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Chapter

Table Tennis and Physics

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Abstract

Table tennis is a fascinating sport with a lot of physics elements. This chapter will concentrate on the forces acting on a table tennis ball. Insights from molecular dynamics simulations clarify the basic properties of table tennis balls interacting with surfaces and their high coefficient of restitution. The table tennis ball trajectory is discussed considering the different force balance contributions. The sum of the gravitational force and the forces from aerodynamics, namely drag and lift, determine the flight path. Extensive numerical modeling is used to study the impact of changes in ball size and net height to the game characteristics. Half a billion different initial conditions like hitting location, initial spin and velocities were analyzed to reach sufficient statistical significance for the different cases. An advanced statistical analysis of the database generated by the simulation is presented.

Keywords: physics modeling, trajectory analysis, statistics, rule changes

1. Introduction

In this chapter, some aspects of physics in table tennis will be discussed. Insights from molecular dynamics and finite element simulations clarify the basic properties of table tennis balls interacting with surfaces and their high coefficient of restitution. The table tennis ball trajectory is calculated considering the different force balance contributions. Statistical analysis allow a better prediction of the consequences on the game for possible rule changes.

Historically, there exists a strong link between physics and table tennis. Werner Heisenberg was an enthusiastic table tennis player. He is one of the founders of quantum physics and established around 1930 in his working group at the University of Leipzig regular meetings where scientific discussions, which could not be resolved, were decided by table tennis [1]. Other members of the group were less motivated, for example Werner Hückel, a famous chemist, did not like table tennis, “because one had to pick up the ball in the corners of the room” [2]. Werner Heisenberg was once beaten by an Asian PhD student. This motivated him so much that on the boat trip to China in 1929 for some lectures he spent most of the time to improve his table tennis skills. He intended to be able to compete against the Chinese players and wanted no further defeat by his PhD students anymore [3].

2. Interaction of table tennis balls with surfaces

In this section, physics effects of the interaction of table tennis balls with surfaces will be discussed. A table tennis ball can be described as a hollow sphere with uniform material properties. Vibrational mode frequencies and coefficient of restitution (CoR) were studied for a hollow elastic sphere with molecular dynamics simulations of spherical clusters of up to 13,500 atoms [4].

In the molecular dynamics simulations, a hollow sphere was studied. This thin but rigid shell represents a table tennis ball and experiences a gentle, long duration of the collision, which does not allow vibrational modes to be excited. A thicker shell shows some excitation of vibrational modes, which corresponds to an energy loss. Collision time is speed-dependent for a solid sphere, but it appears to become independent of speed for a thin spherical shell. Acceleration and consequently force are linearly proportional to compression c for a thin shell sphere. This is much different from the situation for a solid sphere where one gets in agreement with Hertz a force proportional to compression $c^{3/2}$ [5].

Scaling MD simulations to a table tennis ball, the duration of its bounce with a hard table should be 0.7 ms with dominant Fourier components up to about 1400 Hz. The gap between this frequency and the lowest vibrational frequency of the ball (5300 Hz) means that excitation of vibrational modes is not an important mechanism for energy loss, which explains its high CoR with values above 0.8. The CoR values reduce as the drop height is increased, i.e. as the impact speed increases [6].

A spinning ball struck head-on does not rebound along its incident path [7, 8]. The rebound angle and spin depend in a nontrivial manner on the coefficient of friction between the ball and the surface and on the elastic properties of the ball and the surface.

Studies of elastoplastic collisions are of interest also in engineering [9, 10]. Finite element simulations calculated the CoR for a multi-body system involving a revolute joint to simulate the influence of different material yield strength ratios on the CoR [11]. This established a new CoR model with a maximum error of 5% compared with experiment.

This model calculates the CoR as

$$C_r = 1 - 0.1 \ln \left(\frac{V_1}{V_y} \right) \left(\frac{\frac{V_1}{V_y} - 1}{59} \right)^{0.156} \quad (1)$$

with the initial drop velocity V_1 and the yield velocity V_y as

$$V_y = 3.194 \left(\frac{(1.61K\sigma_y)^5 R^{*3}}{E^{*4} m^*} \right)^{\frac{1}{2}} \quad (2)$$

with the coefficient K

$$K = \begin{cases} \frac{k-1}{2+k^{1.65}} & \sigma_{\text{plate}} < \sigma_{\text{ball}} < 3\sigma_{\text{plate}} \\ 1.246 & 3\sigma_{\text{plate}} < \sigma_{\text{ball}} \\ 1 & \sigma_{\text{ball}} < \sigma_{\text{plate}} \end{cases} \quad (3)$$

in which $k = \frac{\sigma_{\text{ball}}}{\sigma_{\text{plate}}}$. σ is the yield strength of the materials, $m^* = \frac{m_1 m_2}{m_1 + m_2}$ the equivalent mass, and $R^* = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$ the equivalent radius. The elasticity moduli E of both colliding bodies are combined with their Poisson's numbers μ to $E^* = \left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right)^{-1}$. Applying this model to the situation of plastic and celluloid table tennis ball impinging on the table gives the fit functions shown in **Figure 1** together with the experimental data of [12]. The material properties used for the calculations are $E_1 = 1400$ MPa, $\mu_1 = 0.3$, and $\sigma = 50$ MPa for the celluloid table tennis ball [4], $E_1 = 1197$ MPa, $\mu_1 = 0.24$, and $\sigma = 51$ MPa for a table tennis ball made from ABS plastic [13] and $E_2 = 5992$ MPa and $\mu_2 = 0.3$ for the table [11].

The model describes quite well the experimental results for plastic balls. For celluloid balls, the model does not match so well because one observes a kind of transition in the data. The celluloid balls behave at smaller initial velocities similarly to plastic balls, but their CoR drops more quickly for initial velocities larger than 20 km/h.

Measurements of the impact force acting on table tennis balls [14] show that the small spherical cap surrounding the initial contact point plays a major role in the interaction. Buckling of the cap in a table tennis ball occurs for impact speeds above about 5 m/s, but that is a relatively small speed compared to typical speeds of about 20–30 m/s in the game. Due to the impact force on the cap large amplitude oscillations are excited. The vibrations of the cap when it buckles to reduce the CoR from 0.9 to 0.8 or even below. This is expected from the MD simulations because energy losses appear. The vibration frequency is relatively high, typically around 10 kHz, and impact duration increases with ball speed. For most other ball types, the impact duration decreases as the ball speed increases. One possible explanation for the different results for the celluloid balls is the onset of stronger, non-linear buckling, which then leads to the larger drop of the CoR at higher velocities compared to plastic balls.

Also, the friction coefficient was found higher for plastic balls when the initial horizontal contact point velocities are lower [12]. Hence, a service with back-spin,

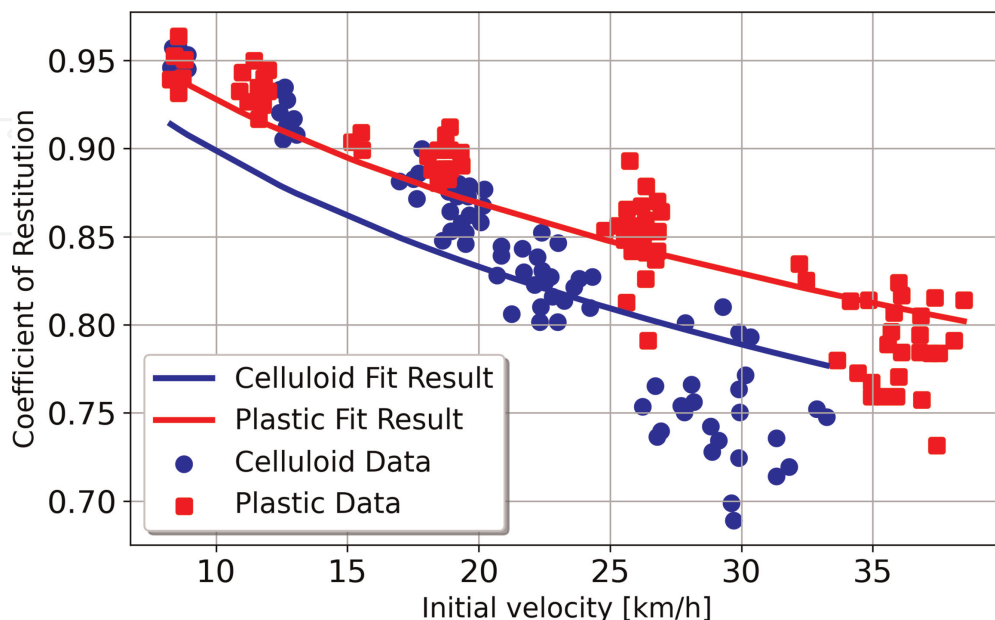


Figure 1. Coefficient of restitution as a function of initial velocity for plastic and celluloid balls (from [12]). The fit functions calculated from the model of [11] are also shown.

which has lower speed, is experiencing more deceleration upon collision with the table than celluloid balls. On the contrary, fast plastic balls with a lot of topspin accelerate more upon collision.

Even more complex than the interaction of the table tennis ball with the table is the interaction with different rubbers. These are optimized for different purposes, for example maximum speed and spin, minimum speed and spin, spin variations, and others. This optimization was done originally, mostly experimentally, to quantify the rubber effect (e.g. [15]). Later, more and more 3D finite element analysis was done to improve the understanding of the interaction. A 3D finite element model was developed where the polymeric time-dependent dissipating compliant behavior was measured with dynamic mechanical analysis and compression tests. The calculations confirm that the friction between the ball and the polymeric layer plays a key role in the non-linear energy dissipation process, alongside the rate-dependent behavior and architecture of the polymeric constituents [16, 17].

Obviously, the interaction of table tennis balls with the table and the racket is a rather complex mechanism. Different materials and deformations play important roles and create a non-linear, multi-scale process, where microscopic interactions determine the macroscopic behavior.

3. Table tennis ball trajectories

In this section, the trajectory of table tennis balls defined by the forces of gravitation and aerodynamics will be discussed. After introducing the basic physics, a statistical approach will be used to study the influence of potential rule changes on the characteristics of table tennis.

3.1 Basic physics

The trajectory of a table tennis ball is determined by its equation of motion. The equation of motion needs a mathematical description of the acting forces.

The gravitational force of the earth and aerodynamic forces determine the flight trajectory of a table tennis ball. The gravitational force

$$\vec{F}_G = m \cdot \vec{g} \quad (4)$$

accelerates a ball of mass m towards the center of the earth with the gravitational constant $g = 9.81 \text{ m/s}^2$ and results in a parabolic trajectory. Subsequently, air drag and lift create more complex trajectories. Air drag is the friction force created by the interaction with the background medium and is directed against the direction of the movement of the ball. If a hand is held out of a driving car, this force will push it back by the interaction of the air molecules with the hand surface. The drag force increases with larger velocity and with the cross-sectional area:

$$\vec{F}_D = -0.5 \cdot C_D \cdot \rho \cdot A \cdot v \cdot \vec{v} \quad (5)$$

with the density of air ρ , the cross-sectional area $A = r^2 \cdot \pi$ for a ball with radius r , the ball velocity v , and an air drag coefficient C_D . This coefficient can be experimentally determined, e.g. in wind tunnel experiments.

The second important aerodynamic force, the airlift, was discovered by Heinrich Gustav Magnus (1802–1870). The “Magnus effect” describes the observation that a rotating ball deviates from its flight path. This is caused by the interaction of the flow with the surface of the spinning ball, which creates a co-rotating air layer at the surface of the ball. The ball rotates on one side with the airflow created by the spinning ball, on the other side opposite to it. This creates a pressure imbalance. On the counter-rotation side, the total velocity of the airflow is reduced, because both velocities compensate partly. On the co-rotation side, a larger flow velocity is created, because both velocities add up. Larger velocities in a flow translate into lower pressure and the pressure differences created by this produce the Magnus force, which deviate the ball from its original path. The air lift force with an air lift coefficient C_L is

$$\vec{F}_L = 0.5 \cdot C_L \cdot \rho \cdot A \cdot v \cdot \vec{e}_\omega \times \vec{v}. \quad (6)$$

The airlift force acts perpendicular to the velocity \vec{v} and to the axis of rotation \vec{e}_ω (see **Figure 2**).

The coefficients of air drag and lift for a rotating ball (see **Figure 3**) as a function of the ratio of spinning velocity to translational velocity are implemented into the computer code as a fit of experimental data [18–22].

During a topspin shot with forward rotation, the lift force acts downwards, during a backspin with backward rotation it acts upwards.

Figure 4 shows the parabolic solution for the free flight of a 40-mm table tennis ball with only gravitational force acting. Activating the drag force results in an asymmetric, shorter trajectory. Accounting only for the lift force in the case of topspin flattens the trajectory even more and this is even amplified if both aerodynamic forces act together. In contrast, the lift force in the case of backspin creates a higher, but again asymmetric trajectory, because both drag and lift force act non-uniform.

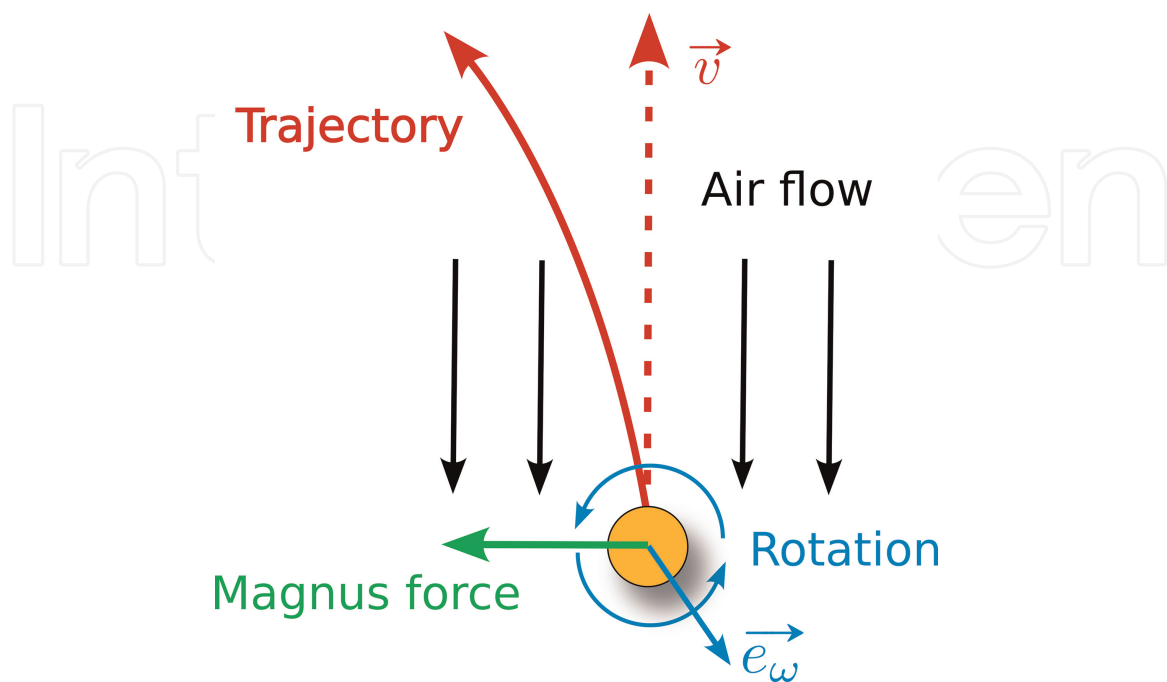


Figure 2.
 Forces on a table tennis ball.

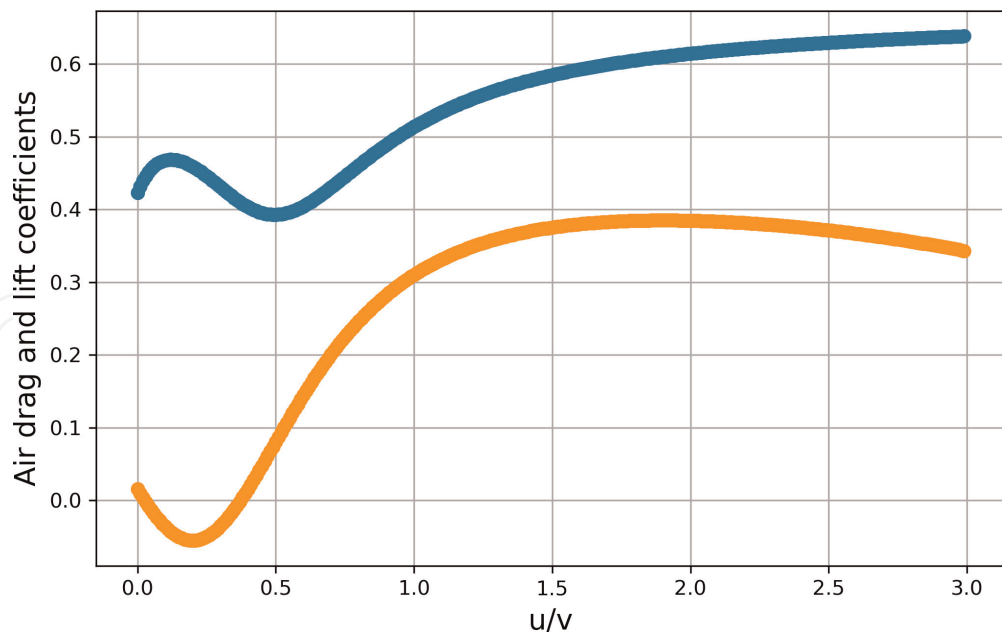


Figure 3. Air drag coefficient C_D (upper blue curve) and air lift coefficient C_L (lower orange curve) as a function of the ratio of spinning velocity u to the translational velocity v .

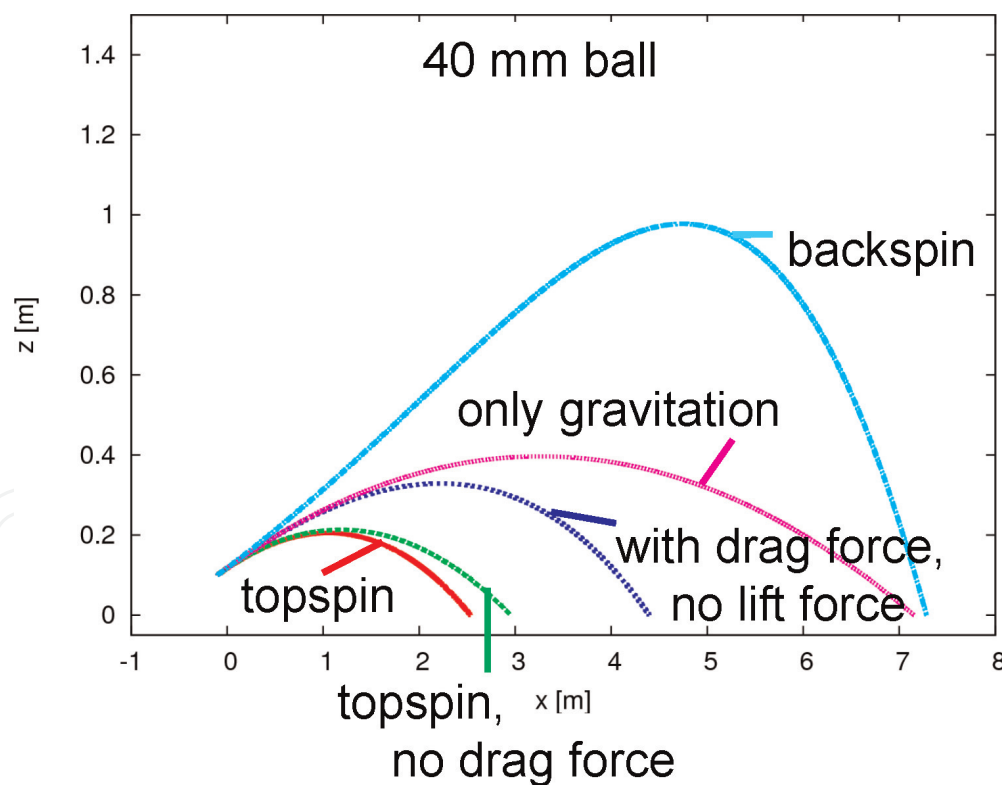


Figure 4. Influence of the different forces on a table tennis ball trajectory.

Figure 3 shows for low spinning velocities a negative value of the air lift coefficient, which is called the inverse Magnus effect. For table tennis balls, the inverse Magnus effect has been experimentally confirmed [23]. During a trajectory of a flying ball, the air lift coefficients can change from positive to negative coefficients and even

back again. This can produce swirling balls, not only discussed in table tennis, but also in soccer and volleyball. For table tennis balls the effect exists, but results only in deviations of some millimeters because the airlift coefficient is only negative where the value of the coefficient itself is already quite small. Table tennis players complain about swirling balls with long pimples, but this is an effect created by the pre-programming of the players in the training. Most players are used to playing against a strongly rotating ball from a normal rubber sponge. However, the long pimples reduce the rotation of the ball and modify its trajectory. The ball has less lift and falls down earlier and by this, the player misses the ball.

4. Statistical analysis of table tennis trajectories

One big problem table tennis faces today is the fact that the speed of the game is nowadays so high that it is very hard for spectators to follow the balls [24–26]. As a consequence, the medial appeal of table tennis seems to go down in terms of TV hours, at least outside Asia. Rule changes to counteract this and to allow a better visual tracking during the rallies, were a larger ball, different counting system, stricter limits for rubbers, and new service rules [24, 25] in order to slow down the game and to reduce the impact of spin on the game. Additional rule changes, like higher nets, are discussed. Such rules or technical changes have strong impacts on the techniques and strategies of the players, because they have to adapt their individual training programs. This creates hesitation of the players towards new rules.

One example for the effect of rule changes is the change of ball size. Today's 40-mm ball is 2 mm larger and 0.2 grams heavier than the 38-mm ball used before. The larger cross sectional area produces a larger air drag and should reduce the maximum velocities [27].

The mass distribution of the larger ball is shifted further away from the center compared with the 38-mm ball. This creates a larger inertial moment and reduces the spin. Measurements show a velocity and spin reduction of about 5–10% [28, 29] for the 40-mm ball.

Interestingly, this is not observed in table tennis match analysis, because the players compensated the effects of the size increase by larger exertions [28, 30].

This put much more emphasis on the fitness of the individual player, because the stroke forces are nowadays not only created by the arms but by the whole body. Especially, the more intense use of the legs compensate the ball size increase in modern table tennis. A similar effect is the more effective wrist usage to support spin production. Using only the forearm gives much less spin for the 40-mm ball compared with the 38-mm ball. The greater forces needed for the compensation of the larger ball effects require much better athletics of the players. The more complex and longer movements for this compensation create also the danger of technical mistakes [31].

Possible changes to further slow-down the game are a larger 44 mm ball or an increased net height (up to 3 cm) for the standard ball of 40 mm. The larger 44-mm ball with small weight could suppress high velocities, but the players could again compensate for this by improving their physical fitness to perform stronger shots.

The second alternative actually discussed is to increase the net height. This is expected to change the general characteristics of table tennis, because it limits fast spins and shots, but also modifies the service. To be able to find a scientifically based decision, a database is missing. Empirical work of short and limited tests with some players exists to study the impact of such changes on the players and the game [32].

One has to be aware of the fact, that the players in these tests have trained for the existing situation (ball size, ball weight, net height) and modifications needed for the new situation take too long for the players to be automatized in the training to be considered. An alternative approach is to solve numerically the equation of motion of table tennis balls for a large number of given initial positions, velocities, and spins in three spatial dimensions (x, y, z) and to establish this a statistical database. Such a trajectory analysis was done for the 38-mm-ball with a weight of 2.5 g, used in tournaments until end of 2000, the actual 40-mm-ball with 2.7 g, and a 44-mm-ball with a weight of 2.3 g, which was tested already in Japan. For the 40-mm-ball an increase of the net height of 1 and 3 cm was analyzed, too.

A first example is given to study differences of the balls for a typical top-spin shot. Initial conditions were chosen as starting point $(-0.7, 0.8, 0.1)$ m, velocities $(10.0, 0.1, 2.0)$ m/s and spins $(10, 120, 10)$ 1/s. The difference between a 38-mm and a 40-mm ball is rather small: the 38-mm ball flies only 3 mm shorter. In contrast, the difference between a 40-mm and a 44-mm ball is more pronounced (see **Figure 5**), because the 44-mm ball is lighter and experiences stronger friction forces due to its larger size.

Trying to get to the same trajectory with the 44-mm ball requires—as expected—a larger initial velocity of $(10.7, 0.1, 2.1)$ m/s. The rotation velocities are unchanged. Therefore, in terms of necessary force for the player to execute the same shot, the smallest force is needed for the 38-mm ball, followed by the 40-mm ball (because it is heavier). Due to the large change in initial velocities, despite its smaller mass, the 44-mm ball requires even more force.

A database is created to quantify the influence of such changes. Modifications in technique, tactics, strength and fitness are not considered in this analysis. For a huge number of initial conditions, the effect on successful strokes is studied by solving the equation of motion. This delivers the maximum amount of possible strokes for different conditions in terms of statistical distributions. One obtains the best possible adaptation to the changes, independent of what this would mean for the players in terms of changes in their training.

For our system studied here, the drag and the Magnus force both influence the trajectory depending on the actual velocity. If a particle on a closed path comes back to

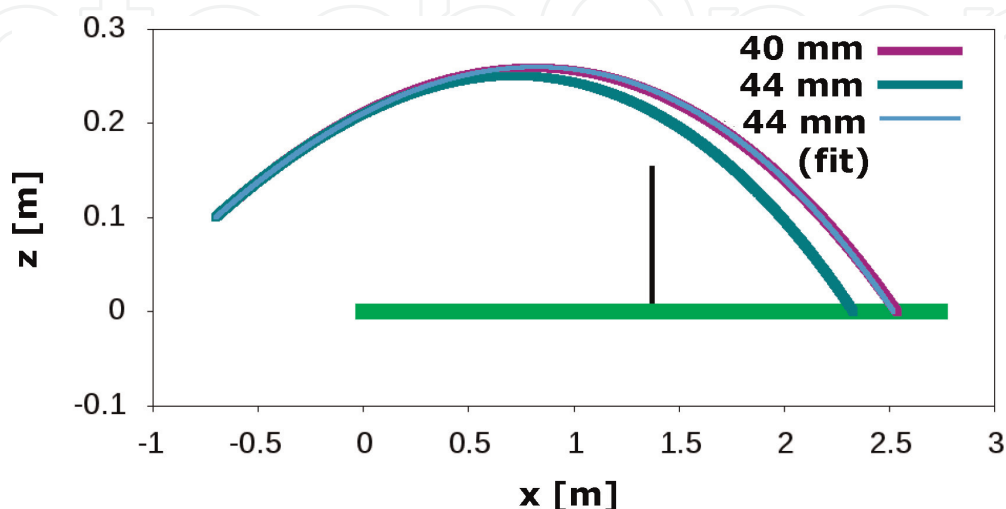


Figure 5. Trajectory of a 40-mm and a 44-mm ball for the same initial conditions. Using a fit algorithm the starting velocities and rotation velocities of the 44-mm ball are modified to get as close as possible to the 40-mm trajectory.

its origin, the drag will slow it down. Energy is not conserved. Therefore, the Euler method was chosen as a fast method for the calculation of half a billion trajectories for each case within the precision limit of 1 mm for the time step of 10^{-4} s.

The Euler solver has also the advantage of algorithmic simplicity, which allows an easy transfer onto a Graphics Processing Unit (GPU) to speed up the code.

Our implementation is described in detail in [33].

For statistical analysis of the effects of ball sizes and net heights on trajectories of table tennis balls, a Monte Carlo procedure was used. Many different initial conditions were solved; hitting locations were varied from 30 cm above the table to 3 m behind the table. The position relative to the table corner was kept constant at 0.381 m, which is one quarter of the width of the table tennis table, because the exact location of the hitting point is not important. The ball was started with an initial height between 0.4 m and -0.4 m. The initial velocity was sampled in the following way: the horizontal angle was varied between the limiting angles of the starting point to the net posts, the elevation angle was chosen randomly. The normalized spin vector was created randomly representing topspin, backspin and sidespin. Experimentally, it was proven that the spinning of the ball is constant during the flight [34].

For the decision on possible rule changes, fast shots are in particular of interest. This was introduced into the algorithm by limiting the height where a ball passes the net to 30 cm. Translational velocities were chosen from 20 to 200 km/h, and spinning velocities from 0 to 150 turns/s (which is equal to 9000 turns/min). These are experimental values for 38-mm balls [35].

In a successful trajectory, the ball stays within the height limit when it passes the net and hits the other side of the table tennis table.

Trajectories for $5 \cdot 10^8$ initial conditions are calculated for the different cases. The total run-time on a Linux Cluster with 32 cores was 640 hours. Using GPU computing with CUDA on a Dell Precision T7500 Desktop with a NVIDIA Quadro FX3800 results in only 3 hours for the same calculation [36].

Calculation of trajectories in table tennis mostly concentrated on individual cases without statistical analysis [12].

New interest in the fast calculation of table tennis trajectories is also motivated by the research on robots [37, 38] and the programming of computer games [39].

5. Analysis of concepts for slowing-down the game

Data sets of successful hits for the different cases (38 mm ball, 40 mm ball, 44 mm ball, 40 mm ball with 1 and 3 cm higher net) are created. **Figure 6** shows as a function of initial kinetic energy the number of successful trajectories.

There is a clear ranking of the different scenarios visible: the largest number of successful hits for the same initial kinetic energy is produced by the 44-mm ball, because it has lighter weight and higher air drag. The 40-mm ball is very similar to the 38-mm ball. Changes of the balls are compensated by other parameter changes. A higher net affects strongly the ball hits above the table limiting there the number of successful trajectories. In **Figure 7** the number of successful trajectories as a function of final velocities are shown. Again, the results for the 38 and 40 mm ball differ only marginally. For the 44-mm ball one gets more successful trajectories compared to the 38- and 40-mm ball for higher initial velocity, the distributions for the final velocities are nevertheless very close again. Very high velocities above 35 m/s are suppressed earlier for the 44-mm ball, because the larger size increases the drag forces. To

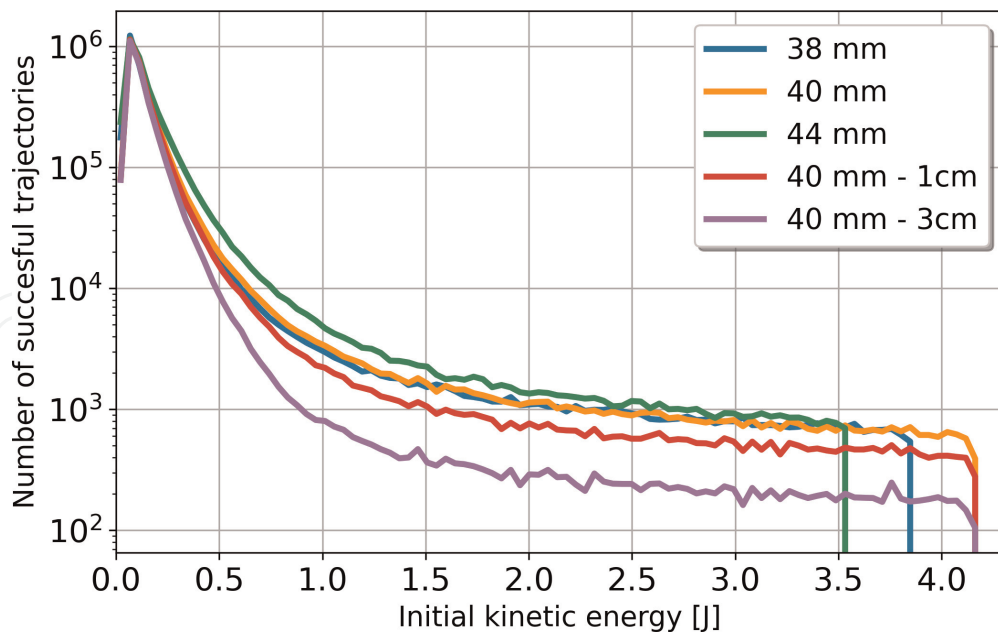


Figure 6. Number of successful trajectories as a function of initial kinetic energy for the different cases studied.

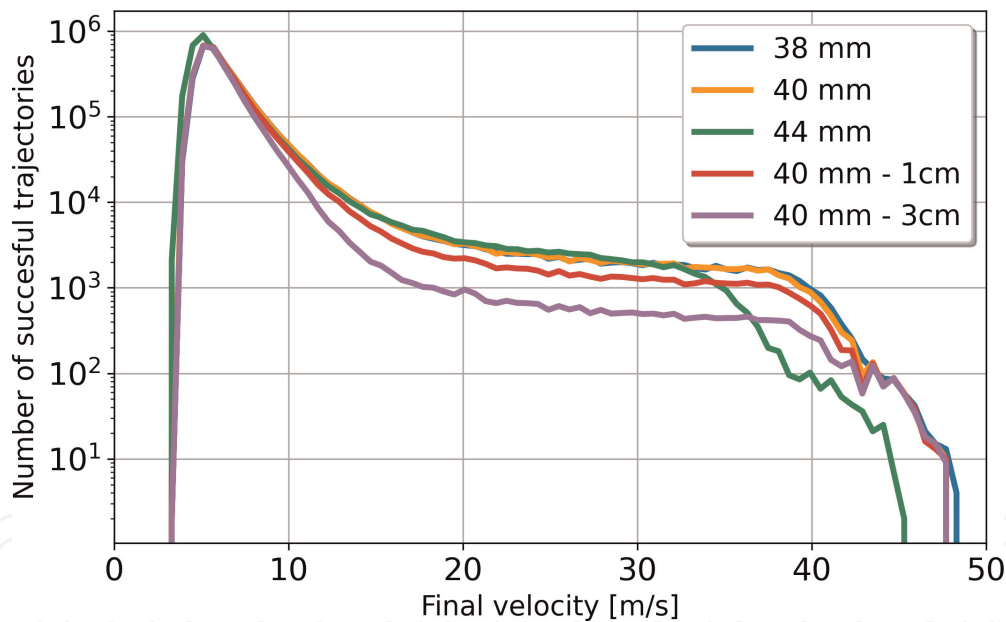


Figure 7. Number of successful trajectories as a function of final velocity for the different cases studied.

influence the trajectories one needs larger spins for the 44-mm ball compared to the 40-mm ball. However, the players could compensate for this again by improving their physical fitness to perform stronger shots. For very low velocities the impact of the air drag is not yet important resulting in larger number of successful trajectories. The strong reduction of successful strokes for higher nets is linked with rather small top spin components, but rather larger sidespin components. This would change the characteristics of long rallies in table tennis. For these cases, not only reduced velocities are important, but the tactical possibilities will be modified by reducing the number of spinless short trajectories. This will not only slow down the game but will increase the importance of diagonal play with longer reaction times for the opponent

than fast parallel balls. This should produce longer and more attractive rallies. Unfortunately, the game characteristics would change strongly, because the players have much fewer technical and tactical alternatives for successful shots. Specifically, the influence of the service will be strongly reduced. The International Table Tennis Federation will have to decide if such strong modifications to the existing sport are worth a possible gain in attractiveness of table tennis to the TV audience.

These results are supported by advanced statistical analysis using the Python packages pandas [40] and scikit-learn [41] of the 21-dimensional data base [42]. A principal component, multi-dimensional histograms, and a cluster analysis confirmed the previous findings and identified the pronounced sidespin component for the cases with higher nets.

6. Conclusions

Table tennis is a fascinating sport, where many elements of physics are important. The interaction of the table tennis ball with the table or the rubber is a highly complex, non-linear, and multi-scale problem.

The trajectories of table tennis balls are determined by the gravitational and aerodynamic forces. Physics based analysis of trajectories allow reliable predictions of possible rule changes for the distribution of successful shots. This is an important input for the discussion, but here also the impact on the changed characteristics of the game on players and the audience has to be considered.


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