GENUS, **LXII** (No. 1), XX-XX

MARIO DI BACCO – LORENZO DEL PANTA – PATRIZIO FREDERIC – GIUSEPPE D'AMORE – NIELS LYNNERUP

The effect of an unbalanced demographic structure on marriage and fertility patterns in isolated populations: the case of Norse settlements in Greenland

1. INTRODUCTION

Can the decline of a small and isolated population occur as a consequence of a *random* imbalance in the sex ratio at birth and/or in the male/female mortality rates in subsequent age groups? We should stress here the use of the term "random". Indeed, the imbalance we refer to is what occurs in a small population as the ultimate result of pure chance.

This hypothesis (of the imbalance of the sex ratio) was studied in particular by the Italian statistician Corrado Gini (1957), although it had previously been put forward by other authors (Livi 1941, 1944). More specifically, Gini explained the extinction of the medieval settlements in Greenland using this hypothesis.

The recent publication of Lynnerup's book (1998)[1] concerning the demography of these settlements, based on the archaeological evidence of skeletal remains, prompted us to re-evaluate Gini's treatise. We wanted to give a statistical contribution – using a Bayesian inference – to evaluate Gini's hypothesis of that extinction. The extinction of the settlements in Greenland is widely regarded as "almost an enigma hinting at some

[1] In order to denominate the people who colonized Greenland, Lynnerup makes use of the term "Greenland Norse". Gini named these colonies "Norman". The long treatise by Corrado Gini we refer to is entitled "Sulla scomparsa delle colonie normanne in Groenlandia" (*On the disappearance of the Norman colonies in Greenland*), and constitutes Chapter IX (pp. 682-715) of the volume: C. Gini, *Corso di sociologia*, Edizioni "Ricerche", Roma, 1957. People coming from Scandinavia (men and women who reached Greenland originated from the Norse territories) were indicated, in the contemporary Latin sources of the overrun countries, by the generic term of *Northmen* (people coming from the North) (see *Encyclopaedia Britannica* ed. 1962 voll.16 (p.494) and 23 (p.150), and also Azzara 1999 p.138). Frisian and English documents, written in vulgar language, instead called the first invaders from Scandinavia *Vikings*, from the ancient Norse term Viking. The term *Normans* today is mainly used in order to identify people who, coming from Scandinavia, settled in the North of France (the present region of Normandy), and starting from there later on conquered the British isles, Sicily and Southern Italy. In our paper, we use the terms *Vikings/Norsemen/Norse* indifferently.

inexplicable and dramatic event veiled by the passage of time" (Lynnerup 1998, p. 8).

The aim of this paper is therefore to check Gini's hypothesis from a Bayesian point of view. In this scheme, the prior distribution is the probability of an imbalance in the sex ratio occurring in the history of a *small* and *closed* population as a result of random and unfavourable alterations of the sex-ratio at birth and at death. In order to evaluate such a probability, a Monte Carlo technique that uses historical, demographic and biometric constants (Livi Bacci 1998) will be outlined. The posterior probability is then updated in the light of some archaeological data.

The paper is structured as follows: in section 2 the history of the colonization of Greenland is briefly outlined. Here we refer extensively to Lynnerup's book (1998) and to Gini's essay (1957). Section 3 deals with the possible causes of the decline and the extinction of the settlements in Greenland. Gini's hypothesis is described in section 4. In section 5 we analyse the main features of the demographic structure of the Greenland colonies, while section 6 describes the archaeological data. In section 7 our Bayesian approach and the evaluation of prior probabilities are described. Finally, a discussion of the results is given in section 8.

2. A SHORT HISTORICAL ACCOUNT OF THE VIKING COLONIES IN GREENLAND

The great wave of Scandinavian emigration spanned the whole period between the ninth and the eleventh century, and took place in various episodes, from single piracy actions, explorations of far-away countries and the colonisation of inhospitable and desolate lands to settlements in areas that were already widely populated and civilized (Azzara 1999, p.139). At least at the beginning, each group of Scandinavians turned to countries which were closer to the areas of departure. Therefore, those coming from present-day Norway sailed towards the West, to the Shetlands, Scotland and Ireland, up to Iceland. The Danes attacked and settled on the southern and eastern coasts of England, as well as in Northern France, while the Swedes mainly directed their voyages towards the Baltic areas.

As far as the Vikings from Norway are concerned, the actual colonization of new lands first involved the previously nearly deserted Faeröese islands, which they had reached by AD 825. Starting from AD 870. groups of Vikings arrived in Iceland, where they found only a few hermits from Ireland. The Vikings settled there, and then completed the colonization in about two generations (Atlante storico mondiale 1986, p.111). The Icelandic population coming from Norway may have increased to about

35'000 inhabitants within one hundred years, giving rise to a prosperous culture. At the end of the Xth century, setting out from Iceland, the Vikings reached Greenland (see map 1). Greenland was practically deserted, the Inuit of the Thule culture not yet having spread out along the southern coastlines. A significant Norse Viking presence is documented at least until the XVth century. Small groups of Vikings from Greenland may even have occasionally reached the coasts of Newfoundland (which they named Vinland), but no permanent colonies were set up there.

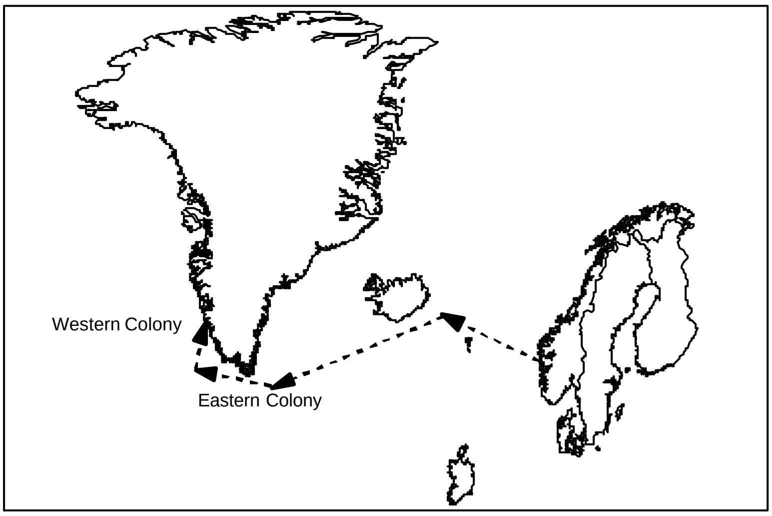
According to the tradition of the Icelandic sagas (Lynnerup 1998, p.8; Gini 1957, p.683), a group of Viking colonists from Iceland, under the leadership of Eric the Red, founded a first settlement on the southern coasts of Greenland in 986. The size of this first group (which was followed by others, coming from Iceland and perhaps also directly from Norway) may be estimated at a few hundred individuals (perhaps 500). The first settlement, usually called the Eastern Settlement, was followed by a second one, called the Western Settlement. This was situated at about 400 kilometres north of the Eastern one but never reached the size of the first one.

Lynnerup (1998, p.8) notes that the environment that Eric the Red and his followers had to face did not differ much from that of Iceland or the Northern territories of Norway. Around the fjords there were grass-lands and the climate was relatively mild. The colonists could also rely on the resources of the sea. In short, it seems that a relative prosperity of the Viking settlements can be documented. In the period of greatest flourishing, they could probably count on the activity of 280 farms (90 in the Western settlement, the remaining 190 in the Eastern one). 16 churches were probably built, and the contacts with Iceland and Norway are also supported by the consecration, in AD 1124, of a bishop who took over in the Eastern colony. In AD 1261 the colonies were submitted to the Norwegian Crown, but contacts with Iceland and Norway seem to have progressively loosened, although they never ceased entirely. Text sources would seem to attest to the definite abandonment of the Western colony around the year AD 1360[2]. As far as the Eastern colony is concerned, the last written evidence (Lynnerup 1998, p.8) dates back to AD 1408, but the definite abandonment probably dates to the end of the XVth or to the first decades of the XVIth century.

We shall deal later on with the estimated demographic size of these colonies during their existence, which lasted several centuries, since these evaluations are not irrelevant with reference to the specific object of this paper. At this point we note that demographic modelling based on the

[2] See Lynnerup 1998, p.8; however Gini (1957, p.684) points out that in 1342 the Western settlement no longer existed.

Map 1 – *Vikings itinerary*



anthropological material suggests as the most likely evaluation a maximum of not much more than 2'000 inhabitants (Lynnerup, 1996). This maximum could probably have been reached in the course of two centuries since the date of the first settlements. Gini, on the other hand, quotes several evaluations ranging from 2'000 to 11'000 inhabitants (the maximum size possibly reached by the Viking settlements on the whole), but tends towards an intermediate figure, maybe closer to the upper limit.

3. SEVERAL HYPOTHESES ON THE DECLINE AND THE DISAPPEARANCE OF THE COLONIES

The question of the possible causes of the decline of the Viking colonies of Greenland has been tackled by many authors[3]. In the past, several authors were in favour of hypotheses linked to traumatic situations or events, such as attacks from pirates, violent oppression by Eskimos, sudden and serious epidemics or even rapid extinction due to "degeneration" of these human groups as a consequence of prolonged isolation. The outcome of more recent studies tends to disregard the more dramatic hypotheses (Lynnerup, 1998). Both Lynnerup and Gini find it unlikely that violent action by pirates coming from European countries could have destroyed the

[3] A review of these studies was produced by Lynnerup (1998)

Viking settlements, and a violent eradication by Eskimos is also very unlikely. Likewise, there is no anthropological or genetic evidence of assimilation (Lynnerup, 1998).

At present, the most convincing explanation emphasizes the interaction of several factors, such as climatic change, excessive exploitation of the land which were available for agriculture and stock-farming, and also external economic causes linked to the trade monopoly instituted in AD 1294 by Norway, which the Greenland colonies were forced to accept in AD 1261. Both Gini and Lynnerup especially point to the worsening of living conditions that, in the long run, must have occurred as a consequence of the progressive climate change. In fact, a decrease of the mean temperatures is documented approximately from AD 1300 onwards (Fricke *et al.*, 1995). Climate change, combined with a probable excessive exploitation of the land, must have led to a progressive decrease in agricultural production. There is archaeological and anthropological evidence (Arneborg *et al.*, 1999) of a change in diet towards fishing production, but also of a certain worsening of the nutritional level, which would have caused an adaptive reaction in terms of a reduction in height. The cooling of the climate must also have had negative implications for connections with Iceland and Norway, leading to an increased cultural and commercial isolation.

One can also add to this the great upheaval provoked by the plague which devastated most of Europe, including Norway and Iceland, from the middle of the XIVth century onwards. In fact, as a consequence of the depopulation of the areas affected by the epidemic, the demographic pressure was drastically reduced and therefore permanence in the marginal and inhospitable sites which had been settled in a period of high relative overpopulation became far less convenient.

Here one ought to underline the temporal gap between the disappearance of the Western settlement (around the middle of the XIVth century) and that of the Eastern one, approximately 100 – 200 years later. In fact, a very convincing interpretation – put forward by Lynnerup – of the progressive crisis of the Eastern settlement, after the middle of the XIVth century (which cannot be applied to the Western colony, for chronological reasons) points to the great upheaval caused in Europe by the plague, starting with the great pandemic of AD 1347-51, as already stressed. We cannot exclude that before the definite abandonment of the Western settlement, a migratory flow may have been generated from the Western settlement towards the Eastern one. Furthermore, emigration towards Iceland (and perhaps also towards Norway or other territories colonized by Vikings) may have started even before the middle of the XIVth as a consequence of the worsening climatic conditions, but probably intensified owing to the drastic reduction (around 30 per cent, according to Lynnerup) of the Icelandic population caused by the plague. We can easily understand the apparent indifference of the contemporary written sources to this back migration (probably diluted over a long period), if we consider that the size of the population of Iceland, even though dramatically reduced in comparison to the 70'000 inhabitants assessed before the plague, was incomparable with the very low number of people who probably came every year from the Greenland colonies.

4. GINI'S HYPOTHESIS

Gini's treatise, after reviewing the different hypotheses, dwells upon the discussion of a possible "intrinsically" demographic explanation of the decline and vanishing of the colonies, which refers to the problem of the "population minimum"[4]. Gini puts forward his hypothesis: in a small and closed demographic group, the natural variability in the number of the demographic events (births and deaths) which may occur every year, and a possible disproportion in the sex ratio at birth (also resulting from the limited size of the group) may generate a strong distortion in the demographic (age and sex) structure. As a consequence, the normal functioning of the marriage market can be impaired or even blocked, especially where strict monogamous rules exist. In such a situation, the whole demographic reproduction system may be disrupted. As we will specify later on, the evidence inferred from archaeological research is consistent with the hypothesis – at least as far as the Western colony is concerned – of a strong disequilibrium in the sex ratio (with a high prevalence of females in reproductive age). This disequilibrium could have led – in a condition of isolation – to the extinction of the group, preventing for a long period the maintenance of a net reproduction rate at least equal to one. Even if we do not have at our disposal, obviously, any direct information about the marriage patterns of these communities, it seems clear that, in a situation like the one that can be hypothesized – from a certain time on – for the Western colony, the difficulties of marriage arrangements, in a situation of isolation, could have been the direct cause of the rapid extinction of this small demographic group.

Also, according to Gini, it should be stressed that the Vikings were strictly monogamous (they were Catholic, and it may be noted that both a monastery and nunnery were established). Consequently, if a sex-ratio

[4] The question of the "population minimum" is extensively discussed also by L. Livi, 1941 (in particular, see chapters 4, 5, 6 of the first volume)

imbalance occurred in the reproductive age, the number of married couples decreased significantly.

Putting aside for the time being the possibility that the uncertain and very incomplete information which can be inferred from the skeletal findings may not accurately represent the real age and sex demographic structure of the whole group, we still have to answer the following question: what may have generated such a strong imbalance in the sex ratio at the reproductive age? Gini, quoting other cases of sex imbalance, found in small, closed demographic groups[5], is inclined to think that this imbalance was not caused by traumatic external events, such as – for instance – a systematic higher male mortality from violent causes (wars, shipwrecks), but rather by a random alteration of the sex ratio at birth or by a similarly random imbalance between male and female mortality risks, as a consequence of the reduced size of the group.

5. HYPOTHESES ON THE MAIN FEATURES OF THE DEMOGRAPHIC SYSTEM IN THE VIKING COLONIES

With reference to the whole population of the Greenland settlements, Lynnerup considers likely a demographic pattern characterised by an initial long period of growth (about 300 years, up to the beginning of the XIVth century). During this long period, an initial group of about 500 people[6] progressively increased and reached or just overtook the threshold of 2'000 inhabitants. This growth was the result of fertility and mortality levels (*total fertility rate* and *life expectancy at birth*) that, in the long run, would have allowed the maintenance of a marginally positive growth rate, combined with a positive immigration rate. Considering the two settlements as a whole, this demographic group should then have progressively decreased, with the hypothesis of a slight increase of mortality (caused by a worsening in living conditions) combined with a migration rate which became slightly negative (*i.e.* emigration). The persistence of these demographic conditions would lead the colonies to extinction in about 200 years. In fact, Lynnerup suggests

[5] In the research carried out by the CISP (Comitato italiano per lo studio dei problemi di popolazione) on isolated ethnic groups, we can cite, for instance, the case (during the first half of the twentieth century) of the Samaritans of Palestine. This small group was at risk of extinction due to a strong and lengthy sex imbalance at birth, which caused difficulty in finding a marriage partner without violating the strict rules which prohibited marriage with the neighbouring Jewish population (Gini 1957 p. 701-02)

[6] This figure seems to be consistent with historical reports on the size of the first groups of colonists who arrived in Greenland before 1000 (Lynnerup 1998, p.115).

several possible models, some of them obtained with deterministic elaborations, others with a stochastic approach. In any case, all of them seem to confirm the framework of the demographic evolution that we have described. We must also stress the fact that the demographic models elaborated by Lynnerup are consistent with the evaluations that can be inferred from archaeological research. In both cases it is possible to evaluate, for the whole period of about four or five centuries of survival of the colonies, a total figure of around 26'000 burials, while different hypotheses (suggested by other authors) on the demographic size of the colonies do not seem to be consistent.

In fact, these estimates concern the entire population of the two settlements, which had indeed quite different dimensions (the Western settlement was far smaller) and, above all, a very different chronology. The early extinction of the Western group – as Lynnerup underlines – must probably be related to its higher "demographic frailty", due to its lower size. This is the very idea which is resolutely upheld by Gini, as we stressed at the beginning of this paper. In fact, our aim is simply to discuss the frailty of a small demographic group, whose environmental conditions were certainly very difficult.

In order to evaluate the likelihood of an extinction of the Western settlement due to purely demographic causes, we are obliged to assume a previous condition of stability[7]. Namely, we have assumed that a small demographic group, in the absence of migratory flows, was characterized by fertility and mortality levels that would be consistent with the historical age and with environmental conditions[8]. According to a deterministic approach, which implies a strict maintenance of stability conditions, any population, even a small one, characterized by a demographic (age and sex) structure consistent with those fertility and mortality levels, would have grown over time, even if very slowly. The purpose of the simulation we have produced was to demonstrate that a small demographic group, owing to a very high variability in terms of the occurrence of demographic events (birth and deaths), may have undergone a very different evolution.

[7] According to Lotka's theory, the stability implies, in addition to the absence of migrations, that fertility and survival functions do not change over time. If these conditions may be assumed, the annual growth rate of the population will be constant over time.

[8] In the simulation be presented in the next paragraph, we have assumed the hypothesis that stability derived from an expectation of life (for both sexes) of 25 years and from a total fertility rate of 5.5 children per woman. If we consider the North model of Coale and Demeny (1983) on stable populations, we can verify that those parameters are consistent with an annual growth rate of about 2.5 per thousand.

6. THE ANTHROPOLOGICAL-ARCHEOLOGICAL DATA

Here we describe the anthropological data that will characterize the likelihood function of section 8. Such data concerns the human remains of adult individuals with available sex diagnosis uncovered in W51 Sandnes, a Western Norse settlement site (also known as Kilaarsarfik in the Eskimese language). W51 Sandnes is located near the shore of a small peninsula that is being progressively eroded and submerged by the sea. The specimens come from an archeological area comprising a church and a churchyard. Direct radiocarbon dating were performed on the human bones for the purpose of dating the skeletal remains in a chronological framework. Concerning the six individuals dated, Lynnerup notes that "…all the skeletons lay in chronological and stratigraphical order; one uppermost, two just below, side by side; and below these another three, one below another." (Lynnerup 1989, pp. 46.47), that is, they were not found scattered in a large area, but were representative of a whole chronological time span. Dating results place the individuals' death within the time period 1297-1413 AD (calibrated age; range including extremes with ±1 sigma: 1275-1432 AD) (Lynnerup 1998, p.147, Table V). This means that they died in a later phase of the whole Greenland Norse settlement chronology.

The W51 Sandnes site was first explored and excavated by Thorhallesen in 1765, then by Møller in 1840 and by Jansen in 1885 (Lynnerup, 1998). All of these pioneer archeologists reported the presence of, and maybe recovered some, human bones; but these finds were subsequently lost. The first modern archaeological survey was carried out by Bruun in 1903. He excavated the ruins of a church and a churchyard close to the shoreline; these ruins emerged from the the sea only at low tide. Along the cliff by the beach he found the human remains of several individuals. Nørlund and Roussel continued the excavations in 1930; they were forced to build a dike along the beach because by then the ruins were completely submerged. They excavated numerous burials, both individual and multiple (presumably family units), and recovered 42 complete human skeletons and other less complete individuals represented by mixed bones. More recent excavations were made by Meldgaard in 1976 and by Arneborg in 1984; professional physical anthropologists were engaged in the recovery of human bones, although not many osteological remains were retrieved (among them a skull from a stable wall foundation and a pelvis from a kitchen midden). In summary, a total of 190 individuals were found within the churchyard area, *plus* another 2 from a less usual context. They are very heterogeneous in terms of their state of preservation and completeness; some are represented by complete skeletons, others only by skulls or other isolated bones. Among these, 89 individuals could be reliably sexed: 33 males and 56 females, ranging from approximately 20 to 50 years of age (Lynnerup, 1998). Age at death was assessed using standard anthropological methods: degree of tooth formation and growth; long bone length; epiphyseal fusion; cranial suture closure; auricular surface of the ilium; sternal rib end changes; and dental erosion (Lynnerup, 1998). It may be noted that while adult – non-adult discrimination is usually considered very reliable, estimation of age at death of an adult produces large margins of error (see for example Rinaldi *et al.* 2003). This may be amended by the use of broader age categories (age intervals). Regarding the Greenland Norse, the adult individuals were classed in 2 conventional anthropological categories: "Adultus" *sensu strictu*, or young adults (18/21-35 years), and "Maturus", including old and "Senilis" adults (>35 years), *plus* a third class "Adult" without further precise age determination. It was further attempted to class the individuals in a more precise way by grouping them in 5 year-intervals, cohorts by sex. In the resulting age and sex distribution (the population "pyramid"), a certain disproportion of the sex-ratio in many cohorts is apparent, namely in the ones of 20-25, 25-30. 40-45 and 45-50. Overall we find that this latter classification produces a strong fragmentation of data, and as noted, may be associated with larger error margins. Hence, in this study, we consider only the previous two large classes "Adultus" and "Maturus" with definite age intervals, *plus* the sub-adult class of "Juvenilis" with an age interval of 12/14-18/21 years, that is, an age cohort comprising, at least, some individuals at their puberty, therefore already potentially in the reproductive age.

Concerning the sex of individuals we do not have any available "external" information to improve sex diagnosis (for example archaeological information, from the burial typology or from the presence of sex-specific grave goods). Therefore sexing is purely based on the usual anthropological criteria, based on sexually dimorphic morphological features, although allowing for their limitations due to inter-population variability. Standard methods of morphometrical assessment, usually employed in anthropological practice, such as multiple discriminant function analysis on measurements, or scoring systems of traits and subsequent averaging and weighting of scores, were not used, because such methods rely heavily on a "normal" reference population, so they can produce unreliable results "when used on a quite different body of material" (Lynnerup 1989, pp.41-42).

To summarize, we employ in our analysis data from the W51 Sandnes locality only, because this data set is the only one of a relevant size available from a Western settlement. It is also probably from a more isolated locality than the Eastern settlement and it pertains to a later period of the whole

Greenland Norse settlement history, when the Western settlement was presumably starting to undergo a fatal demographic reduction. The data are shown in Table 1.

Table 1 – *Archaeological finds data set grouped by sex and age*

Sex	Age			
	<21	21-35	>35	**Total**
Male	3	11	12	**26**
Female	6	26	15	**47**
Total	**9**	**37**	**27**	**73**

7. BAYESIAN INFERENCE

Gini's hypothesis has been tested following the Bayesian paradigm. To achieve this the Random Variable (*RV*) "sex ratio", and "total amount of individuals" has been investigated. A prior Probability Distribution (*PD*) has been provided. Then a likelihood, using the anthropological data described in section 6, has been modelled. Finally, via Bayes' theorem, a posterior *PD* has been obtained.

7.1 The prior

Let name (M,F) be the *RV* "number of males and females in the reproductive age 300 years after the foundation of the colony". The goal is to evaluate P_R, and P_W, the *PD* of the *RV* $R = M/(M+F)$, and $W = M+F$.

Firstly consider P_{MF}, the *PD* of the *RV* (M,F). To evaluate this, it has been assumed that the colonizers comprised 100 couples in reproductive age. Further more it has been assumed that, for three centuries after the first settlement:

A1. the colony was demographically isolated,
A2. fertility and mortality were constant (see section 5),
A3. people were strictly monogamous and any form of incest was forbidden,
A3. no exceptional event intervened.

Actually P_{MF} was evaluated via simulation. Such procedures are nowadays rather popular in demographic studies (Bertino *et al.* 1988, Bertino 1990. Smith J.E. and Oeppen J. 1993, Smith P.W. *et al.* 1996, Zhao 2001). Some pioneering work is also worthy of note (see Pielou 1977).

Here some basic ideas will be outlined, for details see section 9.

The first step of the simulation consists in obtaining 100 couples by randomly coupling the 200 colonizers in reproductive age. Then the reproductive history of each couple is simulated. To do so: each couple, exactly nine months later, experiences a birth event. Then sample the random events *"child is male"*, *"child is female"*, *"he/she reaches the initial fertile age"*, *"couple is dissolved by death"* with some given probabilities (see points C1 and C2 in section 9).

If the couple survives, after a random period of time (see C3 in section 9), we see a new birth event and sample his sex and survival as above.

Observe the population after a given lapse of time ?. Do not select coupled individuals in the reproductive age, and, according to A3, create new couples and simulate their history as above.

Observe the population evolution in 300 years to obtain $(m^{(1)}, f^{(1)})$: the number of males and females in reproductive age at the simulation (1)

Repeat the procedure K=100'000 times to obtain

$$(\mathbf{m},\mathbf{f}) = \{(m^{(1)}, f^{(1)}),\ldots,(m^{(K)}, f^{(K)})\}$$

K independent samples from P_{MF}.. Use (**m,f**) to estimate P_{MF}.

Finally we obtain P_{WR} and P_{MF} via numerical method. For computational purposes, P_{MF}, and P_{WR} have been interpolated with a continuous Probability Density Function (*PDF*), respectively, f_{MF}, and f_{WR}.

7.2 The likelihood function

Remember $R = M/(M+F)$ is the sex ratio of males in reproductive age in the population of living individuals. Actually, according to anthropological data collected in table 3, available data are s_M = 11 male and s_F = 26 female skeletons in reproductive age. The goal is to model the likelihood function $P_{s|R}$, the conditional *PD* of s given R.

Let $D = (D_M, D_F)$ be the set of male and female colonizers who died in a rather short period around 1300 A.D. In accordance with Lynnerup (1998 pages 145-150), it has been assumed:

B1. $s = (s_M, s_F)$ is a random sample of D, *i.e.* the available skeletons are considered a sample of deaths.

Furthermore it has been assumed that:

B2. s is a small part of D;

B3. no sexual selection in the burial is present;

B4. the probability of death at a given age is the same for males and females (see del Panta and Rosina (2002) for a discussion on this topic).

Now define:

p be the probability an individual chosen at random in D is male;

s be the probability that, in an excavation, a male skeleton is founded.

Given B1 and B2 it is natural to model $s|s$ with a binomial model:

$$P_{s|\sigma}(s \mid \sigma) = \binom{s_M + s_F}{s_M} \sigma^{s_M} (1-\sigma)^{s_F} .$$

By points B3 and B4 we obtain $s = p = R$.

Finally we obtain the following likelihood model:

$$P_{s|R}(s \mid R = r) = \binom{s_M + s_F}{s_M} r^{s_M} (1-r)^{s_F} .$$

7.3 The posterior distribution

Now, via Bayes theorem, it is possible to compute the posterior distribution $f_{R|s}$, the conditional *PDF* of *R* given *s*:

$$f_{R|s}(r \mid s) \propto f_R(r) P_{s|R}(s \mid R = r) .$$

Furthermore it has been computed $f_{W|s}$, the conditional *PDF* of *W* given *s*, via numerical integration:

$$\begin{aligned} f_{W|s}(w|s) &= \int_0^1 f_{WR|s}(w, r \mid s) dr \\ &= \int_0^1 f_{W|R,s}(w \mid r, s) f_{R|s}(r \mid s) dr \\ &\propto \int_0^1 f_{W|R,s}(w \mid r, s) f_R(r) P_{s|R}(s \mid R = r) dr \end{aligned} .$$

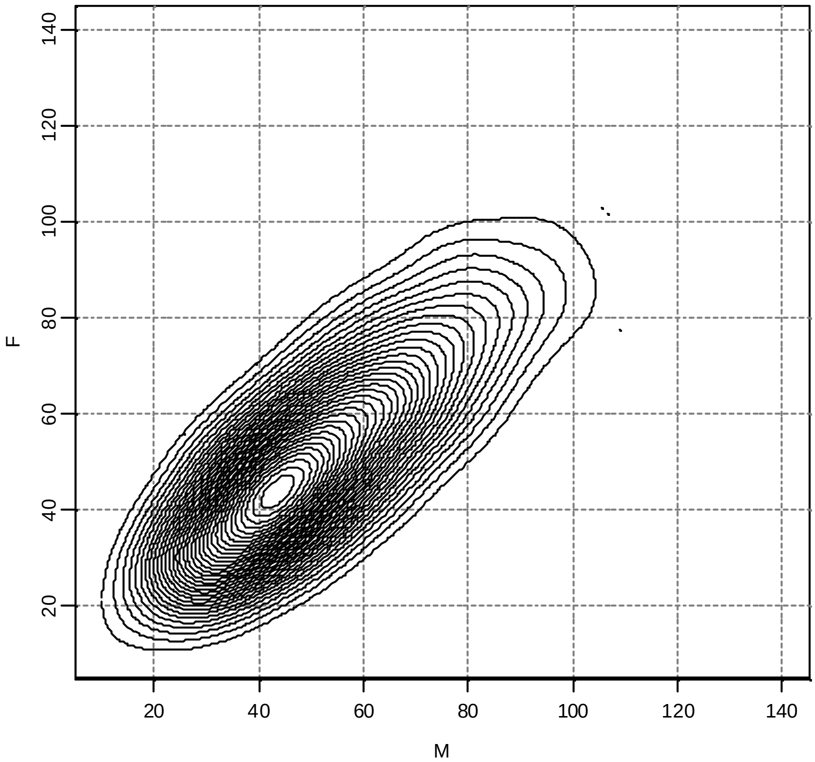
If one assumes that *W* and *s* are conditionally independent given *R*, we obtain:

$$f_{W|s}(w|s) \propto \int_0^1 f_{W|R}(w \mid r) f_R(r) P_{s|R}(s \mid R=r)dr$$
$$\propto \int_0^1 f_{WR}(w, r) P_{s|R}(s \mid R=r)dr$$

7.4 Results

Figure 1 shows the prior *PDF* of (*M*,*F*): the symmetry of the surface shape is evident. Figure 2 shows the prior *PDF* of (*W*,*R*). The graphic shows that such variables are not independent: if *W* increases, the variability of *R* around 0.5 decreases.

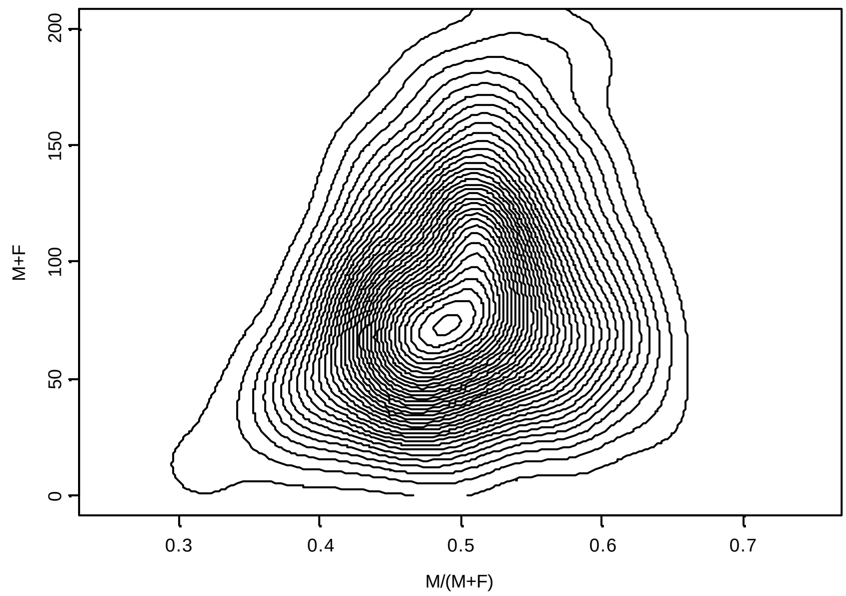
Figure 1 – *Contours of the joint prior density distribution for the number of males and females in reproductive age*



In figure 3 the prior, the normalized likelihood, and the posterior *PDF* for *R* are plotted. The prior is symmetrically distributed around 0.5; the likelihood function is strongly asymmetrically distributed and with mode

0.33; finally the posterior is weakly asymmetric and the mode is in 0.42. The added prior information considerably reduced its variance. Figure 4 shows the related cumulative probability function.

Figure 2 – *Contours of the joint prior density distribution for ratio and sum*



Finally figures 5 and 6 show, respectively, the *PDF* and the cumulative probability distribution of *W*. Prior and posterior distributions are represented. The mode of the prior is around 70 while the mode of the posterior is around 55. In figure 6 it is clear that the prior probability stochastically dominates the posterior one. For example, consider the probability that the total number of people W is less than 50 (a very critical value). That is 0.20 according to the prior and 0.35 according to the posterior.

8. DISCUSSION AND CONCLUSION

In describing the Norse society, archaeologists and historians have relied mainly on the analysis of archaeological data and historical sources respectively. Directly relating to a discussion of isolation is also the question

Figure 3 – *Likelihood (dashed line), prior (solid line) and posterior (point-dashed line) probability density function for ratio*

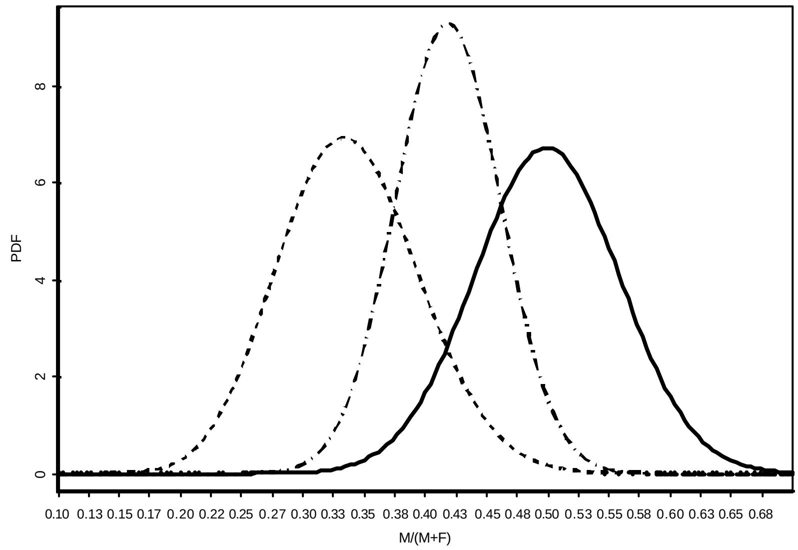


Figure 4 – *Likelihood (dashed line), prior (solid line) and posterior (point-dashed line) cumulative probability function for ratio*

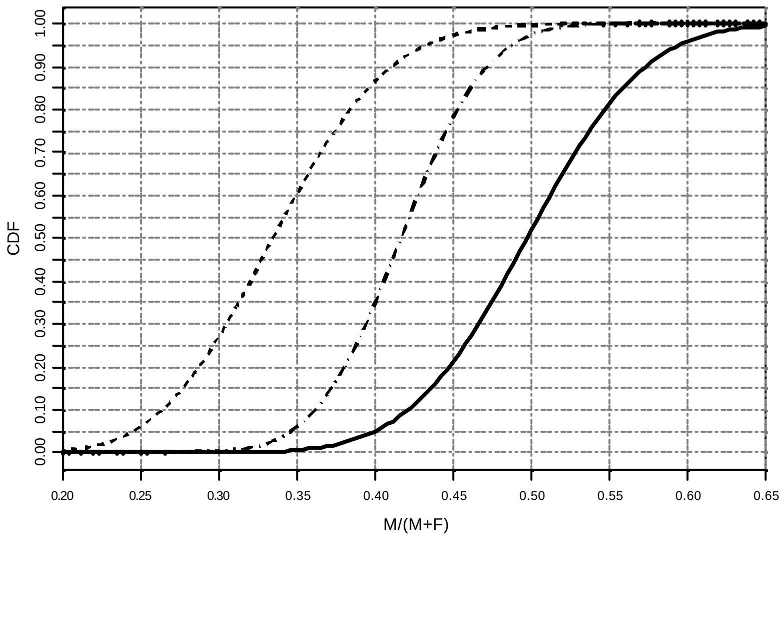


Figure 5 – *Prior (dashed line), and posterior (solid line) probability density function for total number of reproductive people*

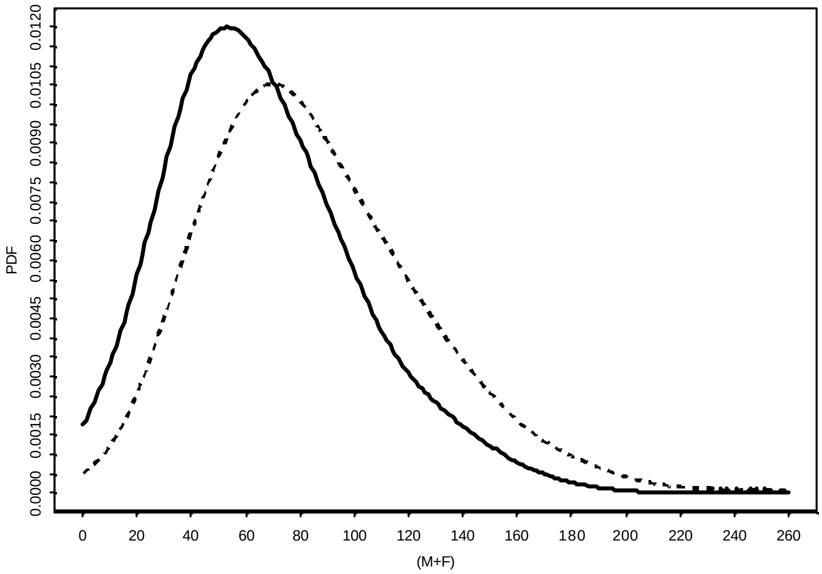
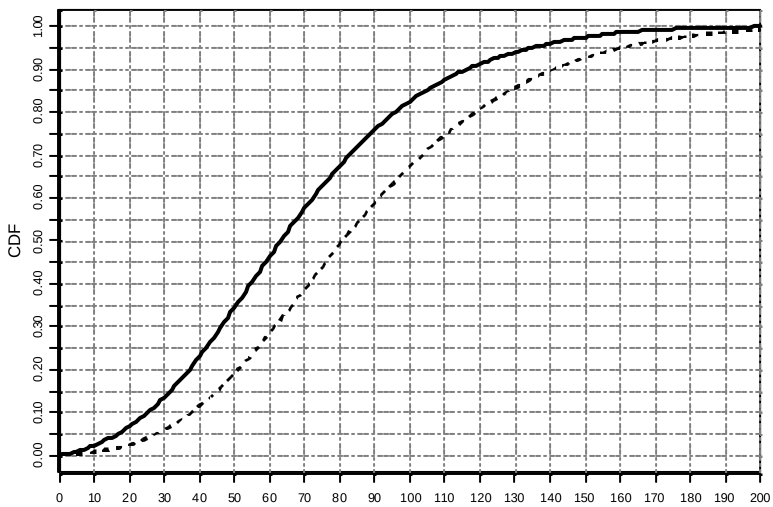


Figure 6 – *Prior (dashed line), and posterior (solid line) cumulative probability function for total number of reproductive people*



of population size and, as addressed in this paper, the demographic stability of the population.

This is really a central question when one wants in some way to reconstruct past societies. Various explanations on the demise of the Norse settlements very much hinge upon how large the population was: if it was small then even slight perturbations may rapidly have brought the population to below sustainable levels, whereas a large population would have had a better "buffering" effect. Estimating the population size may be made by, e.g., analysing the number and size of farms and correlating this information with a population size. Indeed, several archaeologists have done so and estimates have ranged from an average population of about 3'000 to 5'000-6'000 (see Lynnerup 1998 for a review). However, very few farms have been dated in terms of functional periods, and the Norse Greenlanders probably used the transhumance system, where the livestock was moved to outlying grasslands for part of the grazing season. This means that the archaeological values probably represent maximum figures. But if these figures do not seem very large, it must be remembered that they represent the population number at a given point in time. Such population levels accumulate into much bigger figures when the total number of deaths over the settlement period is calculated. For example, over a 500-year period a population of around 5'000 people will "produce" maybe some 70'000 deaths.

This paper explores the possibilities of population frailty, *i.e.* how vulnerable the population was to stochastic fluctuations in fertility and mortality. Here the classical Bayesian paradigm was adopted to test Gini's hypothesis (Gini, 1957). The prior probability was evaluated simulating the amount of a population, starting with a small set of reproductive couples, 300 years later. To evaluate the likelihood function (conditioned by the excavated skeletal remains) a simple binomial model was used. Finally, combining prior and likelihood, in accordance with Bayes theorem, a posterior probability distribution on the distribution male-female was obtained. Simple transformations allowed the evaluation of the posterior distribution of the sex disequilibrium. Such a probability combines the prior information on a possible random disequilibrium in a small population and the evidence given by the likelihood. Specifically, we focused on the Western settlement, where some anthropological data is available. As shown in section 7, data gave evidence to a decrease in the expected ratio R. Furthermore, a slightly higher probability that the population, after 300 years, consisted of less than 50 individuals was pointed out.

Of course, we cannot exclude the concurrence of other possible factors, which may have contributed to the demise of the Viking Greenland colonies, and which must be considered at least as far as the Eastern settlement is concerned.

References

ARNEBORG J., HEINEMEIER J., LYNNERUP N., NIELSEN H.L., RUD R., SVEINBJÖRNSDÓTTIR Á.E. (1999), "Change of diet of the Greenlandic Vikings determined from stable carbon isotope analysis and ^{14}C dating of their bones", *Radiocarbon*, 41, 157-168.

AZZARA C. (1999), "*Le invasioni barbariche*", Il Mulino, Bologna.

BERTINO S. (1990), "Un modello aleatorio per lo studio, mediante simulazione, dell'evoluzione di popolazioni chiuse", *Bollettino di demografia storica*, 11, 37-42.

BERTINO S., PINNELLI A., VICHI M. (1988), "Two models for microsimulation of family life cycle and family structure", *Genus*, XLIV(1-2), 1-23.

COALE A.J., DEMENY P. (1983), *Regional Model Life Tables and Stable Populations*, Academic Press, New York.

DEL PANTA L., ROSINA A. (2002), "Mortalità adulta per genere in Italia prima di Pasteur: variabilità territoriale ed ipotesi esplicative", in DEL PANTA L. *et al.* (eds.), *Dinamiche di popolazione, mobilità e territorio in Italia, secoli XVII-XIX*, Udine, Forum, 181-196.

ENCYCLOPAEDIA BRITANNICA (1962), Chicago-London.

FRICKE H.C., O'NEIL J.P., LYNNERUP N. (1995), "Oxygen Isotope Composition of Medieval Human Tooth Enamel from the Medieval Greenland: Linking Climate and Society", *Geology*, 23, 869-872.

GINI C. (1957), "Sulla scomparsa delle colonie normanne in Groenlandia", in *Corso di sociologia*, Edizioni "Ricerche", Roma.

ISTITUTO GEOGRAFICO DE AGOSTINI (1986), *Atlante storico mondiale*, Novara, (original ed. *The Times Atlas of World History*, Times Books Limited, London).

LIVI BACCI M. (1998), *Storia minima della popolazione del mondo*, Il Mulino, Bologna.

LIVI L. (1944), *Prime linee per una storia demografica di Rodi e delle isole dipendenti dall'età classica ai nostri giorni*, Firenze, G.C. Sansoni Editore.

LIVI L. (1974), *Trattato di demografia*, 2 voll., Cedam, Padova, (reprinting of 1941 edition).

LYNNERUP N. (1996), "Paleodemography of the Greenland Norse", *Arctic Anthropology*, 33, 122-137.

LYNNERUP N. (1998), "The Greenland Norse. A Biological-anthropological Study", *Meddelelser om Grønland, Man & Society*, 24, Copenhagen, 1-149.

PIELOU E.C. (1977), *Mathematical Ecology*, Wiley.

RINALDI A., PACCIANI E., DI BACCO M. (2003), "Age at death diagnosis by cranial suture obliteration: a Bayesian approach", in DI BACCO M., D'AMORE G., SCALFARI F. (eds.), *Applied Bayesian Statistical Studies in Biology and Medicine*, Dordrecht: Kluwer Academic Publishers, 239-249.

SILVERMAN B.W. (1986), *Density Estimation*, Chapman & Hall, London.

SMITH J.E., OEPPEN J. (1993), "Estimating numbers of kin in historical England using demographic microsimulation", in REHER D.S., SCHOFIELD R. (eds.), *Old and new methods in historical demography*, Oxford, Clarendon Press, 280-317.

SMITH P.W., MC DONALD J.W., FORSTER J.J., BERRINGTON A.M. (1996), "Monte Carlo exact methods used for analysing interethnic unions in Great Britain", *Journal of the Royal Statistical Society*, series C: applied statistics, 45(2), 191-202.

ZHAO Z. (2001), "Chinese genealogies as a source for demographic research: a further assessment of their reliability and biases", *Population Studies*, 55(2), 181-193.

Appendix: technical details on prior assignment

We will now describe the algorithm to compute the prior *PD*. Split the time-interval [T_0=1000AD,…, T_{401}=1300AD] in 400 interval of ?=9 months each (gestation time), and define $T_n=T_0+n?$. Let $M(T_n)$ and $F(T_n)$ be, respectively, the number of males in reproductive age (*MRA*), and females in reproductive (*FRA*) age, at time T_n.

The goal is to sample from the conditional *PD* of $M(T_{401})$ and $F(T_{401})$ given $M(T_0)$ and $F(T_0)$:

$$P\left\{M\left(T_{401}\right),F\left(T_{401}\right)\mid M\left(T_0\right)=100,F\left(T_0\right)=100\right\}.$$

To do so the evolution of an artificial population has been simulated.

The simulation starts by setting up 100 couples of a randomly chosen age. Their history is recorded for 400 iterations (300 years). The procedure has been repeated 100’000 times.

The algorithm can be synthesized as follows: at time T_n the *i*-th individual is characterized by:

1. an identification code;
2. his sex;
3. his age $Age_i(T_n)$;
4. the identification code of his parents;
5. the identification code of his partner if he has one, zero if not;
6. if his alive or not.
7. the time to the next pregnancy $t_i(T_n)$ (for females only).

At the iteration n+1 the following actions hold:

i. <u>coupling step</u>
 if a partner is available, each single individual of fertile age is coupled. Couples are subject to ties of consanguinity;

ii. <u>generation step</u>
 for each coupled female *j* **do**:
 $t_j(T_{n+1})=t_j(T_n)-\Delta$;
 if $t_j(T_{n+1})<0$ **then** *j* generate a new individual **and** extract a new value for $t_j(T_{n+1})$ from a gamma distribution (see notes C3. below).
 end;

iii. aging and death step

for each alive individual i **do**:

$Age_i(T_{n+1}) = Age_i(T_n) + \Delta$;

if $Age_i(T_{n+1}) \geq 18$ years **then** increase the set of the incoming fertile individuals by 1;

if $Age_i(T_{n+1}) > 40$ years **then** increase the set of the outgoing fertile individuals by 1;

evaluate $p_i(Age_i(T_{n+1}))$ the probability of death (see notes below);draw death of individual i with probability $p_i(Age_i(T_{n+1}))$;

end;

iv. counter step

update N_M and N_F by:

$$\begin{cases} N_M(T_{n+1}) = N_M(T_n) - In_M(T_{n+1}) + Out_M(T_{n+1}) \\ N_F(T_{n+1}) = N_F(T_n) - In_F(T_{n+1}) + Out_F(T_{n+1}) \end{cases}$$

where $In_M(T_{n+1})$, $In_F(T_{n+1})$, $Out_M(T_{n+1})$, and $Out_F(T_{n+1})$, represent, respectively, the number of incoming *MAR*, the number of incoming *FAR*, the number of outgoing *MAR*, and the number of outgoing *FAR*, at step $n+1$.

It is worth noticing that:

C1. the reproductive age-interval for each individual has been set at (18;40) years old.

C2. All other demographics parameters have been deduced by the "Regional Model Life Tables and Stable Populations" (Coale and P. Demeny (1983)) supposing the third level of mortality (with population increment less than 1%).

C3. The inter-genetic interval t_j for female j (the time to the next pregnancy) has been modelled using a gamma distribution with parameters a and b ($\Gamma(a,b)$). To fix a and b, we first imposed the average number of sons in the whole reproductive age is 6 (*i.e.* one each $(40-18)/6 = 6.3\Delta$ on the average) and obtained $a/b = 6.3\Delta$. Then we imposed $\Pr\{4.8\Delta < t_j < 7.9\Delta\} = 0.95$ and obtained $a = 62.7$ and $b = 9.9$.

Now repeat steps i. to iv. 400 times to obtain the first sample from $P\{M(T_{401}), F(T_{401}) \mid M(T_0) = 100, F(T_0) = 100\}$ *i.e.* the couple $(m^{(1)}, f^{(1)})$.

Once the whole process has been repeated K=100'000 times the data points $\{m^{(k)}, f^{(k)}; k=1,...,K\}$ have been interpolated using the Gaussian kernel density estimation with standard Cross Validation bandwidth selection (see Silverman (1986) for details).

The program to simulate M and F were written in Fortran language. Any other tasks were processed using the statistical package R (http://www.r-project.org/).