"? It is now understood that the correct notion of "process" working with a "finite number of operations" is a *Turing machine*: an idealized, precise, and rigorous mathematical model of a computer. These theoretical objects were introduced by Alan Turing in 1936, in his paper [22]. For the purpose of clarity, the reader can safely think of Turing machines as "computers with no randomness".

Long after Hilbert posed the problem, in 1961 Martin Davis, Hilary Putnam, and Julia Robinson proved the following astonishing result in [5]:

Theorem 2 (DPR) — *The Diophantine problem for exponential Diophantine equations is unsolvable:*

There is no Turing machine that takes as input an exponential Diophantine equation and, no matter how complicated the equation is, after a finite number of steps decides correctly whether the equation has a non-negative integer solution or not.

Here, an *exponential* Diophantine equation is an equation involving polynomial expressions as well as exponential terms of the form 2^x, whose solutions are required to be *non-negative* integers. This is certainly a very interesting class of equations of number theoretical relevance, but unfortunately not quite the same class of equations about which Hilbert wondered in his Tenth Problem. Nevertheless, the authors were convinced that they were on the right track: as we explained before, it doesn't really make a crucial difference whether we use all integers or just non-negative integers. With respect to the exponential, it was expected that the exponential terms of the form 2^x could also be expressed in terms of polynomial Diophantine equations.

Here we make a brief pause to clarify an important point. As the reader is surely aware, the function 2^x is *not* a polynomial. However, the hope was that the solutions of a polynomial in several variables would suffice to "describe" this function. This idea of using polynomials to "describe" functions or sets

[3] Here, "rational integers" simply means "integers". The word "rational" is used to distinguish them from other generalizations of integers studied by mathematicians.

that don't quite look like coming from polynomials will be explained in detail in the next section. For the moment let us simply say that Davis, Putnam and Robinson were absolutely right: about a decade later, Yuri Matiyasevich in [12] used an ingenious construction based on properties of Fibonacci numbers to show that the required exponential function can indeed be expressed by polynomial Diophantine equations! The result thus obtained was:

Theorem 3 (Matiyasevich) — *Hilbert's Tenth Problem is unsolvable:*
There is no Turing machine that takes as input a Diophantine equation and, no matter how complicated the equation is, after a finite number of steps decides correctly whether the equation has an integer solution or not.

3 Diophantine sets

What can we express using Diophantine equations? An example is the property of being non-negative: as mentioned at the end of Section 1, by Lagrange's Four Squares Theorem we see that the property that an integer n be non-negative can be expressed by requiring that the Diophantine equation

$$n = x_1^2 + x_2^2 + x_3^2 + x_4^2$$

has an integer solution.

Can you express the condition "$m \leq n$" using Diophantine equations? Of course! We can write the condition as "$0 \leq n - m$" and this is the same as requiring that the Diophantine equation

$$n = m + x_1^2 + x_2^2 + x_3^2 + x_4^2$$

has an integer solution. Here are some more examples of properties that we can express using Diophantine equations:

- "n is a square" : this is the same as requiring that $x^2 = n$ has an integer solution.
- "n is odd": this is the same as requiring that $2x + 1 = n$ has an integer solution.
- "n is an odd positive number": this is a bit trickier. We already know how to express "n is odd" and how to express "$n \geq 0$". Can we do both at the same time? We (momentarily) leave it to the reader to understand why the following Diophantine equation does the trick:

$$(2x + 1 - n)^2 + (x_1^2 + x_2^2 + x_3^2 + x_4^2 - n)^2 = 0.$$

- "n is a square or is odd (or both)": similarly, we already know how to express squareness and oddness. For the union of these two properties, we (momentarily) leave it to the reader to understand why the following Diophantine equation works:

$$(x^2 - n) \cdot (2y + 1 - n) = 0.$$

(See the lemma at the end of this section to understand the general pattern in the last two examples.)

At this point it is appropriate to make a definition. As usual, the symbol $\mathbb{Z}$ denotes the set of all integers.

Definition 2 (Diophantine set) — *A set of integers $S \subseteq \mathbb{Z}$ is* Diophantine *if there is a polynomial $F(t, x_1, ..., x_r)$ with integer coefficients such that the set S is precisely the set of integer values n (for the variable t), for which the Diophantine equation*

$$F(n, x_1, ..., x_r) = 0$$

has an integer solution in the variables $x_1, ..., x_r$.

If you are familiar with the properties of Pell equations, here is a fun exercise for you:

Exercise — *Show that the set of non-square integers is Diophantine.*

Diophantine sets have several nice properties. For instance, we have the following:

Lemma 1 — *Let A and B be Diophantine sets. Then the intersection $A \cap B$ and the union $A \cup B$ are Diophantine too.*

Proof. Since A and B are Diophantine, there are polynomials $F(t, x_1, ..., x_r)$ and $G(t, y_1, ..., y_s)$ defining them. Then $A \cap B$ is Diophantine because $n \in A \cap B$ if and only if both equations

$$F(n, x_1, ..., x_r) = 0 \text{ and } G(n, y_1, ..., y_s) = 0$$

have integer solutions, which is the same as requiring that the single equation

$$F(n, x_1, ..., x_r)^2 + G(n, y_1, ..., y_s)^2 = 0$$

has integer solutions. (Why? two integer squares add up to 0 if and only if each one is 0.)

Similarly, $A \cup B$ is Diophantine because $n \in A \cup B$ if and only if at least one of the equations

$$F(n, x_1, ..., x_r) = 0 \text{ or } G(n, y_1, ..., y_s) = 0$$

has integer solutions, which is the same as requiring that the single equation

$$F(n, x_1, ..., x_r) \cdot G(n, y_1, ..., y_s) = 0$$

has integer solutions. (Why? the product of two integers is 0 if and only if at least one of them is 0.) Q.E.D.

4 More about Turing machines

For simplicity, let's think about Turing machines as deterministic (that is, non-random) computers, with as much memory for their computations as needed. Also, let's not worry about the running time. Our question is simply the following: what can such a computer do? It should be able to do basic arithmetic. It should also be able to do brute force search, although there is no guarantee that it will ever stop if we don't instruct it to search only up to a predetermined bound. Also, it should be able to compute using "sub-routines": if a function can be computed by such a computer, then we can freely use this function in other computer programs. It is also natural to allow the program to do computations recursively (that is, by repeated application of a function).

When spelled out in a rigorous mathematical formalism, the previous ideas lead to a solid notion of computability called *Turing-computability*, nowadays accepted as the standard notion of *algorithm*.

Definition 3 (Listable set) — *A set of integers $A \subseteq \mathbb{Z}$ is* listable *if there is a Turing machine which, if left running forever, prints every element of A and no element outside A. (We allow the machine to be slow, to print the elements of A in any order, or even to print repetitions!)*

In simple terms, one can say that any set of integers for which there is an algorithm for writing down its elements is by definition listable (even if your algorithm is very slow). For instance the primes, the squares, the powers of 2, those integers whose digits are odd, and more.

Note that the definition doesn't require that we know how to write down the elements outside a listable set A; it suffices to know how to produce all the elements of A. The following definition, instead, requires knowledge about *both* A and its complement.

Definition 4 (Recursive set) — *A set of integers $A \subseteq \mathbb{Z}$ is* recursive *if there is a Turing machine which computes the characteristic function of A. That is, the machine takes as input an integer n, and after a finite computation it prints* "0" *if n is not contained in the set A and* "1" *if n is contained in A.*

In simple terms, A is recursive if there is an algorithm to decide whether an integer is a member of the set A.

Every recursive set is listable, as we now explain. If A is recursive and there is a Turing machine T that decides membership in A, then here is an algorithm to list the elements of A: use T to decide whether $0 \in A$, in which case print 0, otherwise skip 0 and test 1 using T. If $1 \in A$ print 1, otherwise skip 1 and go to -1. If $-1 \in A$ print -1, otherwise skip it and go to 2. Continue in this fashion with 2, -2, 3, -3, and so on. In this way, you have an algorithm to print the elements of A, so A is listable.

However, *not every listable set is recursive.* There are bizarre sets of integers A for which one knows how to print its elements, but for which it is *impossible* that some algorithm decides membership in A. Here, the word "impossible" doesn't simply mean that we have not found the algorithm; it actually means that mathematicians know a rigorous proof that the required Turing machine doesn't exist. This is a foundational theorem by Alan Turing.

Theorem 4 (Turing) — *There is a set of integers H which is listable, but it is not recursive.*

The set H constructed by Turing arises from the *halting problem*, introduced in his paper [22]. We will not review here the construction of H, and for our discussion is suffices to know that not every listable set is recursive.

5 Diophantine sets again

Let C be the set of all integers that can be written as the sum of three integer cubes. Here are some examples:

- $0 \in C$. This is because $0 = 0^3 + 0^3 + 0^3$.
- $-1 \in C$. This is because $-1 = (-1)^3 + 0^3 + 0^3$.
- $6 \in C$. This is because $6 = 2^3 + (-1)^3 + (-1)^3$.

We note that C is Diophantine: an integer n belongs to C if and only if the Diophantine equation

$$x_1^3 + x_2^3 + x_3^3 = n$$

has integer solutions. As mentioned before, it is an open problem whether or not the previous equation has integer solutions for $n = 42$, so, as of today, we don't know whether 42 is an element of C or not.

Despite the fact that it is complicated to decide (with a rigorous proof) whether an integer is in C or not, here is a completely rigorous way to *list* the elements of C:

Evaluate $x_1^3+x_2^3+x_3^3$ over the triples of integers in $\mathbb{Z}$ in a sequential systematic way – for instance, first at $(0,0,0)$, then at $(1,0,0)$, then at $(-1,0,0)$, then at $(0,1,0)$, and continue taking all possible triples of integers in some order. Print the values. That (infinite) list of printed values consists precisely of the elements of C, in some order, possibly repeated.

We have just described an algorithm to list all the elements of the Diophantine set C by using the polynomial that defines it. So, now we know that the Diophantine set C is *listable*. A similar idea works in complete generality, and one can prove:

Lemma 2 — *Every Diophantine set of integers is listable.*

This is somehow not very surprising: the definition of a listable set is extremely broad. Basically, any set of integers that we can describe in a sensible and constructive way is going to be listable, so it is not a surprise that Diophantine sets are of this kind.

However, behold the results of the next section!

6 The DPRM theorem: the good news, the bad news, and the bizarre news

Now we are in a position to state a more accurate version of the theorem proved by the works of Davis, Putnam and Robinson [5] and by Matiyasevich [12].

Theorem 5 (DPRM) — *Every listable set of integers is Diophantine. (Therefore, in view of Lemma 2, listable sets and Diophantine sets are the same thing!)*

The statement of this theorem is short, but it takes some more space to actually understand what it really means.

First, by Turing's Theorem 4, this implies that there is a Diophantine set which is not recursive. Unravelling the definitions of "recursive set" and "Diophantine set", this means the following: there is some polynomial $F(t, x_1, ..., x_r)$ with integer coefficients such that there is no Turing machine that takes n as input and prints 1 if the Diophantine equation

$$F(n, x_1, ..., x_r) = 0$$

has integer solutions in the variables $x_1, ..., x_r$, and prints 0 otherwise.

Or in simpler terms: there is a family of Diophantine equations such that there is no algorithm that can correctly decide existence of integer solutions for all equations in this family.

In particular, *there is no algorithm that can correctly decide existence of integer solutions of each Diophantine equation.* In this way we see that the

DPRM theorem implies the negative solution to Hilbert's Tenth Problem, which was stated before as Theorem 3.

Good news. Hilbert's Tenth Problem is solved (negatively): there is no algorithm as the one that Hilbert wanted.

Bad news. Any method to study solvability of Diophantine equations is known in advance to be incomplete, in the sense that there will be Diophantine equations beyond the scope of that method.

Bizarre news. Wait a minute. So the theorem really says that *every* listable set is Diophantine? Yes, it does. So, for instance, the prime numbers form a Diophantine set: thus, they can be described using a polynomial. Similarly the factorials, the powers of 2, or any strange non-polynomially-looking set that you can construct is a Diophantine set. In fact, there is a whole paper [4] written by Davis, Matiyasevich, and Robinson explaining all sorts of unexpected consequences.

A word about the proof of DPRM. The basic idea is that the functioning of a Turing machine can be completely described by the application of three basic types of functions (successor, constants, and projections) and three basic computation schemata (minimalization, composition, and recursion). Thus, one "only" needs to simulate these six kinds of operations by using Diophantine equations. This requires a number of ingenious applications of methods from elementary number theory. These turn out to be rich enough to simulate the necessary parts of a Turing machine by using only Diophantine equations. See [3] for a self-contained accessible presentation of the proof.

7 Other directions

Surprisingly, the analogue of Hilbert's Tenth Problem for the field of rational numbers $\mathbb{Q}$ remains open. There is a natural strategy: one tries to show that $\mathbb{Z}$ is Diophantine in $\mathbb{Q}$ (with the obvious adaptation of the definition of "Diophantine set" in $\mathbb{Q}$ by requiring a polynomial with rational coefficients), and then a general argument transfers the negative solution of Hilbert's Tenth Problem from $\mathbb{Z}$ to $\mathbb{Q}$.

However, there is some skepticism about this approach to the problem, and it is now a conjecture (believed by many) that $\mathbb{Z}$ is *not* Diophantine in $\mathbb{Q}$. Some research articles that elaborate on this are [2, 13, 19]. Remarkably, the *non-integers* are known to form a Diophantine set in $\mathbb{Q}$, as proved by Koenigsmann [11].

There is another natural extension of the problem to the context of "rings of integers of number fields". Here we are talking about generalizations of the integers $\mathbb{Z}$, such as the ring of Gaussian integers $\mathbb{Z}[i] = \{a + b\mathrm{i} : a, b \in \mathbb{Z}\}$. In the particular case of the Gaussian integers, it is known that the analogue of Hilbert's Tenth Problem also has a negative solution, see [6]. In fact, there are several families of rings of integers of number fields where the same is known, see [7, 8, 16, 20, 23]. But the problem in full generality remains open.

Nevertheless, assuming certain *algebraic* conjectures about elliptic curves (an important type of cubic equations), Mazur and Rubin proved a negative solution in complete generality [14]. On the other hand, if one assumes certain *analytic* conjectures about elliptic curves, then one arrives at the same conclusion in a different way, by work of Murty and yours truly [15]. Let us mention that the connection with elliptic curves is due to Denef [7], Poonen [17], Cornelissen, Pheidas, and Zahidi [1], and Shlapentokh [21].

Several other problems remain open in this very active area of research (see for instance [18, 10]) and it seems that new ideas – maybe *your* ideas, dear reader – are necessary to make further progress on these fundamental questions.

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DOI
10.14760/SNAP-2019-003-EN

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ISSN 2626-1995

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