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ANALYSIS OF COSMOLOGICAL BIAS WITHIN SPHERICAL COLLAPSE MODEL

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Abstract

The goal of our research work is to analyze cosmological bias parameter. Parametric equations of spherical collapse model are used to calculate the values of spherical collapse over density and mass variance, which is further used in bias formulae to find the values of cosmological bias. Spherical collapse over density has been calculated in the range of redshift 0 to 1. Also, it is compared with the value according to the spherical collapse model. Bias is one of the parameters which are utilized to infer cosmological parameters. Extracting the cosmological parameters is very much useful to know and understand about the birth and evolution of our universe. As there is no direct probe to get the idea about the existence of dark matter. Bias factor helps to analyze about dark matter. The bias coefficient of higher order terms in Taylor series expansion are found to be in ascending order. Increasing values of bias indicate the large-scale structure formation at current epoch is more and more clustered. Values of bias are discussed in result. Also, bias values have been analyzed for redshift in the range 2 to 0. The graph has been plotted bias versus redshift. Let's found bias decreases with decrease of redshift. That means bias evolves with redshift. Bias value less than one and negative value of bias implies that structure formation is in linear region and higher values of bias indicates the structure formation occurs in nonlinear region. Negative value of bias is also called as antibias. That means the structure formation has not started yet. It is still in linear region. The bias value nearly equal to one indicates that the structure formation has been transformed from linear region to nonlinear region. So, the result showing bias values greater than one indicates that evolution of structure formation occurs in nonlinear region.

Keywords: cosmological bias, large-scale structure, dark matter halo, galaxies, spherical collapse model.

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1. Introduction

Basically, the goal is to find the values of cosmological bias analytically by using spherical collapse model. A halo is defined as a virialized system having mean density 200 times the critical density of the universe. Halos are the structural unit of the cosmos. Dark matter halos are the non-linear units of cosmology as cosmic structure and galaxies condensed within their cores. [1] worked on halo model to find statistical behaviour of galaxies and dark matter properties also, which it is followed in our research work. One of the applications of halo model is spatial distribution of galaxies. [2] argued that each galaxy is made up of distinct clusters and have a particular size. Since galaxy is a discrete object, its statistical properties can be studied. So, the luminous galaxies that it is possible to see today may be the bias tracers of the dark matter distribution. The non-linear evolution of the dark matter distribution has been studied by numeric simulations of the large-scale structure clustering process. These simulations show that initially smooth matter distribution evolves into a complex network of sheets, filaments, and knots. The dense knots are often called as dark matter halos. It is assumed that dark matter halos are in virial equilibrium. This assumption helps one to estimate the physical properties of a given halo. Spherical collapse model is first studied by [3]. This is the simplest non-trivial model for the way an object like galaxy or a cluster of galaxies breaks away from the general expansion. In the model the universe is spherically symmetric about one spot and the matter is an ideal fluid with zero pressure in the initial stage. The spherical region having wavelength more than Jean's wavelength can collapse to form large scale structure.

Let's extend spherical collapse model to find spherical collapse density for our research work at some redshift. The density of distribution of galaxies is different from the density of underlying dark matter. So, the galaxy density is a function of dark matter density [4] which is called as bias. [5] introduced bias. Bias can be written as the ratio of density fluctuation of galaxy to the density fluctuation. Bias depends on mass of dark matter halo as well as epoch of galaxy formation was determined by [6] using extended [7] theory and [8] mass function. Our research work analyses the dependency of bias on halo mass and redshift. Calculation of bias is done by using [7, 8] halo mass function. According to [9] bias less than one is antibias and greater than one is biased, highly clustered.

2. Materials and methods

Considering a spherical region of uniform density row and radius R the parametric equations [10] for a bound system are:

$$\frac{t}{t_m} = \frac{1}{\pi}(\eta - \sin\eta), \tag{1}$$

where t is initial epoch, t_m is maximum epoch, η is development angle.

$$\frac{R}{R_m} = \frac{1}{2}\times(1-\cos\theta), \tag{2}$$

where R is initial size of halo; R_m is maximum size of halo, θ is zero to 2 pi.

Where t represents initial to maximum epoch, R represents initial size to maximum size of the halo; eta represents development angle (initial angle 0 to turnaround at pi to collapse at 2 pi).

Let's calculate the value of spherical collapse density delta as a function of eta.

Differentiating (1) and (2) with respect to η:

$$dt = \frac{2}{\pi}\times\frac{t_m}{R_m}\times R d\eta. \tag{3}$$

The relation between the redshift z and scale factor a and time factor are as follows: redshift $1+z$ is equal to ratio of scale factor at current epoch to scale factor at any epoch. That is equal to reciprocal of scale factor at any epoch:

$$1+z = \left(\frac{t_0}{t}\right)^{\frac{2}{3}} = \frac{1}{a} \Rightarrow dz = -\frac{1}{a^2}\frac{da}{dt}dt, \tag{4}$$

where t_0 is current epoch; t is any epoch; a is scale factor.

For matter dominated era, the scale factor is:

$$a(t) = \left(\frac{t_0}{t}\right)^{\frac{2}{3}},\ \dot{a} = -\frac{2}{3}\frac{(t_0)^{\frac{2}{3}}}{(t)^{\frac{5}{3}}}, \tag{5}$$

where a is scale factor and function of time.

Using the relations from (4), (5):

$$\left[-\frac{3}{2}\times(t_0)^{-\frac{2}{3}}\times\frac{(t_0)^{\frac{5}{3}}}{(1+z)^{\frac{5}{2}}}dz\right] = \left[\frac{2}{\pi}\times\frac{t_m}{R_m}\times R d\eta\right], \tag{6}$$

where z is redshift at current epoch.

Integrating on both sides of (6):

$$\left[-\frac{3}{2}\int_{z_i}^{z_m}(1+z)-\frac{5}{2^{dz}}\right] = \left[\frac{2}{\pi}\times\frac{t_m}{R_m}\int_0^{\pi} R\mathrm{d}\eta\right], \Rightarrow -\frac{3}{2}\left[\frac{(1+z)^{-\frac{3}{2}}}{-\frac{3}{2}}\right]_{z_i}^{z_m} = \left[\frac{2}{\pi}\times\frac{t_m}{R_m}\int_0^{\pi}\frac{R_M}{2}(1-\cos\eta)\mathrm{d}\eta\right],$$

$$\Rightarrow\left[(1+z)^{-\frac{3}{2}}-(1+z_i)^{-\frac{3}{2}}\right]=\frac{t_m}{\pi}\left[\eta-\left(\eta-\frac{\eta^3}{3!}\right)\right],$$

where z_i is initial redshift.

$$\Rightarrow\left[\left(\frac{1}{z_i}+\frac{z}{z_i}\right)^{-\frac{3}{2}}-\left(\frac{1}{z_i}+\frac{z}{z_i}\right)^{-\frac{3}{2}}\right]\times\frac{1}{(z_i)^{\frac{3}{2}}}=\frac{t_m}{\pi}\times\frac{\eta^3}{6}. \tag{7}$$

At $z=0$ and using Binomial theorem to the second term in LHS to the second term:

$$\left[\left(\frac{1}{z_i}\right)^{-\frac{3}{2}}-\left(1-\frac{3}{2}\frac{1}{z_i}\right)\right]\times\frac{1}{(z_i)^{\frac{3}{2}}}=\frac{t_m}{\pi}\times\frac{\eta^3}{6}. \tag{8}$$

For redshift equal to 1:

$$\left[1-\left(1-\frac{3}{2}\right)\right]\times 1=\frac{t_m}{\pi}\times\frac{\eta^3}{6},\ \Rightarrow\eta^2=\left(\frac{3}{2}\right)^{\frac{2}{3}}\times\frac{(6\pi)^{\frac{2}{3}}}{(t_m)^{\frac{2}{3}}}. \tag{9}$$

From (2):

$$\frac{R}{R_M}=\frac{\left[1-\left(1-\frac{\eta^2}{2!}\right)\right]}{2}=\frac{\eta^2}{4}. \tag{10}$$

Again, the density within a region is $(R_0/R)^3=(1+\delta)$, where, R_0 is initial size of the spherical halo; R is the subsequent size of the spherical halo:

$$R=R_0\left(1-\frac{\delta}{3}\right)\Rightarrow\delta(\eta)=3\left(1-\frac{R}{R_0}\right), \tag{11}$$

$\delta(\eta)$ is spherical collapse density.

Using (9) and (10) in (11):

$$\delta(\eta)=3\left(1-\frac{R_M}{R_0}\times\frac{\eta^2}{4}\right). \tag{12}$$

According to definition, scale factor $a(t_m)=R_m/R_0$. Again, scale factor with respect to time:

$$a(t_m)=\left(\frac{t_m}{t_0}\right)^{\frac{2}{3}},$$

where $a(t_m)$ is scale factor at maximum epoch.

So, (11) becomes:

$$\delta(\eta)=3\left[1-\frac{22.206}{(t_0)^{\frac{2}{3}}}\right], \tag{13}$$

$$\Rightarrow\delta(\eta)=1.793.$$

At current epoch the value of time factor in (13) is considered as 13.799 giga year.

Since spherical density greater than one, spherical collapse occurs to form large scale structures like galaxies and clusters of galaxies. According to spherical collapse model spherical collapse density value is 1.686.

Now mass variance is:

$$\sigma^2(m) = \int \frac{dK}{K} K^3 \left[P^{lin}(K)/2\pi^2 \right] \left| W(KR) \right|^2, \tag{14}$$

where $\sigma^2(m)$ is mass variance; $P^{lin}(K)$ is linear power spectrum, $P = AK^n$, $W(KR)$ is window function, K is wave vector:

$$P^{lin}(K) = AK^1,$$

here A is normalization factor.

According to power law, linear power spectrum P is directly proportional to wave vector K. Where A = normalization factor; $n = 1$ = power index; P = power spectrum. If the real space smoothing window is a Gaussian, then:

$$\left| W(KR) \right|^2 = e^{-K^2R^2/2}.$$

Let

$$\frac{K^2R^2}{2} = X, \Rightarrow K^2 = \frac{2X}{R^2}, \Rightarrow K = \frac{\sqrt{2}X}{R}, \Rightarrow dK = \frac{dX}{KR^2},$$

$$\sigma^2(m) = \int \frac{dX}{KR^2} \frac{2X}{R^2} \frac{(2X)^{n/2}}{R^n} e^{-X} \frac{A}{2\pi^2},$$

here $X = \frac{K^2R^2}{2} \Rightarrow \sigma^2(m) = \frac{A}{\pi^2} \frac{2^{(n-2)/2}}{R^{n+3}} \int dX X^{(n+1)/2} e^{-X}$.

Here n is spectral index, mathematical variable related to K.

$$Let \frac{n+1}{2} = t-1, \Rightarrow t = \frac{n+3}{2}, \Rightarrow n = 2t-3.$$

Here t is a mathematical function related to n.

$$\sigma^2(m) = \frac{A}{\pi^2} \frac{2^{t-2}}{R^{2t}} \int dX X^{t-1} e^{-X},$$

$$\Rightarrow \sigma^2(m) = \frac{A}{\pi^2} \frac{2^{t-2}}{R^{2t}} \Gamma(t).$$

Here $\Gamma(2)$ is gamma factor.

For normalization factor, $A = 0.9$; recent CMB measured with WMAP [11]:

$$\sigma^2(m) = \frac{0.9}{\pi^2} \times \frac{2^{2-2}}{R^4} \Gamma(2). \tag{15}$$

Since R is proportional to $1/(1+z)$, for $z = 0$, mass variance = 0.182. Γ is called as gamma factor. For spectral index $n = 2, 3, 4\ldots$, mass variance = 0.257, 0.364, 0.515… . But mass variance = 0.182 (for $n = 1$) in the calculation of bias is used.

Halo density as a non-linear function of matter density in a large scale can be written as:

$$\delta_h(m,z) = \sum_{k\geq 0} b_k(m,z) \delta^k, \tag{16}$$

δ_h is density of halo; k is natural numbers; b is bias parameter.

Again:

$$\delta_h(m,z)=\frac{N(M)}{n(m)V}.$$

Here $N(M)$ is average number of haloes each of mass M within a volume V which is virialized to another redshift; $n(m)$ is halo mass function per volume having mass between m and dm at certain redshift; δ_h is density of halo which is a function of mass of halo and redshift.

Where, halo mass function is:

$$n(m,t)=\frac{\bar{\rho}}{m^2}\upsilon f(\upsilon), \tag{17}$$

where υ is peak height, $f(\upsilon)$ is halo mass function, $\bar{\rho}$ is mean density.

Fraction of a mass of a halo is:

$$f(\upsilon)=\sqrt{\frac{2}{\pi}}\upsilon e^{-\frac{\upsilon^2}{2}}.$$

Peak height is $\upsilon=\delta_{sc}^2(z)/\sigma^2(m)$. Small en is halo mass function per volume having mass of mass dm at certain redshift; Capital en is average number of haloes each of mass M within a volume V which is virialized to another redshift. (17) is called as Press-Schechter formalism, which is extended by [8] by setting two free parameters p&q to calculate halo mass function. Using value of peak height in (17) and then equalizing with (16) linear bias equations have been reproduced as follows:

$$b_1(m,z)=1+\in_1+E_1,$$

b_1 is first order bias, ϵ_1 is eigenvalue of element one, E_1 is eigenvalue of energy one.

$$b_2(m,z)=2(1+a_2)(\in_1+E_1)+\in_2+E_2,$$

b_2 is second order bias, a_2 is mathematical coefficient, ϵ_2 is eigenvalue of element two, E_2 is eigenvalue of energy two.

$$b_3(m,z)=6(a_2+a_3)(\in_1+E_1)+3(1+2a_2)(\in_2+E_2)+\in_3+E_3,$$

b_3 is third order bias, a_3 is mathematical coefficient, ϵ_3 is eigenvalue of element three, E_3 is eigenvalue of energy three.

$$b_4(m,z)=24(a_3+a_4)(\in_1+E_1)+12\left[a_2^2+2(a_2+a_3)\right](\in_2+E_2)+4(1+3a_2)(\in_3+E_3)+\in_4+E_4.$$

b_4 is fourth order bias; a_4 is mathematical coefficient, ϵ_4 is eigenvalue of element four, E_4 is eigenvalue of energy four.

Where, bias equations contain various terms like mathematical coefficients and eigenvalues. The values of mathematical coefficients are 17/21, 341/567, 55805/130977 respectively [1]. The linear biasing equations are considered as (18). The equations for eigenvalues are represented as in (19) (eigenvalues means the terms present in the bias equations), as follows [1]:

$$\in_1=\frac{q\upsilon-1}{\delta_{sc}(z)},\ \in_2=\frac{q\upsilon}{\delta_{sc}(z)}\left(\frac{q\upsilon-3}{\delta_{sc}(z)}\right),\ \in_3=\frac{q\upsilon}{\delta_{sc}(z)}\left(\frac{q\upsilon-3}{\delta_{sc}(z)}\right)^2,\ \in_4=\left(\frac{q\upsilon}{\delta_{sc}(z)}\right)^2\left(\frac{q^2\upsilon^2-10q\upsilon+15}{\delta_{sc}(z)}\right).$$

Here q is Seth-Tormen parameter, δ_{sc} is spherical collapse density; ϵ_1, ϵ_2, ϵ_3, ϵ_4 are the eigenvalues.

$$E_1=\frac{2p/\delta_{sc}(z)}{1+(q\upsilon)^p},\ E_2=\left[\frac{1+2p}{\delta_{sc}(z)}+2\in_1\right]E_1,$$

$$E_3 = \left[\frac{4\left(p^2-1\right)6pq\upsilon}{\delta_{sc}^2(z)} + 3\epsilon_1^2 \right] E_1,$$

$$E_4 = \left[\frac{2q\upsilon}{\delta_{sc}^2(z)} \left(\frac{2q^2\upsilon^2}{\delta_{sc}(z)} - 15\epsilon_1 \right) + 2\frac{1+p}{\delta_{sc}^2(z)} \left(\frac{4\left(p^2-1\right)+8(p-1)q\upsilon+3}{\delta_{sc}(z)} + 6q\upsilon\epsilon_1 \right) \right] E_1.$$

Here p, q are Seth-Tormen parameters.

3. Result and discussion

Using the formulae in (19), the eigenvalues and are calculated and tabulated in **Table 1**. Then bias values are also calculated and tabulated in **Table 2**. Let's use different values of redshift from 0 to 2 in (8) and found value of eta square, which is further inserted in (12) to find spherical collapse density. Then first order bias is calculated from (18) and tabulated in **Table 3**.

Table 1

Parameters for eigenvalues

Eigenvalues	$q = 0.75, p = 0.3$	$q = 1, p = 0.5$	Eigenvalues	$q = 0.75, p = 0.3$	$q = 1, p = 0.5$
Element one	7.666	10.390	Energy one	0.091	0.143
Element two	54.395	102.196	Energy two	1.468	3.115
Element three	362.159	958.704	Energy three	104.137	48.559
Element four	3986.657	15901.087	Energy four	175.53	581.379

The various eigenvalues for the set of free parameters p, q

Table 2

Parameters for cosmological bias

Bias	For $q = 0.75, p = 0.3$	For $q = 1, p = 0.5$
First order bias	7.895	10.346
Second order bias	58.826	109.334
Third order bias	188.457	798.871
Fourth order bias	1556.748	11078.015

The values of bias factors for the set of free parameters p, q

Table 3

Parameters for first order bias

Redshift	Spherical collapse density	Peak height $\upsilon = \delta_{sc}^2(z)/\sigma^2(m)$	First order bias for $q = 0.75, p = 0.3$	First order bias for $q = 1, p = 0.5$
0.5	–0.258	0.363	–0.433	0.95
1	1.793	17.567	7.895	10.346
1.5	2.080	23.771	9.175	12.028
2	2.135	25.045	9.411	12.339

Analysis of first order bias factor with redshift range 2 to 0.5 for the set of p, q

Fig. 1 the solid green line represents first order bias values for $q = 1$, $p = 0.5$; the solid blue line represents first order bias values for $q = 0.75$, $p = 0.3$. This graph shows the evolution of bias with redshift. Bias decreases with decrease of redshift in the small range of redshift.

The peak height depends on mass of the halo. If the mass tends to infinite, then spherical collapse density tends to zero. Bias is also dependent on peak height so as on halo mass. So, if the halo mass is greater than cut off mass for halo to collapse then haloes are said to be biased. The calculation shows that peak height is greater than one, which further implies that haloes are massive, biased with respect to the underlying dark matter. So, bias is greater than one in **Table 2**. So, let's obtain the values of coefficients of bias from first order to that of higher orders for two set of values of free parameters p, q as shown in **Table 2**. The spherical collapse density as 1.793 and mass variance as 0.182 for calculation of bias are calculated. The eigenvalues are calculated by using set of (19) and tabulated in **Table 1**. Let's found the set of bias values within the range of redshift $z=0$ to 1 and at spectral index $n=1$ as shown in **Table 2**. Bias depends on halo mass and redshift but independent of cell size. The bias values from first to fourth order for $q=0.75$, $p=0.3$ are 7.895, 58.826, 188.457, 1556.748. The bias values are increasing by increasing the values of q, p, so, for $q=1$, $p=0.5$ the bias values are 10.346, 109.334, 798.871, 11078.015. The higher values of bias represents that the matter collapse to form large scale structures like galaxies and clustered galaxies which are more and more complexed.

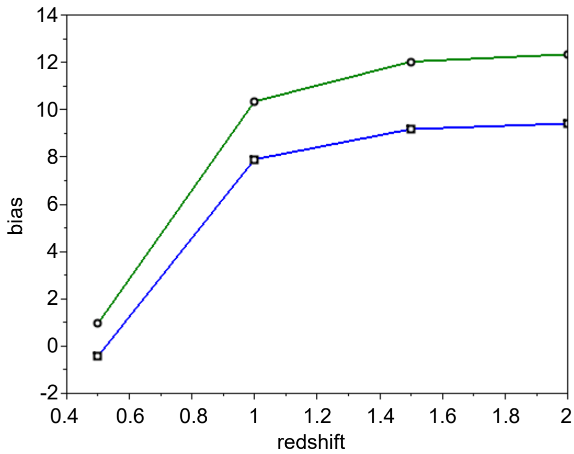

Fig. 1. The bias versus redshift graph

Fig. 1 shows that bias values exponentially decrease for initial redshift 2 to 0.5. The data from **Table 3** are used to plot **Fig. 1**. In **Table 3**, peak height is less than one, which implies the bias values are less than one. That means structure formation is in linear region. At redshift equal to 0.5 the spherical collapse density value is negative which indicates the zero-density region. This is one of the factors responsible for high biasing. In zero-density region the biasing value is negative means it is antibias and structure formation is still in linear region. Bias is –0.433 for $q=0.75$, $p=0.3$ and bias is 0.95 for $q=1$, $p=0.5$. Bias value 0.95 indicates that bias is approaching one, where structure formation enters from linear to non-linear region.

The bias values are $b=6, 4, 2$ for SCDM, ΛCDM, OCDM models respectively found by [12]. The values of bias found in simulations by [13] agrees well with LBGS (Lyman-break galaxies) clustered at $z=3$ having bias values $b=4.5$ for flat model. In our research work first order bias is 7.895 for $q=0.75$, $p=0.3$ and first order bias is 10.346 for $q=1$, $p=0.5$.

The bias values are greater than one means the matter collapse to form the structures in nonlinear region. Again, the bias coefficients in higher order terms are having high values [14] which indicates the large-scale structures formed are more and more clustered [15, 16] and complex also. The graph obtained is shown in **Fig. 1** where, bias decreases from higher redshift 2 to lower redshift 0.5. Bias value less than one in **Table 3** implies structure formation is in linear region. Negative bias value in **Table 3** indicates that it is antibias. So, no structure formation occurs there. The significant point here is that structure formation [17, 18] can be analyzed by calculating cosmological bias values mathematically.

In our research work we are addressing the issue of linear bias. But on small scale bias becomes nonlinear. So, this analytical method is approximately extrapolated for smaller scale. These results can be compared with observed or simulation results.

Our research work is closer to understanding the nature of relationship between the formation of galaxies and evolution of dark matter [19, 20]. Bias factor found here for mass variance calculated at spectral index equal to one. Also, it is possible to calculate mass variance at spectral index equal to two, three, four, which can be utilized to calculate bias factor. Then bias factor at various epoch can be compared.

4. Conclusions

Spherical collapse density was found to be 1.793. The mass variance was calculated as 0.182 for spectral index equal to 1. The bias values from first to fourth order for $q = 0.75$, $p = 0.3$ are 7.895, 58.826, 188.457, 1556.748 and for $q = 1$, $p = 0.5$ the bias values are 10.346, 109.334, 798.871, 11078.015.

The results found explained that the biasing is more at current epoch [21, 22]. That means the difference between baryonic matter density and dark matter density is huge. There are additional processes in the nonlinear evolution of galaxies [23, 24] which leads to a different behaviour compared to that of dark matter.

Cosmological bias values are useful to get the information on the composition of the universe [25, 26], properties of dark matter, dark energy, gravity and the process which produced initial seeds of structure. Bias is the key factor to interpret the observed large-scale structure.

Bias factor depends on luminosity of galaxies, redshift, density of matter [27, 28], power spectrum and gravity. Bias factor is more prominent on small scale but its uniform in large scale. Bias parameter quantitatively relates the difference between distribution of matter and galaxies. Distribution of galaxies can be calculated by counting the number of galaxies at specific redshift [29, 30]. The formation of galaxies depends on gravity and density perturbation. The density of matter must exceed critical density to form galaxy.

Conflict of interest

The authors declare that there is no conflict of interest in relation to this paper, as well as the published research results, including the financial aspects of conducting the research, obtaining and using its results, as well as any non-financial personal relationships.

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