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BUILDING A MATHEMATICAL MODEL TO PREVENT HYDRATE FORMATION IN GAS PIPELINES

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Abstract

Development of mathematical models of laminar gas flow in certainty and uncertainty conditions were considered. All factors that influence to character of flow of gas in pipeline and conditions of arising of hydrate inside of pipeline wall are analyzed. Results of analyze were used for development mathematical model of gas flow in pipeline that allow to calculate main parameters of hydrate deposition process. Model of gas flow consist of three non-linear differential equations that can be solved by exist soft wares. Two and three-dimension characteristic obtained, that describe of quantity of hydrate deposited at given process depending on time.

Besides, the effectiveness of using DELPHI 7 software to determine the preparation of gas for transportation and the hydrates formed during transportation and its prevention based on the results of the application software was discussed. As a result, the change in cross-sectional area of the pipeline of hydrates formed on the inner surface of the pipeline is shown in 3D. Hydrate formation and elimination are visually represented by graphs. The results of theoretical and practical studies of changes in the inner surface of the pipeline depending on temperature and pressure have been identified. All this was investigated during quasi-stationary gas flow in the pipelines and the results were obtained.

The assumes regarding calculation of parameters of gas flow were determined and necessary recommendations for applying of developed model in different conditions with taking account of temperature and pressure variation and depending on time of hydrate deposition were presented. The diagram of gas-hydrate separation boundary considered for detailed analysis of process.

Keywords: natural gas, mathematical model, laminar mode, hydrate formation, pipeline, inhibitor, pressure change.

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1. Introduction

Mathematical model of hydrate formation process during the movement of the inhibitor-added gas in pipelines requires a joint study of algebraic dependencies that take into account changes of thermodynamic state of it. These algebraic dependences are the change in the humidity of natural gas depending on pressure and temperature, the change in the equilibrium of hydrate formation depending on pressure and temperature, the dependence of the transition point of hydrate formation temperature on the amount of methanol and the equation of state of real gas.

The movement of natural gas mixtures in fixed and variable cross-section pipelines has been extensively studied under the following simplification conditions [1–3]:

1. The movement of gas inside the pipe was studied by one dimensional gas dynamics methods.

2. The gas-hydrate phase transition is investigated through the Stephen problem statement.

3. The velocity of the moisture and the inhibitor are taken to be equal to the velocity of the gas itself.

4. The increase in the thickness of the solid layer over time along the axis of the pipe is assumed to be very small, i.e. the motion is assumed to be quasi-stable.

5. It is assumed that the heat transfer in the direction along the axis is constant.
6. Convective heat transfer is accepted, so conductive transfer is not taken into account.
7. The thermal resistance of the pipeline wall was not taken into account.
8. The physical properties of gas and hydrate are considered stable.
9. It is assumed that the reagent used is instantly dissolved in water.
10. It is accepted that the inhibitor is not included in the hydrate as a component.

Mathematical model of hydrate formation process during the movement of the inhibitor added gas in the pipelines requires differential equation of the movement of the mixture inside the pipe, equation describing the change in the thermodynamic state of gas-hydrate system, balance equation describing the distribution of methanol and water along the gas flow (three differential equations), as well as some the joint study of algebraic dependencies taking into account the change of some parameters. These algebraic dependences include the change of humidity of natural gas depending on pressure and temperature, the change of equilibrium of hydrate formation depending on pressure and temperature, dependence of hydrate formation temperature transition point on the amount of methanol, and real state equation of the gas.

Our researches were conducted in quasi-stationary mode due to the need for scientific investigations on hydrate formation and their prevention during the movement of gases in pipelines in quasi-stationary modes. The use of mathematical modeling of the obtained results will play an important role in eliminating the problem.

Taking into account all conditions of above noted, let's consider the hydrate formation process during the movement of gas inside the pipeline without an inhibitor.

2. Materials and methods

For study of movement of gas flow regarding to gas mass conservation law in certain V volume let's use the diagram of part of pipeline that limited by fixed close surface F (**Fig. 1**) [4]. In cases when an inhibitor add let's use equation that taken into account variation of some parameters of flow.

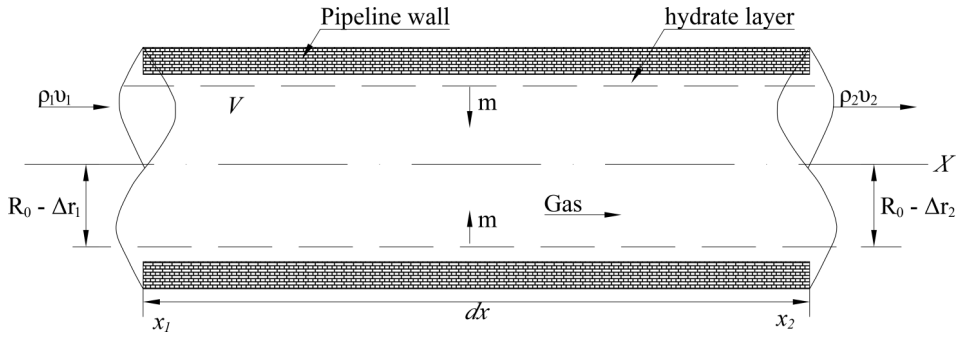

Fig. 1. Part of pipeline

The transportation process in the section of the main gas pipeline is described by the characteristics of the gas flow, which, due to its spatial extent in terms of properties, can be attributed to a continuous medium [5, 6]. Such processes are described in the theory of hydromechanics, hydro- and gas dynamics, and the equations describing these processes are partial differential equations defined for two-dimensional functions of space and time.

For solution of the problem of «slow» subsystem for quasi-stationary motion of gas, when forms hydrate accumulates in the inner wall of the pipe, and changes in the working cross section of the pipeline and the physical and chemical characteristics of gas (velocity, temperature, etc.). Let's add to the system of equations presented in [5, 7] the expression for the function change the cross section of the pipe $S = S(x)$. In the initial case, $S = S(x)$ is known, i.e. it is a constant for individual parts. From the moment t_0 of time, hydrate begins to form, and after a certain time it deposits to the inner surface of the pipe, creating a certain relief (roughness). The situation that can occur is shown schematically in **Fig. 2.**

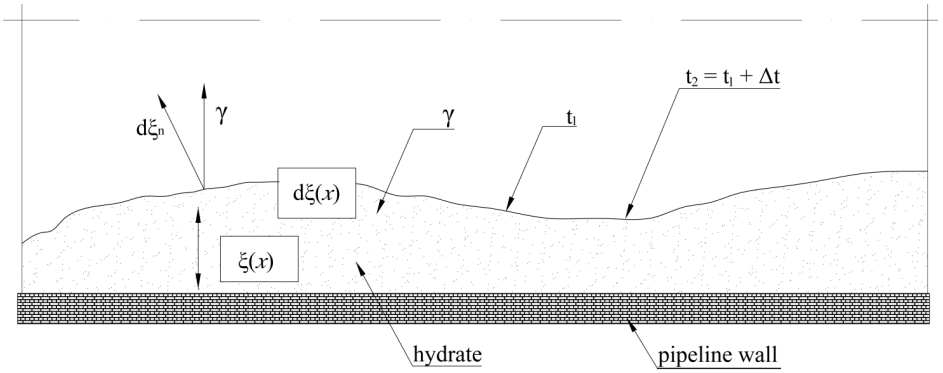

Fig. 2. Scheme of gas-hydrate precipitation phase boundary

3. Results and discussion

Let's write an energy conservation equation for gas-hydrate phases boundary, that shown in **Fig. 2**:

$$L\frac{\partial\xi}{\partial t} = \lambda^2 L_r \rho_r \frac{\partial\xi}{\partial t} = \lambda_r \frac{\partial T_r}{\partial n}\Big|_{r=R_0-\xi} + \alpha\left(T_f - T\right). \tag{1}$$

Where α – the heat transfer coefficient of the hydrated phase of the gas; L_r – latent heat for hydrate formation, ρ_r – density of hydrate, λ_r – thermal conductivity of hydrate, ξ – thickness of hydrate layer, n – external norm of hydrate layer surface (directed inside the pipe); $T_f(P,T)$ – phase transition temperature, T_r – hydrate temperature, T – gas temperature, P – pressure of gas, and R_0 – the radius of the clean pipe.

If to write equation (1) in the projection in the direction of the radius, the following is obtained:

$$L_r\rho_r \frac{\partial\xi}{\partial t} = \sec^2\gamma\left(-\lambda_r \frac{\partial T_r}{\partial r}\Big|_{r=R_0-\xi} + \alpha(T_f - T)\cos\gamma\right). \tag{2}$$

Here γ – angle between the inner surface of the hydrate layer and the normal of the inner surface of the pipe. As well as $\cos\gamma = \left(\sqrt{1+\left(\partial\xi/\partial t\right)^2}\right)^{-1}$ and $\sec^2\gamma = 1/\cos^2\gamma$, taking into account of above relation [8], after a series of transformations, the following expression is obtained:

$$L_r\rho_r \frac{\partial\xi}{\partial t} = -\left(1+\left(\frac{\partial\xi}{\partial t}\right)^2\right)\lambda_r \frac{\partial T_r}{\partial r}\Big|_{r=R_0-\xi} = \alpha\left(T_f - T\right). \tag{3}$$

In real natural conditions $\gamma\to 0$, so $\partial\xi/\partial t < 1$, then (3) transform to next form:

$$L_r\rho_r \frac{\partial\xi}{\partial t} = -\lambda_r \frac{\partial T_r}{\partial r}\Big|_{r=R_0-\xi} + \alpha\left(T_f - T\right). \tag{4}$$

To present of expression:

$$\frac{\partial T_r}{\partial r}\Big|_{r=R_{0-\xi}},$$

in a more appropriate form, let's consider the equation of heat transfer in a cylindrical coordinate system in an area with hydrate [9]:

$$\frac{\partial T_r}{\partial r} = \tilde{a}\left(\frac{\partial^2 T_r}{\partial r^2} + \frac{1}{r}\frac{\partial T_r}{\partial r} + \frac{\partial^2 T_r}{\partial x^2}\right).$$

Here $\tilde{a}$ – temperature transfer coefficient of the hydrate.

Experiments show that the velocity at the gas-hydrate phase boundary is about 100 times less than the velocity of isotherms [10]. Using this, if to assume that the temperature distribution is quasi-steady:

$$\frac{\partial^2 T_r}{\partial r^2}+\frac{1}{r}\frac{\partial T_r}{\partial r}+\frac{\partial^2 T_r}{\partial x^2}=0. \tag{5}$$

Given that the temperature drop in the radial direction is greater than in the axial drop, it is possible to ignore the III member in (5), then let's obtain next:

$$\frac{\partial^2 T_r}{\partial r^2}+\frac{1}{r}\frac{\partial T_r}{\partial r}=0. \tag{6}$$

Let's solve this equation at next boundary conditions:

$$T_r\big|_{r=R_0}=T_0;\, T_r\big|_{r=R_0-\xi}=T_f.$$

Applying replacement $\partial T_r/\partial r=\theta,$ let's obtain:

$$\frac{\partial\theta}{\partial r}+\frac{1}{r}\theta=0. \tag{7}$$

Solution of this equation let's search in form $\theta=B/r$: $B=\text{const}$ determined from boundary conditions. In this case:

$$T_r(r)=B\cdot\ln r+\mathrm{C}. \tag{8}$$

From boundary conditions (7) obtain next:

$$B=\frac{T_0-T_f}{\ln\dfrac{R_0}{R_0-\xi}};\; C=\frac{T_f\ln R_0-T_0\ln(R_0-\xi)}{\ln\dfrac{R_0}{R_0-\xi}}. \tag{9}$$

Finally, these expressions take into account in (6):

$$T_r(r)=\frac{T_f\ln\dfrac{R_0}{2}+T_0\ln\dfrac{r}{R_0-\xi}}{\ln\dfrac{R_0}{R_0-\xi}}, \tag{10}$$

or in compact form:

$$T_r(r)=T_0+\frac{T_f-T_0}{\ln\dfrac{R_0}{R_0-\xi}}\ln\frac{r}{R_0}. \tag{11}$$

Then:

$$\frac{\partial T_r}{\partial r}\Big|_{r=R_0-\xi}=\frac{T_f-T_0}{(R_0-\xi)\ln\left(1-\dfrac{\xi}{R_0}\right)}.$$

If to take into account this expression in (4) and use for calculation of square of hydrate layer next formula:

$$S=\pi(R_0-\xi)^2,$$

let's obtain finally following expression:

$$\frac{\partial \xi}{\partial t} = \frac{1}{2\pi(R_0 - \xi)} \frac{\partial S}{\partial t}.$$

From this expression let's obtain:

$$\frac{\partial S}{\partial t} = \frac{-4\pi a_q (T_f - T_0)}{\rho_r L_r \ln \frac{S}{\pi R_0^2}} - \frac{2\sqrt{\pi} S \cdot \alpha_0 (T_f - T_0)}{\rho_r L_r}. \quad (12)$$

The integration of this equation gives a change in the cross section of the hydrate in the inner wall of the tube over time, which allows $S(x, t)$ to be controlled by keeping P and T constant for each time interval considered. Thus, the proposed mathematical model reflects the dynamic development of the hydrate layer and the associated changes in pressure and temperature fields over time and along the axis of the pipeline.

Let's look at the results of numerical modeling of the solution of this problem. To solve the proposed system of nonlinear ordinary differential equations and construct graphs in 3D format standard forms of application of the IV order Runge-Kutta method were used. Based on the model presented above, proposed a calculating algorithm that consists of following equations:

$$\frac{dP}{dx} = -\rho g \sin\varphi - \frac{\lambda M^2}{4\rho S^2 S_0^2} \sqrt{\frac{\pi}{ss_0}},$$

$$\frac{dT}{dx} = \varepsilon \frac{dp}{dx} + \frac{\pi D \alpha}{c_p M}(T_0 - T) \frac{g}{c_p} \sin\varphi, \quad (13)$$

$$\frac{dS}{dt} = b_2 \frac{T_0 - T_f(\rho)}{1 - b_2 \ln S} - b_1 \sqrt{s}\,(T_f(\rho) - T).$$

Here $\rho = p/zRT$ – density of natural gas, R – gas constant, $z = 1 + 0.07P/T(1 - G/T^2)$ – incompleteness coefficient of natural gas determined by formula of Bertlo, $S(t)$ – changing in the cross section of the pipe at distances $x = x_k$, depending on the accumulating of hydrate, $T(x)$, $P(x)$ – changing of temperature and pressure in the pipe along the pipe at a given specific moment in time. φ is the angle of inclination of the pipe with respect to the horizontal plane.

$\varphi = 0$ can be used for a completely horizontal position, and $\varphi = \pi/2$ for vertical pipes and wells (with some approximation, because in this case the geometric gradient must also be taken into account). Here b_1 and b_2 are defined as follows:

$$b_1 = \frac{D_0 \alpha_1}{4\lambda_h},\ b_2 = \frac{D_0 \alpha_2}{4\lambda_h}.$$

α_1, α_2 are respectively the coefficients of heat exchange between gas and hydrate layer, between the hydrate layer and pipeline wall; τ is non-dimension time:

$$\tau = \frac{\lambda_h T_{kr} t}{\rho_h a_h D_0^2},$$

where index «h» are parameters refer to hydrate, index «0» are initial values of indexed parameters, λ_h – thermal conductivity of the hydrate, T_{kr} – critical temperature of the gas, α_h – specific heat of hydrate formation.

The data required for solution of test task are given below. The initial conditions are as follows:

$$P(0) = P_0,\ T(0) = T_0,\ S(0) = S_0.$$

As seen from (13) equation system has strong non-linear. Such non-linear differential equation system where other variables are parametrical, can be solve by existing program complexes that allow to obtain of two dimension – surface and 3D of distribution diagram of $P(x,t_k)$, $T(x,t_k)$, $S(x,t_k)$ functions. For solving test task used next initial data (**Table 1**).

Table 1

Initial data for calculation

$a = 5.82 W/(m^2 \cdot K)$	$l_h = 2 W/m \cdot K$	$L = 2550$ m
$D_0 = 0.1m$	$l = 510000$ J/kg	$T_{eo} = 328$ K
$y = 0.02$	$c_p = 2300$ J/(kg·K)	$G = 0.0256$ K/m
$b_1 = 3.4$	$p_0 = 240 \cdot 10^5$ N/m^2	$x < L - 400m$ at $T_e = T_{e0} - T_x$
$b_2 = 0.017$	$T_0 = 323$ K	$L > x > L - 400m$ at $T_e = 273.15$
$R = 520$ J/(kg·K)	$T_c = 205.239$ K	–
$r_h = 920$ kg/m^3	$p_c = 46.573 \cdot 10^5$ N/m^2	–

For application of proposed algorithm, the computing program in algorithmic language DELPHI 7 prepared that written in accordance with presented model.

The proposed mathematical model reflects the dynamic development of the hydrate layer and, in connection with it, the change in the pressure and temperature with time along the axis of the pipeline. The calculation algorithm was proposed on base of this model.

All the formulas of the mathematical notation of problem solving approaches are not presented here, the main features of the method are to control the flow parameters at the inlet of pipelines in such a way that hydrate formation in the pipe and along the inner surface is able to be minimized. That is, it is possible to prevent hidrate formation in pipelines by several methods such as creating the stability and structure of the flow and bivouacs, equipping the pipeline with structural forms, etc, which allow to prevent inhibitor consumption.

Calculation results are presented in surface form and curves in following (**Fig. 3–5**).

From **Fig. 3–5** temperature of gas decreases to approximately 280 K, and pressure drop of gas is 2×10^6 Pa in modeling distance from compressor equal to 100 km approximately after 41.6 hours.

Fig. 6–9 are shown changing of thickness of hydrate layer along pipeline in dependence on time.

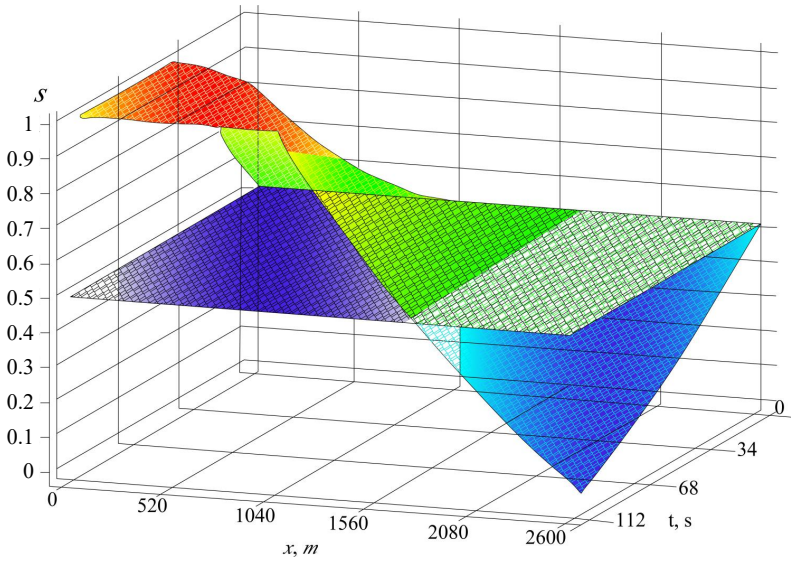

Fig. 3. Changing of $S(x_k,t)$ function

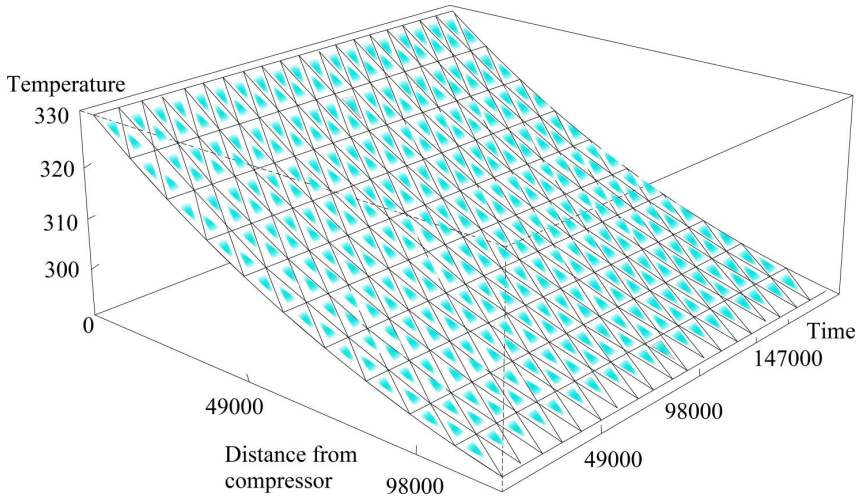

Fig. 4. Changing of temperature of gas in dependency on distance from compressor station and time

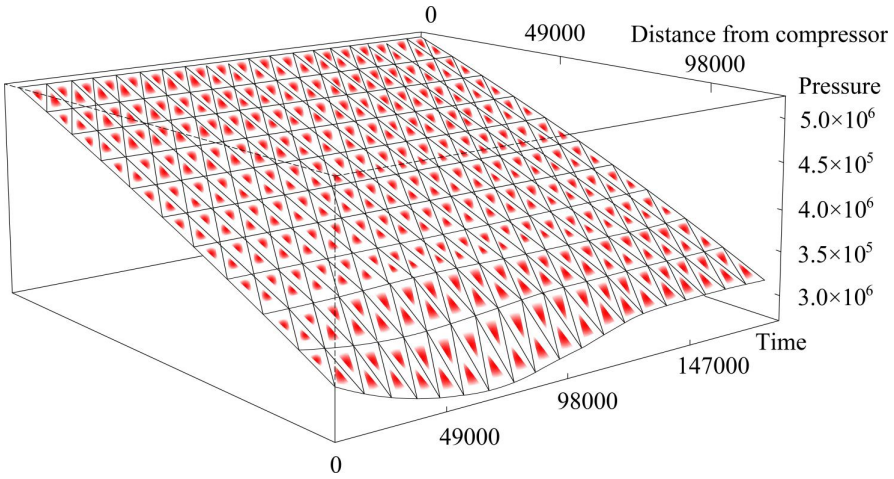

Fig. 5. Changing of pressure of gas in dependency on distance from compressor station and time

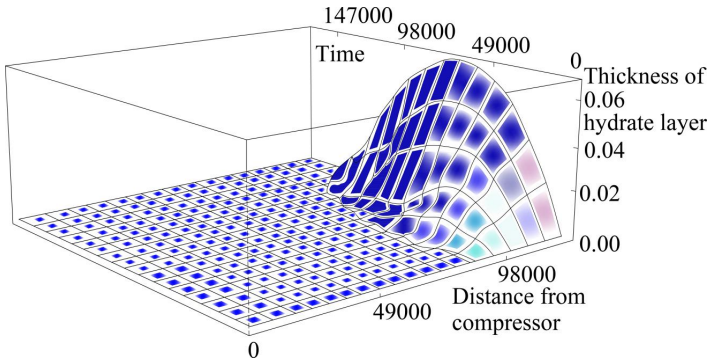

Fig. 6. Changing of thickness of hydrate layer in dependence on distance from compressor station and time

As seen from presented figures (**Fig. 3–9**) all characteristics are shown the changing of different parameters, but do not directly taking into account the time parameter, that complicated of analysis of dynamic of process.

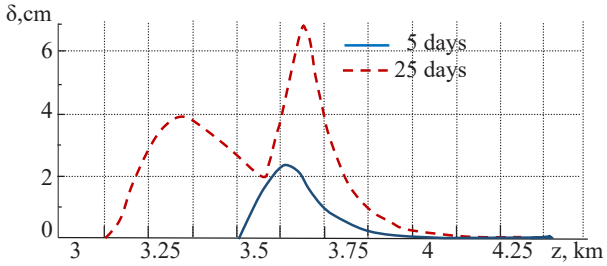

Fig. 7. Thickness of hydrate layer in dependence on pipeline length

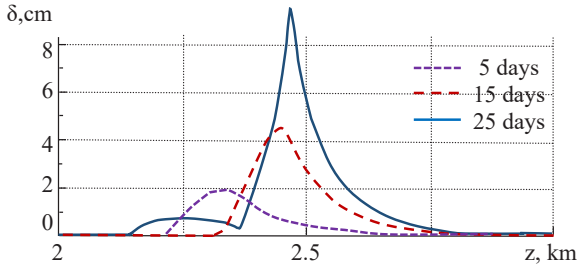

Fig. 8. Changing of thickness of hydrate layer and pressure in dependence on pipeline length and time

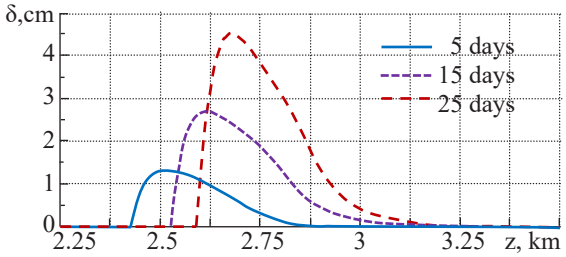

Fig. 9. Changing of thickness in dependence on pipeline length

According to specified formulas, calculations show that in these cases, gas approaches the environment temperature since the temperature changes along the pipelines. At this time, hydrate formation stops. All these are graphically presented in the figures, thus, as a result of changes in temperature and pressure, the increase of hydrate formation depending on time along the diameter and length of the pipeline, and the results of prevention and elimination of hydrate formation due to inhibitor application are graphically presented. All this can be seen from the graphs. The subsequent temperature of the gas does not change, and the pressure continues to decrease. For example, as a result of the calculation, the gas temperature in an underwater pipeline with a diameter of 200 mm and pipe capacity of 800.000 m^3/day becomes equal to the water temperature at a distance of $x_0 = 0.5$ km. In pipelines according to those parameters it becomes $x_0 = 17$ km. In the proposed research option, the process calculation algorithm is presented in a more accurate approach, which will allow to determine the gas movement in the pipeline more accurately.

4. Conclusions

The results of the application of software are presented in the figure. The proposed model describes the process of hydrate formation in the gas flow depending on the pressure, temperature and distance from the compressor and the length of the pipeline without taking into account the processing time, which allows to analyze the process with sufficient accuracy.

For increasing of accuracy required to put the time parameter in finally model of gas flow that complicates the model and its solution.

Adequate model for gas flow, taking into account of informative parameters and the time is possible if to use fuzzy approach to modeling of given process.

The problems stated in the article have been solved with their relevance and the prevention of the consumed reagents usage and hydrate formation and their elimination have been shown as principals by means of a mathematical model.

Conflict of interest

The authors declare that there is no conflict of interest in relation to this paper, as well as the published research results, including the financial aspects of conducting the research, obtaining and using its results, as well as any non-financial personal relationships.

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