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Cover Page Footnote

The authors would like to acknowledge the mentors of the Aerospace Department of Sandip University Nashik, India, and the mentors of the Aerospace Propulsion Department of Cranfield University United Kingdom, who always had been a source of inspiration and ignited the fire of research in the hearts of authors. The authors also wish to show gratitude towards ANSYS for providing with the wonderful tool for CFD analysis.

Experimental and Computational Investigation of Divergent Thrust Losses In Rocket Nozzles

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***Abstract*— The nozzle is a critical component of any rocket engine because it transforms high pressure and high temperature (with sluggish net velocity) within the combustion chamber into high velocity but low pressure. Individually, the molecules in the mixed propellants have rapid velocity in the combustion chamber, but directions are randomly pointing in all directions. A convergent-divergent nozzle, also known as a CD nozzle, is fashioned like a tube with two bulging ends and a thin part in the center. This structure/design provides the nozzle with a balanced shape. By turning the flow's heat energy into kinetic energy, the nozzle accelerates hot, pressured gas traveling past it in the axial (push) direction. Whereas the nozzle provides the acquired incremental thrust to the rocket engine, it also results in unwanted thrust losses. Out of which divergent losses in the divergent section are the primary foci of our research. Moreover, in this research paper, we produced verified results to reduce the thrust losses at the divergent sections of the nozzle by optimizing the nozzle with different half divergent angles.**

Keywords- Ansys Workbench 2022, CFD, Design, Nozzles, Nozzle losses, Pressure, Mach number, Oblique shocks, Half divergent angle, SpaceClaim modeler 2022, Rockets

Nomenclature:

ṁ	***Mass flow rate***
V_j	***Efflux jet velocity***
Θ	***Half divergent angle***
λ	***Divergence loss coefficient***
Δ	***Not available thrust***
Pa	***Pascals***
CFD	***Computational fluid dynamics***

I. INTRODUCTION

All Simply put, a nozzle is a mechanical device that regulates the properties of a flow as it departs a chamber and enters a receiving medium[2]. C-D nozzles as shown in Fig. 1 and Fig. 2.

For say while holding a hose that is connected to the water tap and the water is gushing out through the hose we may experience a certain force that is developed by the nozzle cross-section, thus changing the momentum of the flow[1].

Now we also know that with previous nozzle studies the angle of the convergent section can be any angle since the flow at the throat will have to be Mach Number-1 to accomplish its required task and so any thrust loss occurring in the nozzle will be in the divergent section. Thus, the principle motive of this paper are to investigate the divergence losses in nozzles using both experimental and computational method to obtain optimum angle for the C-D nozzle. Secondly, to validate the results whether both the methods are concurring or not. Lastly to depict the fundamental working of the nozzle how pressure, temperature and velocity relates to one another in convergent and divergent sections respectively.

In addition we performed all the CFD results using Ansys Workbench Student Version 2022 on 2-D Convergent-Divergent Nozzles to validate our primary objectives. Therefore we also briefed about the other nozzle issues in our paper.

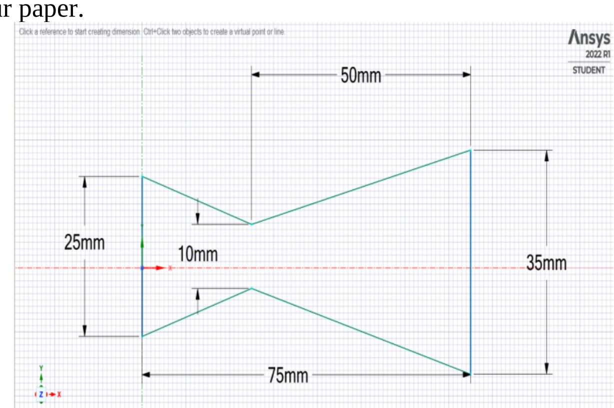


Figure 1. Schematic design of a standard C-D nozzle

Figure 2. A real life C-D or (De Laval) Nozzle [4]

II. NOZZLE THEORY

The De Laval Nozzle was designed by Swedish inventor Gustaf de Laval. The converging-diverging nozzle is typically exploited to provide supersonic jet velocity at the nozzle's outlet. The pressure of the exhaust gases will rise in the convergent part of the nozzle as the hot gases expand through the diverging section at high speeds. When compared to the ambient pressure (pressure outside the nozzle), the pressure in the combustion chamber declines as the flow propagates towards the outlet. This generates maximal expansion, often known as optimal expansion[5], as shown in Fig. 3.

However, a nozzle is a simple axisymmetric tube that tends to primarily focus to manipulate the outflow by getting the desirable thrust or force. As significant, the oxygen is to fuel, so it is the nozzle to rockets and missiles because it is this devise that converts potential energy(thermal energy) into kinetic energy of the exhaust.

The isentropic model is considered good for the nozzle's preliminary design. Since we know that the flow in a nozzle is swift and has very little frictional losses[1].

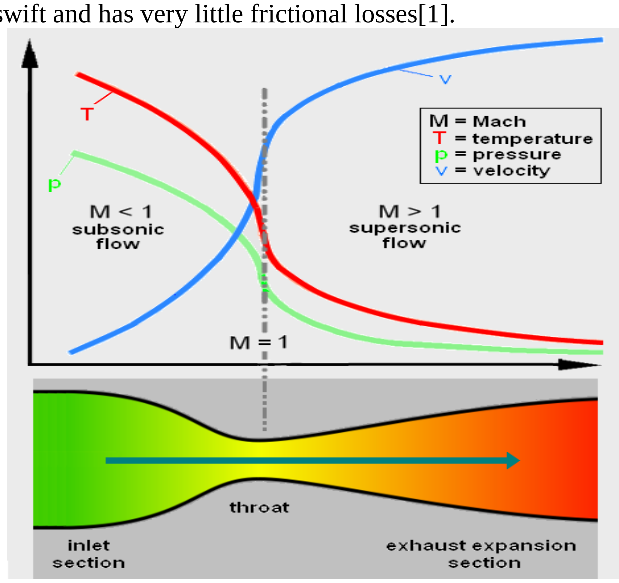


Figure 3. Graphic represents gas flow via a de Laval nozzle used to achieve supersonic velocity of rocket engine exhaust gases[3].

III. NOZZLE DESIGN ISSUES

There are various nozzle configuration conundrums, hence are the mentioned below[6].

A. *Equilibrium*

i. Mechanical:
- Needed to define an equilibrium pressure
- Very fast compared to the other time scales

ii. Thermal:
- The internal degrees of freedom of the gas is linked to relaxation times.
- When compared to previous periods, the rotational relaxation time is quick.
- When compared to rotational relaxation, vibrational relaxation is slower. In rocket applications, this might be critical.
- The premise of a calorie-perfect gas fails.

iii. Chemical:
- Chemical kinetics in finite time (changing temperature and pressure)

Three scenarios are frequently considered:
- Shifting equilibrium due to fast kinetics in relation to residence time (The chemical composition of the gases corresponds to the local equilibrium dictated by the nozzle's local pressure and temperature.)
- Frozen equilibrium due to slow kinetics in relation to residence time (The chemistry of the gases is considered to remain constant.)
- The nozzle flow equations are calculated concurrently with the chemical kinetics equations in nonequilibrium kinetics.

B. *Calorifically Perfect Gas*

Chemical composition variations, temperature shifts, and vibrational non-equilibrium are all examples of vibrational non-equilibrium.

C. *Effects Of Friction*

i. Unless shock waves are present inside the nozzle, there is a favorable pressure gradient.

ii. The effects can be listed as:
- The thickness of the displacement layer affects the distribution of the nozzle area.
- The skin friction force has a direct impact.
- Shock-induced separation due to boundary layer contact.
- Viscous effects are small and generally ignored for shock-free nozzles.

D. *Effects of the 3D flow field*

Velocity at the exit plane is not parallel to the nozzle axis, because of the conical flow field.

E. *Types of nozzle geometries*

i. Conical Nozzles
- Simple design and construction
- Typical divergence angle is 15 degrees (~2% Isp loss)

$$F = mv_j(1+\cos\Theta/2)+(p_e-p_a)A_e \quad (1)$$

- 3D thrust correction can be significant

ii. Perfect Nozzle
- Method of characteristics to minimize 3D losses
- Perfect nozzle is too long

iii. Optimum nozzle (Bell-shaped nozzles)
- Balance length/weight with the 3D flow losses

iv. Plug nozzle and Aerospike nozzle
- Good performance over a wide range of back pressures.

IV. DIVERGENT THRUST LOSS CALCULATIONS FOR CONVERGENT DIVERGENT NOZZLE

Nozzle thrust losses are one of the setbacks for the nozzle to work in its best effective state. We can estimate such losses and increase their efficacy by adding correction factors to the ideal performance parameters of an ideal nozzle. Divide the momentum of all integrated axial components of the diverging flow over the nozzle exit by the momentum of the flow of an ideal nozzle where all of the exit flow is axial to determine correction factors for the non-axial flow of diverse divergent jets. We'd want to create geometry-dependent equations for the nozzle divergence coefficient to help analysts with non-ideal thrust estimations. Thus,

$$C_{\Theta}= (FG_{AXIAL}/FG_{IDEAL}) \quad (2)$$

Where C_{Θ} is one of several possible correction factors for the ideal gross momentum thrust[7].

A. Experimental Calculations

Divergence losses are normally minimal (on the order of one percent) for most nozzles, but they can account for a major portion of overall nozzle performance loss and can be fairly significant in nozzles with high degrees of divergence. The following is a traditional, mathematical way of calculating non-axial thrust losses using basic vector calculus.

Consider the axisymmetric supersonic nozzle which is adapted meaning ambient pressure is equal to chamber pressure.

However, in a nozzle, an average value of flow leaves at an angle of Θ/2. Whereas Θ is the half divergent angle. Therefore, the flow from the nozzle is not always axial, and the flow towards the wall is Θ. The axial gross momentum thrust may be calculated by multiplying the mass flow rate by the axial exit velocity or jet velocity.

$$F=\dot{m}V_j. \quad (3)$$

Assuming mass flowing along the axis;

$$F = \dot{m}\cos(\Theta/2)*V_j\cos(\Theta/2) \quad (4)$$

Therefore the thrust due to the divergence being at an angle Θ will be;

$$F=\dot{m}v_j*\cos^2(\Theta/2) \quad (5)$$

This shows that an average flow is Θ/2 and therefore the mass component along the axis is equal to mcos(Θ/2), and the velocity on an average/axial velocity is equal to $V_j\cos(\Theta/2)$. Further solving and putting trigonometric function we get:

$$F= \dot{m}V_j*(1+\cos\Theta/2) \quad (6)$$

Thus, (1+cosΘ/2) is the loss due to the divergence or also known as the divergence loss coefficient and it is denoted by λ.

$$F= \dot{m}V_j\lambda. \quad (7)$$

Now if I have to have a loss, the loss should be something different and λ is due to non-availability[9].
Not available thrust:

$$\Delta= 1-\lambda. \quad (8)$$

Now analytically verifying the losses for different divergent angles in Table I and Table II.

TABLE I. ANALYTICAL RESULTS

Half divergence angle (Θ)	Divergence loss coefficient (λ)	Loss which I have in the nozzle (Δ)
0	1	0%
5	0.9988	0.12%
10	0.9924	0.76%
15	0.9830	1.7%
20	0.9699	3%
25	0.9537	4.63%
30	0.933	6.7%

We observe [Table-1] that at the half divergent angle 20, the loss is already 1.75 times more than the half divergent angle 15.

In other words if we were to increase the angles the losses would be massive.

Now this brings us to a stage where we say that if increasing the angle would create massisve losses then why not have minimum divergent angles. Hence why not have minimum angles is stated below.

The following figure (4) shows that nozzle has a centre line and the throat radius is R_t, the exit value is R_e and Θ is the angle between L_d and (R_e-R_t)[9].

Therefore R_t is the throat radius of the nozzle, R_e is the exit radius of the nozzle, L_d is the length of the divergent section and Θ is the half divergent angle.

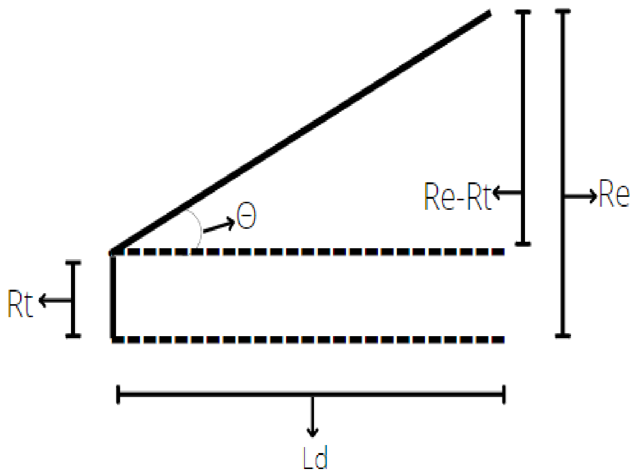


Figure 4. Divergent section of the Nozzle

In other words the value of (R_e-R_t) divided by L_d is equal to the tangent of Θ.

$$(R_e-R_t)/L_d = \tan\Theta \qquad (9)$$

Meaning if we have a small angle nozzle for the same exit diameter we will have a much longer one, if the angle is zero the length of the nozzle would be infinite [Table-2]. Thus the weight of the entire rocket will drastically increase. Which is one of the factor we keep in mind during the manufacturing of rockets that the weight should be at its minimum.

TABLE II. DIVERGENT THRUST LOSSES WITH LENGTH OF DIVERGENT SECTION

Half divergence angle (Θ)	Divergence loss coefficient (λ)	Loss which I have in the nozzle (Δ)	Cot Θ= $L_d/(R_e-R_t)$
0	1	0%	Infinite
5	0.9988	0.12%	11.43
10	0.9924	0.76%	5.67
15	0.9830	1.7%	3.73
20	0.9699	3%	2.75
25	0.9537	4.63%	2.14
30	0.933	6.7%	1.73

Therefore based on this divergence analysis we can easily determine that a conical nozzle will normally have a semi divergence angle of 15 which is the optimum angle.

V. COMPUTATIONAL METHOD

For this computational analysis the SST k- omega turbulence model was devised. Hence this model is a two equation eddy viscosity model that is used for aerodynamic applications[8].

SST k-omega Governing Equations are as follows;

Turbulence Kinetic Energy

$$\partial k/\partial t + U_j(\partial k/\partial x_j) = P_k - \beta^* k\omega + (\partial/\partial x_j)[(v + \sigma k v_T)^*(\partial k/\partial x_j)]$$

(10)

Specific Dissipation Rate

$$\partial\omega/\partial t + U_j(\partial\omega/\partial x_j) = \alpha S^2 - \beta\omega 2 + (\partial/\partial x_j)[(v + \sigma\omega v_T)^*(\partial\omega/\partial x_j)] + 2(1-F_1)\sigma\omega 2(1/\omega)^*(\partial k/\partial x_i)^*(\partial\omega/\partial x_i)$$

(11)

F1 (Blending Function)

$$F_1 = \tanh\{\{\min[\max((k^{1/2}/\beta^*\omega y),(500v/y^2\omega)),(4\sigma_{\omega 2}k/CD_{k\omega}y^2)]\}^4\}$$

(12)

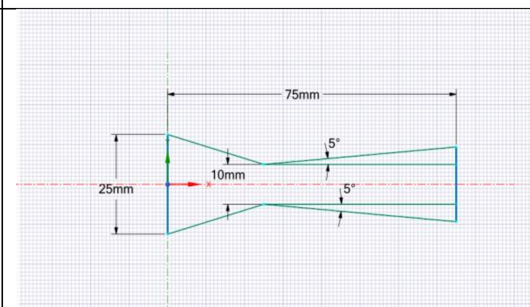
VI. ANSYS WORKBENCH STUDENT VERSION 2022

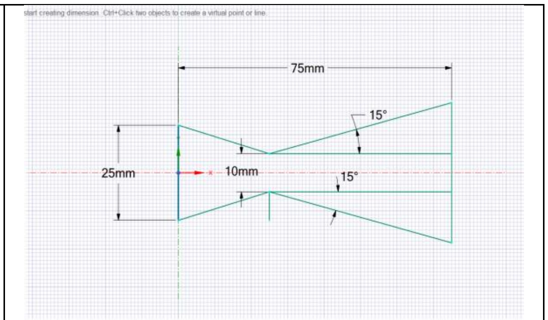
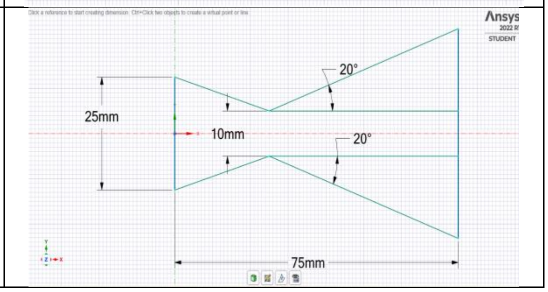
Ansys Workbench Student Version 2022 software was used to design and produce the computational results for the different half divergent angles.

A. *Design Methodology*

The modelling of the De Laval rocket nozzle is based on earlier research. The spaceclaim geometry modeler of Ansys workbench 2022 student version was used to design the nozzles[5, 10] as shown in Table III.

TABLE III. NOZZLE DESIGNS

Half Divergent Angles (Θ)	Design
5	

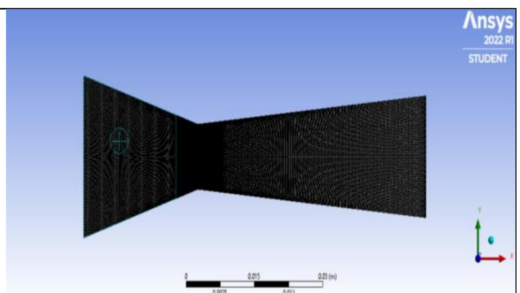
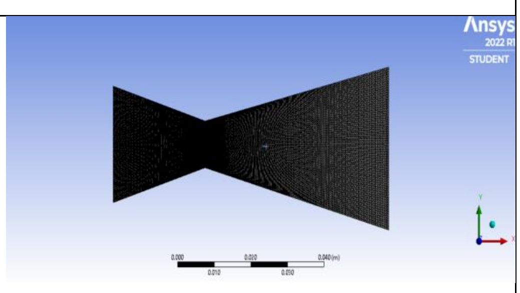
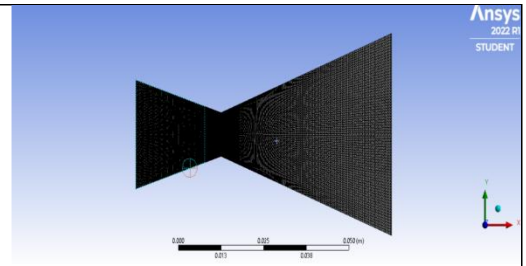
15	
20	

B. MESHING AND SETUP

Edge sizing and face meshing type was implemented to get a perfect structured mesh and to further put up the setup details to achieve solutions smoothly.

In TableIV, we have mentioned all the mesh and setup details.

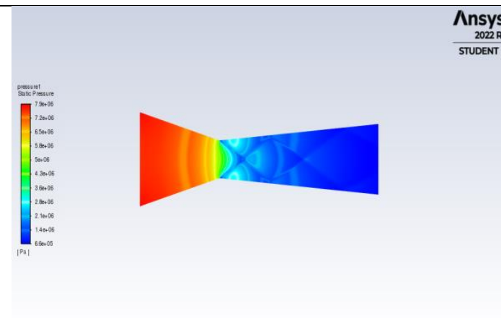
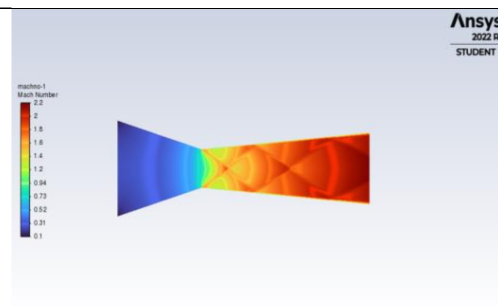
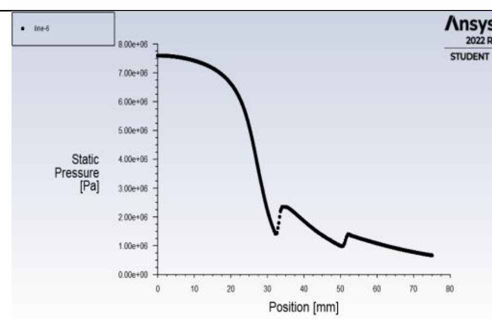
TABLE IV. MESHING AND SETUP

Meshing for 5 Angle Nodes-22176 Elements-43594 Meshing Method-Triangles: Best Split	
Meshing for 15 Angle Nodes-22104 Elements-43452 Meshing Method-Triangles: Best Split	
Meshing for 20 Angle Nodes-22032 Elements-43310 Meshing Method-Triangles: Best Split	
Setup	K- ω Model Energy Equation: On
Material	Air-Ideal Gas
Boundary Conditions	Pressure Inlet- 80 Bar Pressure outlet- 12.987 Bar Temperature- 1583 Kelvin

C. Computational Results

The results produced after the CFD analysis for the nozzles at different half divergent angle (Θ) 5, 15, 20 are represented in Table V, Table VI and TableVIIrespectively.

TABLE V. FOR HALF DIVERGENT ANGLE 5

Pressure Contours	
Mach No. Contours	
Pressure Graph Maximum Pressure: 7.9e+06 Pa Minimum Pressure: 6.6e+05 Pa	

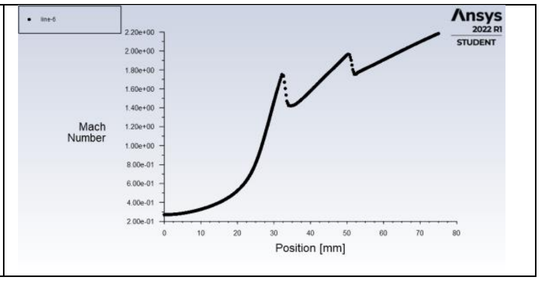
Mach No. Graph Maximum Mach Number: 2.2	

TABLE VI. FOR HALF DIVERGENT ANGLE 15

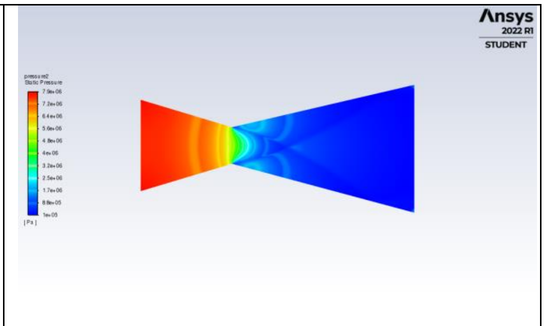
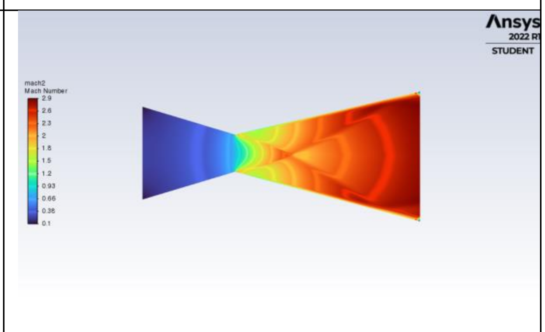
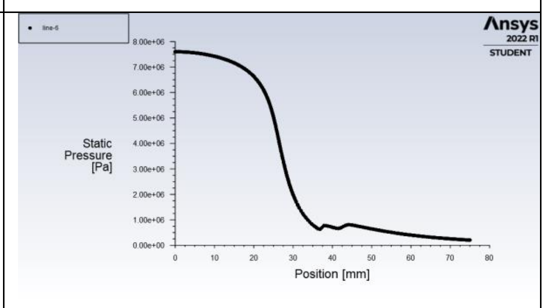
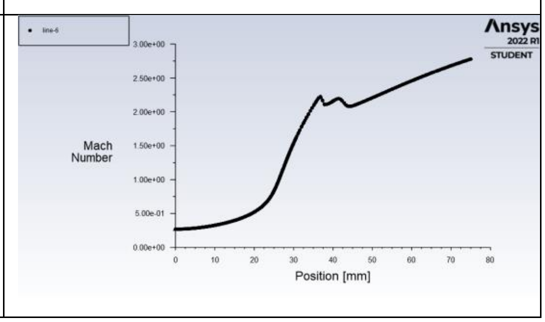
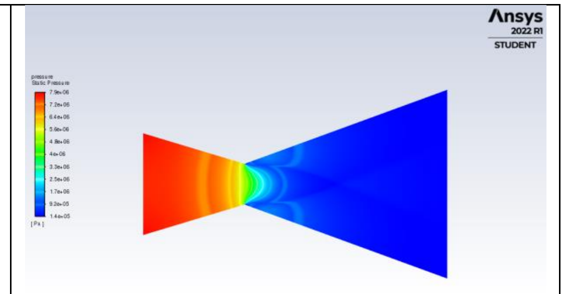
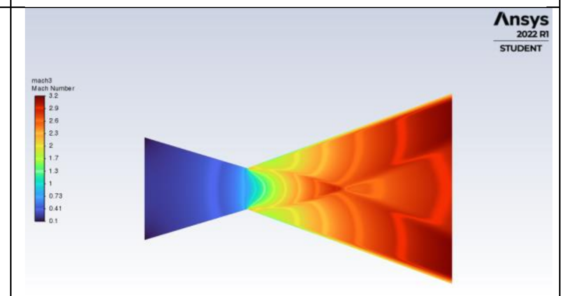
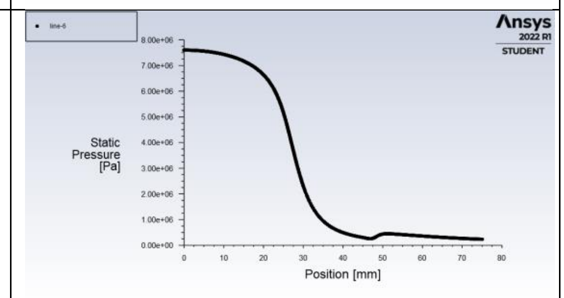
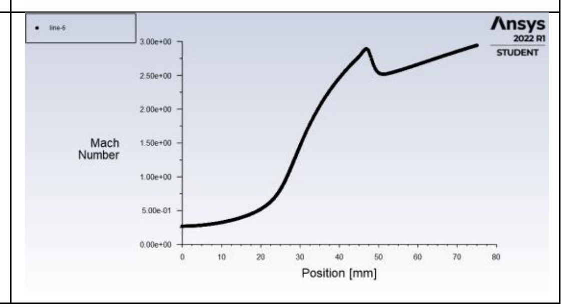
Pressure Contours	
Mach No. Contours	
Pressure Graph Maximum Pressure: 7.9e+06 Pa Minimum Pressure: 1e+05 Pa	
Mach No. Graph Maximum Mach Number: 2.9	

TABLE VII. FOR HALF DIVERGENT ANGLE 20

Pressure Contours	
Mach No. Contours	
Pressure Graph Maximum Pressure: 7.9e+06 Pa Minimum Pressure: 1.4e+05 Pa	
Mach No. Graph Maximum Mach Number: 3.2	

VII. CONCLUSION

The following results firstly showcase the flow through the nozzle which successfully satisfies the working of the nozzle meaning that pressure and velocity relation in convergent and divergent sections respectively. Therefore, showing that the pressure and temperature of gases are more in the convergent section whereas velocity is less in the convergent section. However, when the flow reaches the throat of the nozzle the velocity of the flow is already at Mach-1 or at the speed of the sound hence the velocity of gases in the divergent section is more, and pressure and temperature are less in the divergent section.

We also observed that as the divergent angles were increased the Mach number was also increased hence distinguishing that if the angle would be more the velocity acquired would be more, therefore, compromising the losses at the divergent sections, meaning loss of thrust as the angles were incremented.

Lastly, the half divergent angles(Θ) at 20 is showing promising results but contradict the thrust losses at divergent angles. Then we come to nozzles with lesser half divergent angles like Θ=5 and so, the computational result verifies that if nozzles with lesser angles were used, they would not achieve greater velocity because of the cycle of oblique shock waves generated, and when there is the formation of oblique shock waves there is sharp fall in the velocity. In addition to that, if we kept on minimizing the angles, the length of the nozzle will keep on increasing thus increasing the entire weight of the rocket which is not a sufficient outcome.

Hence this paper verifies using the experimental and computational methods that half divergent angle(Θ) at 15 is the optimum angle at the divergent section. Thus both methods are in accord.

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The authors also wish to show gratitude towards ANSYS for providing with the wonderful tool for CFD analysis.

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