Research in Emerging Financial Technologies

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RESEARCH IN EMERGING FINANCIAL TECHNOLOGIES

by

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Abstract

In chapter one we investigate the price clustering of non-fiat cryptocurrency exchange rates or the pricing of items in cryptocurrency such as bitcoin, which has been accepted as payment at a growing list of companies. For litecoin, a non-fiat currency, priced in terms of satoshi, one hundred millionth of a bitcoin, over 35% are priced at 100 satoshi increments, providing support for the negotiation hypothesis. There is also strategic pricing at 1 satoshi below or above the 100 satoshi increments. At the transaction level, we find that prices are mainly formed due to negotiations and strategic trading, instead of based on psychologically appealing numbers in the order of 0, 5, and others.

In the second chapter we examine commonality in returns and liquidity (trading volume) for Bitcoin-fiat currency pairs, each trading on an exchange in a country with a single time zone. We find evidence that one common factor explains about 54% of the variance in hourly trading volume. We find strong support for the presence of a microstructure-noise volatility multiplier. Volume is higher on local exchanges during local working hours, reflecting a pattern also seen in forex markets, and supporting the view that trading patterns depend on the location of trade rather than the location of the asset being traded.

In the final chapter we use the distribution from Benford's Law to investigate whether fake volume is reported for five bitcoin exchanges that are either regulated by the US Department of Treasury or have licenses from the State of New York and three exchanges that are not so regulated. Using counts of first digits, counts of second digits, and sums of numbers beginning with the same first two digits, we find that the distribution of minute-level volume of regulated

exchanges deviate less from Benford's expected distribution than the remaining three exchanges. We find that the proportion of first digits deviate less for the Bitstamp, Coinbase, and ItBit exchanges, justifying their use as the basis for the index price for CME Bitcoin Futures contracts (BTCA).

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Chapter 1:

Intra-day Price Behavior of Cryptocurrencies

1. Introduction

Price clustering, the non-uniform distribution of the last digit(s) of prices, is observed in many markets. Over 10% of bitcoin USD daily prices end with 00 (Urquhart (2017)). Clustering is reported for stocks (Osborne (1962)), gold (Ball, Torous and Tschoegl (1985)), foreign exchange (Sopranzetti and Datar (2002)), and derivatives (Schwartz, Van Ness and Van Ness (2004)).

Unlike fiat money, cryptocurrencies are independent of a central authority and use encryption methods and a distributed ledger to control currency generation and fund transfers.

We extend previous research by examining the price clustering of non-fiat cryptocurrency exchange rate pairs for the first time. In addition to non-fiat currency pairs, we examine intraday exchange rates between non-fiat currency and fiat currencies other than USD. And in contrast to the coarser daily closing prices used in Urquhart (2017), we use transactions data. Through detailed analysis of millions of intraday transactions, we shed light on clustering in both fiat and non-fiat currencies.

For intraday transactions of the cryptocurrency litecoin, over 35% are at 100 satoshi increments.[1] There is also strategic pricing at 01 and 99 and frequencies at round numbers such 10 through 90 are elevated. The first-order clustering is on 100s, the second-order clustering is on strategic pricing (01 and 99), and the third-order clustering is on rounded 10s.

2. Hypotheses and Data

We test three hypotheses about the occurrence of ending digits in trading prices.

[1] The minimum unit for bitcoin is the satoshi, one hundred millionth of a bitcoin. At 1 bitcoin = 10,000 USD, a satoshi is about one hundredth of a penny.

The price negotiation hypothesis posits that round numbers facilitate negotiations by making trading faster and reducing search costs through a coarser price grid (Harris 1991). According to this hypothesis, price clustering increases with price level and pricing uncertainty.

The attraction hypothesis (Wadhwa and Zhang (2015) indicates that people use psychologically appealing round numbers that are easier to recall so that ending digits of 0 followed by 5 are preferred.

The strategic trading hypothesis predicts that traders seek to gain advantage by placing orders immediately above or below round prices (Sonnemans (2006). For example, when prices cluster at 100, strategic traders place orders to buy at 99 and sell at 101.

The Bitctamp exchange provides us the complete transaction history for bitcoin (BTC), litecoin (LTC), and ripple (XRP) trades on their exchange. Our exchange rates and sample periods are LTC/BTC (6/16/2017–2/28/2018), XRP/BTC (2/10/2017–2/28/2018), BTC/USD (8/17/2011– 2/28/2018), and BTC/EUR (4/16/2016–2/28/2018).

3. Results and discussions

Fig. 1 plots the transaction frequency for prices ending in 00 through 99. For all four pairs, the frequency of prices ending in 00 is the highest, followed by prices ending in 01 and 99. We observe higher clustering in round numbers from 10 through 90 for XRP in Fig. 1 (b) relative to the other exchange rates. This anomaly is likely because XRP is priced at an average of 9,119 satoshis, much lower relative to 154,5940 satoshis per LTC. Fig. 2 plots the intraday variations in price clustering by hourly intervals for ending digits 00, 01, and 99 for LTC/BTC. The clustering on these ending digits persists over the 24-hour day.

Table 1 reports the observed frequencies of ending digits of interest and their expected frequencies. Relative to the 1% expected frequency of ending digit 00, the observed frequency is 35.07% for LTC/BTC, 6.68% for XRP/BTC, 18.03% for BTC/USD, and 27.52% for BTC/EUR.

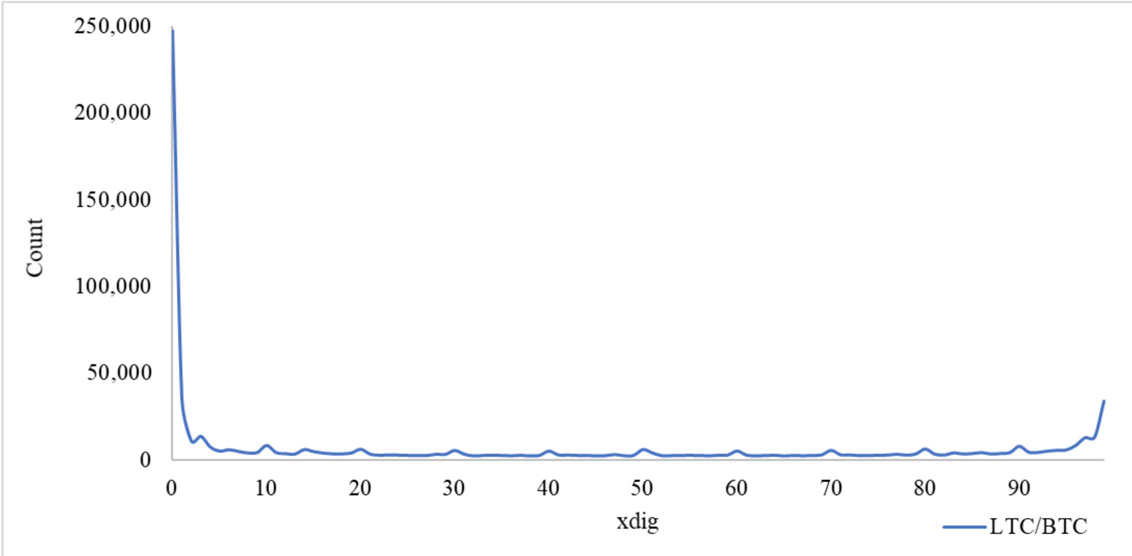
Both these observed frequencies and the frequencies of ending digits 000 and 0000 are statistically significant from the expected values at the 1% level. For example, 21.55% of LTC transaction prices end at 000, over 200 times higher than the 0.10% expected frequency. Therefore, we find significant price clustering at round numbers 00, 000, and 0000, providing support for the negotiation hypothesis. The observed frequencies 1 unit above or below round number 00 are significantly higher than their expected frequencies. For example, for LTC/BTC, the observed frequencies for ending digits 01 and 99 are 4.93% and 4.81%, respectively, over four times higher than their 1% expected frequency. These results support the strategic trading hypothesis. Lastly, we do not observe higher frequencies relative to its expected frequency for prices ending on 5. These results do not support the attraction hypothesis.

As an additional test, we partition the sample into low, medium, and high, based on price level and monthly price volatility, in turn. The negotiation hypothesis predicts higher clustering for higher prices and price volatility while the attraction hypothesis does not. Table 2 reports a monotonic increase in clustering on ending digit 00 as prices increase for all four exchange rates, providing additional support for the negotiation hypothesis but not the attraction hypothesis. For ending digits 01, 99, and 05, we do not find any persistent patterns. The results are similar when we partition the sample based on price volatility.

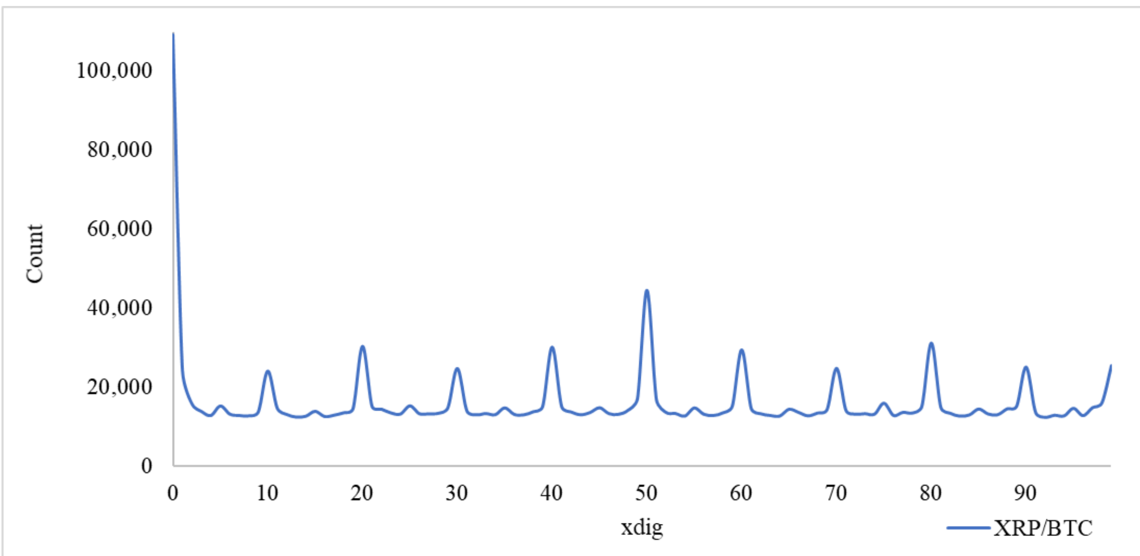
Let xdig be the remainder after dividing prices by 100. Following Ball, Torous and Tschoegl (1985), Table 3 reports the results of regressing the percent of transactions at each xdig against dummy variables, Di, where Di = 1 when i = xdig and 0 otherwise for the following values of i: 00, 01, 05, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 99. The coefficients are statistically significant at the 1% level for D00, D01, and D99 in all four regressions. The coefficients are not significant for D05. These results provide further support for both the negotiation and strategic trading hypotheses, but not the attraction hypothesis.

4. Conclusion

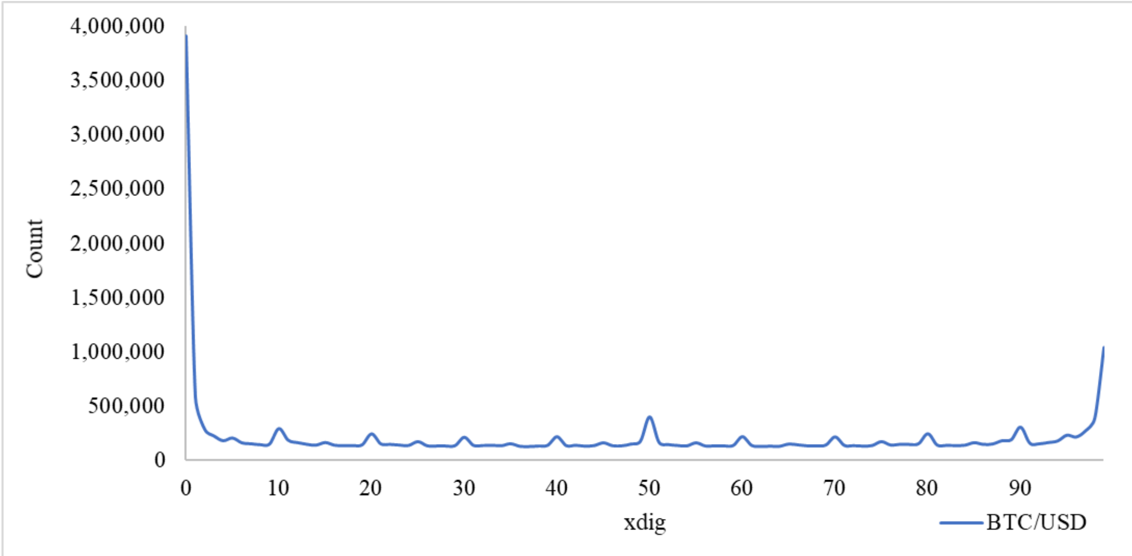
For four pairs of fiat and non-fiat cryptocurrency exchange rates, we document significant price clustering at round numbers 00, 000, and 0000, providing support for the negotiation hypothesis. We also find evidence of strategic trading with high observed frequencies 1 unit above and below 00 ending prices. Our results support the negotiation and strategic trading hypotheses. However, we do not find support for the attraction hypothesis.



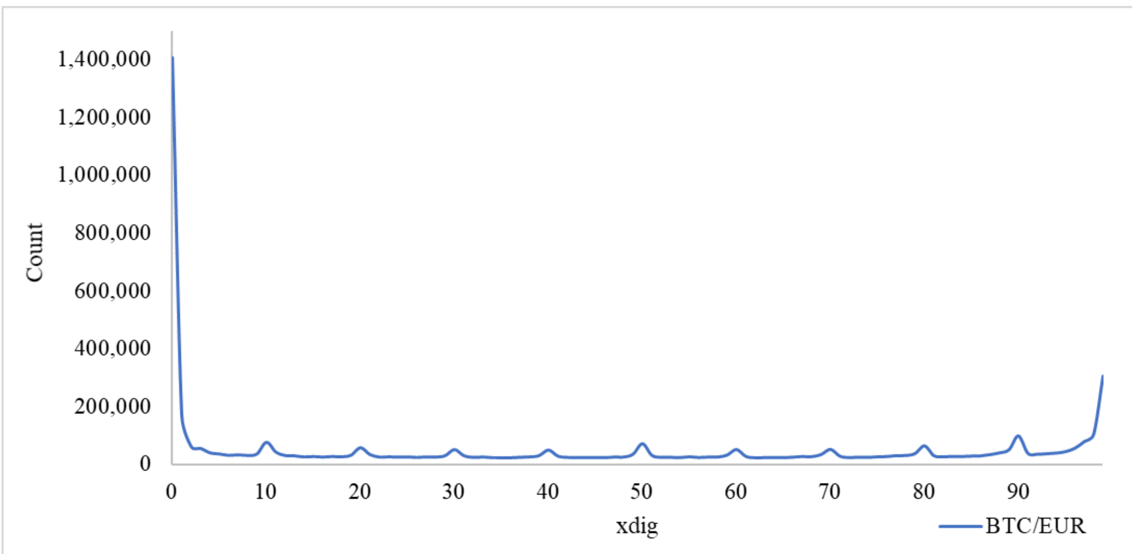
(a) **LTC/BTC**



(b) **XRP/BTC**



(c) BTC/USD



(d) BTC/EUR

Fig. 1. Price distributions for fiat and non-fiat cryptocurrency exchange rates.
We present the ending digits for the following currency pairs: (a) LTC/BTC, (b) XRP/BTC, (c) BTC/USD, and (d) BTC/EUR. Prices for bitcoin are in satoshi and for EUR and USD in cents. xdig is the remainder after dividing prices by 100.

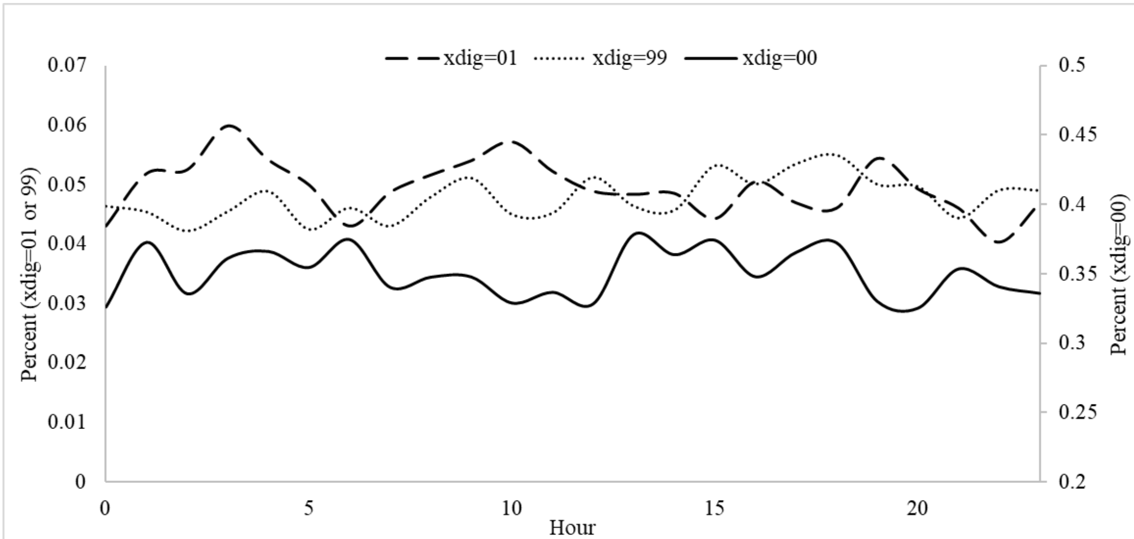


Fig. 2. Intraday variations in price clustering.
For LTC/BTC, for each hour of the day, we plot the proportion of prices with ending digits of 00, 01, and 99. xdig is the remainder after dividing prices by 100.

Table 1
Price clustering for fiat and non-fiat cryptocurrency exchange rates.
We present the percentage of observations for ending digits listed in Column 1 for the following currency pairs: LTC/BTC, XRP/BTC, BTC/USD, and BTC/EUR. Prices for bitcoin are in satoshi and for EUR and USD in cents. xdig is the remainder after dividing prices by 100; 1,000; or 10,000. Column 2 presents the expected percentage for a uniform distribution. All observed percentages except for xdig = 05 are higher than the expected percentage at 99% confidence interval.

xdig	Expected Percent	LTC/BTC	XRP/BTC	BTC/USD	BTC/EUR
00	1.00%	35.07%	6.68%	18.03%	27.52%
01	1.00%	4.93%	1.48%	2.72%	3.27%
99	1.00%	4.81%	1.55%	4.80%	5.96%
05	1.00%	0.72%	0.93%	0.96%	0.69%
000	0.10%	21.55%	1.09%	3.06%	13.02%
001	0.10%	2.54%	0.19%	0.31%	1.30%
999	0.10%	2.61%	0.22%	0.60%	2.56%
0000	0.01%	7.33%	0.13%	1.01%	4.28%
0001	0.01%	0.63%	0.02%	0.08%	0.33%
9999	0.01%	0.79%	0.03%	0.17%	0.73%
Nobs		705,219	1,634,174	21,698,310	5,107,891

Table 2

The impact of price level and volatility on price clustering.

We partition the sample into low, medium, and high, based on price level (Columns 2-4) and monthly price volatility (Columns 5-7). We present the percentage of observations for ending digits listed in Column 1 for the following currency pairs: LTC/BTC (Panel A), XRP/BTC (Panel B), BTC/USD (Panel C), BTC/EUR (Panel D). xdig is the remainder after dividing prices by 100.

	Price			Price volatility		
xdig	Low	Medium	High	Low	Medium	High
Panel A: LTC/BTC						
00	32.63%	35.85%	36.74%	30.12%	32.72%	36.74%
01	5.38%	5.01%	4.39%	4.15%	5.20%	4.61%
05	0.80%	0.69%	0.66%	0.71%	0.80%	0.71%
99	4.22%	4.98%	5.23%	3.82%	6.54%	4.62%
Panel B: XRP/BTC						
00	4.11%	6.23%	9.69%	3.49%	6.96%	10.05%
01	1.21%	1.42%	1.81%	0.96%	1.41%	1.75%
05	0.94%	0.93%	0.92%	0.75%	0.92%	0.96%
99	1.28%	1.53%	1.86%	0.95%	1.56%	1.99%
PanelC:BTC/EUR						
00	26.06%	27.79%	28.72%	20.26%	28.16%	26.86%
01	2.94%	3.03%	3.82%	2.13%	3.19%	3.12%
05	0.72%	0.59%	0.74%	0.72%	0.82%	0.67%
99	6.08%	5.31%	6.50%	6.67%	6.48%	5.82%
Panel D:BTC/USD						
00	10.30%	20.68%	23.12%	4.67%	9.81%	19.67%
01	1.79%	3.08%	3.29%	1.24%	1.72%	2.92%
05	1.07%	0.95%	0.87%	1.81%	1.02%	0.91%
99	2.46%	4.31%	7.63%	2.10%	2.56%	4.55%

Table 3
Test of price clustering.
We test whether prices cluster on particular ending digit for the following currency pairs: LTC/BTC, XRP/BTC, BTC/USD, BTC/EUR. Let xdig be the remainder after dividing prices 100. We regress the percent of transactions at each xdig against dummy variables, Di, where Di = 1 when i = xdig and 0 otherwise for the following values of i: 00, 01, 05, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 99. *, **, and *** indicate significance level at 10%, 5%, and 1% levels, respectively.

Variable	LTC/BTC	XRP/BTC	BTC/USD	BTC/EUR
Intercept	0.0054***	0.0195***	0.2213***	0.0415***
D00	0.3454***	0.1352***	5.3273***	1.9520***
D01	0.0439***	0.0148***	0.6158***	0.1951***
D05	0.0018	0.0021	0.0738	0.0082
D10	0.0065**	0.0146***	0.1962***	0.0657***
D20	0.0034	0.0235***	0.1273**	0.0390**
D30	0.0024	0.0155***	0.0825	0.0298
D40	0.0019	0.0232***	0.0896	0.0273
D50	0.0032***	0.0436***	0.3481***	0.0588***
D60	0.0020	0.0222***	0.0906*	0.0301
D70	0.0025	0.0155***	0.0875	0.0305
D80	0.0036	0.0246***	0.1290**	0.0481**
D90	0.0058*	0.0160***	0.2144***	0.0970***
D99	0.0428***	0.0165***	1.2561***	0.3905***
Adj R^2	0.9916	0.9909	0.9904	0.9911

Chapter 2:

Insights from Bitcoin Trading

Satoshi Nakamoto created Bitcoin, the first and the most widely used cryptocurrency, in 2008 to facilitate and reduce the cost of transferring funds.[1] In ten years, the market capitalization of Bitcoin has grown spectacularly from zero to more than 100 billion USD as of October 2018 and daily 24-hour volume of over 3 billion USD.[2] Business and government leaders such as Bill Gates and Janet Yellen have highlighted the importance of Bitcoin and blockchain technology for the future of the U.S. economy.[3]

Bitcoin markets offer a unique opportunity to test several constructs in microstructure theory related to commonality, liquidity, and price discovery. Bitcoin merits study because it has a number of interesting features in terms of market participants, trading platforms, trading hours, global fungibility, and availability of international trading data that allows us to shed new light on microstructure noise, and market participant preferences and biases that affect price discovery.

Unlike most financial assets that are dominated by institutions, Bitcoin features high levels of retail participation from individuals who are major players in Bitcoin trading. As of October 2018, 48.4% of those engaged with Bitcoin are between the ages of 25 and 34 and about 25% are between the ages of 35 and 44.[4] The median transaction value on 10

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[1]Satoshi Nakamoto, Bitcoin: A peer-to-peer electronic cash system. From https://bitcoin.org/bitcoin.pdf
[2] https://coinmarketcap.com/currencies/bitcoin/ on 10/24/2018.
[3] See: https://www.cnbc.com/2017/12/19/bill-gates-in-2014-bitcoin-is-better-than- currency.html; https://www.coindesk.com/us-fed-yellen-blockchain-impact/
[4] Considering those 18+ only; From https://coin.dance/stats/age

February 2018 was 543 USD.[5] Many individuals trade Bitcoins on peer-to-peer networks that call themselves exchanges. Just one exchange that trades Bitcoin, Coinbase/GDAX, has more than 500 employees and over 13 million accounts,[6] which is more than the large brokerage firm Charles Schwab. Many of the exchanges operate lit limit orders books that enforce price and time priority.[7] Like foreign exchange, but unlike most financial assets, Bitcoin trades 24 hours a day. However, Bitcoin also trades 7 days a week, but there is very little weekend trading in the forex market (Huang and Masulis, 1999). The combination of 24/7 trading and young, individual traders combined with the availability of limit order book data typically seen in equity markets makes Bitcoin an attractive vehicle to revisit several issues previously considered in the finance literature.

Several unique features of Bitcoin affect its commonality of price, returns, and volumes across many fiat currencies. Unlike equities, debt instruments, and forex, Bitcoin has no home market and is not issued by a specific sovereign authority. Storage costs are low and the cost of sending Bitcoins from one location to another around the world are also low (Bitwise Asset Management, 2019).

Initially, we focus on the behavior of Bitcoin prices and returns. We find strong evidence for the propensity of the law on one price to hold. Corwin-Schultz (2019) spreads and Amihud's (2002) illiquidity measure are significantly lower than for equities with comparable trading volume, indicating that the Bitcoin market is tight as discussed by Foucault, Kadan, and Kandel (2005).[8] We find no evidence that Bitcoin prices depreciate

[5] From https://bitinfocharts.com/bitcoin/

[6] Cheng, Evelyn, (2017); https://en.wikipedia.org/wiki/Coinbase

[7] Some exchanges act as dealers so that customers can use market orders without interacting directly with the limit order book.

[8] ~~For reasons we elaborate on below,~~ all of our results are based on trading for four currency pairs: GBP/BTC, JPY/BTC, KRW/BTC, and ZAR/BTC.

during local trading hours and appreciate during non-trading hours as reported for foreign exchange by Breedon and Ranaldo (2013). We find strong support for the proposition of Goettler, Parlour, and Rajan (2005, 2009) that microstructure noise acts as a volatility multiplier.

We also test a number of additional hypotheses related to the number of Bitcoins traded (volume), which plays an important role in commonality, liquidity, and price discovery. Hasbrouck and Seppi (2001) and Korajczyk and Sadka (2008), among others, have shown commonality in the order flows for equities. We find that the correlation of synchronous trading volume between each of our Bitcoin-fiat currency pairs ranges from 46% to 72%. Using factor analysis, we find that our four trading series load on one factor, providing more evidence of commonality for the contemporaneous Bitcoin volume. But, while there is substantial commonality in synchronous trading volume across exchanges around the world, a there is also a substantial local element in trading volume.

We test whether Bitcoin markets exhibit the properties of limit order book (LOB) dynamics as modeled in the price discovery literature for other markets. Parlour (1998) and Goettler, Parlour, and Rajan (2005, 2009) provide theoretical predictions about the dynamics of the LOB in equity markets. We focus on the relation between volume and volatility. Specifically, we test and confirm that there is higher trading volume when asset prices are more volatile, which according to Goettler, Parlour, and Rajan (2009) could reflect that speculators supply less liquidity and instead demand more liquidity under these market conditions.[9]

[9] Microstructure noise is deviation of transaction prices from estimated fundamental value. The need for models that address the unique feature of cryptocurrency markets is illustrated by Goettler, Parlour, and Rajan (2009) who define fundamental value as the expectation of the present value of

Microstructure literature has also highlighted day-of-the-week and time-of-the-day effects in addition to the importance of location of trade for price discovery and commonality. In contrast to the findings of Bollerslev and Domowitz (1993) for forex markets, we reject the hypothesis of equality of mean hourly volume across weekday days, but volume is similar for Saturdays and Sundays for three of our currency pairs. We find significantly more trading on weekdays than on weekends, which is the same pattern found in the forex markets. But, unlike the forex market, there is still substantial volume on the weekend, likely reflecting the large participation by individuals.

Volume of trading is also higher during working hours despite the fact that Bitcoin can be traded 24/7, which is directly relevant for multi-market price discovery and commonality. Our results support the findings of several previous studies, including those of Pirinsky and Wang (2006) who report strong comovement in the stock returns of firms headquartered in the same geographic area. Further, these authors show that when a company changes the location of its headquarters stock price comovements with stocks from the old location decrease and those with stock in the new location increase. Support is also provided for Chan, Hameed, and Lau (2003) who show that trading patterns depend on the location of trade rather than the location of the asset being traded and for Froot and

future cash flows…." Bitcoin does not have expected future cash flows. Nevertheless, we expect many ideas developed for traditional markets to apply to cryptocurrencies.

Dabora (1999) who show that the prices of twin stocks are correlated with the market on which they trade despite the fact that they have common cash flows.

I. Bitcoin background

Satoshi Nakamoto created Bitcoin as a way to transfer funds using blockchain technology. Nakamoto's innovation solved the double-spend problem without the need for a trusted third party.[10] Bitcoin trades are final when they are added to the blockchain. A block is a unit of information about the payer and recipient of Bitcoin transfers. Individuals and firms called miners compete to be the first to solve a mathematical algorithm that allows them to link a new block to the existing chain, which comprises a record of all previous Bitcoin transfers. Miners are compensated through the creation of new Bitcoins when they are the first to add a new block to the chain. Transacting parties can also offer payments as an incentive to miners to add their transaction to the chain. The costs of mining include energy, hardware, and the miner's time.

As of October 2018, the number of Bitcoins in existence totaled more than 17 million with a market cap of over 100 billion USD. Unlike commodities or currencies, there is a hard limit of 21 million Bitcoins that can ever exist. This cap is achieved by making the compensation structure for miners a geometrically declining function of time. Bitcoins are divisible into 100 million units (called satoshi), providing a means for micro transactions. One of the most common ways of trading Bitcoins in on an exchange; the Internet Appendix lists ninety-two exchanges trading Bitcoins against fiat currencies. Exchanges

[10] The double-spend problem refers to the possibility that a counterparty can pay the same coin twice without immediate detection.

are located in scores of countries and many countries have multiple exchanges.[11] The code for Bitcoin is BTC.

II. Data

A. Sample Selection and Preparation

Bitcoin is traded on platforms called exchanges, many of which operate as displayed limit order books. Data on trading of Bitcoin can often be obtained directly from each exchange, from data providers such as Bloomberg, or from other online sites. However, the types of data available are often limited and differ from exchange to exchange. Of course, the data needed depend on the research design.

A major focus of our analysis is on when exchange participants are trading and especially whether local time is important in the timing of trades. Because of our focus on the trading behavior of Bitcoin in local time, we use data for four high-volume BTC/fiat- currency pairs for which most of the trading is likely to be in one time zone. These currency pairs (exchanges) are GBP/BTC (Coinbase Pro), JPY/BTC (Bitflyer), KRW/BTC (Korbit), and ZAR/BTC (Luno).

We do not use data for USD/BTC trading for two reasons. First, there are four time zones just within the continental US. If the work day is 8am to 5pm local time, the US workday would begin at 5am in California (8am in New York) and end at 8am in New York (5pm in California), which is half of the 24-hour day. And this ignores Hawaii and Alaska. In addition, the USD is widely used around the world. Consequently, many exchanges outside the US trade the USD/BTC pair. Furthermore, we believe that the

[11] Source: https://en.bitcoinwiki.org/wiki/Cryptocurrency_exchanges_list

likelihood that non-US persons trade the USD/BTC pair on any given exchange is higher than the likelihood that non-Japanese persons trade the JPY/BTC pair. Similar reasoning applies to the GBP/BTC and ZAR/BTC pairs.

For January-May 2018, we collect the number of Bitcoins traded (volume), high, low, and close time stamped to the nearest minute in Greenwich Mean Time (GMT). These data are from Bitcoincharts.com. Our sample period comprises 151 days. Not all currency pairs trade every hour and each series starts at a slightly different local time due to differences in GMT and local time. The mean hourly trading volume in BTC for each currency pair is JPY, 819.24; KRW, 84.51; GBP, 49.37; ZAR, 30.85; SGD, 1.20; SEK, 0.50, NZD, 0.46,

PLN, 0.25. We drop SGD, SEK, NZD, and PLN because these currency pairs trade no more than 5% of the volume of the remaining currencies.

For our analysis in local time, we adjust GMT by +2 for South Africa and +9 for both South Korea and Japan so that we can look at trading behavior in the respective country's local time. The UK is the only one of these countries that uses daylight savings time during our sample period. For the UK, we use GMT or GMT +1 when appropriate. We number the days of the week beginning with one for Monday for our day-of-the-week analysis and hours in the day beginning with zero for our time-of day-analysis. To identify the workday, for Mondays through Fridays, we divide the day into three eight-hour periods with 9am through the period ending at 5pm representing the workday.[12]

All of our statistical tests are at the 0.01 level unless otherwise stated.

[12] As a robustness check, we also use 8am to 4pm as the workday and obtain similar results.

B. Data Limitations and Fake Trades

In July 2018, Bitwise Asset Management (BAM), filed a registration statement for the first ever cryptocurrency index-based exchange-traded fund (ETF) (Bitwise Investment Advisors, 2018). As part of the registration process BAM made a presentation to the SEC in which they analyzed trading on 81 cryptocurrency exchanges and concluded that as much as 95% of reported volume on some exchanges was fake (Bitwise Asset Management, 2019; Vigna, 2019;).[13] Bitwise's presentation identifies three empirical characteristics of exchanges with fake trade data: (1) trade printing between the bid and ask, (2) multiple hours and days with zero volume, and (3) roughly an identical amount is printed every hour of every day (monotonic trading volume). Only 10 of the 81 exchanges examined did not have suspicious trades considering these three criteria. Hougan, Kim, and Lerner (2019a) updated the Bitwise analysis in an effort to identify fraudulent prints (trades printed on the exchanges tape but without an actual trade occurring) and wash trades (with related parties on both sides of the trade). Using a week of data, these authors examine trade size histograms and volume spike alignment for the 81 exchanges in the Bitwise presentation. These authors propose that the number of trades of a particular size should decline as the trade size increases and present histograms that show this pattern of the 10 "good" exchanges and but unusual trade-size distributions for exchanges with "fake" data. Hougan, Kim, and Lerner (2019a) examine the alignment of trading volume across exchanges. They observe similar patterns for each day across exchanges, especially for May 3, 2019. Exchanges reporting "fake' trades also often have unreasonably wide spreads. In contrast, spreads for the 10 "good" exchanges have low spreads, even as low as

[13] Korean exchanges are excluded from their study because of capital controls in that country.

one cent. Low spreads are possible because the exchange typically have a maker-taker fee structure in which liquidity suppliers are rewarded with negative fees and liquidity demanders pay positive fees.

Nine of the ten exchanges that Bitwise identifies as "good" are regulated by the U.S. Department of the Treasury and six of the exchanges hold a BitLicense from the New York State Department of Financial services. "Good" exchanges also invest in surveillance software.[14] We use data for the following exchanges: Coinbase Pro, Bitflyer, Korbit, and Luno (formerly called BitX). In an earlier version of the paper, we used data Coinsbank. We substituted Coinbase Pro for Coinsbank as our source for the GBP/BTC pair because because Hougan, Kim, and Lerner (2019a) identify the former as a "good" exchange and the later as an exchange with fake trades. Hougan, Kim, and Lerner (2019a) do not examine Korean exchanges because of capital controls in that country. The Luno exchange failed at least one of Hougan, Kim, and Lerner's (2019a) tests for identifying "bad" exchanges. Nevertheless, after extensive robustness testing, we have elected to retain this exchange. For much of our analysis, we present results by exchange. Hence, readers can decide for themselves whether to ignore the ZAR/BTC currency pair from the Luno exchange.

[14] An example of such investment is Binance (2019), a leading cryptocurrency exchange, patterned with Chainalysis. Good exchange tend to implement software that uses "pattern recognition, proprietary algorithms and millions of open source references to identify and categorize thousands of cryptocurrency services to raise live alerts on transactions involved in suspicious activity." (PR Newswire, 2018).14 More recently, Binance patterned with IdentityMind who's "platform enables digital currency exchanges to comply with Know your customer (KYC) and AML regulations worldwide." IdentityMind's risk and compliance platform allows real-time onboarding, transaction monitoring, and case management for digital currency exchanges. These efforts reduce manipulation and improve data quality.

III. Bitcoin Prices and Returns

A. Hypotheses

The law of one price facilitated by arbitrage should ensure that prices of Bitcoins are the same across exchanges in synchronous clock time after considering exchange rates. However, several studies find differences in price levels across exchanges. For the two years ended in February 2018, in Korea, Bitcoins traded at an average premium of 4.73% (Choi, Lehar, and Stauffer 2018), which is referred to as the "Kimchi premium." Kroeger and Sarkar (2017) show persistent, statistically significant differences between USD Bitcoin prices across exchanges. Makarov and Schoar (2018) find that there are large and recurrent price deviations, especially across countries and in times of large bitcoin price appreciation.

There is also evidence that the prices of Bitcoin are manipulated. Gandal, Hamrick, Moore, and Oberman (2018) analyze suspicious trading on the Mt. Gox Exchange that resulted in the loss of 600,000 BTC worth about $188 million to fradusters. These authors conclude that suspicious trading around this event was the cause of the spike in the USD/BTC exchange rate from about $150 to over $1,000 in late 2013. Griffin and Shams (2018) present evidence that there are suspicious purchases of BTC using Tether, a digital currency pegged to USD, following price decreases. These authors conclude that their findings are consistent with the view that Tether is used to provide price support and manipulate cryptocurrency prices. "Using algorithms to analyze the blockchain data, they find that purchases with Tether are timed following market downturns and result in sizable increases in Bitcoin prices. Less than 1% of hours with such heavy Tether transactions are associated with 50% of the meteoric rise in Bitcoin and 64% of other top cryptocurrencies. The flow clusters below round prices, induces asymmetric auto-correlations in Bitcoin, and suggests incomplete Tether backing before month-ends. Hougan, Kim, and Lerner (2019b) also discuss the SEC's concerns about manipulation of Bitcoin trading. Manipulation can break commonality or law of one price.

We investigate these competing concepts by testing whether bitcoin prices across exchanges react similarly to information in the following hypothesis:

H1: Bitcoin returns exhibit commonality across exchanges.

As pointed out by Foucault, Kadan, and Kandel (2005), a market is tight when spreads are small and deep when price impact is minimal. In limit order markets, the unique equilibrium entails efficient risk-sharing and competitive spreads according to Biais, Foucault, and Salanie (1998). Based on these insights, we test the following two hypotheses:

H2: Bitcoin markets are liquid; spreads are small and Bitcoin limit order book is deep relative to comparable assets.

We test the following hypothesis on home bias affecting price discovery patterns based of the findings of Breedon and Ranaldo (2013):

H3: For weekdays, Bitcoin prices depreciate during local work hours and appreciate outside local work hours.

In an ideal, frictionless market, all trades should occur at the fundamental value, and the microstructure noise should be identically zero. Thus, the volatility of the microstructure noise is a measure of the level of trading frictions in the market. Based on the ideas discussed in Parlour (1998) and Goettler, Parlour, and Rajan (2005, 2009), we test the following hypothesis related to liquidity and price discovery:

H4: Microstructure noise acts as a volatility multiplier.

B. Results

To test Hypothesis 1 on commonality, we calculate hourly returns as the change in log closing price from period t-1 to t and then sum to obtain daily returns. Table 1 presents Pearson correlation coefficients for each pair-wise comparison of returns in the indicated currency. All of the correlations are highly significant and exceeds 0.88, confirming that there a strong correlation in the way prices across exchanges react to information, supporting commonality of Bitcoin prices across fiat currencies.

In Table 2, Panel A, using daily low and high prices for January 2018, we estimate spreads for each of our four currency pairs using the method of Corwin and Schultz (2012).[15] This gives us thirty daily spread estimates for each currency. The Corwin-Schultz is especially suited for thinly traded assets. If markets are integrated, we expect spreads to be similar across our currency pairs. We jointly rank the estimated GBP/BTC and JPY/BTC spreads and perform a t-test on the means. This is equivalent to a Wilcoxon rank sum test. As shown in Table 2, Panel B, we cannot reject the null hypothesis of equality for any of the six currency pairs, suggesting commonality in Bitcoin spreads.

For our tests of Hypotheses 2 on liquidity, we construct comparison samples as follows. We download monthly volume for January 2018 for all regular equities from Compustat. Let GBP/BTC and JPY/BTC dollar volume be VG and VJ, respectively. For our two comparison samples, we select the thirty stocks with the smallest values of abs(V – VG) and the smallest values of abs(V – VJ), which gives us two thirty-stock comparison samples. For each sample, we download the time series of trade prices and associated volume from DTAQ. We omit the KRW/BTC and ZAR/BTC from this analysis due to thin

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[15] We do not adjust for overnight returns because Bitcoin trades 24/7.

trading at the minute-to-minute level. We estimate Schultz-Corwin spreads with adjustments for overnight returns.

In Table 2, Panel C, we present the mean and standard deviation of the Corwin-Schultz spreads for our two equity samples compared with the Bitcoin spreads from Panel A. The mean spreads are GBP/BTC, 0.0274; GBP matching equities, 0.0064; JPY/BTC, 0.0235, JPY matching equities, 0.0072. Both BTC currency pairs have significantly smaller spreads than their matching samples, supporting Hypothesis 2 that Bitcoin is more liquid and has lower spreads relative to comparable equities.

We use the Amihud (2002) illiquidity measure as our proxy for the depth of the limit order book. To compute the Amihud measure, we begin by calculating daily returns, N, as $abs(\ln(P_t) - \ln(P_{t-1}))$ where in this case P_t is the last trade price each day. Let V represent daily volume. Our Amihud measure, A, is N/V. Using these data, for each day, we calculate the Amihud measure for GBP/BTC, JPY/BTC and for each comparison sample. We present our results in Table 2, Panel D. Our mean Amihud measures (multiplied by 10 billion) are GBP/BTC, 65.28; GBP matching equities, 28,350; JPY/BTC, 2.64, JPY matching equities, 6,680. Both BTC currency pairs have significantly smaller Amihud measures than their matching samples, supporting Hypothesis 2 for depth.

To test Hypothesis 3 on Bitcoin price depreciation during local hours, we designate work hours as the eight-hour period beginning at 9am and ending at 5pm for Monday- Friday. Non-work hours are the remaining hours excluding weekends. We calculate returns as $(\ln P_t - \ln P_{t-1})$ where P_t is the Bitcoin price in local currency at the end of the workday and non-workday periods, respectively, each day. Table 3 presents the results of our analysis. We cannot reject the null hypothesis of equality of returns for any of the currency pairs. Thus, we find no evidence to support the notion of price depreciation during local trading hours.

We turn now to test Hypothesis 4 on microstructure noise. For each hourly period, let H_t, L_t, and C_t, respectively, be the highest, lowest, and last or closing price during hour t. We use closing price during the previous hour C_{t-1} to the first or open price during hour t. We exclude the first hour from our calculation to exclude the effects of overnight gap returns. Define $HLD = H_t - L_t$ and $CLD = |C_t - C_{t-1}|$. We provide the mean and standard deviations for HLD and CLD for our four currency pairs in Table 4. CLD captures the change in fundamental value during the hour. HLD includes the change in fundamental value, but also the multiplier effect of microstructure noise. For each currency pair for each hour, we subtract the value of (2 X CLD) from the value of HLD.[16] Table 4, Columns 8 and 9, report the results of the test of whether the mean of this difference is significantly different from 0 using a matched pairs t-test[17]. For each of our currency pairs, we reject the equality of HLD and adjusted CLD values. For three of our currency pairs, HLD is higher than CLD at the 0.05 level of statistical significance. Our results clearly show the presence of a volatility multiplier due to microstructure noise in these data, supporting Hypothesis 4.

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[16] See the discussion of the answer to the question "Why is the ratio of high-low range to open- close range close to 2?" https://quant.stackexchange.com/questions/1157/why-is-the-ratio-of-hi- low-range-to-open-close-range-close-to-2 While the value is not exactly equal to 2 use of 2 in this analysis should give us reasonable approximation given the sizes of the t statistics. We have reproduced this site below in the note for referee.

[17] For camparison, we also report the results comparing HLD and CLD in Table 4, Columns 6 and 7.

The law of one price is concerned with returns so that one can expect that price changes will be similar worldwide in synchronous time.[18] But the law of one price says nothing about other aspects of trading. While arbitrage affects volume of trading across exchanges, local demand can also affect volume locally regardless of what is happening on other exchanges. Hence, in the next section, we turn our attention to volume.

IV. Bitcoin Volume

Trading volume is an important barometer for an asset's liquidity. In this section we test several hypotheses on patterns of Bitcoin trading volume.

A. Volume hypotheses

Covrig and Ng (2004) review a number of studies of equity trading and conclude that it is widely documented that volume exhibits high serial correlation. Hence, we test the following hypothesis:

H5a: Bitcoin trading exhibits positive serial autocorrelation.

Using principal components analysis, Hasbrouck and Seppi (2001) provide evidence of common factors in order flows for equities. Korajczyk and Sadka (2008) provide evidence of commonality in turnover measured as the ratio of trading volume to shares outstanding. Mancini, Ranaldo, and Wrampelmeyer (2013) find strong co-movements across liquidity in different currencies and conclude that liquidity shocks affect the forex market as a whole rather than individual exchange rates. Chordia, Roll, and Subrahmanyam (2000) argue that commonality in liquidity arises from a variety of sources, including market-wide responses

[18] And this is also the reason that we expect spreads to be similar across exchanges after taking currency spreads into account.

to price movements, co-movement of trading costs, and volatility. Brockman, Chung, and Perignon (2009) find that local, exchange-level sources account for 39% of a firm's liquidity commonality and that global sources account for an additional 19%. We test the following hypothesis:

H5b: Volume of trading exhibits commonality across Bitcoin, fiat-currency pairs.

Next, we consider several additional determinants of trading volume. Bitcoin markets are volatile and use limit order format. Foucault (1999) predicts increased activity in the limit order book with more order submissions, but a lower fill rate with higher volatility. Using a microstructure framework, Andersen (1996) develops a return volatility-trading volume model in which informational asymmetries and liquidity needs motivate trade in response to information arrivals (volatility).

Based on these trade-offs, we test the following hypotheses:

H5c: Volume of Bitcoin trading is directly related to price volatility and returns.

Bollerslev and Domowitz (1993) report "remarkably similar" trading patterns across days of the week. Hence, we investigate the several patterns relative to the day and time of trading in Hypotheses 6a-6d:

H6a: There is no difference in trading volume across weekday days.

H6b: There is no difference in trading volume across weekend days.

Although forex can be traded 24/7, there is little trading on the weekend. In fact, Huang and Masulis (1999) drop weekend trades from their study due to low trading volume. Goodhart and Demos (1991) also show that there is very little forex trading on the weekend. If Bitcoin trading follows the forex pattern, we expect the following hypothesis to hold:

Bitcoin can also be traded 24/7, but unlike the forex market where individual investors are scarce, individual investors are a major factor in Bitcoin trading. For US equity markets, Lakonishok, and Maberly (1990) argue that individuals make trading decisions over the weekend and trade when the markets open on Monday. Of course, because the Bitcoin market operates 24/7 there is no need for individuals to wait to trade. We investigate whether differences in types of traders between forex and Bitcoin markets result in differences in weekday versus weekend trading, leading to the following alternate hypothesis:

H6c: Trading on weekend days is greater than trading on weekday days.

Two studies investigate the switch in the listing of Jardine Group from Hong Kong to Singapore. Lau and McInish (2003) find that individual-firm volume changes after the switch and is most closely associated with the market on which the shares are traded. Chan, Hameed, and Lau (2003) examine whether the comovement of Jardine stock returns with the Hong Kong and Singapore markets changes after the switch in trading location. To the extent that international financial markets are integrated, there would be no change. But if these markets are segmented, trading co-movement could depend on where the shares are traded. These authors find that Jardine Group stocks are correlated less (more) with the Hong Kong (Singapore) market after the switch, suggesting that location of trade matters. Froot and Dabora (1999) investigate "Siamese twin" stock that trade around the world and have pooled cash flows so that there is no difference in fundamental value. Nevertheless, the stock prices of these stocks are correlated with the markets on which they are traded most. Bodurtha, Kim, and Lee (1995) find that the stock prices of foreign country funds traded in the U.S. are heavily influenced by U.S. market movements despite the fact that their asset values are not.

Because of home bias, we argue that most of the trading in GBP, JPY, KSW, and ZAR likely occurs in the home country of each of these fiat currencies. Home bias is the propensity of individuals to invest more of their assets in their home market than elsewhere. Lau, Ng, and Zhang (2010) show that home bias exists in every country and significantly increases countries' cost of capital. Moreover, The Vanguard Group recognizes that investors have "A preference for the familiar."[19] Hence, Japanese are more likely to hold JPY than are non-Japanese so that trading JPY/BTC would be more natural for Japanese and their transaction costs trading JPY/BTC would likely be lower.

We have already discussed the literature that shows that there is very little forex trading on the weekend because forex trading is dominated by larger banks, central banks, and other institutions.

We conjecture that for our single-time-zone-currency-pairs trading is likely to be higher during the normal workday than at other times. Hence, we test the following hypothesis:

H6d: For weekdays, local volume of Bitcoin trading is higher during working hours.

C. Results for Volume

We present summary statistics for volume in Table 5. There is substantially more volume on Japanese exchange than on the other exchanges. And there is substantially more

[19] https://personal.vanguard.com/pdf/icrrhb.pdf

trading of Bitcoins in Korea than in the UK. Great Britian and South Africa have the smallest volume of Bitcoin trading. Table 6, Panel A, shows that there is substantial and statistically significant correlation of trading volume among our currency pairs. Yet, there is also substantial variation in volume among countries in synchronous time.

We turn to our tests of Hypothesis 5a. In Table 6, Panel A, we report the correlation of hourly volume for each of our four currency pairs. All of the correlation coefficients are highly significant. However, the largest coefficient is 0.7248, indicating that even for this pair there may be significant local effects determining trading volume.

In Table 6, Panel B, using minute-to-minute data for our GBP/BTC and JPY/BTC currency pairs, we present autocorrelation coefficients for lags 1 through 4, all of which are statistically significant at the 0.01 level. [20] Similarly, we present autocorrelation of lags for hourly and daily volume for our four currency pairs. Again, all of the coefficients are statistically significant at the 0.01 level. These results support serial correlation Hypothesis 5a.

To gain insight concerning Hypothesis 5b on commonality, we use factor analysis to determine the minimum number of factors that can adequately explain the variance in the hourly-trading-volume time series of our four fiat-currency/BTC pairs. We present the results in Table 7. Using the conventional eigenvalue cutoff of 1.0 yields, one factor that explains 68% of the variation, which provides strong evidence of commonality in volume across Bitcoin exchanges. We re-estimate the factor analysis forcing the retention of four factors and use a varimax rotation so that each factor is orthogonal. In this case, each

[20] There are insufficient observations at the minute level to estimate autocorrelations for the KRW/BTC and ZAR/BTC pairs.

currency loads primarily on a separate factor. We believe that this result strengthens the view that there is one primary factor that captures commonality in volume.

To test Hypotheses 5c on the relation between volume, volatility and returns, we use MKTRSK as our proxy for market risk. For each hour, we calculate Parkinson's (1980) statistic for volatility as ln(high trade price/low trade price). Daily market risk for each currency is the mean of the 24 values of hourly volatility. MKTRSK for day t is the sum of the daily risk for the four currency pairs divided by 4. Our second RHS variable is daily market return, MKTRTN. We begin with hourly return, which is the change in the natural log of the closing price from the end of the previous hour to the end of the current hour. The daily return for each currency pair is the sum of the 24 hourly returns. We calculate MKTRTN for day t as the sum of these four daily returns divided by 4. We estimate the following regression:

$$MKTVOL_t = b0\ MKTRSK_t + MKTRET_t + MKTRSK_{t-1} + MKTRET_{t-1} + \varepsilon_t \qquad (1)$$

both including and excluding the two terms for t-1 and where ε is a random error term. MKTVOL is the sum of the number of Bitcoins traded each day for our four currency pairs—GBP/BTC, JPY/BTC, KRW/BTC, and ZAR/BTC.

We present our results in Table 8. Variables are standardized using /STB in SAS, which standardizes the intercept to be zero. The coefficient of $MKTRSK_t$ is statistically significant at the 0.01 level or better in both estimations, but the coefficient of $MKTRSK_{t-1}$ is not significant when included. These results provide support for Hypothesis 5c for risk. The coefficients of $MKTRET_t$ and $MKTRET_{t-1}$ are statistically significant at the 0.01 level. These results support Hypotheses 5c.

Turning our attention to trading patterns across days, for each of our BTC-fiat currency pairs, we present the mean of hourly trading volume for each weekday (Monday through Friday), in Table 9, Panel A, and for Saturday and Sunday in Table 9, Panel B. For each currency pair, we test for equality of means for weekdays using ANOVA. For weekdays, we cannot reject the hypothesis of equality for any of the pairs at the 0.05 level, supporting Hypothesis 6a. We repeat the analysis for the two weekend days. For Saturday and Sunday, we reject the hypothesis of equality for KRW. However, we cannot reject the hypothesis of equality for the remaining three currency pairs.

Next, we investigate whether the volume of trading is the same for weekdays versus weekends. For our four currency pairs combined, we calculate the hourly volume for each day of the week. In Table 10, we present the means and standard deviations of these hourly volumes for Monday through Friday combined and for Saturday and Sunday combined in local time.[21] Unlike the forex market, there is substantial trading of Bitcoin on weekends, likely reflecting high levels of retail participation. We jointly rank the approximately (24 hours X 150 days =) 3,600 hourly volumes and test the null hypothesis of equality of the mean ranks for the weekday days and weekend days. This is equivalent to a Wilcoxon rank sum test. We reject the hypothesis of equality of volumes on weekdays and weekends.

We identify the eight hours from 9am until 5pm as working hours and the remaining hours, excluding Saturday and Sunday, as non-working hours.[22] We sum the volumes

[21] In other words, we sum across the first hour after midnight, the second hour after midnight, and so forth. We obtain similar results using GMT.
[22] Breedon and Angelo Ranaldo (2011) use a similar approach to measure working hours.

across the four currencies to produce V. For each day of the week, for V, and for each of our four currency pairs, in Table 11, we present the mean and standard deviation of hourly trading volume for working and non-working hours. We jointly rank the mean hourly values and test for equality of ranks of working and non-working hours using a t-test. For V, JPY, KRW, and ZAR, we reject the hypothesis of equality at the 0.05 level for all 20 of our currency-weekday pairs. These results provide strong support for Hypothesis 6d.

We take a closer look at these 8-hour patterns in Figure 1. For each currency, we standardize the time series of hourly trading volume by subtracting the mean and dividing by the standard deviation. We calculate the mean hourly volume for our three 8-hour periods of the day, beginning at midnight, 8am, and 4 PM, respectively. We also multiply the values for GBP by 8 for convenience in presentation. All of the currency pairs exhibit substantially reduced trading from midnight until the beginning of the workday, providing substantial support for the view that trading patterns are influenced by location of trade. The BTC/ZAR currency pair follows a pattern with high trading during the workday, substantially less trading from midnight until after 6am, and reduced, but still substantial, trading from the end of the workday until midnight. For ZAR trading during the workday and after work is more of less the same. Both the GBR and JPY pairs have high trading volume during the trading day, but even higher volume after the workday. This pattern of intra-day volume indicates high participation in Bitcoin trading by retail participants.

V. Conclusions

Similarities and unique features compared with equities and forex make the Bitcoin (BTC) secondary market an interesting vehicle to use in testing findings that have previously appeared in the literature. Like forex, Bitcoin trading occurs 24 hours a day, 7 days a week, and is worldwide.

Differences include the domination of the Bitcoin market by individuals, unlike equities and forex, which are dominated by large institutions. Also unlike other financial assets, Bitcoin has no home market.

We investigate hourly and monthly prices, returns, and trading volume for four BTC, fiat currency pairs—GBP/BTC, JPY/BTC, KRW/BTC, ZAR/BTC —and minute-to-minute observations for the first two of these. Taking advantage of these similarities and differences, we test several hypotheses.

The correlation of daily returns exceeds 0.88, indicating that there is a strong propensity for the law of one price to hold. Corwin-Schultz spreads for Bitcoin are significantly smaller than for equities with comparable trading volume. The Amihud illiquidity measure is also significantly smaller for Bitcoin than for equities with comparable trading volume. Hence, we conclude that the Bitcoin market is tight in the sense of Foucault, Kadan, and Kandel (2005) and others. We find no evidence that Bitcoin prices depreciate during local trading hours and appreciate outside these hours as found in the forex markets by Breedon and Ranaldo (2013). We find strong support for the presence of a microstructure-noise, volatility multiplier as described in Goettler, Parlour, and Rajan (2009).

We find significant correlations in trading volume for each possible combination of these four currency pairs, a further indication of commonality across markets. However, the cross correlations do not exceed 60% and are as low as 9%, indicating that there are also substantial local effects on trading volume.

Using factor analysis and the conventional cutoff of eigenvalue = 1, we find that there is only one factor and that factors explains about 54% of the variation synchronous variability in Bitcoin volume. Again, there is both a substantial common component and a substantial local component in Bitcoin volume.

We find statistically significant differences in trading volume across weekdays, but not between Saturday and Sunday for three of our currency pairs. Trading volume is higher during the week than on weekends. But unlike currency markets, there is substantial trading on Saturday and Sunday, which we believe is likely due to the trading of individuals. In addition, we find strong support for the findings of Pirinsky Wang (2006), Chan, Hameed, and Lau (2003), and Froot and Dabora (1999) that location of trade is an important determinant of trading patterns regardless of the location or characteristics of the asset traded. Bitcoin markets provide a unique opportunity to test this location hypothesis and several other microstructure theories that researchers need to revisit in the machine age Fintech world according to Easley et al. (2019). Some extensions of our study with future availability of order level data may include tests of theories (e.g., Foucault, 1999) on order placement strategies.

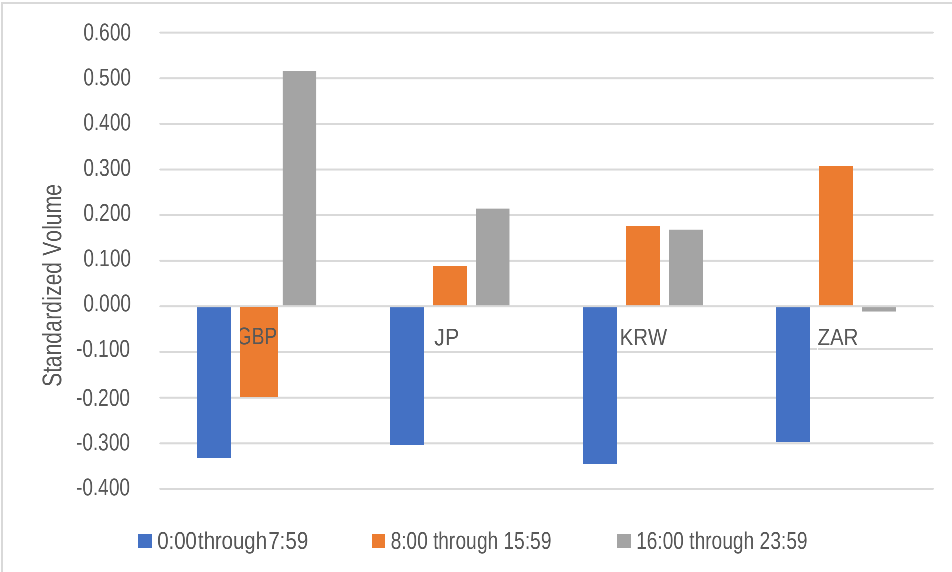

Figure 1. Standardized Hourly Volume, by Period of the Day

We examine mean standardized hourly trading volume for three 8-hour periods of the day, beginning at midnight and ending at 8am. For each currency pair—GBP/BTC, JPY/BTC, KRW/BTC, and ZAR/BTC , we standardize the time series of hourly trading volume by subtracting the mean and then dividing by the standard deviation. We calculate the mean hourly volume for our each currency pair. Hourly data is retrieved from bitcoincharts.com for January 2018-May 2018. We also multiply the values for GBP by 8 for convenience in presentation.

Table 4. Correlation of Daily Returns

We present the Pearson correlation coefficients of daily returns among each possible combination of four BTC/fiat-currency pairs—GBP/BTC, JPY/BTC, KRW/BTC, and ZAR/BTC—along with p values. Hourly returns are the change in the natural log of the closing price. We obtain daily returns by summing the hourly returns over the 24 hours of the day. n = 151. Hourly data is retrieved from bitcoincharts.com for January 2018-May 2018. Note that all of the correlation coefficients are significant at the 0.01 level.

	JPY	KRW	ZAR
GBP	0.9720*	0.8923*	0.9382*
JPY		0.9186*	0.9421*
KRW			0.8916*

↑Significant at the <0.0001 level.

Table 5. Tests of Bitcoin LOB

Panel A presents the means and standard deviations of daily Corwin and Schultz (2012) spreads for the last thirty days of January 2018 estimated using daily high/low prices. We use the same data from bitcoincharts.com. Because Bitcoin trades 24/7, we make no adjustments for overnight returns. In Panel B, we test for equality of mean spreads for each pair of Bitcoin-currency pair. We joint rank the GBP and JPY spreads and perform a t-test on the ranks. This is equivalent to a Wilcoxon rank sum test. We present the p- values in Row 2. We repeat this analysis for each of the remaining five currency pairs. In Panel C, we compare the GBP/BTC to spreads for 30 matched equities selected by minimizing the squared difference between BTC dollar volume and the equity's dollar volume during the month. We jointly rank the daily spreads and test for equality of mean ranks using a t-test. This is equivalent to a Wilcoxon rank sum test. t-statistics and p-values are presented below the means and standard deviations. We repeat the analysis for the JPY/BTC pair. Panel D presents the mean and standard deviations of the Amihud (2002) illiquidity measure. GBP/BTC and JPY/BTC prices are converted to USD for comparison to the matched US-equities. Again, we test for equality of daily means using a t-test of ranked values and present the t-statistics and p-values below the means and standard deviations.

Panel A: Corwin-Schultz daily spreads

	GBP		JPY		KRW		ZAR	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
	0.0274	0.0271	0.0235	0.0329	0.0276	0.0419	0.0169	0.0253

Panel B: Tests of equality of means for Bitcoin for Corwin-Schultz daily spreads

	GBP with			JPY with		KRW with
	JPY	KRW	ZAR	KRW	ZAR	ZAR
	65.28	59.37	28,350	43,700	2.6438	1.6427
p value	0.6233	0.9812	0.1257	0.6782	0.3825	0.2350

Panel C: Tests of equality of means for Bitcoin and Equities for Corwin-Schultz daily spreads

	GBP/BTC		Matching equities		JPY/BTC		Matching equities	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
	0.0274	0.0271	0.0064	0.0091	0.0235	0.0329	0.0051	0.0072
	t = 5.53		p = <0.0001		t = 3.01		p = 0.0005	

Panel D: Amihud measure†

GBP/BTC	Matching equities	JPY/BTC	Matching equities

	Mean	STD	Mean	STD	Mean	STD	Mean	STD
	59.63	59.37	28,350	43,700	2.6438	1.6427	6,680	10,220
	t = 65.28		p = <0.0001		t = 37.12		p = <0.0001	

†All values in Row 1 multiplied by 10 billion.

Table 6. Currency Depreciation

We define working hours as 9am to 5pm for weekdays and non-working hours as the remaining hours, excluding weekends. We present the mean and standard deviations of returns for the working and non-working hours for each of the four BTC currency pairs indicated in Column 1. We test whether the difference in mean returns for working and non-working hours are significantly different by performing a t-test of the jointly ranked observations. This is equivalent to a Wilcoxon rank sum test. The p-values are presented in the last column.

	Working hours		Non-working hours		Significance of Difference	
BTC/	Mean	STD	Mean	STD	t-value	Pr > |t|
GBP	0.00018	0.0152	-0.00045	0.0125	-0.39	0.6962
JPY	-0.00027	0.0119	-0.00031	0.0136	-0.77	0.4403
KRW	-0.00123	0.0148	0.00011	0.0130	1.32	0.1865
ZAR	-0.00065	0.0179	-0.00025	0.0154	1.34	0.1802

Table 7. Microstructure Noise Multiplier

We define HLD as the difference between the highest and lowest trading price during hour t and CLD as the absolute difference between the closing price for hours t and t-1. For each currency pair, we present the mean and standard deviation of the time series of hourly HLD and CLD values. Each hour has an observation for both HLD and CLD. We use these observations to conduct a matched pairs t-test of whether the mean values of HLD and (2 X CLD) are equal. We present the statistics for this test in the last two columns.

	HLD		CLD		Significance of (CLD)– HLD)		Significance of (2 X CLD) - HLD	
	Mean	STD	Mean	STD	t-value	Pr > |t|	t-value	Pr > |t|
GBP	110	101	58	70	-54.46	<0.0001	-3.20	0.0014
JPY	16,5623	15,220	8,176	10,586	-55.18	<0.0001	-1.00	0.3165
KRW	199,472	232,254	96,090	144,022	-42.10	<0.0001	-2.50	0.0126
ZAR	1,705	2,261	1008	15612	-33.71	<0.0001	11.09	<0.0001

Table 8. Statistics for Volume

For number of Bitcoins traded (volume), we present the number of observations, the first four moments, and maximum, median, and minimum values for two of our currency BTC pairs for minute-to-minute observations Columns 2 and 3. We present the same variables for all four of our currency pairs for hourly observations Columns 4-7 and Daily observations in Columns 8-11. We do not present minute-to-minute observations KRW/BTC and ZAR/BTC due to many missing observations.

	Minutes		Hours				Days			
	GBP	**JPY**	**GBP**	**JPY**	**KRW**	**ZAR**	**GBP**	**JPY**	**KRW**	**ZAR**
N	182,056	215,088	3,624	3,624	3,624	3,624	151	151	151	151
Mean	0.37	13.80	18.55	819.24	84.51	30.85	445.2	19,642	2,028	740
STD	0.83	16.09	21.98	547.37	84.46	43.22	345.1	8,940	1,247	665
Skew	11.32	4.487	5.01	1.98	3.13	5.56	3.05	1.65	1.41	3.38
Kurt	338.76	67.58	40.46	7.86	17.55	49.93	11.04	4.70	2.22	14.14
Max	62.53	757.4	334.74	6,667.77	1,028.34	645.55	2,152	63,537	7,062	4,403
Median	0.14	8.88	12.86	692.10	59.89	18.92	352.01	18,033	1,721	558
Min	0.00	0.01	0.00	35.32	0.00	0.10	97.52	5,696	374.4	157

Table 9. Correlation and Autocorrelation of Hourly Volume

In Panel A, we present the Pearson correlation coefficients of hourly volume among all the possible pairwise combinations of four BTC/fiat-currency pairs—GBP/BTC, JPY/BTC, KRW/BTC, and ZAR/BTC. In Panel B, we present the autocorrelations of volume for lags 1 – 5. Note that all of the correlation coefficients are statistically significant at the 0.01 level. n ≈ 3,624.

Panel A: Correlation of Volume between Currency Pairs

	JPY	KRW	ZAR
GBP	0.5902	0.4611	0.7248
JPY		0.5917	0.5475
KRW			0.5110

Panel B: Autocorrelation

	Autocorrelation of Volume				Autocorrelation of Volume			
	Lag1	Lag2	Lag3	Lag4	Lag1	Lag2	Lag3	Lag4
				Minutes				
		GBP/BTC				**JPY/BTC**		
Lag0	0.7640	0.5950	0.5231	0.4767	0.5633	0.4592	0.4200	0.3970
Lag1		0.7641	0.5950	0.5231		0.5633	0.4592	0.4200
Lag2			0.7641	0.5950			0.5633	0.4592
Lag3				0.7641				0.5633
				Hours				
		GBP/BTC				**JPY/BTC**		
Lag0	0.7640	0.5950	0.5231	0.4767	0.6968	0.5658	0.4760	0.4578
Lag1		0.7641	0.5950	0.5231		0.6969	0.5657	0.4975
Lag2			0.7641	0.5650			0.6970	0.5657
Lag3				0.7641				0.6969
		KRW/BTC				**ZAR/BTC**		
Lag0	0.6747	0.4833	0.4129	0.3803	0.7720	0.6208	0.5278	0.4553
Lag1		0.6746	0.4832	0.4128		0.7720	0.6208	0.5278
Lag2			0.6746	0.4832			0.7720	0.6208
Lag3				0.6745				0.7720
				Days				
		GBP/BTC				**JPY/BTC**		
Lag0	0.6498	0.3159	0.3012	0.3261	0.6839	0.4088	0.3756	0.3853
Lag1		0.6486	0.3141	0.2990		0.6816	0.4053	0.3706
Lag2			0.6478	0.3121			0.6800	0.4012
Lag3				0.6470				0.6783
		KRW/BTC				**ZAR/BTC**		
Lag0	0.6761	0.5017	0.4888	0.4731	0.6703	0.3217	0.2551	0.3017
Lag1		0.6732	0.4980	0.4855		0.6698	0.3209	0.2536
Lag2			0.6710	0.4950			0.6693	0.3192
Lag3				0.6693				0.6689

Table 10. Factor Analysis of Hourly Volume

In Panel A, we present the eigenvalues for a factor analysis of the time series of hourly volume of Bitcoins traded for four currency pairs—GBP/BTC, JPY/BTC, KRW/BTC, and ZAR/BTC. Only one factor is retained using the usual cutoff of eigenvalue = 1. I The first factor explains 67.9% of the variance (Row 1, Column 5). In Panel B, we present the results of estimating the factors using a varimax rotation to produce orthogonal factors. The factor loadings from the one retained factor are presented in Panel B, Column 2. We also force the retention of four factors and present the results in
Columns 3-6.

Panel A: Statistics for factor analysis

Factor	Eigenvalue	Difference	Proportion	Cumulative
1	2.7168	2.0996	0.6792	0.6792
2	0.6172	0.2111	0.1543	0.8335
3	0.4061	0.1463	0.1015	0.9350
4	0.2598		0.0650	1.0000
Sum	4			

Panel B: Factor patterns

	Factor Pattern	Rotated Factor Pattern			
Factor	1	1	2	3	4
GBP	0.8494	0.1852	0.2655	0.3556	0.8768
JPY	0.8274	0.2886	0.8979	0.2163	0.2525
KRW	0.7664	0.9276	0.2647	0.2038	0.1671
ZAR	0.8506	0.2301	0.2247	0.8783	0.3539

Table 11. Regression Analysis for Determinants of Bitcoin Market Volume

Our LHS variable, MKTVOL, is the number of Bitcoins traded (volume) for each day aggregated over our four currency pairs—GBP/BTC, JPY/BTC, KRW/BTC, and ZAR/BTC. Our first RHS variable proxies for risk. For each currency pair, hourly risk is log (high trade price/low trade price). Daily MKTRSK is the mean of hourly risk across the four exchanges. Our second RHS variable is return. For each currency pair, hourly return is the change in the natural log of the closing price and daily return is the sum of these hourly returns over 24 hours. MKTRET is the sum of these four daily returns divided by 4. We regress $MKTVOL_t$ on $MKTRSK_t$, $MKTRET_t$, $MKTRSK_{t-1}$, and $MKTRET_{t-1}$ where the t subscript represents each day. n = 150.

	Coefficien t	Pr > |t|	Std. Coefficien t	Coefficien t	Pr > |t|	Std. Coefficien t
Intercept	9,575	<0.0001	0.0000	9,186	<0.0001	0.0000
$MKTRSK_t$	36,638	<0.0001	0.8960	40,504	<0.0001	0.9933
$MKTRET_t$	32,045	0.0002	0.1630	36,977	<0.0001	0.1886
$MKTRSK_{t-1}$				-2,296	0.3598	-0.0561
$MKTRET_{t-1}$				23,285	0.0084	0.1188
Adj. R-sq.	0.7504			0.7617		

Note: The results are qualitatively the same if we equally weight the volume for each of the four exchanges or if we omit the KRW/BTC and ZAR/BTC currency pairs.

Table 12. Volume of Hourly BTC Trading, Local Time

We present statistics for the number of Bitcoins traded each hour for four currency pairs— GBP/BTC, JPY/BTC, KRW/BTC, ZAR/BTC. Panel A presents data for each individual weekday. We test the null hypothesis that the means are equal across weekdays using ANOVA. Panel B repeats the analysis for Saturday and Sunday. n≈515 for each day.

Variable	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD
Panel A: Hourly volume by weekday days										
/BTC	Monday		Tuesday		Wednesday		Thursday		Friday	
GBP	17.43	22.12	21.68	29.03	21.70	26.02	20.53	20.32	20.08	24.35
JPY	755.2	488.5	826.1	590.0	879.1	597.0	902.3	610.4	933.9	564.8
KRW	72.93	59.59	83.81	81.01	93.66	89.91	98.08	108.5	100.9	87.33
ZAR	27.88	34.00	38.61	62.82	39.01	54.00	36.67	40.91	32.61	45.12

Test of equality

	FValue	Pr>F
GBP	2.67	0.0305
JPY	7.31	<0.0001
KRW	8.60	<0.0001
ZAR	4.93	0.0006

Panel B: Hourly volume by weekend days

	Saturday		Sunday	
	Mean	STD	Mean	STD
GBP	14.10	10.54	13.98	13.02
JPY	742.4	438.1	698.2	468.3
KRW	79.31	87.06	63.38	59.95
ZAR	21.35	21.76	18.99	19.36

Test of equality

	FValue	Pr>F
GBP	0.02	0.8800
JPY	2.41	0.1210
KRW	11.55	0.0007
ZAR	3.38	0.0664

Table 13. Volume of BTC Trading, Weekdays versus Weekends, Local Time

We aggregate hourly trading volume for our four currency pairs—GBP/BTC, JPY/BTC, KRW/BTC, ZAR/BTC—for each day of the week. We present the mean and standard deviation of hourly volume for weekday days (Monday-Friday) and for weekend days (Saturday and Sunday). We jointly rank the mean hourly volumes for all days and test for equality of the means of the ranks for Monday-Friday and Saturday-Sunday using a t-test. This is equivalent to a Wilcoxon rank sum test. $n \approx 3{,}600$ for weekday days and 1,000 for weekend days.

Monday through Friday		Saturday and Sunday	
Mean	**STD**	**Mean**	**STD**
1004.3	661.0	824.9	504.5
t = -7.57 Pr > F <0.0001			

Table14. VolumeduringWorking and Non-workingHours, Local Time

We identify the eight hours from 9am until 5pm as working hours and the remaining hours as non- working hours. V is the mean hourly volume for the aggregate of our four currency pairs— GBP/BTC, JPY/BTC, KRW/BTC, ZAR/BTC. For each currency pair for each weekday, we present the mean and standard deviation for working and non-working hours. For each weekday, in turn, we jointly rank the hourly observations of V and test for equality of mean ranks using a t-test. This is equivalent to a Wilcoxon rank sum test. We repeat the analysis for each currency pair, in turn. n $\approx$ 175 for working hours and 344 for non-working hours.

Hours:		Working (W)		Non-working (N)		Stat. sig. of N - W	
	Variable	Mean	STD	Mean	STD	*t*-stat.	*p* values
Monday	V	960.4	419.1	816.7	569.7	960.4	<0.0001
	GBP	21.8	19.3	15.3	23.1	21.8	<0.0001
	JPY	813.6	397.5	725.2	527.2	813.6	<0.0001
	KRW	81.8	47.5	68.4	64.5	81.8	<0.0001
	ZAR	43.3	34.9	20.1	30.8	43.3	<0.0001
Tuesday	V	1,133.7	695.8	888.5	662.1	-3.74	<0.0001
	GBP	26.0	28.2	19.5	29.2	-5.31	<0.0001
	JPY	942.4	592.6	768.0	580.8	-4.79	<0.0001
	KRW	107.2	89.2	72.1	74.0	-6.43	<0.0001
	ZAR	58.1	87.7	28.9	42.5	-10.05	<0.0001
Wednesday	V	1,104.0	569.0	998.2	715.8	-2.42	0.0013
	GBP	27.2	31.1	18.9	22.6	-2.99	<0.0001
	JPY	916.0	555.2	860.6	616.7	-2.19	0.0430
	KRW	109.0	85.5	86.0	91.2	-4.53	<0.0001
	ZAR	51.8	60.7	32.6	49.2	-9.51	<0.0001
Thursday	V	1,145.1	479.4	1012.2	754.7	3.68	<0.0001
	GBP	26.9	28.2	17.3	17.5	-4.82	<0.0001
	JPY	948.2	490.8	877.9	661.3	-3.24	0.0015
	KRW	119.9	127.4	87.0	95.8	-4.76	<0.0001
	ZAR	50.0	42.8	29.9	38.2	-11.07	<0.0001
Friday	V	1,286.0	630.5	975.3	632.7	-4.71	<0.0001
	GBP	26.9	36.3	15.8	13.4	-3.40	<0.0001
	JPY	1,086.8	541.0	846.8	562.1	-6.51	<0.0001
	KRW	120.4	85.6	89.5	86.6	-5.49	<0.0001
	ZAR	50.1	57.5	23.9	34.4	-12.75	<0.0001

Chapter 3:

Fraudulent Bitcoin Volume

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Fraudulent Bitcoin Volume

I. Introduction

Many media outlets have recently published articles with headlines such as "95% Of Reported Bitcoin Trading Volume Is Fake, Says Bitwise."[1] In July 2018, Bitwise Asset Management (BAM) filed a registration statement for the first ever cryptocurrency index- based exchange-traded fund (ETF) (Bitwise Investment Advisors, 2018). As part of the registration process BAM made a presentation to the SEC in which they analyzed trading on 81 cryptocurrency exchanges around the world and concluded that as much as 95% of reported volume on some exchanges was fake (Bitwise Asset Management, 2019; Vigna, 2019). Nine of the ten exchanges that Bitwise identifies as good are regulated by the U.S. Department of the Treasury and six of the exchanges hold a BitLicense from the New York State Department of Financial services. Good exchanges also invest in surveillance software.[2]

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[1] https://www.forbes.com/sites/cbovaird/2019/03/22/95-of-reported-bitcoin-trading-volume-is- fake-says-bitwise/#6758ef8d6717 (For another example, see: Vigna, P., 2019, August 7, "Most Bitcoin Trading Faked by Unregulated Exchanges, Study Finds," Wall Street Journal, At: https://www.wsj.com/articles/most-bitcoin-trading-faked-byunregulated-exchanges-study-finds- 11553259600

[2] An example of such investment is Binance (2019), a leading cryptocurrency exchange, patterned with Chainalysis. Good exchanges tend to implement software that uses "pattern recognition, proprietary algorithms and millions of open source references to identify and categorize thousands of cryptocurrency services to raise live alerts on transactions involved in suspicious activity." (PR Newswire, 2018).2 More recently, Binance patterned with IdentityMind who's "platform enables digital currency exchanges to comply with Know your customer (KYC) and AML regulations worldwide." IdentityMind's risk and compliance platform allows real-time onboarding, transaction monitoring, and case management for digital currency exchanges. These efforts reduce manipulation and improve data quality.

If these allegations are true, many academic papers present results derived from data that contains high levels of fake trading. We investigate these allegations of fake volume using a tool that has been used previously to identify fraudulent accounting data— conformity to Benford's Law. In his dissertation, Nigrini (1992) uses Benford's Law to detect accounting fraud. First discovered by Simon Newcomb in 1881, forgotten, and rediscovered and popularized by Robert Benford (1938), Benford's Law asserts that digits of naturally occurring numbers conform to distributions based on logarithms. For example, the occurrence of the digit 1 as the first digit is expected to be [(LOG10(1+(1/1)) ≈] 0.3010. Although Benford's Law was discovered more than eighty years ago, most research using this law has occurred in recent years. The cumulative number of papers citing the law was 50 in 1975, 150 in 2000, and more than 1,500 in 2015 (Nigrini 2012).

Benford (1938) showed that the law subsequently named after him applied to many types of data including area of rivers, population data from the census, some mathematical sequences such as Fibonacci numbers, atomic weights, and a totally random pick of numbers from the newspaper. Moreover, Benford distributions are scale invariant. If a dataset in meters is Benford then its distribution in feet is also Benford. However, not all data conforms to Benford's Law. According to Nigrini (1996) numbers fabricated by human thought are likely to deviate from the Benford distribution. The series of prime numbers is not Benford.

To investigate whether there is evidence of exchanges reporting fake Bitcoin volume, we focus on five exchanges that we believe are less likely to have fake volume because they are either regulated by the U.S. Department of the Treasury and or hold a BitLicense from the New York State Department of Financial Services. In addition, we examine three exchanges that are listed by Bitwise as bad exchanges and not regulated by New York State and three additional exchanges, including one Chinese exchange. We include the Chinese exchange because the Chinese government has periodically taken steps that have disrupted cryptocurrency trading.

For our eight exchanges mentioned above, we find evidence that the regulated or licensed exchanges have fewer deviations from Benford's Law than the other exchanges.

II. Literature review

A. Fake Volume

Jain, McInish, and Miller (2019) find that commonality of volume is determined by both global and local factors and provide results that directly contradict the volume spike test performed by Bitwise. These authors find that bitcoin trading volume is generally higher during the local workday (9am-5pm) and lower in the late night and early morning. The structure of Bitwise's expectations influences their analysis of volume spikes, making this test biased towards the exchanges which trade a fiat currency located in the same time zone as their expected good exchanges, specifically exchanges that predominantly trade BTC/ USD. Liquidity also has been known to be partially driven by local factors as documented in Brockman et al. (2009) so we should not expect spread patterns to be consistent around the world. Since at least two of the four tests that Bitwise uses are not empirically sound, we provide an alternative measure to detect fake volume—conformity to Benford'sLaw.

The Blockchain Transparency Institute published a report in April 2019 showing that 17 of the largest 25 exchanges on coinmarketcap.com, a common source for many Bloomberg and Wall Street Journal articles, had more than 99% fake volume. (Bitwise 2019). The Institute describes five potential ways to exaggerate volume: 1. posting trades when none occurred; 2. simultaneously buying and selling bitcoins with itself or an accessory trader; 3. paying market makers to make wash trades; 4. paying traders in an alt coin to compensate for trader fees; and 5. incentivizing more trading by paying a rebate or giving a discount to clients who reach certain thresholds of trading activity.

Bitwise's presentation identifies three empirical characteristics of exchanges with fake trade data: (1) trade printing between the bid and ask, (2) multiple hours and days with zero volume, and (3) roughly an identical amount printed every hour of every day (monotonic trading volume). Only 10 of the 81 exchanges examined passed all three tests. Hougan, Kim, and Lerner (2019a) updated the Bitwise analysis to identify fraudulent prints (trades printed on the exchanges tape but without an actual trade occurring) and wash trades (with related parties on both sides of the trade). Using a week of data, these authors examine trade size histograms and volume spike alignment for the 81 exchanges in the Bitwise presentation. These authors propose that the number of trades of a particular size should decline as the trade size increases and present histograms that show this pattern of the 10 good exchanges and but unusual trade-size distributions for exchanges with fake data.

Hougan, Kim, and Lerner (2019a) examine the alignment of trading volume across exchanges. They observe similar patterns for each day across exchanges, especially for 3 May 2019. Exchanges reporting fake trades also often have unreasonably wide spreads. In contrast, spreads for the 10 good exchanges have low spreads, even as low as one cent. Low spreads are possible because the exchange typically have a maker-taker fee structure in which liquidity suppliers are rewarded with negative fees and liquidity demanders pay positive fees.

Bitcoin price manipulation also may spill over to derivative and investment fund markets as these financial vehicles base their price on the price of Bitcoin on a specific exchange or on a weighted index composed of trading prices on multiple exchanges.

B. Applications of Bedford's Law

Nigrini (1996) first tested if the nonrandom element of human behavior allows analyst to use Benford's Law to detect tax evasion and finds that low-income taxpayers evade more than high-income taxpayers. To provide a useful testing procedure for assessing conformity to Benford's Law, Nigrini and Mittermaier (1997) suggest using three categories of tests— preliminary (proportion of first-digits, second-digit, and first-two digits), secondary (sums of digits and replication), and advanced (chi-squared and Sum of Squared Differences (SSD))

Using conformity to Benford's Law to investigate the effect of the implementation of the Sarbanes Oxley Act, using data for 2001-2010, Fatima (2013) examines financial statements for regulated versus less regulated exchanges and finds that regulated exchanges better comply with the Law. Fatima (2013) also investigates the effect of monitoring by comparing firms that are audited by big four accounting firms and firms that they expect to experience less stringent oversight and show that

.

Durtschi, Hillison, and Pacini (2004) examine a plethora of datasets both true and fraudulent to provide an outline of what accounting datasets can be examined using Benford's Law and provide a way to determine a base rate for fraud in a given dataset.

C. Mt Gox and OKCoin

Gandal, Hamrick, Moore, and Oberman (2018) find that the demise of the leading bitcoin exchange, Mt. Gox, was due to fraudulent trading stemming from two accounts. The trading of these accounts happened in two distinct time periods, only a couple days apart, and both accounts were making uneconomic bitcoin transactions to successfully raise Bitcoin's price. The first suspicious account had "?" as an entry for user country and user fields and those were among the first red flags. The account made many trades that were identical during the same day. These authors discovered that the account did not pay for bitcoins it acquired or transaction fees associated with its trades.

The second account was able to trade when the rest of Mt. Gox trading was disabled. In the 90 minutes the API was offline on 7 January 2017 the account was active, trading 10- 19 Bitcoins every 6-20minutes.

Gandal, Hamrick, Moore, and Oberman (2018) show that the strategies used by the owners of these two accounts fooled investors on the Mt. Gox exchange defrauding them of 600,000 bitcoins worth $188 million at the time. Not only were these Bitcoins not paid for with fiat currency, but the trades also raised the price of bitcoin from $150 to $1,000 in the two months of suspicious trading.

Another exchange that is well known to have had fake volume is the Chinese yuan (CNY) cryptocurrency exchange OKCoin, which, on December 19, 2013, recorded a daily volume of 9 million Litecoins—the third most widely used cryptocurrency at the time— when there were only 20 million Litcoins in existence. From 23 January 2017 to 24 January 2017 the OKCoin Exchange experienced an 80% drop in volume overnight after the implementation of a 0.02% trading fee. We believe that this huge change in volume with the addition of a small fee shows that most of the trading was for the purpose of increasing the exchange's volume.

BTCChina

III. Hypothesis development

A. Background and description of Benford's Law

Benford's Law was originally discovered by Simon Newcomb in 1881 and was rediscovered and popularized by Robert Benford in a 1938 paper titled "The Law of Anomalous Numbers." Contrary to naive belief that the first digits of numbers such as the number of shares in a trade should be equally likely—or 11.11% each for digits 1-9— Benford proposed that larger digits are rarer and the distribution of first digits is equal to 1/ln(N). The equation for the expected proportion of first digits is:

$$\text{Prob}(d1=d1) = \log(1+1/d1);\ d < \{1,2,\ldots,9\}$$

or: 1, 30.103%; 2, 17.609%; 3, 12.494; 4, 9.691%; 5, 7.918%; 6, 6.695%; 7, 5.799%; 8, 5.115%; and 9, 4.576%. Note that the first digit cannot be zero. Benford originally noticed this phenomenon when he saw that the first pages of his logarithmic tables starting with 1's and 2's were more worn than the latter pages where logs started with 8's and 9's.

The second digits of numbers range from 0-9 and follow their own distinct pattern, which is more uniform than the distribution of first digits.

$$p_{d_k} = P(D_k(X) = d_k) = log_{10}\left(1 + d_k^{-1}\right) \forall k \in \mathbb{N}^+$$

$$D_k(x) = \lfloor |x| \cdot 10^{(-1 \cdot \lfloor log_{10}|x| \rfloor + k - 1)} \rfloor$$

$d_k \in \{10^{k-1}, 10^{k-1}+1, \ldots, 10^k - 1\}$ (cf. Hill, 1995, p. 354).

The expected distribution of second digits is as follows: 0, 11.968%; 1, 11.389%; 2; 10.9XX%; 3, 10.433%; 4, 10.031%; 5, 9.668%; 6, 9.337%; 7, 9.035%; 8, 8.757%; and 9, 8.5XX%.

The expected proportions of the first two digits is given by: Prob(D1D2=d1d2) =

$$\log(1+1/d_1d_2);\ d < \{10,11\ldots,99\}$$

B. Hypotheses

Our first hypothesis is:

H1: Trading volume for regulated exchanges conform better to Benford's Law. Our second hypothesis is:

H2: The volume of exchanges located countries that regulate and tax bitcoin conform better to Benford's Law.

We test out first hypothesis in the following ways:

Test 1: First digits conform better to Benford's Law. Test 2: Second digits conform better to Benford's Law. Test 3: The distribution of the sums more closely conforms to Benford's Law

H2a

- The most duplicated minutely volume amounts on good exchanges are in line with the most expected values using Benford's Law

H2b

- The most duplicated minutely volume amounts on bad exchanges are in with volume maximization, likely medium size amounts, similar to stealth trading

H3

- The sums of outlying digits will be larger on bad exchanges

H4

- Exchanges that are more in line with natural occurring numbers using Benfords law are more likely to pass ad hoc tests performed by Bitwise Asset Management

H5

- Negotiating of trades, prices just below $1.00 increments, are only found on the good exchanges

IV. Data

Because of the entry and exit of exchanges, the number of exchanges with contemporaneous data is limited. We obtain minute-level data for December 1 through December 14, 2018 because the availability of contemporaneous data from regulated exchanges. We also limit our sample to exchanges that trade the BTC/USD pair. Also, to increase the likelihood that our data are comparable across exchanges, we use one data source— bitcoincharts.com. Our sample comprises data for the following exchanges: Bitstamp (Luxemburg, Luxemburg/ San Francisco, CA/ London, UK), Coinbase Pro, Kraken (San Francisco, United States), Bitflyer (San Francisco, United States), ItBit (New York, United States), Coinsbank (Tallinn, Estonia) CEX.IO (London, United Kingdom), and Coinsbit (Estonia).

In addition to testing whether there is fake volume of the above eight exchanges, we examine whether our tests detect fake volume of the two exchanges known to have fake volume—Mt. Gox and Okcoin.

We conduct a separate study on the Chinese exchange BTCChina (Hong Kong, China) to see how Bitcoin exchanges behave when there is an implicit ban making it illegal to facilitate banking transactions with cryptocurrency exchanges

We investigate two additional exchanges where Bitcoin is partially banned as the case for Vietnam (VBTC) who have a ban on Cryptocurrency transactions and Venezuela (SurBitcoin) were mining is banned but had potential data issues. We use VBTC and Surbitcoin to illustrate data issues in testing Benford's Law. Our data source provides minute-level volume in hundredths with 0.00 as the minimum. These data can present a serious issue because minimums and maximums disrupt the distribution of digits. Our source also rounds to 0.00 or 0.01 proving a near guaranteed fail of our tests as first digit 1s and second digit 2s are greatly distorted. In Venezuela trading is legal, but mining is illegal, and remittances are regulated. In Vietnam trading is legal, but it is illegal to use bitcoin as a paymenttool.

V. Methodology

The Benford's Law tests we use can be categorized into three groups- primary tests, advanced tests, and associated tests. The two primary tests we use are tests of 1) first digits,

2) second digits, and 3) first two digits (first order).

To examine the distribution numbers (1-9) individually for our first digits tests we compare the expected frequency from the Benford's distribution with our actual frequencies. To test the overall goodness of fit for our primary tests, we use a chi square statistics and Sum of Squared Deviations. We also look at contribution to chi-square statistic to examine specific values that contribute the most to the chi-square. In other words, what digits primarily make the goodness of fit of first digits from a given exchange differentiate from the expected distribution. We follow same approach for second digits and the first two digits.

The summation theorem (Nigrini 1992) finds that the sums of numbers per digit have approximately the same distribution as the count of each digit. The summation test is the most crucial test for detecting fraud once primary tests are complete. It allows practitioners to examine which digits drive the desired outcome. In our case, the exchanges benefit from higher volumes so being able to identify how much of the total volume is contributed by the volume of a specific digit is an important in identifying fraud. We take the sum of all volume amounts that start with the same first two digits for digits 10-99. For example, let's take the number twelve. 1.2, 12, and 120 all shave 12 as the first two digits and if we took the sum of these three numbers, we get 133.2. We then take 133.2 and divide it by the sum of volume for our sample to determine what portion of our volume is attributable to volume. This differs from testing the count of the first two digits as the count would be 3 whether the number are 1.2, 12, and 120 or 1.2, 1.2, and 1.2. We use the summation test to look for abnormally large trades. When used in conjunction with the repetition test, which we will discuss next, it helps us identify wash trading strategies. There is no direct statistical test for the summation test. Due to space constraints and difficulty to read 90 digits for multiple exchanges we present our results in Figure 2.

All statistical tests are at the 0.01 level unless otherwise stated.

VI. Results

A. Regulated versus unregulated exchanges

In Table 1 for each possible first digit 1-9, we present the expected logarithmic proportions from Benford's Law (Column 2), and the actual distributions and p-values for the regulated exchanges (Columns 3-7), and the unregulated exchanges (in Columns 8-10). We reject the null hypothesis of equality for seventeen of the eighteen tests for the Coinsbank and Cex.io Exchanges so that these exchanges conform the least to Benford's Law. For the regulated exchanges, we cannot reject the hypothesis of equality for nineteen of forth-five tests. Overall, the regulated exchanges conform to Benford's Law much better than the unregulated exchanges. The non-conformity of the regulated exchanges to Benford's Law may be due in part to our use of minute-level data rather than trade-level data.

Table 2 presents chi-square and SSD statistics for our test of first digits. Or each exchange, in turn, we test first digits of jointly (simultaneously). Based on chi-square statistics, we reject the null hypothesis of equality for all exchanges. The regulated exchanges (Columns 2-6) have uniformly lower chi-squared values than the unregulated (Columns 7-10).

chi squared statistics having less power for smaller sample sizes. To subjectively test how much the actual proportions of first digits differ from the expected proportions of first digits, we use the SSD test. The SSD does not take sample size into account. The Kraken Exchange (Column 4) most closely conforms to Benford's Law (SSD = 0.001), with the Bitstamp Exchange (Column 2) second (SSD = 0.003), the ItBit Exchange (Column 3) third (SSD = 0.004), and the Coinbase Exchange (column 6) forth (SSD=0.006). The exchanged that deviates the most from logarithmic is the Coinsbank Exchange (Column 10) (SSD=0.208), the second worst is the Cex.io Exchange (SSD=0.144).

Figure 2 presents the results of our plots of the sums of the first two digits. In contrast to the plots for Mt. Gox presented in Figure 1, the plots for the regulated exchanges (Panels a-e)—aside from BitFlyer (Panel d)—are like to the expected Benford distribution (Figure 1, Panel a).

The graphs for Coinsbit and Coinsbank show strong skewness of large numbers (10s and 100s of bitcoins) starting in the 10, 20s, and 30s digit with a sharp drop for first two digits beginning with 31-99,[3] indicating that the exchanges that performed poorly in the first digits count test likely have fake volume of larger trade sizes.

As seen in Figure 3, overall the exchanges that we expect to be good, bars 1-5, deviate much less from the expected proportions when examining the first digits of minute volume. This is with exception of the Bitflyer. The Bitflyer also does not follow the expected pattern well for the sums test of first two digits and looks fairly like the sums pattern we find for Mt. Gox when we know fraud was occurring. Overall, exchanges lacking the United States Department of Treasury license deviate more from the expected proportions of numbers for first digits. Coinsbank consistently performs the worst across counts of first digits, sums of first two digits, and deviations of first digits from Benford's Law, and deviations of proportions of first digits from other exchanges.

Examining Table 3, compared to other exchanges, the Bitflyer exchange (Column 6) deviates the most from the expected proportion of 1s for the second digit, with more than

[3] In the context of Benford's Law these are small numbers, large numbers would be 80 and 800 or 90 and 900.

25% of second digits beginning with 0, more than double the proportion expected based on Benford's Law. The second digit test results are noisy and inconclusive when examined independently. We are unable to identify a distinct pattern among good or bad exchanges. All exchanges are roughly similar as we reject conformity to Benford's Law at the 0.01 level for most digits. The main insight from looking at these digits independently is to identify outlying digits. Examining Table 4, based on a chi square test, we reject conformity to Benford's Law for all exchanges. Like first digits, Bitstamp, Itbit, and Coinbase have among the smallest sum of squared deviations, but surprisingly Coinsbank, one of our possible bad exchanges, has the lowest SSD of any exchange in our sample. One possibility is that this exchange generates fake trades using a random number generator and they are not rounded trade amounts thought of by a human.

Examining Table 5, we find that our good exchanges exhibit little deviation from Benford's Law among themselves when examining the distribution of first digits. When comparing good exchange first digit distributions to the first digit distributions of bad exchanges, we find they deviate much more than when we compare good exchange to good exchange. Bad exchanges' distributions deviate from other bad exchanges' distributions the most. The Coinbase and Itbit distributions of first digit minute volumes are the most similar with the sum of their absolute value of deviations of each proportion of digits 1-9 being only 0.0762, Bitstamp and Coinbase are third/fourth with a sum of absolute value of deviations of 0.0842. Bitstamp and Itbit with a sum of absolute value of deviations of 0.0918. We interpret these results as showing that Bitstamp, Coinbase, and ItBit volume having a similar level of quality, with minimal fake volume. It's likely no coincidence that the CME futures contracted monitored by the Chicago Board of Options Exchange (CBOE) primarily use these three exchanges to set the futures price. The Cex.io Exchange exhibits relatively high deviations from first digit proportions on other exchanges, and not surprisingly, the sum of the absolute value of deviations is the highest for Cex.io to Coinsbank. The Coinsbank Exchange has the largest deviations in proportions when compared to the other exchanges in our sample, and comes in last place similar to our other first digits tests , leading us to believe its volume numbers contain a higher amount of fabricated, uneconomic trades than the other exchanges we test.

B. Countries where bitcoin trading is illegal

In Appendix 1, we present a list of 97 countries and the periods that cryptocurrencies were legal/illegal and unregulated/regulated in the respective countries. We can obtain bitcoin data for one country where bitcoin became illegal—China. As shown in Table 6, for BTCChina, we find results like the other unregulated exchanges in our sample, which could be due to it based in Hong Kong rather than in mainland China, like the OKcoin Exchange that we discuss above.

We examine the first digits of the Chinese bitcoin exchange ChinaBTC (Table 6) and find a high proportion of the first digit is comprised of 1s, 0.4351, which is higher than any other exchange in our sample except for Bitflyer. Partially due to this high proportion of 1s, the first digit of all other numbers are less than the expected proportion, with exceptionally low proportions of 7s, 8s, and 9s. When testing first digits jointly, we get a chi square value of 571, which is higher than all our good, regulated exchanges, but lower than the unregulated exchanges in our sample. The SSD is higher than the SSD of all exchanges in our primary sample aside from Bitflyer and both Bitflyer's and ChinaBTC's high SSD are mostly due to their disproportionate amount of 1s.

VII. Limitations of applications of Benford's Law

One needs to be careful when choosing a correct dataset to test using Benford's Law. It is preferable that the data set covers multiple magnitudes (1s,10s, 100s), covers a full range of magnitude instead of temperature (in Fahrenheit) ranging from 30 to 95 degrees, and that the data is not averaged. It is also critical that the numbers are not rounded or have minimums or maximums. Many bitcoin exchanges trade relatively low volume and for those exchanges rounding and minimums are problematic. Our data source, bitcoincharts.com, rounds volume to 0.01 increments, which can distort results, especially when a significant portion of minutes have volume near or less than 0.01 bitcoins. Values less than 0.005 are rounded to 0 and values 0.005-0.014 are presented as 0.01. This is especially problematic for the test of second digits because volumes of 0.01 show a second digit of 0 using our tests. We examine many exchanges to identify unregulated and illegal exchanges that have significant volume and also several exchanges that we believe have issues related to rounding.

Distributions that are expected to follow Benford's Law include transactions-level data (ex: sales, trade size), numbers that result from a combination of numbers—quantity*price. Data sets for which the mean is greater than the median are also more likely to follow Benford's Law. Numbers that are not expected to obey Benford's Law include numbers that are assigned sequentially (such as IDs), prices, and numbers with minimums or maximums (Kossovsky 2019).

We illustrate the difficulty in applying Benford's Law to rounded data by examining two exchanges—SurBitcoin and VBTC—that we believe have rounded minute-level volumes. The proportions of volume for each first digit number are presented in Table 7, proportions for each second digit number in Table 8, and proportions for first digit numbers tested jointly in Table 8. These results differ substantially from the expected Benford values and to a much greater extent than for the exchanges regulated by the US Department of Treasury aside from the first digits of the Bitflyer exchange. The amount of deviation of first digits tested independently is similar to the unregulated exchanges in our primary sample, but the test of second digits presented in Table 7 shows how rounding can really be an issue. Over 70% if Surbitcoin minute volume has a second digit of 0 and the same is true for about 58% of VBTC exchange minute volumes. We find that this is driven by trades of 0.01 as this second digit is 0.010.

Using a chi square test, we jointly test the actual distributions of first digits against the expected distribution based on Benford's Law and obtain the following chi-square statistics: Surbitcoin, 209; VBTC, 10,874. We reject the null hypothesis of equality for VBTC, but not for Surbitcoin.

Next, we use the SSD to measure the distance between our observed distributions and a logarithmic distribution. The lower the SSD the closer the data are to Benford's Law. To subjectively judge an SSD value, we compare the SSD of a dataset in question with the SSD of a known honest dataset. SSD is the same as Sum of Squared Errors for residual analysis when examining regressions. Our results are based on minute-level data for the first two weeks on January 2017. We obtain the following results for SSD: Surbitcoin, 0.0068; VBTC, 0.2428. The SSD of VBTC is greater than any exchange in the sample presented in the body of our paper.

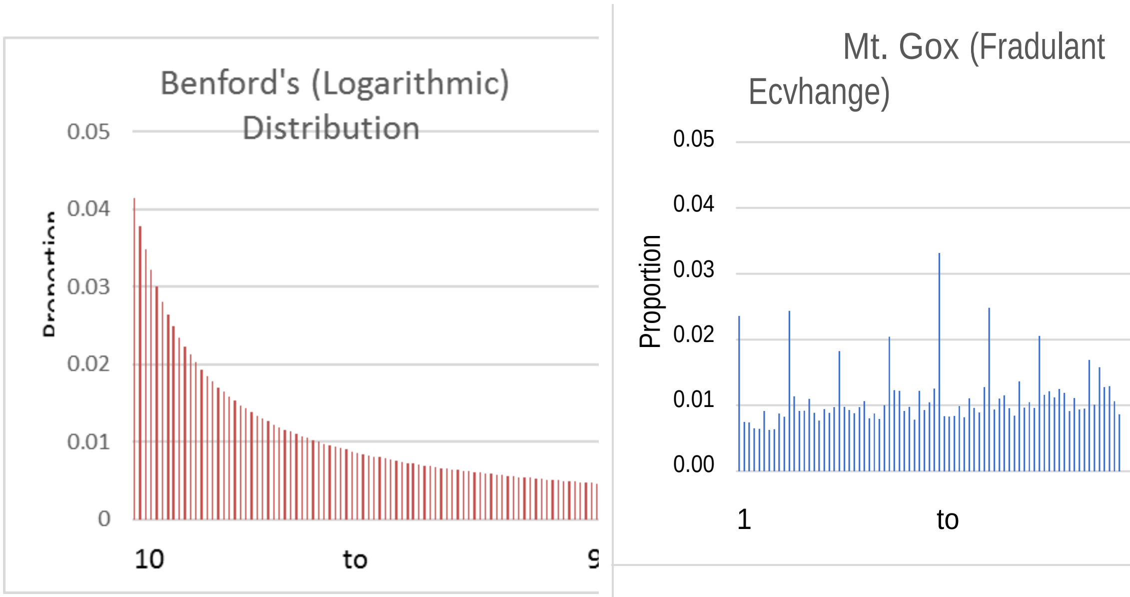
VIII. Conclusion

We investigate whether data from trading on bitcoin exchanges is reliable. Benford's Law has been used to test for data reliability in a variety of contexts including looking for fake results in academic publications and testing for fraud in accounting statements. According to Benford's Law, the probability of the first digit being 1 is log10(1 + (1/d1))

= log10(2) = 0.30103. Expected distribution of first digits, second digits and the first two digits together can be derived from Benford's Law. We use these expected distributions to investigate the possibility of fake reported volume for bitcoin exchanges.

Initially, we examine trading on five exchanges that are either regulated by the US Department of Treasury or have a license from New York State. We compare these exchanges to three exchanges that do not meet either of these requirements, but trade in countries where cryptocurrencies are legal. We find that exchanges that are more heavily regulated conform better to Benford's law, indicating less fake volume. We find that the proportion of first digits deviate less for the Bitstamp, Coinbase, and ItBit exchanges, justifying their use as the basis for the index price for CME Bitcoin Futures contracts (BTCA).

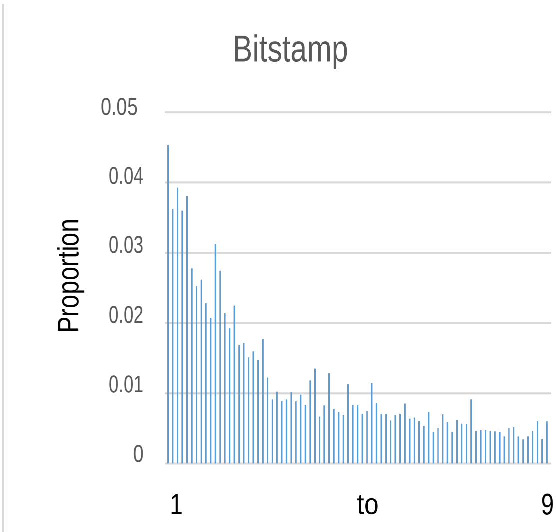
We extend the analysis to the examination of three exchanges where bitcoin trading became illegal. The Chinese exchange, BTCChina is the only of the three that has enough volume to analyze it at the minute level without rounding and minimum increments problems. For BTCChina, we find results like the other unregulated exchanges, which could be due to it being based in Hong Kong rather than in mainland China.



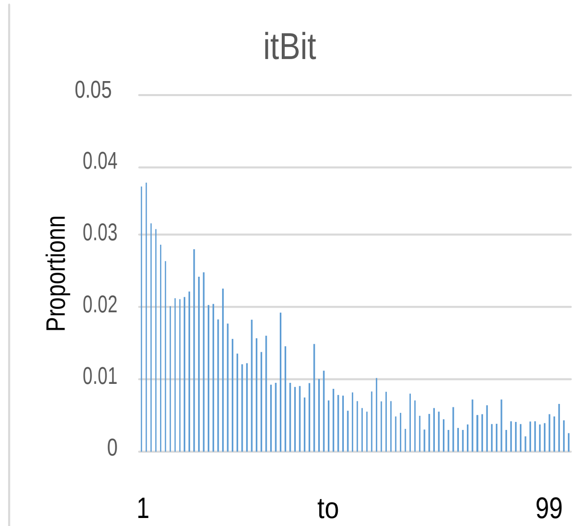
(a) (b)

Figure 4. Benford's and Mt. Gox's distributions of sums

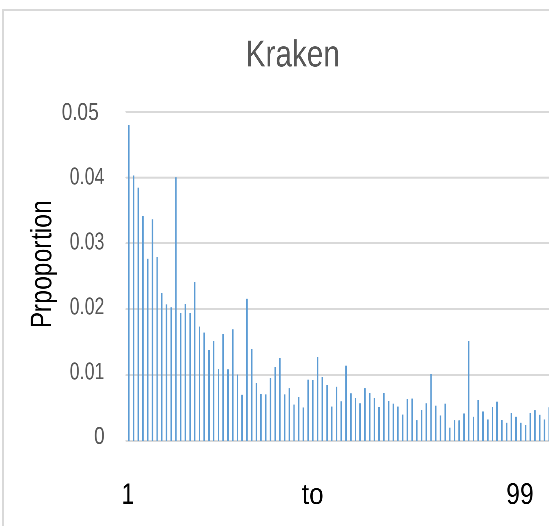
In Panel (a) we present the expected distribution of the sums of the first two digits for Benford's distribution. In Panel (b) we present the actual distribution of the sums of first two digits for the Mt. Gox Exchange.


Bitstamp
Proportion
0.05
0.04
0.03
0.02
0.01
0
1
to
9

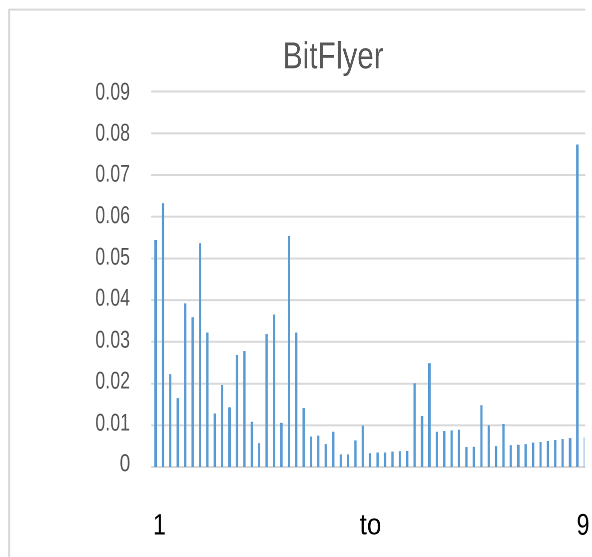
(a)


itBit
Proportionn
0.05
0.04
0.03
0.02
0.01
0
1
to
99

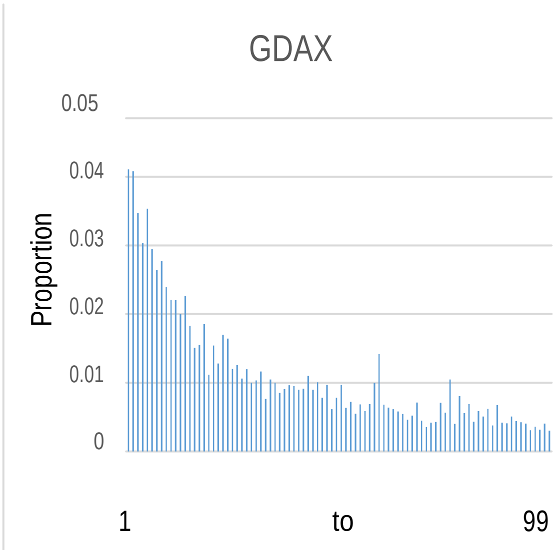
(b)


Kraken
Prpoportion
0.05
0.04
0.03
0.02
0.01
0
1
to
99

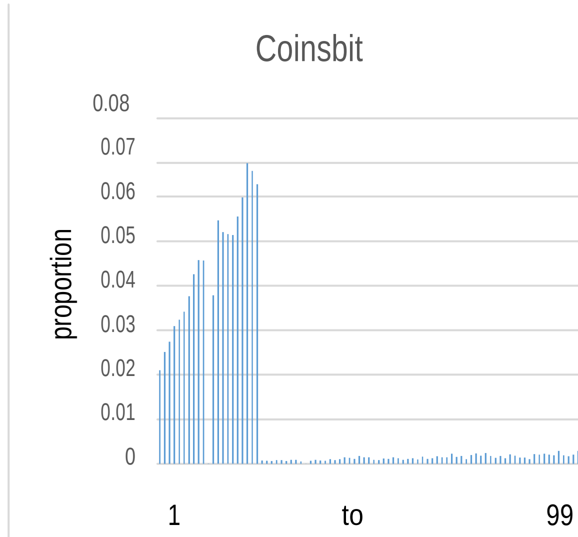
(c)


BitFlyer
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1
to
9

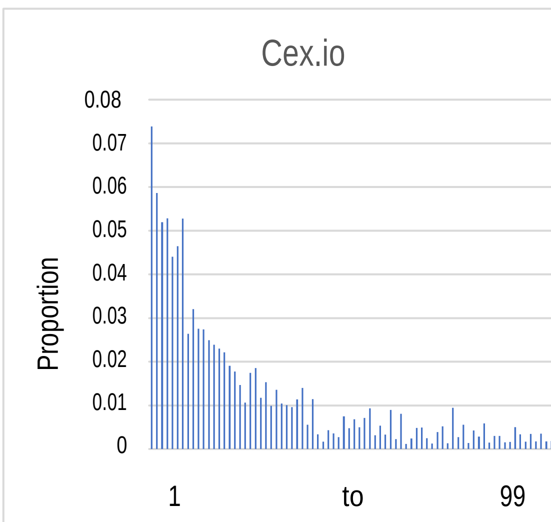
(d)



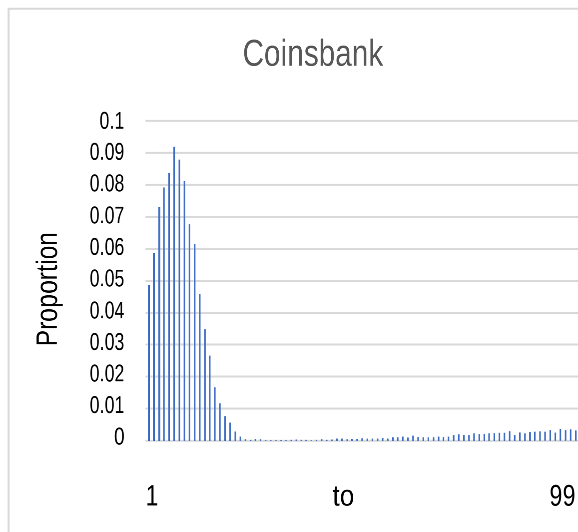
(e)



(f)



(g)



(h)

Figures 5. Distributions of sums for sample exchanges
We present the distributions of the sums of the first two digits for eight exchanges. Exchanges in Panels (a-e) are regulated by the US Department of Treasury and those in Panels (f-h) are not.

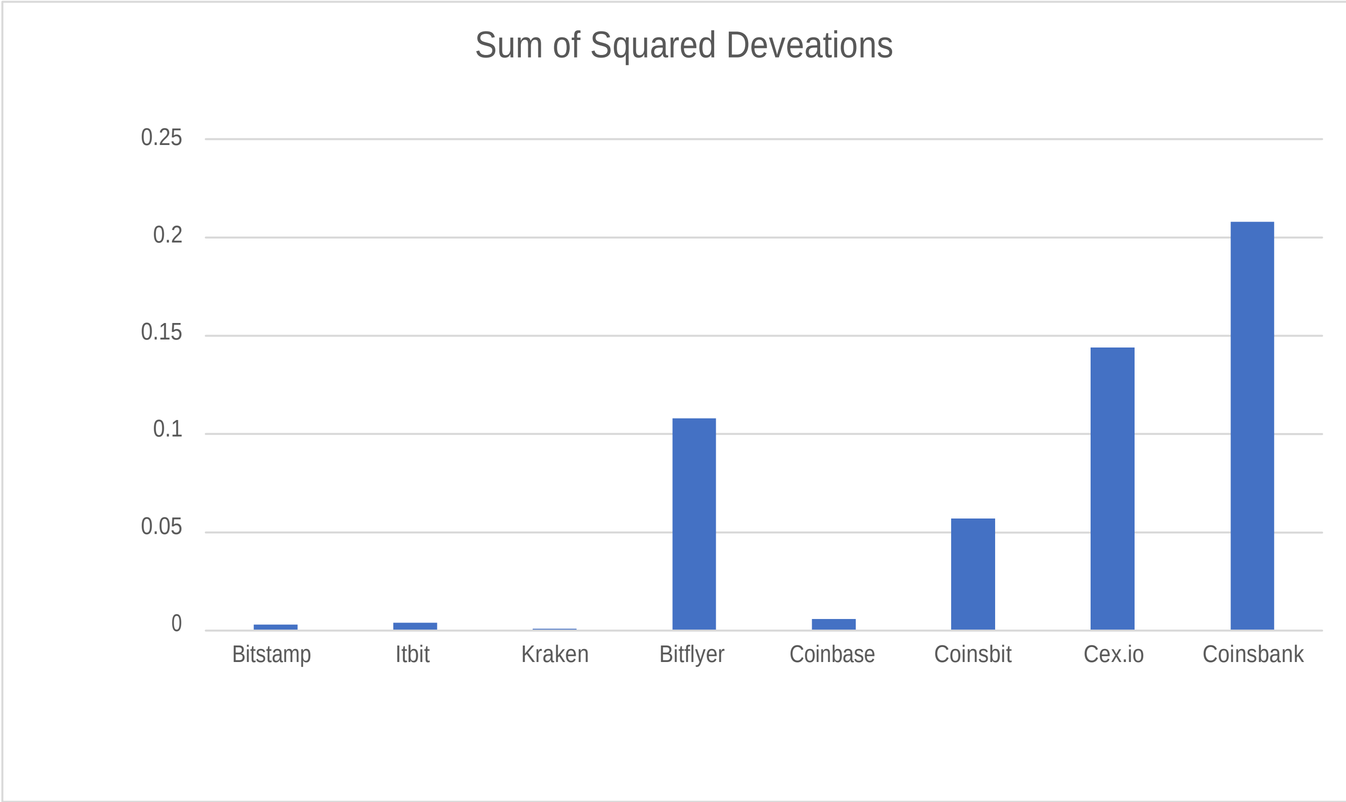


Figure 678. **Sum of squared deviations for first digits**
We present the square of the sum of the difference between the expected proportion of first digits and the observed proportion of first digits for minute volume for eight exchanges.

Table 15. First Digits of Volume, Tested Independently

We examine volume by minute for trades of 1-9.99 Bitcoins. For the first digits listed in Column 1, we present the expected (based on Benford's Law) proportion (Column 2), the actual proportion for each exchange indicated in Columns 3-10. For each exchange, we test whether the actual values equal the expected values and report p-values in parentheses. p-values less than 0.01 are reported as 0.01. The first five exchanges are regulated, and the remaining exchanges are not. We use minute-level data obtained from bitcoincharts.com for the first two weeks of December 2018.

		Regulated Exchanges					Unregulated Exchanges		
Digits	Expected	Bitstamp	Itbit	Kraken	Bitflyer	Coinbase	Coinsbit	Cex.io	Coinsbank
1	0.301	0.252	0.257	0.290	0.607	0.235	0.109	0.655	0.008
		(0.01)	(0.01)	(0.011)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
2	0.176	0.173	0.211	0.185	0.162	0.197	0.105	0.180	0.020
		(0.188)	(0.01)	(0.013)	(0.318)	(0.01)	(0.01)	(0.313)	(0.01)
3	0.125	0.136	0.139	0.139	0.061	0.153	0.116	0.079	0.040
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.145)	(0.01)	(0.01)
4	0.097	0.114	0.105	0.096	0.031	0.113	0.106	0.028	0.061
		(0.01)	(0.011)	(0.367)	(0.01)	(0.01)	(0.096)	(0.01)	(0.01)
5	0.079	0.099	0.080	0.096	0.067	0.083	0.118	0.024	0.095
		(0.01)	(0.363)	(0.01)	(0.293)	(0.060)	(0.01)	(0.01)	(0.01)
6	0.067	0.073	0.073	0.068	0.033	0.071	0.104	0.014	0.124
		(0.01)	(0.021)	(0.383)	(0.032)	(0.041)	(0.01)	(0.01)	(0.01)
7	0.058	0.060	0.053	0.049	0.011	0.058	0.125	0.010	0.186
		(0.183)	(0.036)	(0.01)	(0.01)	(0.468)	(0.01)	(0.01)	(0.01)
8	0.051	0.051	0.046	0.043	0.011	0.050	0.106	0.006	0.211
		(0.480)	(0.037)	(0.01)	(0.01)	(0.254)	(0.01)	(0.01)	(0.01)
9	0.046	0.043	0.036	0.035	0.017	0.040	0.110	0.004	0.255
		(0.483)	(0.01)	(0.01)	(0.029)	(0.01)	(0.01)	(0.01)	(0.01)

Table 16. First Digits, Tested Jointly

We examine volume by minute for trades of 1-9.99 Bitcoins. Using chi-square statistics, we jointly test whether the actual distributions of first digits are like the distribution expected based on Benford's Law and report the results in Row 1. To measure the distance between our observed distributions and a logarithmic distribution, we use the Sum Squares Deviation (SSD) and report the results in Row 2. The lower the SSD the closer the data are to Benford's Law. SSD is the same as Sum of Squared Errors for residual analysis when examining regressions. Our results are based on minute-level data obtained from bitcoincharts.com for the first two weeks of December 2018. * indicates statistical significance at the 0.01 level.

	Regulated Exchanges					Unregulated Exchanges		
	Bitstamp	Itbit	Kraken	Bitflyer	Coinbase	Coinsbit	Cex.io	Coinsbank
chi sq.	168*	132*	99.1*	178*	335*	754*	1,413*	9,535*
SSD	0.003	0.004	0.001	0.108	0.006	0.057	0.144	0.208

Table 17. Second Digits, Tested Independently

We examine volume by minute for trades of 1-9.99 Bitcoins. For the second digits listed in Column 1, we present the expected (based on Benford's Law) proportion (Column 2), and the actual proportion for each exchange indicated in Columns 3-10. For each exchange, we test whether the actual values equal the expected values and report p-values in parentheses. p-values less than 0.01 are reported as 0.01. The first five exchanges are regulated, and the remaining exchanges are not. We use minute-level data obtained from bitcoincharts.com
for the first two weeks of December 2018.

		Regulated Exchanges					Unregulated Exchanges		
Digits	Expected	Bitstamp	Itbit	Kraken	Bitflyer	Coinbase	Coinsbit	Cex.io	Coinsbank
0	0.120	0.143	0.176	0.178	0.251	0.109	0.194	0.172	0.088
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
1	0.114	0.117	0.099	0.112	0.164	0.108	0.085	0.156	0.097
		(0.197)	(0.01)	(0.289)	(0.013)	(0.014)	(0.01)	(0.01)	(0.01)
2	0.199	0.099	0.105	0.109	0.086	0.105	0.103	0.126	0.086
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
3	0.104	0.102	0.092	0.101	0.092	0.103	0.078	0.114	0.096
		(0.274)	(0.01)	(0.143)	(-0.31)	(-0.373)	(0.01)	(0.091)	(0.048)
4	0.100	0.094	0.089	0.088	0.064	0.097	0.092	0.101	0.099
		(0.019)	(0.01)	(0.01)	(-0.04)	(-0.087)	(0.126)	(0.497)	(0.374)
5	0.097	0.103	0.087	0.105	0.092	0.097	0.090	0.087	0.099
		(0.0239)	(0.01)	(0.01)	(-0.45)	(-0.473)	(0.165)	(0.065)	(0.308)
6	0.093	0.084	0.092	0.078	0.081	0.099	0.085	0.077	0.100
		(0.01)	(0.346)	(0.01)	(-0.30)	(-0.016)	(0.108)	(0.01)	(0.078)
7	0.090	0.087	0.089	0.087	0.064	0.101	0.094	0.055	0.105
		(0.097)	(0.357)	(0.162)	(-0.11)	(0.01)	(0.304)	(0.01)	(0.01)
8	0.088	0.088	0.086	0.071	0.045	0.093	0.095	0.064	0.112
		(0.462)	(0.300)	(0.01)	(0.017)	(0.027)	(0.152)	(0.01)	(0.01)
9	0.085	0.084	0.086	0.071	0.061	0.089	0.086	0.050	0.118
		(0.318)	(0.359)	(0.01)	(0.129)	(0.062)	(0.474)	(0.015)	(0.01)

Table 18. Second Digits, Tested Jointly

We examine volume by minute for trades of 1-9.99 Bitcoins. Using chi-square tests, we jointly test whether the actual distributions of first digits are like the expected distribution based on Benford's Law and report the results in Row 1. To measure the distance between our observed distributions and a logarithmic distribution, we use the Sum Squares Deviation (SSD) and report the results in Row 2. The lower the SSD the closer the data are to Benford's Law. SSD is the same as Sum of Squared Errors for residual analysis when examining regressions. Our results are based on minute-level data obtained from bitcoincharts.com for the first two weeks of December 2018. * indicates statistical significance at the 0.01 level.

	Regulated Exchanges				Unregulated Exchanges			
	Bitstamp	Itbit	Kraken	Bitflyer	Coinbase	Coinsbit	Cex.io	Coinsbank
Chi Sq.	539*	520*	694*	101*	595*	196*	219*	161*
SSD	0.011	0.013	0.012	0.037	0.009	0.016	0.013	0.004

Table 19. Sum of absolute deviations matrix

We present the sum of the absolute value of deviations of the proportion of each first digit for all possible combinations of exchanges. * indicates significance at the 0.01 level.

	Itbit	Kraken	Bitflyer	Coinbase	Coinsbit	Cex.io	Coinsbank
Bitstamp	0.0918	0.1074	0.7103	0.0842	0.4746	0.8206	1.0984
Itbit			0.7011	0.0762	0.552	0.7044	1.1662
Kraken			0.6351	0.1352	0.5676	0.7298	1.1644
Bitflyer				0.7449	1.1099	0.1675	1.5245
Coinbase						0.8396	1.138
Coinsbit						1.2422	0.6624
Cex.io							1.892

Table 20. First digits, ChinaBTC Exchange

For the first digits listed in Column 1, we present the expected proportion—based on Benford's Law—in (Column 2), the actual proportion for the ChinaBTC Exchange (Column 3) and p-values for the difference between the actual and expected values in parentheses (Column 4). p-values less than 0.01 are reported as 0.01. Our results are based on minute-level data for the first two weeks of January 2017.

Digits	Expected	ChinaBTC	p-values
1	0.30103	0.4351	(0.01)
2	0.17609	0.1776	(0.49)
3	0.12494	0.0798	(0.01)
4	0.09691	0.0918	(0.38)
5	0.07918	0.0739	(0.36)
6	0.06695	0.0339	(0.01)
7	0.05799	0.0399	(0.05)
8	0.05115	0.0299	(0.02)
9	0.04576	0.0379	(0.23)

Table 21. First digits, bad rounding

For the first digits listed in Column 1, we present the expected proportion—based on Benford's Law—in (Column 2), the actual proportions (Columns 3-4), and p-values of for the difference between the actual and expected values (in parentheses). p-values less than 0.01 are reported as 0.01. Our results are based on minute-level data for the first two weeks of January 2017.

Digits	Expected	SurBitcoin	VBTC
1	0.30103	0.374	0.405
		(0.01)	(0.01)
2	0.17609	0.191	0.171
		(0.01)	(0.5)
3	0.12494	0.118	0.09
		(0.03)	(0.17)
4	0.09691	0.089	0.099
		(0.01)	(0.32)
5	0.07918	0.072	0.081
		(0.01)	(0.46)
6	0.06695	0.046	0.045
		(0.01)	(0.23)
7	0.05799	0.042	0.045
		(0.01)	(0.35)
8	0.05115	0.037	0.018
		(-.01)	(0.09)
9	0.04576	0.032	0.045
		(0.01)	(0.43)

Table 22. Example of limitation due to rounding, second digits

For the first digits listed in Column 1, we present the expected proportion—based on Benford's Law—in (Column 2), the actual proportions (Columns 3-4) and the p-values of for the difference between the actual and expected values (in parentheses below each observation). p- values less than 0.01 are reported as 0.01. Our results are based on minute-level data for the first two weeks of January 2017.

Digit	Digit	SurBitcoin	VBTC
0	0.1197	0.7045	0.5856
		(0.01)	(0.01)
1	0.1139	0.0439	0.0541
		(0.01)	(0.03)
2	0.1090	0.0426	0.045
		(0.01)	(0.02)
3	0.1043	0.0404	0.036
		(0.01)	(0.01)
4	0.1003	0.0301	0.045
		(0.01)	(0.04)
5	0.0967	0.0359	0.0541
		(0.01)	(0.09)
6	0.0934	0.0275	0.036
		(0.01)	(0.03)
7	0.0904	0.0252	0.045
		(0.01)	(0.07)
8	0.0876	0.0246	0.036
		(0.01)	(0.04)
9	0.0850	0.0252	0.0631
		(0.01)	(0.26)

References

Andersen, T.G, 1996, "Return Volatility and Trading Volume: An Information Flow Interpretation of Stochastic Volatility," Journal of Finance 51, 169-204.

Amihud, Y., 2002, "Illiquidity and Stock Returns: Cross-Section and Time-Series Effects," Journal of Financial Markets 5, 31–56.

Ball, Clifford A., Walter N. Torous, and Adrian E. Tschoegl, 1985, The degree of price resolution: The case of the gold market, Journal of Futures Markets 5, 29-43.

Benford, R., 1938, The Law of Anomalous Numbers, American Philosophical Society (Proceedings) 4, 551-572.

Bitwise Asset Management, 2019, Presentation to the U.S. Securities and Exchange Commission March 19.
At: https://www.sec.gov/comments/sr-nysearca-2019-01/srnysearca201901-5164833-183434.pdf

Binance, 2019, March 25, "Binance Partners with IdentityMind to Enhance Compliance and Data Security Measures," At: https://www.binance.com/en/blog/317231329092952064/Binance-Partners-with-IdentityMind-to-Enhance-Compliance-and-Data-Security-Measures

Biais, B., T. Foucault, and F. Salanie, 1998, "Floors, Dealer Markets and Limit Order Markets," Journal of Financial Markets 1, 253-284.

Bitwise Asset Management, 2019, March 19, "Presentation to the U.S. Securities and Exchange Commission." At: https://www.sec.gov/comments/sr-nysearca-2019-01/srnysearca201901-5164833-183434.pdf

Bitwise Investment Advisers, 2018, July 24, "General Form for Registration of Securities under the Securities Act of 1933." Acc-no: 0001493152-18-010390 (33 Act). Securities and Exchange Commission, Bitwise ETF Trust, Form S-1.
At: https://www.sec.gov/Archives/edgar/data/1746379/000149315218010390/forms-1.htm

Bollerslev, T. and I. Domowitz, 1993, "Trading Patterns and Prices in the Interbank Foreign Exchange Market," Journal of Finance 48, 1421–1443.

Breedon, F., and A. Ranaldo, 2013, "Intraday Patterns in FX Returns and Order Flow, Journal of Money, Credit, and Banking 45, 953-965.

Brockman, P., D.Y. Chung, and C. Perignon, 2009, "Commonality in Liquidity: A Global Perspective." Journal of Financial and Quantitative Analysis 44, 851–882.

Chan, K., A. Hameed, and S.T. Lau, 2003, "What if Trading Location is Different from Business Location? Evidence from the Jardine Group," Journal of Finance 58, 1221–1246.

Cheng, E., 2017, "Bitcoin Exchange Coinbase Has More Users than Stock Brokerage Schwab." https://www.evernote.com/shard/s385/sh/3196901a-4cd9-4c46-8654-c2449fc5dd09/c22e4e78467a26cd72a3293dda3de971

Choi, K.J., A. Lehar, and R. Stauffer, 2018, "Bitcoin Microstructure and the Kimchi Premium," University of Calgary Working Paper.

Chordia, T., R. Roll, and A. Subrahmanyam, 2000, "Commonality in Liquidity." Journal of Financial Economics 56, 3–28.

Corwin, S.A. and P. Schultz, 2012, "A Simple Way to Estimate Bid-Ask Spreads from Daily High and Low Prices," Journal of Finance 67, 719 – 759.

Covrig V. and L. Ng, , 2004, "Volume Autocorrelation, Information, and Investor Trading," Journal of Banking & Finance 28, 2155–2174.

Durtschi, C., C. Hillison, and C. Pacini, 2004, The Effecive Use of Benford's Law to Assist in Fraud in Accounting Data, Journal of Forensic Accounting, 5, 17-34.

Fatima A., 2013, Benford's Law: Analyzing a Decade of Financial Data, Journal of Emerging Technologies in Accounting, 10, 1-39.

Gandal, N., J.T. Hamrick, T. Moore, and T. Oberman, 2018, Price Manipulation in the Bitcoin Ecosystem, Journal of Monetary Economics 95, 86-96.

Hougan, M., H. Kim, H., and M. Lerner, 2019a, May 24, Economic and Non-Economic Trading in Bitcoin: Exploring the Real Spot Market for the World's First Digital Commodity, Bitwise Asset Management. At: https://www.sec.gov/comments/sr-nysearca-2019-01/srnysearca201901-5574233-185408.pdf

Hougan, M., H. Kim, and M. Lerner, 2019b, Annotated Commentary on the Winklevoss Order: A Comprehensive Analysis of How the SEC's July 26, 2018, Disapproval of the Winklevoss, Bitcoin Trust Does and Does Not Apply to the Bitwise Bitcoin ETF Trust Proposal., June 11. At: https://www.sec.gov/comments/sr-nysearca-2019-01/srnysearca201901-5663768-185796.pdf

Easley, D., M. López de Prado, M. O'Hara, and Z. Zhang, 2019, "Microstructure in the Machine Age," NBER working paper.
Available at http://papers.nber.org/conf_papers/f117415.pdf.

Foucault, T., O. Kadan, and E. Kandel, 2005, "Limit Order Book as a Market for Liquidity," Review of Financial Studies 18, 1171-1217.

Froot, K.A., E.M. Dabora, 1999, "How are stock prices affected by the location of trade?" Journal of Financial Economics 53, 189–216.

Gandal, N., J.T. Hamrick, T. Moore, and T. Oberman, 2018, "Price Manipulation in the Bitcoin Ecosystem," Journal of Monetary Economics 95, 86-96.

Goettler, R.L., C.A. Parlour, and U. Rajan, 2005, "Equilibrium in a Dynamic Limit Order Market," Journal of Finance 60, 2149–2191.

Goettler, R.L., C.A. Parlour, and U. Rajan, 2009, "Informed Traders and Limit Order Markets," Journal of Financial Economics 93, 67–87.

Goodhart, C. and A. Demos, 1991, "Reuters Screen Images of the Foreign Exchange Markets: The Yen/Dollar and the Sterling/Dollar Spot Market," Journal of International Securities Markets no volume, 35–64.

Griffin, J.M. and A. Shams, 2018, "Is Bitcoin Really Un-Tethered?" University of Texas at Austin Working Paper.

Hasbrouck, J. and D.J. Seppi, 2001, "Common Factors in Prices, Order Flows, and Liquidity," Journal of Financial Economics 59, 383–411.

Hougan, M., H. Kim, H., and M. Lerner, 2019a, May 24, "Economic and Non-Economic Trading in Bitcoin: Exploring the Real Spot Market for the World's First Digital Commodity," Bitwise Asset Management. At: https://www.sec.gov/comments/sr-nysearca-2019-01/srnysearca201901-5574233-185408.pdf

Hougan, M., H. Kim, and M. Lerner, 2019b, "Annotated Commentary on the Winklevoss Order: A Comprehensive Analysis of How the SEC's July 26, 2018, Disapproval of the Winklevoss Bitcoin Trust Does and Does Not Apply to the Bitwise Bitcoin ETF Trust Proposal., June 11. At: https://www.sec.gov/comments/sr-nysearca-2019-01/srnysearca201901-5663768-185796.pdf

Huang, R.D. and R.W. Masulis, 1999, "FX Spreads and Dealer Competition across the 24– Hour Trading Day," Review of Financial Studies 12, 61–93.

Korajczyk, Robert A. and R. Sadka, 2008, "Pricing the Commonality across Alternative Measures of Liquidity," Journal of Financial Economics 87, 45–72.

Kossovsky, A., 2019, Benfords Law Theory, the General Law of Relative Quantities, and Forensic Fraud Detection Applications, World Scientific.

Kroeger, A. and A. Sarkar, 2017, "The Law of One Bitcoin Price," Federal Reserve Bank of New York Working Paper.

Lakonishok, J. and E. Maberly, 1990, "The weekend effect: Trading patterns of individuals and institutional investors," Journal of Finance 45, 231–243.

Lau, S.T. and T.H. McInish, 2003, "Trading Volume and Location of Trade: Evidence from Jardine Group listings in Hong Kong and Singapore," Journal of Banking and Finance 27, 1411–1425.

Lau, S.T., L.K. Ng, and B. Zhang, 2010, "The World Price of Home Bias," Journal of Financial Economics 97, 191–217.

Library of Congress, 2019, Regulation of Cryptocurrency Around the World, At: https://www.loc.gov/law/help/cryptocurrency/world-survey.php

Makarov, I. and A. Schoar, 2018, "Trading and Arbitrage in Cryptocurrency Markets," University of Calgary Working Paper.

Mancini, L., A. Ranaldo, and J. Wrampelmeyer, 2013, "Liquidity in the Foreign Exchange Market: Measurement, Commonality, and Risk Premiums." Journal of Finance 68, 1805– 1841.

More, T. and N. Christin, 2013, Beware the Middleman: Empirical Analysis of Bitcoin- Exchange Risk, Financial Cryptography and Data Security, 25-33.

Mittermaier, J. and m. Nigrini, 1997, The Use of Benford's Law as an Aid in Analytical Procedures, Auditing: A Journal of Practice and Theory 16, 2.

Mu, E., 2013, Chinese Bitcoin Exchange Okcoin Accused of Fake Trading Data, Coindesk, December 21. At: https://www.coindesk.com/chinese-bitcoin-exchange- okcoin-accused-fake-trading-data

Nigrini, M., 1992, The Detection of Income Tax Evasion through and Analysis of Digital Frequencies, thesis, University of Cincinnati, Ohio.

Nigrini, M., 1996, A Tax Payer Compliance Application of Benford's Law, Journal of American Taxation Associates, 18, 72-91.

Nigrini, M., 2012, Benford's Law Applications for Forensic Accounting, Auditing, and Fraud Detection, Wiley Corporate F&A.

Nigrini, M.J., 2015, Persistent Patterns in Stock Returns, Stock Volumes, and Accounting Data in the U.S. Capital Markets. Journal of Accounting, Auditing & Finance 30, 541-57.

Osborne, M.F.M., 1962, Periodic structure in the Brownian motion of stock prices, Operations Research 10, 345-379.

Parkinson, M., 1980, "The Extreme Value Method for Estimating the Variance of the Rate of Return," Journal of Business 53, 61–65.

Parlour, C., 1998, Price dynamics in limit order markets," Review of Financial Studies 11, 789–816.

Perkinscoie, 2019, September, Digital Currencies: International Actions and Regulations, At: https://www.perkinscoie.com/en/news-insights/digital-currencies-international- actions-and-regulations.html

PR Newswire, 2018, October 17, Chainalysis Partners with Binance to Tackle Global Cryptocurrency Money Laundering, At: https://www.prnewswire.com/newsreleases/ chainalysis-partners-with-binance-to-tackle-global-cryptocurrency-money-laundering- 300732667.html

Pirinsky, C. and Q. Wang, 2006, "Does Corporate Headquarters Location Matter for Stock Returns?," Journal of Finance 61, 1991–2015.

Schwartz, Adam L., Bonnie F. Van Ness, and Robert A. Van Ness, 2004, Clustering in the futures market: Evidence from S&P 500 futures contracts, Journal of Futures Markets 24, 413-428.

Sonnemans, Joep, 2006, Price clustering and natural resistance points in the Dutch stock market: A natural experiment, European Economic Review 50, 1937-1950.

Sopranzetti, Ben J., and Vinay Datar, 2002, Price clustering in foreign exchange spot markets, Journal of Financial Markets 5, 411-417.

Urquhart, Andrew, 2017, Price clustering in Bitcoin, Economics Letters 159, 145-148.

Wadhwa, Monica, and Kuangjie Zhang, 2015, This number just feels right: The impact of roundedness of price numbers on product evaluations, Journal of Consumer Research 41, 1172-1185.

Vigna, P., 2019, August 7, "Most Bitcoin Trading Faked by Unregulated Exchanges, Study Finds," Wall Street Journal, At: https://www.wsj.com/articles/most-bitcoin-trading-faked- by-unregulated-exchanges-study-finds-11553259600

Appendix 1

Summary of Legal and Regulatory Status of Cryptocurrencies, by Country

We present the legal and regulation status for 97 countries. Columns 2 and 3 present the time cryptocurrencies were legal and illegal and Columns 4 and 5 present the period of time cryptocurrencies were unregulated and regulated. Countries were cryptocurrencies were always legal say "always" in Column 2 and "never" in Column 3. We include implicit bans and partially illegal in the illegal category to capture their change in legality, even if cryptocurrencies were not completely banned. Countries where cryptocurrencies have never been regulated say "always" in Column 4 and "never" in Column 5. Missing values in Columns 5 and 6 are due to lack of data.

	Type of period			
Country	**Legal**	**Illegal**	**Unregulated**	**Regulated**
Algeria	inception-12/26/2017	12/27/2017-present		
Argentina	always	never		
Australia	always	never		
Australia	always	never		
Austria	always	never	always	never
Bahrain	inception-1/6/2018	1/7/2018-present (implicit ban)		
Bangladesh	inception-9/16/2014	9/16/2014-present		
Belgium	always	never	always	never
Bermuda	always	never		
Bolivia	inception-4/18/2017	4/19/2017-present		
Brazil	always	never		
Bulgaria	always	never	inception-11/19/2018	11/20/2018-present
Canada	always	never	inception-4/26/2013	4/26/2013-present
Cayman Islands	always	never	9/19/2017	9/20/2017-present
Chile				
Chile	always	never	always	never

China	inception-12/5/2013	12/5/2013-present (implicit ban)		
Colombia	inception-3/26/2014	3/27/2014-present		
Croatia	always	never	always	never
Cyprus	always	never	always	never
Czech Republic	always	never	Inception - 11/13/2016	11/14/20160-present
Denmark	always	never	Inception-2/26/2018	2/27/2018-present
Dominican Republic	inception-6/27/2017	6/28/2017-present (implicit ban)		
Ecuador	always	never		
Egypt	inception-1/9/2018	1/10/2018/-present		
Estonia	always	never	Inception-November 26, 2017	November 27, 2017
Finland	always	never		
France (many events)	always	never	inception-7/23/2014 (proposed	7/23/2014-present
Georgia	always	never	always	never
Germany	always	never	inception-12/22/2011	12/22/2011-present
Greece	always	never	always	never
Hong Kong	always	never		
Hungary	always	never	always	never
Iceland	always	never		
India	inception-4/5/2018 (implicit ban)	4/6/2018-present		
Indonesia	inception-11/28/2017	11/28/2017-present		
Iran	inception-12/29/2017	12/30/2017-present		
Iraq	inception-12/2/2017	12/3/2017-present		

Israel	always	never	inception-1/16/2018	1/17/2018-present
Italy	always	never		
Jamaica	always	never	always	never
Japan	always	never		
Jordan	always	never		
Kazakhstan	inception-3/29/2018	3/30/3018-present implicit ban		
Kenya	always	never		
Kuwait	inception-12/17/2017	12/18/2017-present (implicit ban)		
Latvia	always	never		
Lebanon	always	never		
Lithuania	inception-10/10/2017	10/11/2017-present		
Luxembourg	always	never		
Malaysia	always	never		
Malta	always	never		
Mexico	always	never		
Morocco	inception-11/19/2018	11/20/2018-present		
Nepal	Inception-8/12/017	8/13/2017-Present		
Netherlands	always	never		
New Zealand	always	never		
Nigeria	always	never	always	never
Norway	Always	never	incpetion-11/10/2013	11/11/2013-present
Oman	always	never		
Pakistan	Inception-4/5/2018	4/6/2018-present		
Panama	always	never		
Peru	always	never	always	never
Philippines	always	never		

Poland	always	never (announcement considered making illegal on 1/1/2018)	inception-4/3/2018	4/4/2018-present
Portugal	always	never	always	never
Qatar	incpetion-7/1/2018	7/2/2018-present		
Romania	always	never	incpetion-3/3/2018	3/4/2018-present
Russia	always	never		
Russia	inception-1/11/2016	1/12.2016-present (implicit ban)		
Saudi Arabia	inception-8/11/2018	8/12/2018-present (implicit ban)		
Saudi Arabia	inception-6/3/2017	6/4/2017-present (implicit ban)		
Senegal	always	never		
Serbia	always	never		
Singapore	always	never		
Slovakia	always	never	inception-3/22/2013	3/23/2013-present
Slovenia	inception-12/23/2013	12/24/2013-present		
South Africa	always	never	inception-4/5/2018	4/6/2018 or earlier-present
South Korea	always	never		
South Korea	inception -1/8/2019	1/9/2019-present (implicit ban, no IPOs)		
Spain	always	never		
Spain	always	never	inception-4/31/2018	5/1/2018-present

Sweden	always	never	inception-4/23/2015	4/24/2015-present
Switzerland	always	never	inception-11/29/2017	11/30/2017-present
Switzerland	always	never	inception-6/24/2014	6/25/2014-present
Taiwan	inception-1/5/2014	1/6/2014-present (implicit ban)		
Thailand	inception-2/12/2018;3/14/2018-present	2/13/2018-3/13/2018		
Tunisia	always	never		
Turkey	always	never	always	never
U.K.	always	never	always	never
UAE	Inception- 12/31/2016; 3/28/2018-present	1/1/2017-3/27/2018	always	never
Ukraine	always	never	always	never
US	always	never		
Venezuela	always	never		
Vietnam	Inception-10/27/2017	10/28/2017-Present	always	never
Zambia	always	never		
Zimbabwe	incpetion-5/13/2018	5/14/2018-present (implicit ban)		

Sources: https://www.loc.gov/law/help/cryptocurrency/world-survey.php#argentina

https://www.perkinscoie.com/en/news-insights/digital-currencies-international-actions-and-regulations.html