261. Meaningful finite element discretization scheme: a case for circular disk subjected to four radial loads

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Abstract. In finite element (FE) modelling, apart from proper element selection, selection of an appropriate discretization scheme is crucial in correctly evaluating the intended variables. Photoelasticity plays an effective role in selecting an appropriate discretization scheme for modelling problems in stress analysis. A meaningful FE discretization is very important in evaluating the desired parameters in FE analysis. The adaptive mesh refinement emphasizes the need for a meaningful discretization of domain. This paper presents a discretization strategy for meaningful discretization of the circular disk subjected to four equal radial loads. The work evolved the generation of software for the disk subjected to four radial loads perpendicular to each other. Due to symmetry of the problem only quarter of the disk is discretized.

Keywords: finite element modelling; mesh discretization; photoelasticity; fringe contours; isochromatics

1. Introduction

The discretization of any domain into finite elements is important as the value of the variables to be found is very much dependent on it. The manual discretization of any domain is very tedious and time consuming. Hence the development of automatic mesh generation software is very important in FE analysis of a problem.

The concept of meaningful discretization has been proposed by Pathak and Ramesh [1]. The basic guideline proposed is that the discretization is meaningful if the fringe pattern observed in a photoelastic experiment [2] is simulated faithfully by FE modeling as photoelasticity is the only optical experimental technique available to study the stresses interior to model. Peindl et al. [3] supported this study [1] in improving, augmenting and validating FE analysis, particularly with respect to establishing appropriate boundary conditions. They worked on photoelastic stress freezing analysis of total shoulder replacement system. Ragulskis and Ragulskis [4] on their study on plotting isoclinics for hybrid photoelasticity and finite element analysis noted the study of Ramesh and Pathak [1] for the detection of FEM meshing problems.

The photoelastic fringe contours can be simulated from FE results by post processing it. Literature provides simple approach for plotting of fringe contours from Finite Elements Results. Recent trend is on development of photoelastic fringe plotting scheme [5] from 3D FE Results.

This paper presents a meaningful discretization scheme for a disk subjected to four concentrated radial loads acting along mutually perpendicular diameters.

2. Factors to be Considered During Discretization

The number, type, shape and density of elements used in a given problem depend on a number of considerations. The basic guidelines [6] to be followed for mesh generation are: (i) it should represent the geometry of the computational domain and load accurately; (ii) the body is discretized into sufficiently small elements so that steep gradients of the solutions can be accurately calculated; (iii) it should not contain elements with unacceptable geometries especially in regions of large gradients.

Generation of meshes of a single element type is easy because elements of the same degree are compatible with each other. The mesh can be refined by subdividing existing elements into two or more elements of the same type. This is called h version mesh refinement. A mesh refinement should meet three conditions. (i) All previous meshes should be contained in the refined mesh. (ii) Every point in the body can be included within an arbitrary small element at any stage of the mesh refinement. (iii) The same order of approximation for the solution may be retained.

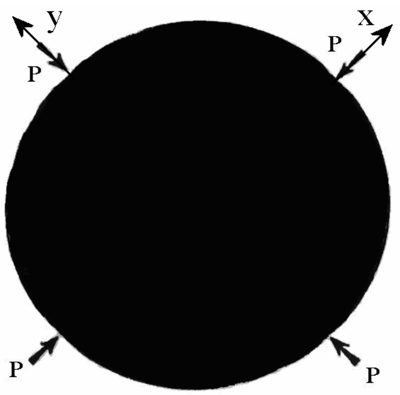
Taking above factors into account we have generated the scheme in which the disk is discretized using eight nodded quadrilateral elements and mesh refinement has been done at the most stress concentration area, i.e. near steep stress gradient area around the point source.

3. Mesh Generation

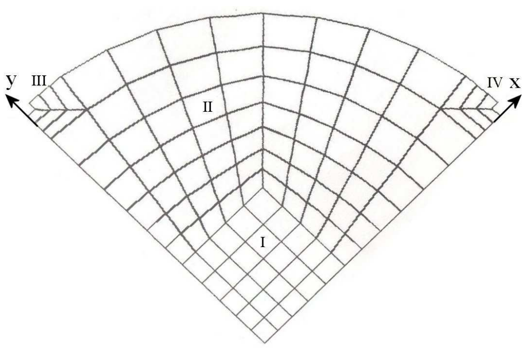
Mesh generation is vital in FE analysis. It refers to generation of nodal coordinates and elements connectivity. It also includes the automatic numbering of nodes and elements based on minimal amount of user supplied data. Automatic mesh generation [7] reduces errors and saves a great deal of user's time, thereby reducing FE analysis cost. Next section presents the proposed discretization scheme.

4. The Discretization Scheme

We consider a circular disk subjected to radial load as it is shown in Fig. 1(a). The circular disk has symmetry about both x and y axis so the discretization and FE analysis has been performed only for a quarter of the disk. The center of the disk is taken as the origin and then the coordinates of the nodes are found with respect to the origin.



a)



b)

Fig. 1. a) disk subjected to four radial loads;
b) meaningful finite element discretization of the disk subjected to four radial loads

In the proposed discretization scheme the disk is divided into four regions as shown in Fig. 1(b). The scheme of discretization of different regions is as follows.

(i) *Region I*

This is a rectangular region of the quarter of the circle as shown in Fig. 1(b). In the software flexibility is provided to select the length and height of rectangular region. User also has the flexibility to choose the number of elements in which the big rectangle is to be discretized. This rectangular region is further divided into eight nodded rectangular elements. The programme then numbers the eight nodded rectangular elements and finds nodes coordinates and their connectivity.

(ii) *Region II*

As shown in Fig. 1(b), this part is adjacent to big rectangle. This part is also discretized into eight nodded quadrilateral elements.

(iii) *Region III and IV*

One of the elements of region II where load is applied is marked as region III and region IV. These regions are discretized into many small elements without affecting the other elements of the region. Final meshing near the load enables to better analyse the stresses and represent the steep stress gradient accurately. This discretization scheme has been adopted from Ramesh and Pathak [8].

4.1. Generation of nodal coordinates:

The basic guideline proposed is to divide the element lengths into geometric progression, and thus find the coordinates of various nodes. Thus the logic for discretization and evaluation of nodal coordinates of various nodes is as follows.

4.1.1 Region I

Let the radius of the disk be R, horizontal length of big rectangle be l_h, vertical length of big rectangle be l_v. The user is first asked to specify the ratio (R/l_h = (ratio)$_h$) and (R/l_v = (ratio)$_v$). Once these ratios are known, then for a given radius of disk the l_h and l_v are found out as:

$$l_h = R/(ratio)_h \quad (1)$$

and $$l_v = R/(ratio)_v \quad (2)$$

Now the user is asked to specify the number of horizontal (n_h) and vertical division (n_v) and geometric ratio in horizontal (r_h) and in vertical direction (r_v). Then the length of the first small rectangle can be found from:

$$l_h = \frac{a_h[1-(r_h)^{n_h}]}{[1-r_h]}$$

$$\Rightarrow\ a_h = \frac{l_h[1-r_h]}{[1-(r_h)^{n_h}]} \tag{3}$$

$$\text{Similarly}\ \Rightarrow\ a_v = \frac{l_v[1-r_v]}{[1-(r_v)^{n_v}]} \tag{4}$$

Here a_h and a_v are the length of the first small element in horizontal and vertical directions respectively. Now multiplying a_h with geometric ratio (r_h) and a_v with geometric ratio (r_v) one can find the length of next rectangular sub elements and by adding theses values to a_h and a_v one can find the horizontal and vertical coordinates of next nodes.

4.1.2 Region II

Similar to region I, region II is also divided in geometric progression and the same formula is utilized for the calculation of nodal coordinates. In both of the regions user has the flexibility to choose a proper geometric ratio so as to continuously increase or decrease the element length or divide it equally by giving geometric ratio as unity.

To discretize the second region, points at the periphery of a disk are located at equal angular intervals. These points are then joined to outer nodes of region I elements as shown in Fig. 1(b). With the help of known angular intervals the coordinates of nodes on the periphery of a disk are found. The line joining these nodes to outer nodes on region I is then divided into geometric progression as done for region I.

4.1.3 Region III & IV

After calculating the coordinates for region I and II, region III is discretized, using the same logic as for region I and II. As the coordinates of end points of quadrilaterals of region III are already found we discretize it as shown in Fig. 2.

Here first the user gives the number of elements in which one wants to discretize the region III, and the geometric ratio. From this information, the program finds out length of small rectangles using the formula:

$$a = \frac{l(1-r)}{(1-r^n)}, \tag{5}$$

and utilizing the same logic the coordinates of the nodes are found.

4.2. Generation of Elements Connectivity

First, all the elements are numbered continuously from left to right. As the node numbering is also from left to right continuously. There is relation between the local nodes and the global nodes. Once the relationship for one element is analysed and formulated the same relationship is used for others. This relation dominates at least for one line and may range to several lines too. When the relation changes the other relations are searched and that is modified. In this way connectivity for region I and II is simultaneously found. For region III and IV separate formulation of relationships are done by analyzing the global and local coordinates.

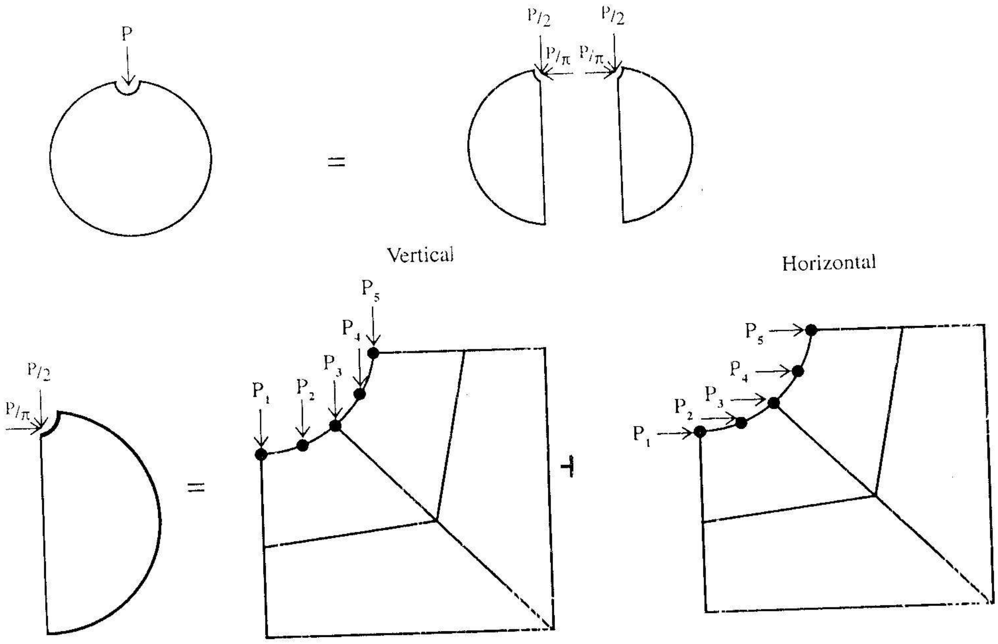


Fig. 2. Specification of concentrated load as nodal forces in horizontal and vertical directions

5. Validation of the Discretization

To validate the discretization the disk is assumed to be subjected to four equal radial loads acting along two mutually perpendicular diameters. The disk diameter, thickness and material fringe value considered are 23.11 mm, 3.61 mm, and 15.06 N/mm/fringe [9] respectively. A load of 253.54 N is applied as shown in Fig.1(a). The problem is analysed as plane stress elasticity problem. The input considered are information of mesh as discussed in section 4, and specification of essential boundary condition (EBC) and natural boundary condition (NBC). The EBC is displacement u and v in x and y directions. The v = 0 is taken along x axis and u = 0 is taken along y axis. The NBC (i.e. nodal forces) are specified using the concept of elasticity and as explained by Ramesh and Pathak [8] and shown in Fig. 2. The distribution of concentrated force as equivalent horizontal and vertical force is shown in Table 1 [8]. Elastic properties such as modulus of elasticity of material and poissons ratio are specified. Using the plane elasticity formulation the values of σ_x, σ_y and τ_{xy} at the nodes are found. From these σ_1-σ_2 are calculated. Now from stress optics law:

$$\sigma_1 - \sigma_2 = N F_\sigma / t \qquad (6)$$

This equation relates the principal stress difference with the fringe order *N*. Here F_σ is the material fringe value and *t* is the thickness of the specimen. Thus to simulate the photoelastic results the contour corresponding to *N* needs to be plotted. Using the simple non iterative approach as explained by Ramesh et. al. [10], the plotted contours are shown in Fig. 3(a). Owing to the symmetrical nature of the loads it follows that the center of the disk is subjected to equal normal stresses and to zero shear stresses. The center of the disk is therefore an isotropic point. It is seen that these contours are similar to those obtained experimentally [9] and shown in Fig. 3(b). This validates our discretization scheme.

6. Conclusions

Contour plotting helps to appreciate the whole field representation of the results, and they are plotted for the nodal values obtained from FE results using appropriate interpolation functions. The thickness of the fringe indicates the gradient of the variable. The fringes are very broad where gradient is small and vice versa. Numerical simulation of the fringe contours is a handy useful tool for visualizing the experimental fringe contours and to identify the stress concentration zones in model under considerations. It is seen that the geometric features of the fringe are captured well in numerical simulation, thus validating the proposed discretization scheme for circular disk subjected to four radial loads. These results can be applied to actual situations for FE discretization of similar domains.

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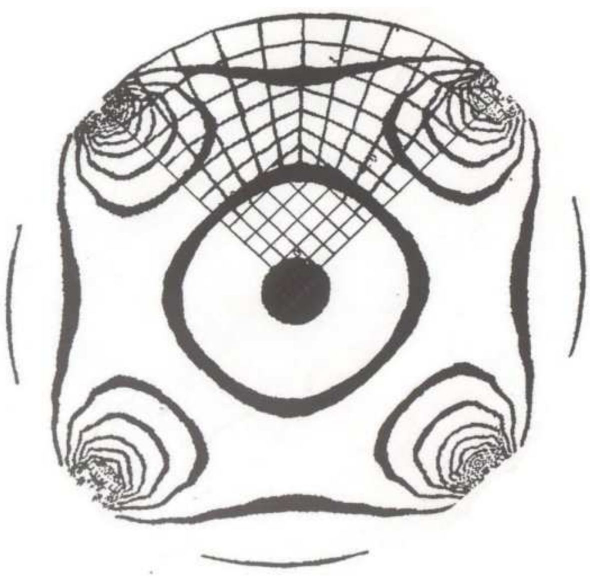
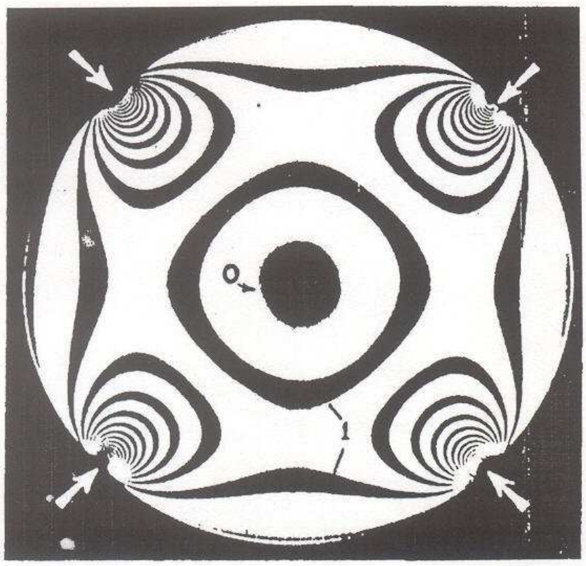


a)



b)

Fig. 3. a) Meaningful finite element discretization of disk subjected to four radial loads; b) Disk subjected to four radial loads

References

1. **Pathak P. M. and Ramesh K.** “Validation of Finite Element Modelling Through Photoelastic Fringe Contours”, Communications in Numerical Methods in Engineering, 1999, 15: 229-238.
2. **Dally J. W. and Riley W. F.** “Experimental stress Analysis (2nd ed.)”, McGraw Hill Inc, 1991.
3. **Peindl R. D., Harrow, M. E., Connor P. M., Banks D. M. and D’Alessandro D. F.** “Photoelastic Stress Freezing Analysis of Total Shoulder Replacement Systems”, Experimental Mechanics, 2004,. 44: 228-234.
4. **Ragulskis M. and Ragulskis L.** “Plotting Isoclinics for Hybrid Photoelasticity and Finite Element Analysis”, Experimental Mechanics, 2004, 44: 235-240.
5. **Babu P. R. D. K. and Ramesh K.** Development of Photoelastic Fringe Plotting Scheme From 3D FE Results, Communications in Numerical Methods in Engineering, 2006, 22(7): 809-821.
6. **Reddy J. N.** “An Introduction to the Finite Element Method (2nd ed.)”, McGraw Hill Inc, 1993.
7. **Zienkiewicz O. C., and Zhu J. Z.** "Adaptivity and mesh generation", Int. J. Numer. Methods Eng. 1991, 32, 783-810.
8. **Ramesh K. and Pathak P. M.** "Role of photoelasticity in evolving discretisation schemes for FE analysis", Experimental Techniques, 1999, 23(4), 36-38.
9. **Frocht M. M.** “Photoelasticity”, Volume I, NewYork, John Wiley & Sons, Inc. 1941.
10. **Ramesh K., Yadav A. K., and Pankhawalla V. A.** “Plotting of Fringe Contours From Finite Elements Results”, Communications in Numerical Methods in Engineering, 1995, 11: 839-849.