283. Mathematical modelling – tool to diagnosis of human apparatus

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Abstract. There is presented a model of human gait based on power developed by muscle joints in this paper. The mathematical model was presented for healthy subjects, patients with gonarthrosis and patients with Anterior Cruciate Ligaments (ACL). The regression function and the stochastic approximation function were used to the modeling. The differences between the values of model's coefficients of people with these two pathologies have been compared to the model's coefficients of healthy subjects. Proposed model is very accurate. The modeling could help to define the level of gait pathology and treatment in a large number of patients.

Keywords: modeling, regression function, stochastic approximation function, human gait.

1. Introduction

Human walking is an example of a well-learned fundamental movement pattern that, in normal situations is performed with a great deal of coefficiency and consistency. Gait analysis and modeling has long been used as a research tool to develop complete understanding of the mechanics control of human movement. A standard physical examination can't provide a complete description of pathology of abnormal human gait. Gait analysis and modeling can. The primary purpose of gait analysis and modeling in clinical application is to provide objective documentation of gait and increase understanding of the mechanism of pathological gait. Mathematical modeling is an important tool for quantifying normal and pathological patterns of locomotion. With this information, more informed decision making about treatment to improve gait function is possible. In this study will be presented the parametric methods for identification of human gait model. There can be many pages explaining what parametric identification method is in depth, but it can be explained as a technique that helps one to structure a mathematical model based on measured data by adjusting parameters within a given model until the simulated output coincides as closely as possible with the measured output. Using parametric identification methods, the dynamic properties of human gait can be identified. Recently, many interactive computer software and hardware packages have been developed that allowed further development of these methods. However, many researchers have attempted the simulation of gait by limiting their model with constraints [10,11,12]. Many of these take an analytical approach to the synthesis of motion, using trial and error, dynamic optimization, or other control schemes to develop the proper force time histories necessary to drive the motion. Onyshko and Winter presented an important model of locomotion in 1980 [1]. In this work, the issue of responsible modeling was stressed in regard to pure synthesis (forward dynamics). One of the few dynamic models of the knee joint was presented in 1988 [2]. The approach involved the development of a 3D model that was later reduced to a 2D model for analysis and simulation of motion. The skeletal model and the origins and insertions were taken from the work of Crownenshield and co-workers [3]. The synthesis of motion was described as a reduction of a second order non-linear system of ordinary differential equations to an algebraic system and then linearized. The tibia was subjected to several dynamic loading situations as different mathematical function shapes, and the resulting dynamic forces were computed. The contact points, joint orientation, and joint and ligament loads were reported, and showed that knee extension

caused elongation of the medial and lateral collateral and anterior cruciate ligaments (ACL), but not the posterior cruciate ligament (PCL). It was also shown that the ACL carried the highest load. This study did not consider the effects of muscles or parametric changes such as the loss of a ligament or the manipulation of the geometry. A later study presented 3D, 7 segment models with revolute joints [4]. This model was driven by inverse dynamic forces and moments obtained from a gait analysis laboratory, and a forward solution were obtained using ADAMS dynamic modeling software. Ground contact was modeled as spring-damper forces on an ellipsoid foot to reflect the progression of the center of pressure. Torsional spring-dampers were placed at each joint to regulate the motion. The model was applied to the swing phase of gait, and joint moments were altered in order to obtain kinematic patterns similar to the original measured motion [4].
The goal of this work is to investigate the prospects of dynamic model as a tool for understanding and treating pathological musculoskeletal motion.

2. Material and method

In this study 45 healthy subjects (the average age 46 yr.), 37 patients with gonarthrosis (the average age 67 yr.), and 36 subjects with Anterior Cruciate Ligaments (ranged between 20 and 37 yr.) were evaluated. Subjects were recruited into Centre of Bioengineeria in Milan. The optoelectronic system Elite-3D was used for the measurements. The system is based on an on-line data processing of signals from a number of TV cameras. It is possible to recognize in the field of view of each TV camera those bright areas which are of interest for motion analysis. Quantization of biomechanical variables and spatio-temporal parameters of walking was performed by means of a computerized system for automatic acquisition of kinematics and ground reaction forces. A working volume 3 m long, 2.5 m high, and 1.2 m wide was calibrated by a precision grid, which was displaced in three different parallel planes. The resultant accuracy was assessed by measuring the movement of a special stick with three retro reflective markers placed on it. Pre-processing of raw data involved a tracking procedure, three-dimensional reconstruction of the marker's coordinates, correction for optoelectronic distortion, and filtering. The frequency of acquisition was set at 50 Hz. All the subjects were analyzed with the S.A.F.Lo protocol. The location of the markers on each subject was the following: two on the Posterior Superior Iliac Spines, one on the Sacrum Bone, two on the Lateral Femoral Condyles, two on the Lateral Malleoli, and one on the Wrist. The inertial parameters were also derived using the measurement and some kinematics data from optimization, according to the adjustments of the Zatsiorsky-Seluyanov's parameters [7]. In this work ground reaction data were collected with two KISTLER platforms placed in these labs. Three forces and three moments relative to each force plates were recorded. Force plates were also calibrated by leaving the special eight markers device on each force plate at the time. The mean distance computed between the two spheres on the stick differed in general from the actual distance (400 mm) by less than 0.3 mm. Using data from the ground reaction platform, the kinematic data (trajectories, joint angles, acceleration etc.) has been combined with ground reaction forces and interia parameters in order to compute the joint moments and powers.

2.1 The power's model of human gait using the regression function

The two parametric identification methods were used to the modeling of human gait: the regression function and the stochastic approximation function.
The computation of power joint is a relatively straightforward application of Newtonian mechanics. The mechanical power associated with joint rotation is computed from the combination of the joint moment and the joint angular velocity (the rotational velocity of one segment relative to another) [8,9]. The power was normalized to the body weight of each subject. The formula of power joint is facilitated through the use of Eqn.1:

$$P_i = \vec{M}_i \cdot \vec{\omega}_i, \qquad (1)$$

where P - joint power, M - joint moment, ω - angular velocity.

The power's model proposed by the author is based on instantaneous power developed by muscle joints. The human gait model using the regression function is determined by Eqn. 2 [8,13,14]:

$$\hat{Y}_n = \underline{u}_n \cdot \underline{a}, \quad n=1,2,\ldots,N, \qquad (2)$$

where:
$\hat{Y}$ – model's output (power joints in the n instant),
$\underline{u}_n$ – model's input (power joints in the n instants before),
$\underline{a}$ – unknown parameters of human gait,
N – sample size.

The unknown vector $\underline{a}$ is determined by Eqn.3:

$$\underline{a} = \left(\underline{\underline{U}}^T \cdot \underline{\underline{U}}\right)^{-1} \cdot \underline{\underline{U}}^T \cdot \underline{Y}, \qquad (3)$$

where $\underline{\underline{U}}$ - the matrix of input data, $\underline{Y}$ - the vector of output data, $\underline{a}$ – unknown parameters of human gait.

2.1 The power's model of human gait using the stochastic approximation function

To identify the value of model's coefficients in time, the stochastic approximation function has been proposed in this paper. The equation of human motion using the stochastic approximation function can be expressed by Eqn.4:

$$\overset{\Lambda}{\boldsymbol{Y}}n = a_1 \boldsymbol{Y}_n + a_2 \boldsymbol{Y}_{n-1} + ... + a_{n-k} \boldsymbol{Y}_{n-k}, \qquad (4)$$

where the vector $\underline{a}$ is expressed as below:

$$\underline{a}_i = \underline{a}_{i-1} + \underline{\underline{P}}_i \underline{U}_i^T \left(y_i - \underline{U}_i \underline{a}_{i-1} \right), \qquad (5)$$

$$\underline{\underline{P}}_i = \underline{\underline{P}}_{i-1} - \frac{\underline{\underline{P}}_{i-1} \underline{U}_i^T \underline{U}_i \underline{\underline{P}}_{i-1}}{\left(1 + \underline{U}_i \underline{\underline{P}}_{i-1} \underline{U}_i^T \right)}. \qquad (6)$$

Where Y_n– model's output (power developed by muscle joints in n-instant), $\underline{U}_n$– model's input (power developed by muscle joints in n-instants before), $\underline{a}$– unknown parameters of human gait, $\underline{\underline{P}}$ - matrix of covariance.

The initial value of vector $\underline{a_0}$ and of matrix $\underline{\underline{P}}_0$ must be given by Eqn. 8-9:

$$\underline{a}_0 = [0 \quad 0 \quad \dots 0]^T , \qquad (7)$$

$$\underline{\underline{P}}_0 = r \underline{\underline{\Lambda}}, \qquad (r \approx 10^2 \div 10^3), \qquad (8)$$

where $\underline{\underline{\Lambda}}$ - is a unitary matrix.

3. Results

Table 1 shows power's model coefficients of healthy subjects into seven phases of human gait: the initial contact (IC), the loading response (LR), the midstance (MSt), the terminal stance (TSt), the initial swing (ISw), the midswing (MSw), and the terminal swing (TSw). The standard deviation value of the model's coefficients is also given in the same table.

Table 1. The power's model coefficients of healthy subjects using the regression function (±SD)

Phase	a_1	a_2	a_3
IC	0,636 ±0,102	-0,068 ±0,003	0,046 ±0,008
LR	1,232 ±0,117	-0,350 ±0,045	-0,076 ±0,007
MSt	1,039 ±0,154	-0,138 ±0,035	-0,091 ±0,009
TSt	0,576 ±0,122	-0,408 ±0,101	-0,057 ±0,005
ISw	0,641 ±0,107	-0,187 ±0,050	-0,097 ±0,021
MSw	0,610 ±0,151	-0,061 ±0,005	-0,051 ±0,008
TSw	1,226 ±0,221	-0,263 ±0,051	-0,151 ±0,080

There are presented model's coefficients of patients with Anterior Cruciate Ligaments and with gonarthrosis in Tab.2-3.

Table 2. The power's model coefficients of patients with gonarthrosis using the regression function (±SD)

Phase	a_2
LR	-0,075±0,050
TSt	-0,800±1,150
Isw	-0,096±1,101

The model's coefficients of patients with gonarthrosis were compared to the model's coefficients of healthy subjects. In the initial contact, the midstance, the midswing and in the terminalswing no difference between model's coefficients. The value of coefficient a_2 is different in the three phases of human gait: the loading response, the terminal stance, and the initial swing.

There are presented model's coefficients for patients with Anterior Cruciate Ligaments in Table 3.

Table 3. The power's model coefficients of patients with Anterior Cruciate Ligaments using the regression function (±SD)

Phase	a_2
IC	-0,193±0,120
LR	-0,146±0,040
TSt	0,533±0,306
ISw	0,418±0,025

There is no difference of model's coefficients in the midstance, the midswing, and in the terminal swing compared to the healthy subjects. The significant difference of coefficient a_2 is in: the initial contact, the loading response, the terminal stance, and in the initial swing.

With the help of a program in MATLAB by the stochastic approximation function the model's coefficients with the best fit to the data were easily computed. Table 4 shows power's model coefficients for the hip joint, the knee joint, and for the ankle joint of patients with gonarthrosis and patients with Anterior Cruciate Ligaments using the stochastic approximation function. The standard deviation is also given in the same table.

Power's model coefficients of these two pathological groups were compared to the power's model coefficients of healthy subjects [10]. Analysis of model's coefficients shows, that there is no difference of coefficient a_1 and a_2 for all joints of human body. The significant difference is noted for the coefficient a_3.

Table 4. The power's model coefficients of patients with gonarthrosis and LCA using the stochastic approximation function (±SD)

Joint	Coeff.	Gonarthrosis	LCA
Hip	a_3	-0,063 ±0,047	-0,045 ±0,015
Knee	a_3	0,143 ±0,121	-0,113 ±0,101
Ankle	a_3	-0,052 ±0,031	-

Statistical analysis was performed on the whole population of healthy subjects and patients with gonarthrosis and those with Anterior Cruciate Ligaments. A characterization of the difference was obtained by computing the following parameters: the standard deviation, correlation, variance, and confidence intervals.

4. Conclusion

Gait Analysis is a good tool for the documentation of human movement. In this study the regression function and the stochastic approximation function were used to identify dynamic properties of human gait of healthy subjects and patients with gonarthrosis and those with Anterior Cruciate Ligaments. The methods showed the most accurate results. The regression function and the stochastic approximation function complete each other. The stochastic approximation function allows to analysis model's coefficients currently. Model's coefficients using the stochastic approximation function have lower values than model's coefficients using the regression function. There is shown that some model's coefficients should not be analysed during research process in this paper. The reason of rejected model's coefficient is no difference of model's coefficients between healthy subjects and presented groups of pathology such as the gonarthrosis and Anterior Cruciate Ligaments. Through the numerical analysis and experiments, it is concluded that these methods are the most suitable for accurately monitoring of the dynamic properties of human gait.

It is very likely that applying kinetics data, especially power joints, helps to define gait pathology and treatment in a large number of patients. With the help of advanced computing software, parametric identification method has proved to be convenient and powerful method for monitoring human gait. The advantage of the proposed technique is possibilities to classify human gait to different groups of pathology. It also provides the information from which a treatment recommendation may be made to enhance the subnormal gait patterns evident in these patients. These methods of identification represent human movement in a very accurate way during walking
a man in sagittal plane.

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