Management Adviser

Volume 9 | Number 2 Article 1

3-1972

Letters

Tappan S. Roy

John W. Caron

John O. Mason Jr.

COMMENTS LETTERS CRITICISMS

Dear Sir:

In their article, Messrs. Mason and Connelly[1] have described and illustrated the use of four self-checking digit techniques. These techniques are useful in detecting various types of coding errors generated when numbers are transcribed from one document to another. For each technique, they also evaluated, by simulation, the conditional probability that a single transposition error would be uncovered given that such an error has occurred. In particular, in Table 1, they list the conditional probability of a Mod 11-Geometric technique uncovering a single transposition error as .90. The conditional probability should have been listed as 1.00 since the Mod 11-Geometric technique can detect all single transposition errors. The proof is as follows: Let the number to be checked be N, a positive integer of any magnitude. That is

$$N = x_m \ldots x_{i+1}x_i \ldots x_3x_2x_1,$$

base 10, where m is the number of digits in N, and x_i is the value of the digit in the i^{th} position. Let N′ represent N after a single transposition error has occurred. That is,

$$N' = x_m \ldots x_ix_{i+1} \ldots x_3x_2x_1.$$

The check digit for N is the complement of the remainder developed when

$$2x_1 + 2^2x_2 + \ldots 2^ix_i + 2^{i+1}x_{i+1} + \ldots + 2^mx_m$$

[1] Mason, John O., and William E. Connelly, "The Application and Reliability of the Self-Checking Digit Technique," MANAGEMENT ADVISER, September-October, 1971, pp. 27-34.

is divided by 11. The check digit for N′ is the complement of the remainder developed when

$$2x_1 + 2^2x_2 + \ldots + 2^{i-1}x_{i-1} + 2^ix_{i+1} + 2^{i+1}x_i + \ldots 2^mx_m$$

is divided by 11. However, if x_i does not equal x_{i+1}, the difference between these two dividends (say D1), when divided by 11, will always have a remainder whose magnitude is greater than zero. Therefore, the check digit for N′ will not equal the check digit for N, and the single transposition error will always be detected. We can show that D1, when divided by 11, will have a remainder whose magnitude is always greater than zero by the following:

$$2x_1 + 2^2x_2 + \ldots + 2^ix_i + 2^{i+1}x_{i+1} + \ldots 2^mx_m$$

minus

$$2x_1 + 2^2x_2 + \ldots + 2^ix_{i+1} + 2^{i+1}x_1 + \ldots 2^mx_m$$

equals

$$2^ix_i - 2^ix_{i+1} + 2^{i+1}x_{i+1} - 2^{i+1}x_i$$

which equals

$$2^i(x_{i+1} - x_i).$$

Now since x_{i+1} and x_i are positive integers, $x_{i+1} - x_i$ must be an integer and lie between -9 and $+9$. Therefore, 11 can never be a factor of $2^i(x_{i+1} - x_i)$. It is only when $x_{i+1} = x_i$ that N and N′ will have the same check digits. But if this were true, then N′ equals N, and there would be no single transposition error.

Tapan S. Roy
John W. Caron
The Travelers
Hartford, Connecticut

Critics are correct

Dear Sir:

Thank you for the copy of the Roy and Caron letter.

Their equations are correct. Due to an error in programing, the conditional reliability of the Mod 11-Geometric method was listed as 90 per cent when instead it should have been reported as 100 per cent. Our revised Table 1 is shown below.

Although we do agree to the modification of Table 1 for the computational error as noted by Messrs. Roy and Caron, we must point out that our conclusions remain virtually unchanged. The one modification to our conclusions is as follows: "The ability to detect errors is greatest in the Mod 11-Geometric method. In all categories the Mod 11-Geometric method detected coding errors as well or better than the Mod 10 methods and the Mod 11-Arithmetic method. There is an extremely small probability that these results were due to chance (less than one in a thousand)."

John O. Mason, Jr.
University of Alabama

TABLE I—Revised

RELIABILITY FACTORS ASSOCIATED WITH SELF-CHECKING DIGIT METHODS (ROUNDED TO THE NEAREST PER CENT)

TYPE OF ERROR / SELF-CHECKING DIGIT METHOD	SINGLE TRANSCRIPTION	SINGLE TRANSPOSITION	DOUBLE TRANSPOSITION	RANDOM SCRAMBLE	SUBSTITUTION OF VALID, BUT INCORRECT NUMBER
Mod 10 — Simple Sum	100%	0%	0%	90%	0%
Mod 10 — Alternate	94%	90%	90%	90%	0%
Mod 11 — Arithmetic	100%	90%	90%	90%	0%
Mod 11 — Geometric	100%	100%	90%	90%	0%