CENTRO STUDI BANCA E FINANZA

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CEFIN Working Papers
No 11

Option based forecasts of volatility: An empirical study in the DAX index options market

by Silvia Muzzioli

May 2008

CEFIN – Centro Studi di Banca e Finanza
Dipartimento di Economia Aziendale – Università di Modena e Reggio Emilia
Viale Jacopo Berengario 51, 41100 MODENA (Italy)
tel. 39-059.2056711 (Centralino) fax 059 205 6927

Option based forecasts of volatility:
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S. Muzzioli[1*]

Abstract

Option based volatility forecasts can be divided into "model dependent" forecast, such as implied volatility, that is obtained by inverting the Black and Scholes formula, and "model free" forecasts, such as model free volatility, proposed by Britten-Jones and Neuberger (2000), that do not rely on a particular option pricing model.

The aim of this paper is to investigate the unbiasedness and efficiency in predicting future realized volatility of the two option based volatility forecasts: implied volatility and model free volatility. The comparison is pursued by using intradaily data on the Dax-index options market. Our results suggest that Black-Scholes volatility subsumes all the information contained in historical volatility and is a better predictor than model free volatility.

Keywords: Implied Volatility, Model free volatility, Volatility Forecasting.

JEL classification: G13, G14.

[1] Department of Economics and CEFIN, University of Modena and Reggio Emilia, Viale Berengario 51, 41100 Modena (I), Tel. +390592056771 Fax +390592056947, e-mail: silvia.muzzioli@unimore.it

[*] The author wishes to thank Giuseppe Marotta for helpful comments and suggestions The author gratefully acknowledges financial support from MIUR. Usual disclaimer applies.

1. Introduction

Volatility estimation and forecasting are essential both for the pricing and the risk management of derivative securities. There have been various contributions aimed at assessing the best way in order to forecast volatility. Among the various models proposed in the literature we distinguish between option based volatility forecasts that use prices of traded options in order to unlock volatility expectations and time series volatility models that use historical information in order to predict future volatility.

Following Poon and Granger (2003), among time series volatility models, we have predictions based on past standard deviation, ARCH conditional volatility models and stochastic volatility models. Among prediction based on past standard deviation we have the simple random walk hypothesis in which the best estimate of future realised volatility is today volatility, methods based on averages, such as historical averages, moving averages and exponential smoothing moving averages, that try to solve the trade off between having as much observations as possible and sampling close to the present time and simple regression models that regress volatility on its past values. ARCH family conditional volatility models (see Bollerslev, Chou and Kroner (1992) for a survey) formulate conditional variance as a function of past squared returns via maximum likelihood. ARCH models have the advantage that the next step forecast is available by their very same construction. In Stochastic volatility models (see Ghysels, Harvey and Renault (1996) for a survey) volatility is driven by a different source of uncertainty from the one of the underlying asset price. Stochastic volatility models are very flexible, but difficult to implement, since they usually have no closed form solution.

Among option based volatility forecasts we have implied volatility, that is a "model dependent" forecast since it relies on the Black and Scholes model, and the so called "model free" volatility, proposed by Britten-Jones and Neuberger (2000), that does not rely on a particular option pricing model.

Implied volatility is usually extracted from a single option, by inverting the Black and Scholes formula, by means of a numerical method such as the bisection method.

Model free Implied Volatility, proposed by Britten-Jones and Neuberger (2000), is based on the observation that the expected sum of squared returns between two dates is completely specified by two sets of options expiring on the two dates. Model free implied volatility is derived by using a cross section of option prices differing in time to maturity, strike prices and option type. Therefore it should be more informative than implied volatility backed out from a single option. Moreover, while the examination of the forecasting power of the Black and Scholes implied volatility is a joint test of model specification and market efficiency, model free implied volatility, being independent from a particular option pricing model, provides a direct test of market efficiency.

The CBOE VIX volatility index is an example of a switch from a Black and Scholes implied volatility to a model free one (for more details see Carr and Wu (2006)). The CBOE volatility index expresses a one-month implied volatility and is deemed as a benchmark for stock market volatility and market fear. Prior to September 2003 it was computed (since then it has been renamed VXO) by using Black and Scholes implied volatility backed out from eight near to the money options (4 calls and 4 puts) written on the S&P100 at the two nearest maturities. From 22 September 2003, the CBOE changed the definition and computation rules of the VIX index. The underlying is now the S&P500 and the computation is based on the model free implied volatility backed out from a cross section of at and out of the money call and put options for the two nearest maturities.

A drawback of using Black and Scholes implied volatility is clearly its dependence on the strike price of the option (the so-called smile effect), time to maturity of the option (term structure of volatility) and option type (call versus put). Many papers have investigated the information content of implied volatility backed out from different option classes. Christensen and Prabhala (1998) examine the relation between implied and realized volatility on S&P100 options, on the time period 1983-1995. They found that at the money calls are good predictors of future realized volatility. Fleming (1998) investigates the implied-realised volatility relation in the S&P100 options market and finds that at the money call implied volatility has slightly more predictive power than put implied volatility. Ederington and Guan (2005) examine how the information in implied volatility differs by strike price for options on S&P500 futures. They point out that implied volatilities calculated from moderately high strike options (moderately out of the money calls and in the money puts) are efficient predictors of future volatility and fully embed all the available information, while implied volatilities calculated from low strikes (out of the money puts and in the money calls) and at the money strikes are biased and less efficient predictors of future volatility. They conclude that the information content in implied volatilities varies roughly in a mirror image of the implied volatility smile.

Nonetheless, at the money Black and Scholes volatility is usually considered as the market's expectation of future realised volatility between now and the expiration date of the option. Even if from a theoretical point of view there is no clear reason for that, since the Black and Scholes model postulates a constant volatility, from an empirical point of view, various papers have demonstrated the soundness of such a choice. In fact, numerous papers have analysed the empirical performance of at the money Black Scholes implied volatility in various option markets, ranging from indexes, futures or individual stocks and find that implied volatility is an unbiased and\or efficient forecast of future realised volatility (see e.g. Christensen and Prabhala (1998) for options on indexes, Ederington and Guan (2002) for options on Futures, Szakmary et al. (2003) and Godbey and Mahar (2005) for options on individual stocks and Blair, Poon and Taylor (2001b) and Bandi and Perron (2006) for the VIX volatility index).

Up to now, very few papers have dealt with the forecasting power of model free implied volatility. From a theoretical point of view, Carr and Wu (2006) highlight that model free volatility should be superior to Black Scholes volatility. In fact, they showed that at the money Black Scholes implied volatility can be considered as a proxy of a volatility swap rate, while model free volatility is a proxy for a variance swap rate. While the payoff on a volatility swap is difficult to replicate, the payoff of a variance swap rate is easily replicable by using a static position in a continuum of European options and a dynamic position in futures. From an empirical point of view, the evidence in favour of the superiority of model free volatility against Black Scholes volatility is mixed. Lynch and Panigirtzoglou (2003) analyse the predictive power of model free implied volatility on four different markets: S&P500, FTSE100, Eurodollar and sterling futures and find model free implied volatility is a biased though efficient estimate of future volatility. Jiang and Tian (2005) investigate the predictive power of the model free volatility in the S&P500 options. They find that model free implied volatility is an efficient forecast of future realised volatility and an unbiased forecast after a constant adjustment and subsumes all the information contained in the Black and Scholes implied volatility. On the other hand, Andersen and Bondarenko (2007) found opposite results. They investigate the forecasting performance of model free implied volatility in the S&P 500 futures market and find that it does not perform better than the simple Black and Scholes volatility.

The aim of this paper is to investigate the unbiasedness and efficiency in predicting future realized volatility of the two option based volatility forecasts: implied volatility and model free volatility. In order to pursue a fair comparison with model free implied volatility, that is derived based on a cross section of option prices, for implied volatility we use a weighted average of implied volatilities backed out from different option classes. The comparison is performed by using intradaily data on the Dax-index options market. The market is chosen for two main reasons. First, the options are European, therefore the estimation of the early exercise premium is not needed and can not influence the results. Second, the Dax index is a capital weighted performance index composed of 30 major German stocks and is adjusted for dividends, stocks splits and changes in capital. Since dividends are assumed to be reinvested into the shares, they do not affect the index value.

The plan of the paper is the following. In section 2 we illustrate the theoretical concept of model free implied volatility and we show the practical problems arising in the implementation. In section 3 we present the data set used, the sampling procedure and the variables definitions. In section 4 we describe the methodology used in order to address the unbiasedness and efficiency of the different volatility forecasts. In section 5 we report the results of the univariate and encompassing regressions and we test for robustness our methodology in order to see if some errors in variables problem may have affected our results. In order to analyze the dependence of model free volatility on the range of strike price used, in section 6 we present an alternative implementation of model free volatility. The last section concludes. In Appendix 1 we discuss some implementation issues for model free volatility.

2. Model Free Implied Volatility

Under mild conditions, Britten Jones and Neuberger (2000) showed how to derive the variance of the asset returns from a set of option prices. Suppose that the underlying asset *S* follows a diffusion process with time varying volatility, does not pay dividends and that the risk free rate is zero. Suppose that a continuum of option prices *C(T,K)* in strikes and maturities is available.

The risk neutral expected sum of squared returns between two dates T_1 and T_2 is completely defined by a set of option prices expiring on the two dates:

$$E^Q\left[\int_{T_1}^{T_2}\left(\frac{dS_t}{S_t}\right)^2\right] = 2\int_0^\infty \frac{C(T_2,K)-C(T_1,K)}{K^2}dK \qquad (1)$$

where the expectation is taken under the risk neutral measure *Q*, *St* is the underlying asset, *C(T,K)* is a call option with strike *K* that expires at time *T*.

As the methodology does not rely on any particular assumption on the underlying stochastic process, (1) is called a “model free” measure of the variance. Therefore, in order to obtain the model free variance is sufficient to have a continuum set of observed call prices expiring on dates T_1 and T_2.

The squared root of the variance is the model free implied volatility σ:

$$s = \sqrt{2\int_0^\infty \frac{C(T_2,K)-C(T_1,K)}{K^2}dK}$$

Note that (Britten Jones and Neuberger (2000)) this introduces an upward bias in the volatility since:

$$E^Q\left[\sqrt{\int_{T_1}^{T_2}\left(\frac{dS_t}{S_t}\right)^2}\right] \le \sqrt{2\int_0^\infty \frac{C(T_2,K)-C(T_1,K)}{K^2}dK}$$

Jang and Tian (2005) introduced several theoretical and practical modifications in order to compute the model free implied volatility. From a theoretical point of view they relaxed the assumptions of no dividends and zero risk free rate. From a practical point of view, as in the options market we observe only a limited number of strike prices, they showed how to cope with the problems of truncation (the strike prices range is limited) and discretization (strike prices are available only at discrete increments) of the strike prices domain.

By taking into account dividends and non-zero interest rates, equation (1) becomes (see Jang and Tian (2005) for the complete derivation):

$$E^Q\left[\int_{T_1}^{T_2}\left(\frac{dS_t}{S_t}\right)^2\right] = 2\int_0^\infty \frac{C(T_2, Ke^{rT_2}) - C(T_1, Ke^{rT_1})}{K^2} dK \qquad (2)$$

where S_t is considered as the observed underlying price minus the expected value of the dividends.

In order to forecast a variance measure between now and time T, taking $T_1=0$ and $T_2=T$ equation (2) simplifies to:

$$E^Q\left[\int_0^T\left(\frac{dS_t}{S_t}\right)^2\right] = 2\int_0^\infty \frac{C(T, Ke^{rT}) - \max(S_0 - K, 0)}{K^2} dK \qquad (3)$$

In this case only one set of options maturing at time T is necessary, in order to specify the model free variance.

Equation (3) requires the availability of a complete set of option prices with a continuum of strike prices. As in the market only options with a limited number of strike prices are traded, we face both truncation and discretization errors. Truncation errors arise when a limited range of strike prices $K \in [K_{\min}, K_{\max}]$ is used, instead of taking $K \in [0, \infty]$. Discretization errors arise when only a finite number of strike prices are used, instead of a continuum of strike prices.

In order to account for the limited and discrete strike price domain, Jiang and Tian (2005) propose the following approximation to equation (3):

$$2\int_0^\infty \frac{C(T, Ke^{rT}) - \max(S_0 - K, 0)}{K^2} dK \approx \sum_{i=1}^{m}\left[g(T, K_i) + g(T, K_{i-1})\right]\Delta K \qquad (4)$$

where $\Delta K = (K_{\max} - K_{\min})/m$, m is the number of abscissas, $K_i = K_{\min} + i\Delta K$, $0 \le i \le m$, $g(T, K_i) = [C(T, K_i e^{rT}) - \max(S_0 - K_i, 0)]/K_i^2$ and the trapezoidal rule for numerical integration has been used.

Moreover, in order to mitigate both truncation and discretization errors Jiang and Tian (2005) propose to apply a curve-fitting method to interpolate implied volatilities between strike prices.

First they translate call and put prices into implied volatilities by using the Black and Scholes formula (out of the money call and put prices are used).

Second, they use a curve fitting method (cubic splines), in order to interpolate implied volatilities.
Third, in order to extend the domain of strike prices they suppose that for strikes below the minimum value the implied volatility is constant and equal to the volatility of K_{min}, for strikes above the maximum value the implied volatility is constant and equal to the volatility of K_{max}, i.e they suppose constant volatility outside the available range of strike prices. It is important to notice that in this way they are introducing a third source of approximation error that is different from both truncation and discretization.
Last, they use the Black and Scholes formula in order to convert implied volatilities into call prices, obtained at the desired strike price frequency.
It is important to remark that the use of the Black and Scholes formula does not affect the model free attribute of volatility, since it is merely used in order to translate options prices into implied volatilities.

3. The Data set, the sampling procedure and the variables definitions.

The data set[2] consists of intradaily data on DAX-index options, recorded from 1 January 2001 to 31 December 2006. Each record reports the strike price, expiration month, transaction price, contract size, hour, minute second and centisecond. As for the underlying asset we use intradaily prices of the DAX-index recorded in the same time period. As a proxy for the risk-free rate we use the one month Euribor rate.

DAX-options started trading on the German Options and Futures Exchange (EUREX) in August 1991. They are European options on the DAX-index, which is a capital weighted performance index composed of 30 major German stocks and is adjusted for dividends, stocks splits and changes in capital. Since dividends are assumed to be reinvested into the shares, they do not affect the index value, therefore we do not have to estimate the dividend payments. Moreover the fact that the options are European avoids the estimation of the early exercise premium. This latter feature is very important since our data set is by construction less prone to estimation errors if compared to the majority of previous studies that use American style options. DAX-index options are quoted in index points, carried out one decimal place. The contract value is EUR 5 per DAX index point. The tick size is 0.1 of a point representing a value of EUR 0.50. They are cash settled, payable on the first exchange trading day immediately following the last trading day. The last trading day is the third Friday of the expiration month, if that is an exchange day, otherwise the

[2] The data source for Dax-index options and Dax index is the Institute of Finance, Banking, and Insurance of the University of Karlsruhe (TH), the risk-free rate is available in Data-Stream.

exchange trading day immediately prior to that Friday. The final settlement price is the value of the DAX determined on the basis of the collective prices of the shares contained on the DAX index as reflected in the intra-day trading auction on the electronic system of the Frankfurt Stock Exchange. Expiration months are the three near calendar months within the cycle March, June, September and December as well as the two following months of the cycle June and December.

Several filters are applied to the option data set. First, we eliminate option prices that are smaller than 1 Euro, since the closeness to the tick size may have affected the true option value. Second, in order not to use stale quotes, we eliminate options with trading volume less than one contract. Third, following Jiang and Tian (2005) in order to use only at the money and out of the money options, we eliminate in the money options (call options with moneyness *(X/S) < 0,97* and put options with moneyness *(X/S) > 1,03*). Fourth, we eliminate option prices violating the standard no arbitrage bounds[3]. Finally, in order to reduce computational burden, we only retain options that are traded between 2.00 and 3.00 p.m.

As for the sampling procedure, in order to avoid the telescoping problem described in Christensen, Hansen and Prabhala (2001), we use monthly non-overlapping samples. In particular, we collect the prices recorded on the Wednesday that immediately follows the expiry of the option (third Saturday of the expiry month) since the week immediately following the expiration date is one of the most active. These options have a fixed maturity of almost one month (from 17 to 22 days to expiration). If the Wednesday is not a trading day we move to the trading day immediately following.

We compute four volatility measures: realized volatility (σ_r), historical volatility (σ_h), Black-Scholes implied volatility (σ_{BS}) and model free volatility (σ_{mf}).

Following Andersen and Bollerslev (1998) and Andersen, Bollerslev, Dieblod and Labys (2001) that showed the importance of using high frequency returns versus the choice of daily returns in order to correctly measure realized volatility, we choose to measure realised volatility (σ_r) by using high frequency data. As Andersen and Bollesrslev (1998) point out that the returns at a frequency higher than five minutes are affected by serial correlation, we choose the five minutes frequency. Therefore, realised volatility is computed as the squared root of the sum of squared returns by using five-minutes frequency index returns over the life time of the option (almost one month) and then it is annualized by multiplying it by $\sqrt{12}$:

$$s_r = \sqrt{\sum_{t=1}^{n}\left[\ln\left(\frac{S_{t+1}}{S_t}\right)\right]^2 * 12} .$$

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[3] No arbitrage bounds are defined as follows: $C \geq \max(S - Xe^{-rT}, 0)$ $P \geq \max(Xe^{-rT} - S, 0)$.

where n is the number of index prices spaced by five minutes in the one month period.

Historical volatility (σ_h) is taken as the lagged (one month before) realised volatility.

As for the Model free implied volatility (σ_{mf}), the following procedure, based on Jang and Tian (2005) has been used in order to compute the right hand side of equation (4). We start from the cleaned data set of option prices that is made of at the money and out of the money call and put prices recorded from 3.00 to 4.00 p.m. We need a set of option prices with strike prices K ranging from K_{min} to K_{max}. As only a limited number of strike prices is available, we need to interpolate option prices in order to generate the missing prices. Due to the non linear relation between option prices and strike prices, we follow Shimko (1993) and Ait-Sahalia and Lo (1998) and we perform a curve-fitting method to interpolate implied volatilities between strike prices, rather than option prices.

We compute call and put implied volatilities by using synchronous prices, matched in a one minute interval, by inverting the Black and Scholes formula. As we are using option prices that are traded in one hour interval, we obtain different implied volatilities for the same strike price, depending on the time of the trade. Therefore, in order to have a one to one mapping between strikes and implied volatilities, we group implied volatilities that correspond to the same strike price by computing the average. In order to have a smooth function, following Bates (1991) and Campa, Chang and Reider (1998) we use cubic splines to interpolate implied volatilities.

In order to extend the domain of strike prices, following Jiang and Tian (2005) we suppose constant volatility outside the available range of strikes: for strikes below the minimum value the implied volatility is equal to the volatility of K_{min}, for strikes above the maximum value the implied volatility is equal to the volatility of K_{max}. The domain of strike prices is extended by using a factor u such that: $S/(1+u) \le K \le S(1+u)$, for the current implementation u has been chosen to be equal to 0,5. In order to have a sufficient discretization of the integration domain, we compute strikes spaced by an interval $\Delta K = 10$. In Appendix 1 we discuss some implementation issues on the truncation (choice of u) and discretization (choice of ΔK) errors of model free volatility, while in Appendix 2 we analyse the extrapolation errors given by the artificial extension of the strike price domain outside the existing range by deriving a model free volatility by using only traded option prices and performing a comparison with the model free volatility obtained following the Jiang ang Tian extrapolation methodology.

Finally, we use the Black and Scholes formula in order to convert implied volatilities into call prices. As we need a single value for the underlying asset, we take the average value of the underlying in the hour of trades.

In order to compute the Black and Scholes implied volatility (σ_{BS}) we use the following procedure. First we compute call and put implied volatilities, with the Black and Scholes formula, for the options closest to being at the money i.e. with strikes one below and one above the underlying price, by using synchronous prices, matched in a one minute interval. As we are using option prices that are traded in one hour interval, we compute the average implied volatility for the two close to the money strike prices. Black and Scholes implied volatility is defined as the weighted average of the two implied volatilities, with weights inversely proportional to the distance to the moneyness (for example if the DAX-index is 5355 and the closest strikes are 5400 and 5350 the implied volatility of the 5400 strike will be weighted 5/50 against the implied volatility 5350 strike which is weighted 45/50). As we need a single value for the underlying asset, we take the average value of the underlying in the hour of trades.

We report descriptive statistics for volatility and log volatility series in Table 1. On average realized volatility is lower and less volatile than both implied volatility estimates. Model free volatility is on average higher and more volatile than Black-Scholes implied volatility. The volatility series are highly skewed (long right tail) and leptokurtic and the hypothesis of a normal distribution is rejected for all the three series. Since the natural logarithm of the volatility series conform more to normality, in line with the literature (see e.g. Jiang and Tian (2005)) we decided to use the natural logarithm of the volatility series instead of the volatility itself in the following empirical analysis. In Table 2 we summarize the correlation matrix of the log volatility series. Black-Scholes volatility is highly correlated with model free volatility. Both Black-Scholes and model free volatilities are highly correlated with realised volatility, but Black Scholes volatility has the highest correlation.

Table 1. Descriptive statistics.

Statistic	s_{mf}	s_{BS}	s_{r}	ln(s_{mf})	ln(s_{BS})	ln(s_{r})
mean	0,290	0,265	0,236	-1,338	-1,431	-1,569
std dev	0,139	0,132	0,125	0,441	0,451	0,497
Skewness	1,094	1,311	1,090	0,396	0,416	0,298
Kurtosis	3,201	4,189	3,305	2,327	2,522	2,206
Jarque Bera	11,660	20,040	11,710	2,613	2,224	2,379
p-value	0,003	0,000	0,002	0,270	0,329	0,304

Table 2. Correlation matrix of log volatility series.

	ln(s_{r})	ln(s_{mf})	ln(s_{BS})
ln(s_{r})	1,000	----	---
ln(s_{mf})	0,906	1,000	---
ln(s_{BS})	0,914	0,987	1,000

4. The methodology.

The information content of implied volatility is examined both in univariate and in encompassing regressions. Even if, from a theoretical point of view, it is good practice to start from the general encompassing regression and analyse in turn the nested regressions, in order to keep the outline of the analysis consistent with the related literature, we examine first the univariate regressions and second the more general encompassing regressions.

In univariate regressions, realized volatility is regressed against one of the three volatility forecasts: Black-Scholes implied volatility (σ_{BS}), model free volatility (σ_{mf}), or historical volatility (σ_h), in order to examine the forecasting ability of each volatility estimator. The univariate regressions are the following:

$$\ln(s_r) = a + b\ln(s_i) + e \qquad (5)$$

where σ_r = realized volatility and σ_i= volatility forecast, *i=h,BS, mf*.

In encompassing regressions, realized volatility is regressed against two or more volatility forecasts in order to distinguish which one has the highest explanatory power and to address whether a single volatility forecast subsumes all the information contained in the others. Therefore we first compare pairwise one option based forecast (Black-Scholes implied volatility, model free volatility) with historical volatility in order to see if one of the option based forecast subsumes all the information contained in historical volatility. Second we compare the two option based forecast: Black-Scholes implied volatility and model free volatility, in order to distinguish which one carries most information on future realised volatility. Third we regress realised volatility on the three volatility forecasts in order to see the relative importance of each forecast.

The encompassing regressions used are the following:

$$\ln(s_r) = a + b\ln(s_i) + g\ln(s_h) + e \qquad (6)$$

where σ_r = realized volatility, σ_i= implied volatility, *i = BS, mf* and σ_h = historical volatility.

$$\ln(s_r) = a + b\ln(s_{BS}) + g\ln(s_{mf}) + e \qquad (7)$$

where σ_r = realized volatility, σ_{BS}= Black-Scholes implied volatility and σ_{mf} = model free volatility.

$$\ln(s_r) = a + b\ln(s_{BS}) + g\ln(s_{mf}) + d\ln(s_h) + e \qquad (8)$$

where σ_r = realized volatility, σ_{BS}= Black-Scholes implied volatility and σ_{mf} = model free volatility and σ_h = historical volatility.

In univariate regressions (5), we test three hypotheses, following Christensen and Prabhala (1998). The first hypothesis is H_0: $b = 0$: if the volatility forecast contains some information about future realised volatility, then the slope coefficient should be different from zero. Therefore we test if $b = 0$ and we see whether it can be rejected. The second hypothesis is H_0: $a = 0$ and $b = 1$ and assesses the unbiasedness of the volatility forecast. If the volatility forecast is an unbiased estimator of future realised volatility, then the intercept should be zero and the slope coefficient should be one. In case this latter hypothesis is rejected, we see if at least the slope coefficient is equal to one (H_0: $b = 1$) and, if confirmed, we interpret the volatility forecast as unbiased after a constant adjustment. Finally if implied volatility is efficient then the error term should be white noise and uncorrelated with the information set.

In encompassing regressions (6) we test two hypotheses. First we test H_0: $g = 0$ i.e. whether one of the two option based forecasts (Black-Scholes implied, model free volatility) subsumes all the information contained in historical volatility. Moreover, as a joint test of information content and efficiency we test in equations (6) if the slope coefficients of historical volatility and one of the option based forecasts (Black-Scholes implied, model free volatility) are equal to zero and one respectively (H_0: $g = 0$ and $b = 1$). Following Jiang and Tian (2005), we ignore the intercept in the latter null hypothesis, and if our null hypothesis is verified, we interpret the volatility forecast as unbiased after a constant adjustment.

In encompassing regressions (7) we investigate the different information content of the two option based forecasts (Black-Scholes implied, model free volatility). To this end we test, in regression (7), if $g = 0$ and $b = 1$, in order to see if Black-Scholes implied volatility subsumes all the information contained in model free volatility.

Finally in encompassing regression (8) we investigate the different information content of the three forecasts (Black-Scholes implied, model free volatility, historical volatility). In order to assess if the slope coefficient of the Black-Scholes implied volatility subsume all the information contained in both model free volatility and historical volatility, i.e. the joint hypothesis H_0: $d = 0, g = 0$ and $b = 1$.

Christensen and Prabhala (1998) compared the information content of Black and Scholes implied volatility with historical volatility in the S&P100 index options market. They run both OLS regressions and EIV regressions in order to correct for potential errors in variables due to the early exercise feature of the options and the dividend yield estimation and found different results. As our dataset consists of prices of options on the DAX index that are European style and are written on a non-dividend paying index, we avoid measurement errors that may arise in the estimation of the dividend yield and the early exercise premium. Moreover we carefully cleaned the dataset by applying rigorous filtering constraints detailed in Section 2 and we use synchronous prices for the index and the option that are matched in a one minute window. Therefore we expect our data to be less prone to measurement errors than the ones of Christensen and Prabhala (1998). Nonetheless, as the computation of the Black and Scholes and the model free volatility has involved some methodological choices deeply described in Section 2, we pursue an EIV procedure in order to see if there is any error in variables in the Black and Scholes or in the model free volatility. The instruments used for Black and Scholes implied volatility (model free volatility) are both historical volatility and past Black and Scholes implied volatility (model free volatility) as they are possibly correlated to the true Black and Scholes implied volatility (model free volatility), but unrelated to the measurement error associated with Black and Scholes implied volatility (model free volatility) one month later. As an indicator of the presence of errors in variables we use the Hausman (1978) specification test statistic[4].

5. The results.

The results of the OLS univariate and encompassing regressions are reported in Table 3 (p-values in parentheses). In all the regressions the residuals are homoscedastic and not autocorrelated (the Durbin Watson statistic is not significantly different from two and the Breusch-Godfrey LM test confirms non autocorrelation up to lag 12[5]), although they are not normal[6]. Some comments are in order. First of all, in all the three univariate regressions the beta coefficients are significantly different from zero: this means that all the three volatility forecasts (Black-Scholes, model free and historical) contain some information about future realised volatility. However, the null hypothesis that any of the three volatility forecasts is an unbiased estimate of future realized volatility is strongly rejected in all cases. In particular, in our sample, realized volatility is on average lower than the two option based volatility forecasts, suggesting that option based forecasts overpredict realised volatility. This is in line with the results found in Jiang and Tian (2005) and Lynch and Panigirtzoglou (2003), that document a positive risk premium for stochastic volatility. As neither one of the forecasts is unbiased we test if at least β is insignificantly different from one. The hypothesis can not be rejected at the 10% critical level for the two option based estimates, while it is strongly rejected for historical volatility. We can therefore consider both option based estimates

[4] The Hausman specification test is defined as: $m = \frac{\left(\hat{b}_{IV} - \hat{b}_{OLS}\right)^2}{Var(\hat{b}_{IV}) - Var(\hat{b}_{OLS})}$ where: $\hat{b}_{IV}$ is the beta obtained through the Two Stage Least Squares procedure, $\hat{b}_{OLS}$ is the beta obtained through the OLS procedure and *Var(x)* is the variance of the coefficient *x*. The Hausman specification test is distributed as a $\chi^2(1)$.

[5] In the regressions that include as explanatory variable the lagged realised volatility, the Durbin's alternative has been computed and it has confirmed the non autocorrelation of the residuals. The results of the Durbin's alternative and of the Breusch-Godfrey LM test are available upon request.

[6] The departure from normality of the residuals is not a result of ARCH effects: in all the regressions a test for ARCH effects on the residuals has been conducted that confirms the absence of autocorrelation in the squared residuals up to lag 12. Rather, it is caused by one outlier that corresponds to the September 2001 crash. In order to eliminate the effect of the outlier, regressions (5), (6), (7), (8) have been re-estimated on the sample period 26 September 2001- 31 December 2005 and the results, that are available upon request, are consistent with the ones reported for the entire sample period.

as unbiased after a constant adjustment given by the intercept of the regression. As for the adjusted R^2, among the two option-based volatility forecasts, the Black-Scholes volatility is ranked first in explaining future realized volatility, strictly followed by the model free volatility, while historical volatility has the lowest forecasting power.

Let us turn to the analysis of the encompassing regressions, in which we compare pairwise two different volatility forecasts in order to understand if one of them subsumes all the information contained in the other. First of all, we can observe that both option based volatility forecasts subsume all the information contained in historical volatility. This is evident by comparing the adjusted R^2 of univariate and encompassing regressions and by looking at the coefficient of historical volatility in the encompassing regressions. In fact, the inclusion of historical volatility does not improve the goodness of fit according to the adjusted R^2 and the coefficient of historical volatility is not significantly different from zero in both regressions. Moreover, both option based volatility forecasts are efficient and unbiased after a constant adjustment given by the intercept of the regression. In fact the slope coefficients of both option based volatility forecasts are not significantly different from one at the 10% level and the joint test of information content and efficiency $g = 0$ and $b = 1$ does not reject the null hypothesis for both option based volatility forecasts.

In order to see if Black Scholes implied volatility subsumes all the information contained in model free volatility, we test in encompassing regression (7) if $g = 0$ and $b = 1$. First of all, we observe that only the slope coefficient of Black-Scholes implied volatility is significantly different from zero, while the slope coefficient of model free volatility is not. Moreover, the joint test $g = 0$ and $b = 1$ does not reject the null hypothesis, providing evidence for the superiority of Black-Scholes implied volatility with respect to model free implied volatility.

For completeness, let us analyze the results of encompassing regression (8) in which we compare all the three volatility forecasts. First of all, the inclusion of both model free volatility and historical volatility doe not improve the goodness of fit given by the adjusted R^2. In fact both the coefficients of historical volatility and model free volatility are not statistically different from zero.

Moreover, also in this case Black-Scholes volatility is both efficient and unbiased after a constant adjustment, as it is evident by looking at the χ^{2c} column that jointly tests if $d = 0$, $g = 0$ and $b = 1$ and does not reject the null hypothesis, providing evidence for the superiority of Black-Scholes implied volatility with respect to both historical and model free volatility.

Finally, in order to test for robustness our results, and see if Black-Scholes implied volatility or model free volatility have been measured with errors, we adopt an instrumental variable procedure (IV) and run a two stage least squares. The Hausman (1978) specification test reported in the last column of Table 3 indicates that the errors in variables problem is not significant both in univariate and encompassing regressions[7]. Therefore we can trust the OLS regressions results.

Table 3. OLS regressions.

Dependent variable: log realized volatility									
Independent variables									
Intercept	**ln(σ_{BS})**	**ln(σ_{mf})**	**ln(σ_h)**	**Adj. R^2**	**DW**	**X^{2a}**	**X^{2b}**	**X^{2c}**	**Hausman Test**
-0,13	1,01***			0,83	1,89	26,917			0,572
(0,06)	(0,00)					(0,00)			
-0,20		1,02***		0,82	1,97	69,04			0,121
(0,01)		(0,00)				(0,00)			
-0,24			0,85	0,69	1,96	6,7127			
(0,02)			(0,00)			(0,04)			
-0,13	1,01***		-0,01^{+++}	0,83	1,89		0,0169		0,085
(0,06)	(0,00)		(0,96)				(0,99)		
-0,21		1,05***	-0,03^{+++}	0,82	1,96		0,168		0,002
(0,00)		(0,00)	(0,83)				(0,92)		
-0,1	0,83***	0,2^{+++}		0,83	1,91		0,395		0,047
(0,05)	(0,01)	(0,55)					(0,82)		
-0,14	0,83***	0,21^{+++}	-0,03^{+++}	0,83	1,90			0,379	0,062
(0,05)	(0,01)	(0,57)	(0,80)					(0,94)	

Note: The number in brackets are the p-values. The χ^{2a} reports the statistic of a χ^2 test for the joint null hypothesis $a = 0$ and $b = 1$ (p-values in parentheses) in the following univariate regressions: $\ln(s_r) = a + b \ln(s_i)$ where σ_r = realized volatility and σ_i= volatility forecast, *i=h,BS, mf.* The χ^{2b} reports the statistic of a χ^2 test for the joint null hypothesis $g = 0$ and $b = 1$ (p-values in parentheses) in the following encompassing regressions: $\ln(s_r) = a + b \ln(s_i) + g \ln(s_h)$, where σ_r = realized volatility, σ_i= implied volatility, *i* = *BS, mf* and σ_h = historical volatility and $\ln(s_r) = a + b \ln(s_{BS}) + g \ln(s_{mf})$, where σ_r = realized volatility, σ_{BS}= Black-Scholes implied volatility and σ_{mf} = model free volatility. The χ^{2c} reports the statistic of a χ^2 test for the joint null hypothesis $d = 0$, $g = 0$ and $b = 1$ (p-values in parentheses) in the following encompassing regressions: $\ln(s_r) = a + b \ln(s_{BS}) + g \ln(s_{mf}) + d \ln(s_h)$, where σ_r = realized volatility, σ_{BS}= Black-Scholes implied volatility and σ_{mf} = model free volatility and σ_h = historical volatility. The superscripts ***, **, * indicate that the slope coefficient is insignificantly different from one at the 10%, 5%, and 1% critical level respectively. The superscripts $^{+++}$, $^{++}$, $^{+}$ indicate that the slope coefficient is insignificantly different from zero at the 10%, 5%, and 1% critical level respectively The last column reports the Hausman (1978) specification test statistic (one degree of freedom) 5% critical level = 3,841.

[7] In encompassing regression (3) the results are reported for the instrumental variable procedure for ln(σ_{BS}).

Overall, these results point to a better performance of Black-Scholes implied volatility versus model free volatility. These results are in line with Andersen and Bondarenko (2007). The better performance of the Black Scholes volatility can be attributed to the fact that it has been computed by using synchronous prices, from a vast set of options in a one hour window and it has been computed as a weighted average of the two volatilities that correspond to the two strikes, one above and one below the underlying asset with weights inversely proportional to the distance of the strike price to the at the money strike. This better computational methodology has probably led the Black-Scholes volatility to an improved forecasting performance with respect to previous papers (see e.g. Christensen and Prabhala (1998)) in which non-syncronous prices were used and Black-Scholes volatility were backed out from a single option. On the other hand, model free volatility that has been computed by using all the cross section of option prices, obtains a good performance that, however, is slightly worse than the simple Black-Scholes volatility. This is probably due to some noise added by the hypotheses on the extrapolation of a continuum range of strike prices that were necessary in order to compute the model free volatility. In particular, the lack of liquid options in the tails of the strike price domain, and the assumption of a constant volatility equal to the one of the nearest option strike may induce errors in the computation of the model free volatility. A possible solution to this problem is investigated in the following section.

6. An alternative implementation of model free volatility.

The extrapolation method of Jiang and Tian (2005) is based on the extension of the domain of strike prices by supposing that for strikes below the minimum value the implied volatility is constant and equal to the volatility of K_{min}, for strikes above the maximum value the implied volatility is constant and equal to the volatility of K_{max}. This assumption may introduce some bias in the model free volatility estimation. Therefore in this section we compute model free volatility without extending the domain of strike prices: we indicate this new estimate with model free volatility 2 (σ_{mf2}). This is also in line with the computation methodology of the VIX volatility index that uses only existing call and put options.

Descriptive statistics are reported in Table 4. We can see that on average σ_{mf2} is greater than σ_{BS} and lower than σ_{mf}, therefore it contains a lower volatility risk premium than σ_{mf}. Given that the natural logarithm of the series conforms more to normality, we use the latter in the regressions.

We run both univariate regression (5) and encompassing regressions (6), (7) and (8) in order to investigate the performance of this different model free volatility estimate, in predicting future realized volatility. The results are reported in Table 5.

Table 4. Descriptive statistics for s_{mf2}.

Statistic	s_{mf2}	$\ln(s_{mf2})$
mean	0,279	-1,370
std dev	0,127	0,427
skewness	1,075	0,309
kurtosis	3,444	2,345
Jarque Bera	11,640	1,960
p-value	0,002	0,375

Table 5. OLS regressions for s_{mf2}.

Dependent variable: log realized volatility								
Independent variables								
Intercept	$\ln(s_{BS})$	$\ln(s_{mf2})$	$\ln(s_h)$	**Adj. R^2**	**DW**	$X^{2\,a}$	$X^{2\,b}$	$X^{2\,c}$
-0,1546		1,032***		0,79	2,09	46,63		
(0,13)		(0,00)				(0,00)		
-0,1248	0,936***	0,0763^{+++}		0,84	1,90		0,25	
(0,07)	(0,00)	(0,64)					(0,88)	
-0,15		0,9504***	0,08^{+++}	0,78	2,10		0,54	
(0,16)		(0,00)	(0,60)				(0,76)	
-0,127	0,9439***	0,0984^{+++}	-0,03^{+++}	0,84	1,88			0,26
(0,06)	(0,00)	(0,63)	(0,82)					(0,97)

Note: The number in brackets are the p-values. The χ^{2a} report the statistic of a χ^2 test for the joint null hypothesis $a = 0$ and $b = 1$ (p-values in parentheses) in the following univariate regression: $\ln(s_r) = a + b\ln(s_{mf2})$. The χ^{2b} report the statistic of a χ^2 test for the joint null hypothesis $g = 0$ and $b = 1$ (p-values in parentheses) in the following encompassing regressions: $\ln(s_r) = a + b\ln(s_{mf2}) + g\ln(s_h)$, where σ_r = realized volatility, σ_{mf2}= model free volatility 2 and σ_h = historical volatility and $\ln(s_r) = a + b\ln(s_{BS}) + g\ln(s_{mf2})$, where σ_r = realized volatility, σ_{BS}= Black-Scholes implied volatility and σ_{mf2} = model free volatility 2. The χ^{2c} report the statistic of a χ^2 test for the joint null hypothesis $d = 0$, $g = 0$ and $b = 1$ (p-values in parentheses) in the following encompassing regressions: $\ln(s_r) = a + b\ln(s_{BS}) + g\ln(s_{mf2}) + d\ln(s_h)$, where σ_r = realized volatility, σ_{BS}= Black-Scholes implied volatility and σ_{mf2} = model free volatility 2 and σ_h = historical volatility. The superscripts ***, **, * indicate that the slope coefficient is insignificantly different from one at the 10%, 5%, and 1% critical level respectively. The superscripts $^{+++}$, $^{++}$, $^{+}$ indicate that the slope coefficient is insignificantly different from zero at the 10%, 5%, and 1% critical level respectively .

We can see that the model free volatility computed by using only existing call and put prices obtains a worse performance than model free volatility computed by extending the strike price domain as suggested by Jiang and Tian (2005). Although model free volatility remains still an efficient and unbiased forecast of future realised volatility because it subsumes all the information contained in historical volatility, it obtains an adjusted R^2 that is inferior to the one of the original model free estimate. Moreover, also in this case, Black Scholes implied volatility subsumes all the information contained in this latter model free estimate. Therefore we can conclude that even if the Jiang and Tian (2005) extrapolation method is based on particular assumptions on the smile shape outside the existing strike price interval, it is superior to a methodology that uses only the existing strike price domain.

7. Conclusions.

The forecasting performance of Black Scholes implied volatility versus a time series volatility forecast has been extensively analysed in various papers. In most cases Black Scholes implied volatility is extracted from at the money options. Only a few papers have addressed the issue of investigating the forecasting performance of a different option based volatility forecast: model free implied volatility. Model free implied volatility is completely specified by a continuum set of option prices expiring on date T. As it is derived by using a cross section of option prices differing in strike prices and option type, it should be, theoretically, more informative than implied volatility backed out from a single option. However, due to numerous practical limitations (limited number of strike prices, that are available only at fixed increments), the evidence about the superiority of model free implied volatility versus Black-Scholes implied volatility is mixed (see e.g. Jiang and Tian (2005), Andersen and Bondarenko (2007)).

In this paper we have investigated the forecasting power of the two option based volatility forecasts: Black-Scholes implied volatility and model free volatility. As Black and Scholes implied volatility differs depending on strike price of the option (the so-called smile effect) and, due to measurement errors, option type (call versus put), which option class yields implied volatilities that are most representative of the markets' volatility expectations is still an open debate. In order to pursue a fair comparison with model free implied volatility, that is derived based on a cross section of option prices, for implied volatility we have used a weighted average of implied volatilities backed out from different option classes.

Two hypotheses have been tested: unbiasedness and efficiency of the different volatility forecasts. Our results suggest that both option based volatility forecasts contain more information about future realised volatility than historical volatility. They are not unbiased, since they contain a substantial risk premium, but they can be considered unbiased after a constant adjustment given by the intercept of the regression. Moreover they are both efficient forecasts of realised volatility in that they subsume all the information contained in historical volatility.

The comparison between the two option based volatility forecasts has highlighted a slightly better performance of Black-Scholes implied volatility versus model free volatility. These results are in line with the finding of Andersen and Bondarenko (2007) that found that Black and Scholes implied volatility obtains a better forecasting performance than model free volatility, being less sensitive to the time variation in the volatility risk premium.

The better performance of the Black Scholes volatility can be attributed to the fact that it has been computed by using synchronous prices, from a vast set of options in a one hour window and it has been computed as a weighted average of the two volatilities that correspond to the two strikes, one above and one below the underlying asset, with weights inversely proportional to the distance of the strike price to the at the money strike. This better computational methodology has probably led the Black-Scholes volatility to an improved forecasting performance with respect to previous papers (see e.g. Christensen and Prabhala (1998)) in which non-syncronous prices were used and Black-Scholes volatility were backed out from a single option. On the other hand, model free volatility, that has been computed by using all the cross section of option prices, obtains a good performance that, however, is slightly worse than the simple Black-Scholes volatility. This is probably due to some noise added by the hypotheses on the extrapolation of a continuum range of strike prices that were necessary in order to compute the model free volatility. In particular, the lack of liquid options in the tails of the strike price domain, and the assumption of a constant volatility equal to the one of the nearest option strike may have induced errors in the computation of the model free volatility. One possible solution to reduce the impact of this latter assumption would be to use only the existing strike price domain, as it is done for the computation of the VIX index. However, this tactic has proved to be inferior, in the present dataset, to the extrapolation methodology proposed by Jiang and Tian (2005). The study of other possible remedies in order to improve the model free volatility performance is left for future research.

Appendix 1.

In order to examine the performance of model free implied volatility when we vary the parameters ***u*** and *DK*, we show some simulations based on the Heston's (1993) stochastic volatility model.

In the Heston model the underlying asset follows the stochastic process:

$$\frac{dS_t}{S_t} = m dt + V_t^{1/2} dW_t$$

The variance of the underlying asset follows the mean reverting process:

$$dV_t = k_v(q_v - V_t)dt + s_v V_t^{1/2} dW_t^v$$

with:

$$dW_t dW_t^v = r\, dt$$

V_t is the variance of the returns of the underlying asset, W_t and W_t^v are the standard Wiener processes for the underlying and the variance with correlation r , q is the long run average of variance, k is the mean reversion rate, s_v is the volatility of variance.

For the present implementation we use the same parameters as in Heston (1993), namely: spot price *S*=100, risk free rate *r*=0, time to maturity *t*=0,5, current variance v=0,01, correlation r =-0,5, mean reversion rate *k*=2, long run average of variance q =0,01, volatility of variance s_v =0,225, price of volatility risk λ=0. The volatility of the stock returns over the life time of the option is 0,071. For the current implementation we generate call and put option prices, by using the Heston model for a range of strike prices *K=[80,120]* equally spaced by *DK=0,5*. For this interval of strikes, we analyse both the truncation and the discretization errors by computing the model free volatility by using different values of ***u*** and *DK* and deriving the percentage error ***(PE)*** in predicting the true volatility of 0,071, as: $PE = \frac{0,071 - s_{mf}}{0,071}$. Recall that the parameter ***u*** determines the range of strike prices used, while *DK* sets the spacing between strike prices.

In order to examine the truncation error we compute the model free volatility by using different levels of *u* and a fixed *DK=0,5*. The results are reported in Table A1.1: *u* determines the strike price range used [K_{min}, K_{max}], s_{mf} is the model free volatility and *PE* is the percentage error in predicting the true volatility of 0,071, that is computed as: $PE = \frac{0,071 - s_{mf}}{0,071}$. We can see that the truncation error is substantially high only if we use a very narrow interval of strikes: increasing the strike price interval the truncation error converges very quickly to zero. In Figure A1.1 we plot the percentage error against *u*. We can see that the percentage error is very low for values of *u* bigger than 0,3. As the spot price is *100*, a strike price interval of *[77,130]* is enough to ensure that the truncation error does not have any impact on the model free calculation. For the implementation on the Dax index data we used *u=0,5*, therefore we are confident that truncation errors are not likely to have affected our results.

Table A1.1. The truncation error for different values of u.

u	K_{min}	K_{max}	s_{MF}	PE
0,05	95	105	0,0579147	0,179678
0,1	91	110	0,0660902	0,063878
0,15	87	115	0,0689055	0,024001
0,2	83	120	0,0700137	0,008304
0,25	80	125	0,0703841	0,003058
0,3	77	130	0,0705385	0,000871
0,35	74	135	0,0705855	0,000206
0,4	71	140	0,0705973	0,000038
0,45	69	145	0,0705994	0,000009
0,5	67	150	0,0706000	0,000000
0,55	65	155	0,0706002	-0,000002
0,6	63	160	0,0706002	-0,000003
0,65	61	165	0,0706002	-0,000003
0,7	59	170	0,0706002	-0,000003
0,75	57	175	0,0706002	-0,000003
0,8	56	180	0,0706002	-0,000003
0,85	54	185	0,0706002	-0,000003
0,9	53	190	0,0706002	-0,000003
0,95	51	195	0,0706002	-0,000003
1	50	200	0,0706002	-0,000003

In order to examine the discretization bias we compute the model free volatility by using different values of *DK* ranging from *0,05* to *3*, and a fixed *u=1*. In Figure A1.2 we plot the percentage error against *DK*: we can see that the discretization error is negligible for *DK* bigger than *1*. As the spot price is *100*, a strike price discreteness of *1%* is enough to ensure an insignificant discretization error. As the Dax index values, for the data set used in the implementation, are typically greater than *2900*, for the implementation on the Dax index we choose *DK=10*.

Figure A1.2. The truncation error for different values of u.

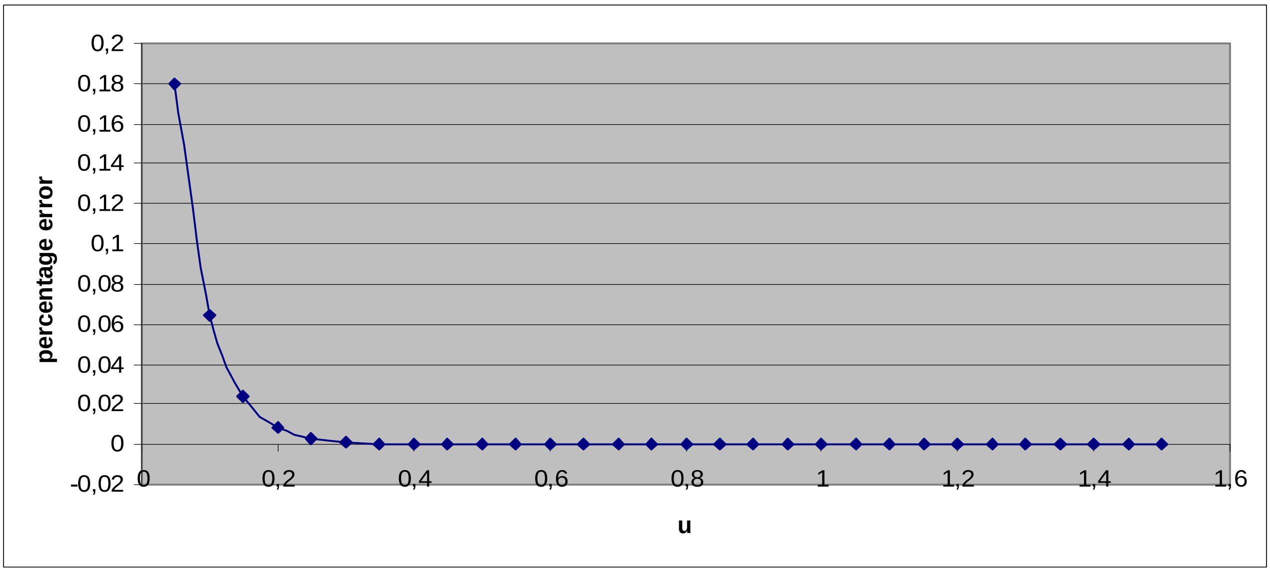
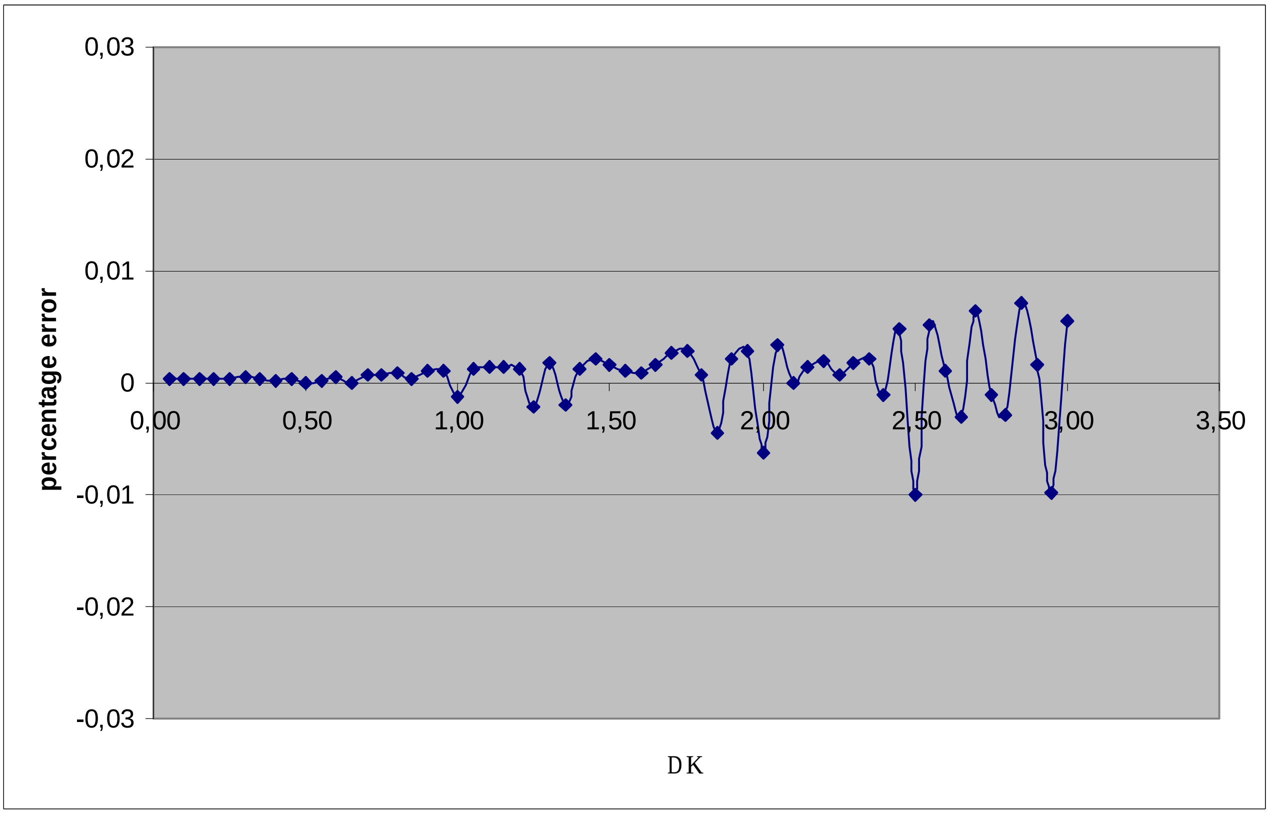


Figure A1.2. The discretization error for different values of *DK*.



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