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The Effects of a Blended Learning Environment on Students' Discourse in a Secondary Mathematics Classroom

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THE EFFECTS OF A BLENDED LEARNING ENVIRONMENT ON STUDENTS'

MATHEMATICAL DISCOURSE WITHIN A SECONDARY MATHEMATICS CLASSROOM

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By

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ABSTRACT

To prepare students for college and careers, the Common Core Mathematical Practice Standards propose teachers engage students in classroom discourse where they make sense of mathematics by working collaboratively and communicating their thinking. The purpose of this mixed methods study was to explore the relationship between a blended learning environment and the discourse students produce while in an AP Calculus AP course. Participants in this study are members of a large high school enrolled in either hybrid or virtual learning environments. Data were collected through recorded observations, coded using the Mathematics Classroom Observation Practices Protocol (MCOP^2) and analyzed using statistical tests and thematic coding. The data in this study indicate there is an association between different aspects of discourse and the learning environment. Numerous themes arose from the 110 groups as they completed five different tasks. As there is a lack of research on the impact of COVID-19 on students' productive mathematical discourse while in a blended learning environment, the findings of this study will contribute to teachers' and educational stakeholders' understanding of how groups can create discourse regardless of their learning modality.

Keywords: Blended learning, productive discourse, hybrid learning, virtual learning, collaborative groups, MCOP^2.

DEDICATION

This dissertation is dedicated to my family who has helped push me through this long, tumultuous process. To my mom, Debbie Blake, you are my best friend, my strength, my knowledge, my motivation. You doubt yourself and your gifts, but I know that I am who I am because of you. To my late grandfather Calvin Blake, who always said I could achieve whatever I put my mind to; I guess you were right, because this was a dream, I never imagined I would achieve. To the rest of my family who prayed for me continuously and taught me to be the woman I am today, this is for you.

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CHAPTER 1: INTRODUCTION

The COVID-19 pandemic has transformed society in multiple, never before experienced ways. To curtail its spread, governments worldwide moved to suspend face-to-face teaching in K-12 and higher education, affecting some 95% of the worlds' student population, resulting in the greatest disruption to education in history (United Nations, 2020). One of the most significant impacts of the pandemic has been the majority of educational providers worldwide forced to rapidly convert to online learning (Basilaia & Kvavadze, 2020). Educational systems are forced to implement newer instructional blended learning modalities that differ significantly from the traditional classroom setting to accommodate the new normal that living in a pandemic provided.

Reiterated by the findings from the U.S. Education Department's (USDOE) meta-analysis on the effectiveness of online and blended instruction (Means, et al., 2009), the practice of blended learning has progressively increased in grade-level and secondary classrooms across the nation. In the analysis, Means et al. (2009) found academically, students performed slightly better while completing courses solely online as compared to their counterparts who were learning strictly face-to-face, but significantly better in studies that merged the two learning environments. However, it is often not practiced in teacher training programs, which still tend to separate pedagogical instruction from technology training (Duhaney, 2012; Pulham & Graham, 2018; Short, Graham, Holmes, Oviatt, & Bateman, 2021), even though many experts expect blended instruction to be standard practice in the K-12 classroom of the future (Rittle-Johnson, Schneider, Star, 2015). It appears that although the benefits of blended learning in a secondary education environment are widely known to scholars and researchers, many educators, particularly in the mathematics classroom, are not adequately prepared or equipped to utilize this instructional strategy in a way that could benefit its students.

The body of research in K-12 blended learning is small but growing exponentially due to the increased implementation of virtual learning throughout the world for the 2020-2021 school year. Although the potential of blended learning is well documented, due to the quickness in which Covid-19 changed education, not all teachers were adequately prepared to implement technology or a new instructional strategy within their classrooms. According to the 2021 EDUCAUSE Horizon Report, "Faculty buy-in, training, and support are essential for the adoption and effective use of technologies and other tools in the classroom" (p. 8).

Background of the Problem

Throughout the summer of 2020, K-12 educators and educational policy experts examined various instructional delivery options to determine which might be the most beneficial for student academic gain and the health safety of students and staff as schools reopened for the upcoming school year. Lieberman (2020) stated that this lengthy examination focused primarily on three forms of internet-based instruction: complete face-to-face instruction, total online learning, and a combination of both. Lieberman asserted that the combination of online and face-to-face instruction, which in this research is termed blended learning, appeared to be the instructional option that was most advantageous for the start of the 2020 school year. In this same *Education Week* article, Lieberman stated instructors prefer "blended learning as it enabled educators to customize learning options based upon each student's academic strengths and weaknesses while maximizing students and staff's safety" (p. 23). This opinion stems from the educational systems' ability to limit the number of students who utilize the face-to-face component on any given school day, ensuring life-threatening issues such as social distancing, proper wearing of face masks, and continuous disinfection of the school facilities were achievable. Schools that choose to implement blended learning can monitor and dictate the number of students who enter the school building and learn face-to-face or stay at home and learn from an online platform. Limiting the number of bodies within a school building provides a component of safety for the students and the teachers and staff.

During the past several decades, there has been an influx of technology usage in Americans' daily lives (Acree, Gibson, Mangum, Kellogg, & Branon, 2017; Pew Research Center, 2018). According to the Pew Research Center (2018), over 89% of all American households, including one out of every ten adults, have access to the internet, mainly through smartphones. Since the vast majority of school-aged children have convenient access to the world wide web, cultivating online identities is as much a child's culture as the one they were genetically born into (Grieco, 2017). Children, particularly teenagers, see technology not as a luxury but as a way of life to communicate, socialize, and engage in their everyday world (Schrum & Levin, 2015). In the last decade, society has been, more than ever before, working and living in a world that creates knowledge from connections beyond our immediate reach. This cultural norm has created a need to integrate technology with teaching and learning in educational institutions to best reach students effectively (Blau, Shamir-Inbal, & Avdiel, 2020).

Blended learning is an educational technique that provides various opportunities for a classroom to become a multidimensional learning environment (Schindel, Hughes, & Sadowski, 2013). According to Wong, Tatnall, and Burgess (2014), blended learning is an approach that incorporates various components of learning through its design in order to reach all students. Blended learning offers a manageable teaching method due to the availability of technology that can be implemented into the classroom during face-to-face instructional time, along with the easy access to course materials outside of the traditional classroom. According to the 2019 K-12 Digital Content Report (2019), 49% of all American school districts have a 1:1 device program. The students enrolled receive a personal school-issued device that they use for their schoolwork. This same study reported that 31% of school districts that did not have a 1:1 device program planned to have one within the next five years. In 2016, merely three years before the Digital Content Report was published, only 26% of school districts had a 1:1 device program, with only 11% of districts planning on implementing one. At the end of the 2020-2021 school year, 16.3% of K-12 students attended school virtually, while 30.6% of K-12 students attended

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hybrid school and 53.1% attended traditional, in-person school, five days a week (Camera, 2021). These virtual students need access to technology, specifically students who do not have the means to purchase a device on their own, depending on their school district to supply the appropriate tools required to learn.

Researchers have written about the challenges of blended and online learning long before the pandemic. The literature on blended learning dates back to more than two decades ago. This growing instructional strategy gained increased popularity in recent years, particularly with the onslaught of the Covid-19 pandemic and the drastic surge in student access to technological devices. The most notable researched shortcomings of blended education include limited socialization and poor personal contact between learners and educators (Maher, 2014), lack of ability to develop a sense of community (Sun & Chen, 2016), and the lack of self-motivation and discipline (Gilbert, 2015). According to Sun and Chen (2016), there is not enough literature from the students' perspective that discusses their experience with online education and how it impacts their levels of discourse and learning outcomes.

Problem Statement

The unexpected urgency for social distancing demanded by the Covid-19 pandemic has led to a rapid and hurried transformation of the traditional education environment to blended learning. Zimmerman (2020) states that "education's current situation presents an excellent opportunity for testing how online teaching works" (p. 18). One of the main problems teachers, schools, and districts face today is the rapid pace at which technology has advanced, causing educators to be thrust into the process of developing and teaching in a blended learning environment prior to receiving the necessary training to be effective (Cuhadar, 2018; Lohnes, 2017). Due to the rapid development and expansion of online learning, Escueta, Vincent, Nickow, & Oreopoulos (2017) concluded that technology advancements have outpaced researchers' ability to conduct meaningful studies from which policy and practices can be produced that maximize the potential of computer-enhanced instruction in K-12. For decades,

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"scholars have noticed that educators have been ill-prepared to teach with technology" (Foulger, Graziano, Schmidt-Crawford, & Slykhuis, 2017, p. 418), yet with the onslaught of Covid-19, instructors all over the world were forced to use an instructional mode that many were not fluent in. There has been a significant shift from learning how to use technology to using technology to learn (Lohnes, 2017; Song & Kapur, 2017).

Methods to facilitate student-to-student mathematical conversations in a blended learning environment are not straightforward. However, research regarding how mathematics teachers can facilitate teacher-to-student discourse in a traditional setting is well-researched. Little research guides practitioners on the specifics of fostering an online environment conducive to high levels of student-to-student discourse without a great deal of guesswork on the part of the teacher (Wagner & Herbel-Eisenmann, 2014). Educators need more researched-based strategies to guide their students toward autonomy, which encourages students to take responsibility for their classmates' learning of mathematics and their own. Those strategies need to include the idea that classroom environments are complicated settings where students negotiate with their peers to find common and correct mathematical understandings (Cobb & Yackel, 1996). A better understanding of best practices must be explored and developed to enhance these instructional techniques with the blended learning approach, pedagogy, and innovation. Without rigorous studies, it is difficult, if not impossible, for K-12 educators and educational policy developers to create and implement research-based online instructional practices that promote productive discourse in K-12. Research regarding the effects of online learning is quite limited, undoubtedly because the phenomenon is still developing in large-scale applications, and observing the outcomes has its challenges. Song and Kapur (2017) stated that additional research needed to be conducted on the pedagogical methods within the blended learning classroom, and Fisher, Prenyi, and Birdthistle (2018) asserted due to the newness of the blended learning classroom, more research needed to be conducted to determine its overall effectiveness on all aspects of the educational setting. Blended learning is not a new notion;

however, it has become a more prominent concept in the 21st century and deserves more attention and adequate investment from the institution, the teacher, and the student.

There is extensive debate about the correct name for blended learning (often been referred to as hybrid learning or technology-based learning in literature), and that debate often includes the educational benefits of blended learning in a secondary educational environment. Means et al. (2014) predicted that blended learning would become standard practice in future K-12 classrooms. The result from their 2011-12 school year study of blended learning raises some doubts as its findings were mixed. Ultimately, more research needs to take place within specific K-12 classrooms so conclusive evidence can be collected about the benefits, or lack thereof, of blended learning. This qualitative research addressed this problem by exploring the discourse that evolved naturally while students collaborated on mathematical tasks in a blended learning environment.

Significance of the Study

This study is significant because the results show that regardless of the learning environment's modality, productive student-to-student discourse can occur. This study informs mathematics education by discovering the amount of productive discourse within a blended learning environment. The results of this research project allow for practitioners to understand that productive discourse can occur within a mathematics classroom, regardless of the type of learning environment in which the students are placed.

Research Questions

The following research question emanated from the research problem:

- How does the student's learning environment impact their mathematical discourse when completing tasks in an online synchronous work environment?

The following questions guided the research:

- How does the student's physical environment affect their level of productive discourse when placed in a virtual group?
- In what ways do the dynamics of the group change because of a blended learning environment?

Definition of Terms

The following terms are defined in the context of this study:

Asynchronous learning: An approach to teaching through online learning resources to exchange information. This sharing of information occurs outside the constraints of time and place among a group of people (Bonk & Graham, 2006).

Blended learning: Learning environments that "combine face-to-face instruction with computer-mediated instruction" (Graham, 2006, p. 41). Additionally, blended learning is commonly known by different terms, such as remote learning, flexible learning, and hybrid learning (Kim, 2013).

Blended learning environments: Learning environments that consist of a mixture of face-to-face instruction and computer-mediated instruction (Bonk & Graham, 2006).

Classroom mathematical discourse: The ways of representing, thinking, talking, agreeing, and disagreeing about mathematical ideas (NCTM, 2000, p. 46); concerns both the process and content of communicating mathematical concepts in a classroom setting (Sherin, 2002).

Collaboration: Collaboration is considered to be how students may work together with the material and help each other understand or share ideas (Garrison & Vaughan, 2013).

Face-to-face learning: Brick-and-mortar locations that take the students away from their homes and bring them together in a predetermined location, typically on campus." The face-to-face classroom environment allows for a real-time meeting of all the students at once with the instructor" (Osguthorpe & Graham, 2003, p. 229).

Group dynamics: "The actions, processes, and changes that occur within groups and between groups" (Forsyth, 2018, p. 2).

Productive mathematical discourse: "Goal-directed discursive exchanges about mathematical objects, relations, and dynamics of relations, including questioning, affirming, reasoning, justifying, and generalizing" (Powell & Alqahtani, 2015, p. 255).

Subgroup: A group of three or four randomly selected students will take the same course within the same class period.

Synchronous learning: "A learning event in which a group of students is engaged in learning simultaneously, either in person or online" (Bower et al., 2015, p. 1).

Traditional instruction methods: Traditional instruction focuses on face-to-face and teacher-centered instruction in the classroom involving teacher-led discussion, teacher imparting knowledge to students, note-taking, and teacher-led activities. Materials used in a traditional classroom include textbooks, lectures, and low-level assignments (Staker & Hom, 2012).

Virtual learning: learning that is "used for remote education through the World Wide Web" (Bri, et al., 2009, p. 33).

Limitations

Limitations to research studies are impossible to avoid; researchers understand this and report study weaknesses as limitations (Leedy & Ormrod, 2010). There were limitations to the generalizability of this study. One limitation was the sample size of this investigation. This study was conducted in one suburban high school and results may not be generalized to another high school in a rural or urban setting. The classroom where the study took place was an advanced course, with students who were top of their class. Another limitation was that the teacher/researcher was the only one who listened to the groups and coded them. This limitation was due primarily to time, or lack thereof. With the stress of Covid-19 effecting every aspect of education, there were no teachers willing to give up their time to listen to 110 recordings and code them. It was proposed that graduate students could listen and code, but that idea was rejected by the administration at the high school in which the study took place, based on the iznitial promise of student anonymity. This study did not incorporate tasks implemented

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throughout the entire school year. Due to the fact the permission to conduct research form was not signed by the proper personnel until months after the 2020-2021 school year started, the researcher had to wait to start her research. Future studies would benefit from recording one task from each unit, starting with unit 1 at the beginning of the school term and ending with the last and final unit of the curriculum.

Conclusion

This study was organized in the following way: Chapter 1 provided the introduction of the study, the statement of the problem, the research questions, the significance of the study, the definition of terms, and the limitations. Chapter 2 reviews the current studies available about student-to-student discourse and blended learning environments. This chapter also goes into depth about the variables of discourse and student-student interaction and indicators of quality learning experiences within a blended learning setting. Chapter 3 provides rationale for use of the selected methodology in this study and elaborates on the components of the methodology including research design, sources of data, data collection procedures, and data analysis. Chapter 4 reveals the results of data analysis and findings. Chapter 5 reviews and further interprets the findings of the study then presents the practical implications of the study, recommendations and conclusion of the study.

CHAPTER II: LITERATURE REVIEW

Educators have seen technology progress firsthand, from a few computers available in the faculty lounge to one computer assigned to each classroom, mobile labs for student use, and finally, to one-to-one devices for every student on campus (Bakir, 2016; Cook & Sonnenberg, 2014). Technology, which is now available at students' fingertips, has opened the doors for blended learning to become a convenient and valuable instructional strategy. According to Staker (2011), approximately 45,000 students took blended learning classes in 2000, but over 4 million students participated in blended learning classes in 2010. This rapid growth increased exponentially during the Spring of 2020 when the world faced COVID-19. Overnight, school districts across the world were thrust into the world of online and distance learning. Teachers only had a few weeks to transform their classrooms into an online learning environment through the use of various digital mediums. Although it is widely believed that blended learning courses will become the new norm for secondary education (Dzuiban, et al,, 2018), to start the 2020-2021 school year, teachers were expected to step into a widely unknown virtual environment and create curriculum and activities conducive to high levels of student achievement and productive student-to-student student discourse (Johnson & Marsh, 2014; Lee & Martin, 2017; Lohnes, 2017).

Evolution of Standards

Twenty years ago, the publication of Principles and Standards for School Mathematics (NCTM, 2000) identified communication as a critical component in attaining mathematical knowledge, and since then, mathematical discourse has been an essential topic within literature. In 2014, NCTM asserted, "Learners should have experiences that enable them to

construct knowledge socially; through discourse, activity, and interaction" (p. 9).

Previously, teachers and parents learned mathematics by memorizing facts, formulas, procedures and repeatedly practicing skills. This way of learning has changed considerably, and according to NCTM (2014), effective math teachers facilitate discourse among students. This discourse builds a shared understanding of mathematical ideas and allows students to analyze and compare ideas. "Effective mathematics teaching engages students in discourse to advance the mathematical learning of the whole class. Mathematical discourse includes the purposeful exchange of ideas through classroom discussion, and through other forms of verbal, visual, and written communication" (NCTM, 2014, p. 29).

In 1980, NCTM began an initiative to move students beyond procedural fluency towards conceptual understanding with a greater focus on the skills required to solve problems. This movement sparked the establishment of the Commission on Standards for School Mathematics in 1986 and the subsequent development of mathematical standards for K–12 learners and teachers (Research Advisory Committee of the National Council of Teachers of Mathematics, 1988), including the Curriculum and Evaluation Standards (NCTM, 1989), Professional Teaching Standards (NCTM, 1991), and Assessment Standards (NCTM, 1995). In 2000, NCTM's Principles and Standards of School Mathematics incorporated these three separate sets of standards into one cohesive collection, emphasizing the need for well-prepared teachers and learners of mathematics in the 21stcentury. This updated vision of K–12 mathematics included a set of five skills and practices that "highlight ways of acquiring and using content knowledge" (NCTM, 2000, p. 29), known as the process standards: Problem-Solving, Reasoning and Proof, Connections, Communication, and Representations.

Twenty years ago, Trends in International Mathematics and Science Study (TIMSS) described mathematics instruction in the United States as teacher-driven, procedural, and inadequate in engaging students in reasoning and explaining (Mullis, Martin, & Gonzalez, 2000). In response to this claim and in an attempt to improve student proficiency, standards-based

educational reform, specifically, the Common Core State Standards (CCSS), has required

teachers to reexamine both what and how they teach. Since improving mathematics education

cannot be achieved entirely by introducing a new set of content standards, the CCCS also

addressed critical mathematical processes in which students engage, identified as the eight

Standards for Mathematical Practice (SMP).

Education standards at the individual state level have been around since the early

1990s. By the early 2000s, all 50 states had created and adopted their own learning standards

that specify what students in grades 3-12 school should be able to accomplish. Unfortunately, at

the individual state level, proficiency was defined in multiple ways. Ultimately, proficiency is the

level at which a student is determined to be sufficiently educated in order to pass on to the next

grade level and ultimately graduate. This lack of standardization was one reason why multiple

states decided to develop the Common Core State Standards in 2009. The development of the

CCSS is often viewed as a success story of meaningful, state-led educational change. Two

years after this process started, in 2011, states were allowed to review and even ratify the

adoption of the CCSS, ultimately using it as a replacement for their existing state standards. At

the conclusion of 2013, 45 states and four territories adopted the CCSS in both mathematics

and literacy (Achieve, 2015). As of today, there are 41 states and five territories that have fully

implemented the CCSS into grades K-12 ELA/math courses (Achieve, 2020).

Within the mathematics CCSS, the eight Standards for Mathematical Practice (SMP)

were created. The Standards for Mathematical Practice describe varieties of expertise that

mathematics educators at all levels should seek to develop in their students. These practices

rest on important "processes and proficiencies" with longstanding importance in mathematics

education. Out of the eight Standards for Mathematical Practice, SMP3 has become a vehicle

for the other seven practice standards. SMP3 in its entirety is as follows:

> *CCSS.MATH.PRACTICE.MP3*: **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine the domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. (NGA & CCSSO, 2010, pp. 6-7)

This standard requires students to communicate mathematically by constructing viable arguments and analyzing the reasoning of others. Students learn best when given opportunities where they are engaged in talking and making sense of mathematical problems. Evidence indicates that collaboration and high-quality discourse are strongly related to positive educational outcomes (Mercer & Littleton, 2007). When students talk, it informs both teachers and peers about a student's mathematical thinking. It leads to a deeper understanding where students explain and justify their solutions with valid mathematical reasoning. This standard provides insight into the critical role discourse plays in the mathematical classroom.

This literature review will focus on three main components: theoretical framework, blended learning, and discourse. Each element presents the background needed for understanding mathematical discourse in a blended learning environment. First, social constructivism and sociocultural theory are introduced as this study's foundation. Then, research regarding the rise of the different blended learning environments is explored. Lastly, the constructs of productive student discourse are discussed.

Theoretical Framework

In this research, two distinct theoretical perspectives were combined, constructivism and sociocultural theory. Many researchers have suggested that sociocultural and constructivist

approaches are not contradictory but, in fact, complementary. Sfard (2001) stated these two theories represent "functional and structural perspectives on understanding" (p. 8). Sociocultural theory "focuses on the activity of the classroom," while constructivism "on what students take with them from the classroom" (Heibert, et al., 1996, p. 17). Cobb (1994) said that each perspective "tells half of a good story" (p. 17).

This phenomenological qualitative research study is grounded in both the sociocultural and constructivist perspectives, each of which gives importance to the contextual nature of learning and construction of knowledge. The current research examines the nature of discourse that supports mathematical knowledge development and identifies what discourse develops in a blended learning environment. This work hinges on the belief that discourse is necessary for constructing knowledge (Vygotsky, 1978; Wertsch, 1986).

Constructivism

Mathematics has traditionally been viewed from a platonic belief that mathematical objects exist in their specific math world outside of reality, this view places mathematics as absolute truth on an objective pedestal. However, Restivo (1993) theorizes mathematics as a social construction. The foundation for his argument lies in the thought that "all talk is social; the person is a social structure; and the intellect is a social structure" (Restivo, 1993, p. 248). From a constructivist perspective, for students to learn mathematics with understanding, they must construct, "build on, and revise their current mental structures through the processes of action, reflection, and abstraction" (Battista, 2011, p. 526-527). According to Milbrant, Felts, Richards, and Abghari (2004), the constructive approach to education is more effective when students are in control of their own learning. Korucu & Cakir (2018) report teachers need to stop directing their students and start facilitating learning through designing interactive lessons, incorporating collaboration between students, and developing activities that encourage critical thinking and creativity.

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The late psychologist Lev Vygotsky first studied and recognized the power of collaboration, and his ideas are currently still taught in child development courses. Vygotsky's beliefs about building understanding through the process of collaboration, also referred to as constructivism, are foundational to current conclusions about how students use discourse as a means to produce knowledge (Wardle, 2009). Developmental psychologist Jean Piaget had similar beliefs about student interactions during learning. Piaget had a strong focus on children's interactions with the physical world, but he also believed social interactions played a crucial role in learning (Mercer, 2002). Piaget also asserted the attainment of new information was constructivist; he believed individuals could gain new knowledge through communication with others, which, at times, results in an individual having conflicting viewpoints. The student must then work through this knowledge gained socially on an individual level to make sense of it (Mercer, 2002).

Acquiring new knowledge to be either assimilated or accommodated is gained through discourse with others; when discourse occurs, the collaborative process of constructing knowledge is happening synchronously (Pfister, 2005). The Swiss psychologist Piaget identified the differences between assimilated or accommodated knowledge in multiple papers. Piaget asserted that knowledge that fits comfortably into our understanding of the world is assimilated while new, opposing ideas cause us to accommodate them by finding ways they can acclimate into our views. While engaging in academic discourse is a viable way for children to construct knowledge, rarely is it explicitly taught (Monaghan, 2005). According to multiple researchers (Ward, 2020; Korucu & Cakir, 2018; Lee & Martin, 2017; Moore et al., 2017), the most effective strategies within a blended learning environment were those that incorporated interactive communication and active and engaging activities.

The concept of constructivism is in sharp contrast to the view that many teachers have towards teaching and learning mathematics. Students do not merely absorb mathematical constructs (Saram & Clements, 2009). Teaching must rely on more than simply being the

transmission of facts, skills, and concepts. Constructivist learning is where students are guided to think and work collaboratively and to question and problem solve through active evaluation. According to Clements and Battista (2009), five substantial ideas characterize constructivist mathematics: First, children actively create, not passively absorb, knowledge from the learning environment. Second, experiences and interactions shape people's interpretations of the world around them. Learning mathematics is a process of adapting to one's world. Third, learning is entirely a social process. Mathematical understanding is at its foundation cooperative. Fourth, children create new mathematical knowledge by reflecting on actions and integrating existing knowledge. Lastly, sensemaking, and not rote learning, is a requirement. If the teacher requires students to use a process, students must have an activity or task to make sense of the method.

Sociocultural Theory

Although Vygotsky's Sociocultural Theory was developed in the early 1900s, it is still relevant to education today. Vygotsky believed children learn how to think and interpret the world around them through personal, social experiences (Wardle, 2009). The Soviet Psychologist believed learning occurs within a context, which is why social interactions are necessary to make sense of new knowledge. He did not feel knowledge is constructed independently but continuously within the context of collaboration. Vygotsky's ideas reinforce the connection between problem-solving and discourse. Vygotsky's beliefs about learning within a context and how children utilize language to do so mirror current research in the field of education. Vygotsky's "zone of proximal development" described the distance between the actual development level and the level of potential development with adult guidance or in collaboration with more capable peers, which provided a way of understanding a child's cognitive capacity.

Zone of Proximal Development. Discourse has been shown to help students make meaning of mathematics, construct mathematical objects and develop a more in-depth understanding. For discourse to be productive, all members of the discourse community must be able to make

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meaning of the communication. Evidence indicates that collaboration and high-quality discourse are strongly related to positive educational outcomes (Mercer & Littleton, 2007). This section looks at how discourse can help construct mathematical knowledge by creating a zone of proximal development (ZPD).

The zone of proximal development refers to the gap between the level of learning a student could achieve unaided and the learning that can be achieved with a teacher or peer assistance (Murphy, Scantlebury, & Milne, 2015). There has been debate regarding how Vygotsky interpreted the zone of proximal development as he provided very little analysis before his untimely death. Vygotsky described the zone of proximal development as a "discrepancy between a child's actual mental age and the level he/she reaches in solving problems with assistance" (Vygotsky, 1934, p.187). For this research, the zone of proximal development refers to the gap between what students could accomplish on their own and the knowledge potentially obtained with their classmates' assistance.

Vygotsky's zone of proximal development has been interpreted in numerous ways. Researchers refer to ZPD as a "gap of knowledge," a maturity level of learning, interactions between students and teachers, and it has even been described as a tool for justifying instructional strategies used in the classroom (Ibrahim & Widodo, 2020). Before his death, Vygotsky could not provide a thorough analysis of the zone of proximal development with studies to support this concept. Therefore, the zone of proximal development has left a platform for researchers to explore, interpret, and expand.

The ZPD is not a physical space, but a metaphorical space created through social interaction. The ZPD is a theoretical construct that can be used as a framework for analyzing learning. Goos (2014) interprets the ZPD as taking three forms, but in this study, we only focused on the second:

(2) The ZPD as *collaboration*: This manifestation of the ZPD follows from the realization that the ZPD can occur through collaboration between students of equal expertise.

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Collaborating with peers gives learners the chance to construct and test new understandings and contribute to each other's knowledge. However, not all peer interactions result in the construction of a ZPD.

This research focuses on Goos' (2014) second form of ZPD: collaboration. The ZPD leaves open the nature of the collaboration required and the specification of what is learned during the instructional interactions of students (Wood & Wood, 1996). Harré (1983) suggests that thought occurs not just as a manifestation of speech, as Vygotsky proposes, but also as a social location (collective or individual). As a manifestation of speech, thought occurs publicly as external speech or privately as inner speech, but Harré views thought as also arising through social interaction, either collectively or individually. Whenever a student is immersed in and engaging with others, group culture is developing, as are language and other ways of communicating and understanding the world together.

Donato and McCormick (1994) state that "sociocultural theory maintains that social interaction and cultural institutions, such as schools and classrooms, have important roles to play in an individual's cognitive growth and development." n the classroom setting, the social activities can take the form of books, textbooks, dialogues, interactions, instruction, and questions. This pedagogical implication of the sociocultural theory is closely consistent with Vygotsky's main area of research where the interaction between a child and other people happens through the dialogue in different discourse forms (Bodrova and Leong, 1998).

Technology Integration in Education

Technology is influencing the way people live and communicate and is one of the greatest influences in the world today (Davidson, Richardson, & Jones, 2014). Young people use technology for many avenues, including education, entertainment, and social media (Delen & Bulut, 2011). Technology in society, as well as education, has undergone vast changes during its history and continues in a state of ongoing change (Daugherty, Klenke, & Neden, 2008). The authors further discussed that necessity is one of the key driving forces behind technological

change, especially in the modern age of education. The first integrations of technology into education were meant to connect humans and create distance learning (Feenberg & Jandric, 2015). Feenberg and Jandric (2015) also found technology is the solution for transforming education in the future.

Technology advancements in education have increased significantly over the past century, but more so in the past few decades. Technology is changing and being developed at a rapid pace, which has a considerable effect on all work fields, especially education (Ozdamli & Asiksoy, 2016). Technologies including videos, radio, instructional television, and computers have been pushed on teachers to make them better and faster when educating the youth of today (Cuban & Jandric, 2015). Since technology is constantly changing and advancing, it is important for teachers to keep up with the trends, use technology wisely, and implement it to enhance their curricula.

Education can help level the playing field for all students regardless of their background, and Shing and Yuan (2017) said educational technology must do the same. The education system should allow the same access to available technology for all students as well as the training they need to lead a successful life (Shing & Yuan, 2017). The constant curriculum change also influences the way technology is integrated into the education system. When making a change in technology in education, it must be centered on the curriculum, classroom structure, and available space (Daugherty, Klenke, & Neden, 2008). When a new curriculum is being planned, technology must be part of the plan. Although today's modern world demands technology be available in every classroom for all students, it can become a financial burden for districts.

Students are influenced by the technology of today's world; therefore, teachers should try to incorporate as much technology as necessary to balance the interest of their students with their implementation of learning objectives. Students today are accustomed to technology because they have grown up using it (Davidson et al., 2014). Chuang (2013) found teachers

need to use technology to enhance their instruction because technology is driving today's learner. In many cases, students are more skilled at technology than the teachers themselves. As a result, teachers can use student technology experts to help integrate technology into a classroom, which also increases student collaboration (Henderson & Honan, 2008). Regardless of how teachers plan to use technology, it is important they find balance when integrating it. Each time a new technology idea occurs, it tends to create a wave in education, and teachers feel they must try to incorporate all the new ideas (Ertmer & Ottenbreit-Leftwich, 2010). Teachers should drive their technology integration based on student interest and use it to enhance their curricula.

A popular technological advancement over the past few decades, especially at the university level, has been online learning and virtual instructional tools. Online instruction has become more popular in research lately because it allows students opportunities to learn in a variety of settings, collaborate with others, and engage in individualized instruction (Chuang, 2013). Online learning also allows students from all over the world access to post-high school education (Todhunter, 2013). There are many benefits of online learning, including an increase in virtual communication skills, many possibilities for student-centered assistance, adaptable curriculum pacing and scheduling, and timely feedback for all students (Marteney & Bernadowski, 2016). Online learning platforms increase opportunities for students to receive an education from anywhere in the world and allows collaboration among students from a variety of diverse backgrounds.

Technology in Mathematics

Technology advancements over the course of time have touched many subjects but have greatly influenced mathematics curricula and their implementations. Technology in mathematics began in the 1960s and included discovery-based instruction along with logic and algebraic constructs (Daugherty, Reese, & Merrill, 2010). Another technology wave occurred in the early 1990s where the National Council of Teachers of Mathematics (NCTM, 2018)

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recommended java programs, geometry software, computer-based algebra, and a variety of calculators with assorted functions. By allowing students to start using calculators, teachers could place emphasis on the students' ability to problem solve and not worry about basic computation (Daugherty et al., 2010). Advancements in math technology have helped teachers execute instruction. The flipped classroom is an instructional method where teachers can implement technology to promote a deeper understanding from students while allowing teachers time to promote problem-solving, collaboration, and rich discourse in their classrooms (Bergmann & Sams, 2012).

When technology was used for fun, math achievement also showed an increase (Dumais, 2009). Technology integration alone is not enough to improve student achievement; however, it helps with student engagement, which does improve student achievement (Anderson, Crawford, & Griffith, 2017). Mathematics teachers need to embrace technological advances and learn to incorporate them into their classrooms to increase student engagement and achievement. Mathematics can be a difficult subject for many children to master, so teachers should create a fun and engaging learning environment by incorporating technology to help strike a mathematics interest in more students (Daugherty et al., 2010). The use of technology in education can boost, assist, and enhance instruction but cannot replace a teacher (Shing & Yuan, 2017).

Technology in society today, mixed with current curricula, often leaves teachers struggling to find a balance that promotes student engagement. Teachers frequently find creating engaging and fun lessons that engage students in math is a hard task to master (Long, 2013). One way to create an engaging and enjoyable lesson is to implement technology. Incorporating technology in mathematics is not as hard as it used to be because there are now instructional videos, educational games, and eBooks available, as well as the flipped learning model to aid in implementing the curriculum (Long, 2013; Bergmann & Sams, 2012). Technology can be easily incorporated when teachers find the available resources easy to use

(Callaghan et al., 2017). Integrating technology can benefit both teachers and students, and when students complete online math exercises, the teachers are able to track student growth which can initiate conversations between the teacher and the student. These conversations could increase student achievement (Callaghan et al., 2017). Advancements in technology, such as eBooks, allow for students to cognitively engage in the learning, as well as track student progress to help aid the teacher when planning instruction (Hwang & Lai, 2017). The primary problems teachers face with student engagement in their math classrooms can easily be removed with the proper application of technology and flipped learning.

Synchronous Learning

When students are emerged in synchronous online courses, they are able to participate in learning activities that look similar to those found within a traditional learning environment (Hrastinski, 2008; Harris et al., 2009; Simonson, Smaldino, Albright, and Zvacek, 2012). Lectures and all other instructional activities in synchronous online courses occur at a specific time with the expectation that students would log in and participate at that time using the appropriate technology or software (Er, Özden, & Arifoglu, 2009). According to researchers Diaz & Entonado (2009) and Er, Özden, and Arifoglu (2009), online course satisfaction directly correlates to student collaboration, and synchronous learning opportunities help to foster increased collaboration.

Synchronous online courses attempt to increase both student-to-student and student-to-teacher interaction and collaboration to encourage student achievement by providing appropriate, real-time learning activities (Bonk & Zhang, 2006; Martinez-Caro & Campuzono-Bolarin, 2011). West and Jones (2007) claimed, in response to the limited student interaction found in asynchronous online courses, some students are asking for more synchronous opportunities to interact and collaborate with one another. Both "instructors and students experience synchronous online learning as more social, as students can get their questions

answered in real-time" (Hrastinski, 2007, p. 29). Despite its overall learning benefits, synchronous online learning has some disadvantages that have contributed to the limited use of synchronous learning activities that include high technology cost to students and school, limited access to a sufficient internet connection for students, and a lack of flexible attendance options with high potential for scheduling conflicts for both student and faculty (Duemer, et al., 2002).

Asynchronous Learning

Virtual asynchronous learning gives students the opportunity to engage with online content at a time of their choosing, according to their individual schedules (Hrastinski, 2007; Hawkes, 2006). Asynchronous online discussion forums are online spaces for students to have discussions by posting messages to an online discussion board and are one of the most commonly used means for promoting collaborative knowledge construction (Schrire, 2006). According to a study done by Vaughan and Garrison (2005), online discussion forums can be constructive for "sustaining the discussion momentum and expanding the scope of students' dialogue" (p. 8). However, participation in discussion forums as such does not guarantee co-construction of knowledge and does not necessarily result in higher-order cognitive processing (Admiraal, et al., 1998; Dillenbourg, 2002; Lockhorst, et al., 2002). A learner needs to actively contribute to the discourse of meaning creation in order to gain an enriched understanding of an issue and acquire knowledge (Higham, Brindley, & Van de Pol, 2014). Gerosa et al. (2010) found that when appropriately applied, asynchronous online discussion promotes dialogue between students, reflection on learned material, construction of knowledge, and self-assessment.

DeWert, Babinski, and Jones (2006) report that asynchronous discussion that includes sharing of thoughts, question-asking, and feedback, is the main way to encourage interaction and community building in the online learning environment. Researchers Collison, et al. (2000) also point out that asynchronous discussions allow students more time to reflect on the

contributions of their peers as well as themselves since they can look back at the discussion threads at any time throughout the course.

Researchers Bocchi, Eastman, and Swift (2004), Hiltz and Shea (2005), and Hirschheim (2005) point out that the most frequent reason students choose the asynchronous online format is because of its flexibility or convenience. Kock, Verville, and Gaza (2007) assert that students may often state that they prefer the asynchronous online format because they would like to avoid the problems that face-to-face courses can have, like traffic and work schedules. Daymont and Blau (2008) emphasize that the anytime, anywhere nature of asynchronous learning allows students the opportunity to choose the times when they can engage in learning rather than being locked down to certain times during the day or week.

Blended Learning

After a few years of experience with both synchronous and asynchronous online education, some institutions felt that elements of the traditional face-to-face model should be combined with new technology to better the learning experience for their students (Friesen, 2012). This combination of asynchronous and synchronous online course elements was described as *blended learning* and began in 1999, and around the same time, this mixed mode type of learning also began to be referred to as *hybrid learning* (2012). Snart (2010) makes a distinction between blended learning and hybrid learning by pointing out that the term "blended" is frequently used to describe a traditional face to face course with online components, while "hybrid" is most often used to describe an online course with face-to-face components.

McGee and Reis (2012) define blended learning as a mix of delivery modalities, including face-to-face and technology-driven instruction, to accomplish learning outcomes in a manner meaningful to the learner. Blended learning "emphasizes the search for best practices by identifying the optimum mix of course delivery to provide the most effective learning experience" (Sharma, 2010, p. 6). Combining multiple instructional techniques empowers students to take ownership and be accountable for their learning while becoming more

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motivated about their work (D'addato & Miller, 2016). In an online learning platform, students are required to express themselves and demonstrate an understanding of their learning both verbally and in writing, which is consistent with a high-performing environment that is student-centered, engaging, and community-oriented (West-Burns & Murray, 2016). Blended learning courses provide the combined format and structure of a classroom as well as the flexibility of online instruction while offering the potential for personalized learning opportunities (Patrick, Kennedy, & Powell, 2013).

Students in traditional classrooms where lecture is the primary mode of instruction lack the ability to interact with the material in a meaningful way (Inaki, Anton, & Prada, 2015; Sousa, 2011; Sprenger, 2010). Blended learning, according to Horn and Staker (2015), is a student-centered pedagogical approach where students learn partially through online learning and partially through in-person teaching with some choice over "time, place, path, and/or pace" of the material (p. 35). When students are learning at their own pace, there is less pressure to keep up with the group (Saltan, 2017). Blended learning allows for individualized learning for all students and permits teachers to meet each student's unique learning needs (Powell et al., 2015). Additionally, this flexible learning environment shifts the role of the teacher into the classroom facilitator, allowing students to take on the bulk of the responsibility of learning (D'addato & Miller, 2016).

Pros and Cons of Blended Learning

On the surface, blended learning appears to be a sustainable enhancement to traditional education. However, some shortcomings prevent its effectiveness. According to Education Week (2015), common obstacles facing the implementation of blended learning practices include the lack of internet access at students' homes and constant concerns raised by teachers regarding their ability to be effective with this new instructional technique. Gilakjani (2013) echoed this sentiment and stated that 55% of the teachers she surveyed for her research felt that if "they did not have confidence in using computers, they would be more inclined not to

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integrate technology into their classrooms" (p. 18). Due to the infancy of blended learning within secondary education, many districts across America are still in the process of developing their blended learning curriculum and often leave the creation of in-class activities up to the teachers' discretion (Ward, 2020). Research states that professional development to train teachers on the pedagogical methods of implementing a blended learning classroom is critical to teacher confidence to develop effective learning activities for student success (Cuhadar, 2018; Nemescu & Petrovici, 2015).

One major deterrent to secondary schools implementing blended learning as a primary learning environment to their students is the effect it has on the retention and drop-out rates. The physical separation seems to reduce students' sense of community. It makes them feel disconnected and isolated, and it lacks personal attention. In traditional schools, higher retention rates have been reported to be related to students' sense of belonging, ownership of their progress, and engagement. Moreover, participating by maintaining relations with peers has been argued to increase student motivation (Mabrito 2006). Similarly, Wegerif (1998) reported that students who did not collaborate and engage in social interaction felt disassociated with the course.

Much of the research on blended learning involves post-secondary institutions with little focus on the secondary classroom, specifically the secondary mathematics classroom and the student-to-student discourse, which could potentially occur. Rarely have studies examined secondary classrooms, and the few that did found that properly implemented blended learning environments positively affect students, namely their self-efficacy and academic performance (Cheung, 2018; Kazu & Demirkolb, 2014; Yapici & Akbayin, 2012). Carlson et al. (2017) discovered that even though blended learning could increase student performance outcomes in secondary mathematics, the majority of teachers were not comfortable implementing this teaching strategy within their own classrooms.

Blended learning environments are growing at such a rapid rate, thereby requiring institutions to aggressively establish procedures to develop effective pedagogy methods for their blended learning courses (Staker, 2011; Hilliard, 2015). The growth is attributed to the increased access and flexibility for scheduling, real-world experiences for students, and most recently, to the COVID-19 pandemic (California Office of Education, 2020).

Blended Learning as an Alternative to the Traditional Classroom

During the 2009-2010 school year, enrollment in K-12 distance learning courses was estimated at being over 1,800,000, of which almost all are online learning courses. Over 60% of this enrollment was comprised of high school credit recovery students (INACOL, 2013). This is a new form of teaching and learning that high school teachers encounter when transitioning their teaching to an online/blended format.

One of the biggest hurdles that students face when transitioning from a traditional classroom to a blended learning environment is their autonomy. Shea and Bidjerano (2010) have found in their research that student self-regulation is essential for student success in virtual courses. Self-regulation includes managing time and setting goals (Shea & Bidjerano, 2010). Shea and Bidjerano (2010) also state that for students to have self-regulation, they must have a strong self-efficacy, and a personal belief they can succeed. In this way, students have a 'learning presence'.

A virtual setting may be a better environment for some students to learn rather than a traditional face-to-face class. When students take online courses, they do not have distractions from other students as they do in traditional instruction (Oliver & Kellogg, 2015). The student in Barbour and Siko's (2012) case study engaged well during synchronous virtual class time. However, students may have challenges such as motivation and sufficient access to technology at home (Barbour & Siko, 2012; Oliver & Kellogg, 2015).

The Role of Social Interaction in Blended Learning

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One essential facet of blended learning that continues to be overlooked by schools is social interaction. Whether students are interacting with their teachers or with each other, these encounters offer an "outlet for them to express their opinions and a way to recognize their potential" (Zilka, Cohen, & Rahimi, 2018, p. 12). Asynchronous and synchronous communication is characterized by different discourse features and may thus be used for different pedagogical purposes (Romiszowski & Mason, 2004). Sparks (2019) argues that relationships are a fundamental part of a students' learning experience. Two additional factors closely associated with students' social interaction are teacher presence and social presence. Teacher presence refers to a teacher's personality, ability to facilitate social interactions within the class and teaching style. Social presence refers to a teacher's ability to create a collaborative learning community online. Shea and Bidjerano (2009) describe social presence as involving instructors and students feeling comfortable with communicating with each other online. Social presence provides the foundation for students to feel comfortable interacting. Teaching presence is provided when instructors are actively involved with discussions and incorporating scaffolding questions (Shea & Bidjerano, 2010). This presence may be integrated into a high school online learning course in a multitude of ways. Students benefit from social presence both in their virtual and physical environments. Online instructors and facilitators need to find and incorporate ways to make their virtual courses a comfortable social environment for students.

Teacher presence is the most important of these two components as it shapes student discourse and social processes (Zilka et al., 2018). Although many supporters of virtual learning cite the fact that online learning is flexible, economical, and convenient, cynics maintain that attending school online is at best a weak substitute for actual in-person exchanges between instructors and peers inside a classroom environment (Asif, 2013). One study found that when learner-to-learner engagement or interaction is present, the blended learning experience of students will improve and ultimately increase their satisfaction and achievement in the course

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(Kurucay & Inan, 2017). Similarly, Chen and Wang (2009) found in their study that student coordination and social discussions positively impact course content learning. According to Picciano (2002), "both students and faculty typically report increased satisfaction in online courses depending on the quality and quantity of interaction" (p. 22). Most high school students experience online learning within a mixture of their online and physical space. They tend to engage in online courses in brick-and-mortar high schools. In de la Varre, Keane, & Irvin's (2010) study, onsite facilitators were present in the classrooms while students participated in courses with online instructors. This process allows educators to engage the students socially (de la Varre, et al., 2010).

Group Dynamics

Group dynamics is roughly a century old. Although scholars have long contemplated the nature of groups, the first scientific studies of groups were not carried out until the 1900s. In one of the first ever written reviews on the origins of group dynamics, Cartwright and Zander (1968) wrote that the slow developing history stemmed in part from several unfounded assumptions about groups. Many in the scientific community felt that the dynamics of groups was a private affair, and ultimately should not be laid out for public scrutiny. Other scientists felt that human behavior was too complex to be studied and that this complexity was magnified when groups of interacting individuals became the objects of interest.

In the late 1800's social sciences started to emerge as their own unique disciplines, and the dynamics of groups became an immediate topic of concern for each. In 1895 social theorist Gustave Le Bon, publisher of *Psychology of Crowds,* claimed that individuals are transformed when they join a group (Forsyth, 2018). Then psychologist Wilhelm Wundt (1916) started to study groups extensively and published his book *Volkerpsychologie*, which is loosely translated to "group psychology" (Alderfer, 1983). These two primary works laid the groundwork for the scientific study of groups. Lewin, who many have argued is the founder of the movement to study groups experimentally, chose the word dynamic to describe the activities, processes,

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operations, and changes that transpire in groups. This word suggests that groups have a profound impact on individuals; they shape actions, thoughts, and feelings (Lewin, 1951). Groups can also change their members by prompting them to change their attitudes and values as they come to agree with the overall consensus of the group (Newcomb, 1943).

Once scientists and researchers started to explore groups at multiple levels, they immediately disagreed on which level of analysis to take; group-level or individual-level. Sociological researchers tended to prefer group-level analysis while psychological researchers preferred the individual-level (Forsyth, 2018). Psychologist Floyd Allpart (1924) chose the individual in the group as the unit of analysis when he wrote, "the actions of all are nothing more than the sum of the actions of each taken separately" (p. 5). Groups that undertake extreme actions under the encouragement of charismatic leaders fascinate both the public and researchers alike (Jahoda, 2007) and studies are continuously being executed to determine why and how one person can create an impact on its group.

Group dynamics consist of elements that ensures togetherness in an already formed group which ensures smooth and coherent functioning of a group to accomplish a goal or task. In social context, group dynamics have been a complex phenomenon which is defined by the interpersonal relationships that develops in a small group set to accomplish a task (Forsyth, 2018). A recent study of reviews from the literature on group dynamics in various educational settings have identified five key group dynamic elements that support effective small group learning in educational settings; support, openness, engagement, style of dominant behavior, and quality of communication (Merlin, et al., 2020).

Discourse and Mathematical Learning

In 1991, the National Council of Teachers of Mathematics (NCTM) published Professional Standards for Teaching Mathematics that identified discourse as a key feature in three of the six standard areas. This included *Standard 2: The Teachers' Role in Discourse*, *Standard 3: Students' Role in Discourse*, and *Standard 4: Tools for Enhancing Discourse*. This

brought new interest to research on discourse-based teaching as the research community took notice that these standards, while outwardly practical, lacked grounding in scientific evidence. Accordingly, the number of studies examining discourse has increased dramatically since the early 1990s.

The quality of discourse called for in the NCTM teaching standards is substantially different than traditional teacher-led questioning. According to NCTM (1991) guidelines:

> Students should engage in making conjectures, proposing approaches and solutions to problems, and arguing about the validity of particular claims. They should learn to verify, revise, and discard claims on the basis of mathematical evidence and use a variety of mathematical tools. Whether working in small or large groups, they should be the audience for one another's comments; that is, they should speak to one another, aiming to convince or to question their peers. Above all, the discourse should be focused on making sense of mathematical ideas, on using mathematical ideas sensibly in setting up and solving problems. (p. 45)

Mathematical classroom discourse includes the persistent exchange of ideas through classroom discussion and through other forms of communication (NCTM, 2014). The word discourse is complex and multi-faceted, therefore the phrase *mathematical discourse* is also complex and used in multiple ways. Mathematical discourses are different from other discourses in relation to the words, visual mediators, narratives, and routines (Sfard, 2008) that are used. Mathematical classroom discourse is compiled by not only mathematical conversations but also by more generic classroom discourses, for instance, *nonmathematical discourses* (Moschkovich, 2002; Setati, 2005) *social norms* (Yackel, Cobb, & Wood, 1991) and *everyday discourse* (Evans, 2006). Researchers have argued that nonmathematical discourse is not always irrelevant for understanding mathematical concepts, and meaningful math discussions can be created within almost any type of discourse.

Mathematics-talk communities are places where students can have meaningful math discussions to construct knowledge and support the learning of others in the group. Throughout the cycle of discourse, several types and levels of discourse occur (Weaver et al., 2005). Answering a question, for example, is Level 1 discourse, whereas relating one's thinking to

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another's Level 6 discourse (Table 1). Answering, making a statement or sharing, explaining, questioning, challenging, relating, predicting or conjecturing, justifying, and generalizing represent the different levels of mathematical discourse that can occur throughout the cycle of discourse (Weaver et al., 2005, p. 2–3).

Table 1

Levels of Mathematical Discourse

Level	Definition	Explanation
1	Answer	A student gives a short answer to a direct question from an instructor or peer.
2	Making a Statement or Sharing	A student makes a simple statement or assertion without an explanation of how or why.
3	Explaining	A student explains a mathematical idea or procedure by stating a description of what they did. Still, the explanation does not provide explicit justification of the validity of the procedure or idea.
4	Questioning	A student asks a question to clarify his understanding of a mathematical idea or procedure.
5	Challenging	A student makes a statement or asks a question in a way that challenges the validity of a mathematical idea or procedure.
6	Relating	A student makes a statement indicating he or she has made a connection or sees a relationship to some prior knowledge or personal experience.
7	Predicting or Conjecturing	A student makes a prediction or conjecture based on their understanding of the mathematics behind the problem.
8	Justifying	A student justifies the validity of a mathematical idea or procedure by explaining the thinking that led him to the concept or procedure.
9	Generalizing	A student makes a statement that is evidence of a shift from a specific problem to a general case.

Adapted from "Oregon Mathematics Leadership Institute's classroom observation protocol" by D. Weaver, T. Dick, K. Higgins, K. Morromgelle, L. Foreman, N. Miller, 2005 *RMC Researcher Corporation,* p. 1-16.

Here, discourse is conceived broadly as a means through which students can gain insight into how a procedure works, pose questions to peers and compare their own

perspectives to those of others. In other words, discourse is a vehicle for constructing knowledge; by using concepts interactively, the concepts themselves become clearer and more defined through the practice of relevant language. A classroom atmosphere that supports collaborative inquiry among students can also help students to bridge the difficulty of applying concepts to new problems and situations. Thus, developing students' interest has to be a main educational objective for schools as well as individual teachers. Interested learners develop more differentiated domain-specific knowledge (Renninger, et al., 2011), are more focused and have better attention (Ainley, Hidi, & Berndorff, 2002), and pursue mastery rather than performance goals (Harackiewicz, et al., 2008). As students get more comfortable within the mathematics environment, they become more willing to conjecture and try out potential strategies without fear of being wrong, and eventually begin to participate in a process of mathematical discovery. This experience, in turn, enables students to understand mathematics as a field of exploration and discovery rather than rote memorization and right answers. Ultimately, if teachers can actively engage their students in productive classroom discourse, they are likely to engage them in more meaningful and sustained learning experiences (Walshaw & Anthony, 2008).

Successfully Engaging in Academic Discourse

Researchers believe there are several reasons why students engaging in academic discourse is beneficial to learning. Several components go into lesson planning for this type of discourse, such as learning outcomes, explicit teaching, the complexity of the task, lesson structure, scaffolding, and community building. Although there is a substantial amount of research pertaining to the need for students to engage collaboratively in meaningful discourse, studies suggest teachers are still doing most of the talking during learning activities (Monaghan, 2005). When this research study was completed, there was no literature present about the extent of meaningful student discourse without the presence of their instructor. In a blended

learning environment, there are many occasions where students can collaborate, without their teacher posing questions or influencing behavior or discourse.

When students experience a sense of community within their classroom environment, they have a tendency towards a heightened level of enjoyment, academic success, along compassion for their peers. Dwyer et al. (2004) stressed that "Fostering a positive climate and sense of community for students in educational settings has been linked with retention and academic success. Students who report feelings of community report greater academic motivation, an affinity for school, empathy to help others, better conflict resolution skills, greater enjoyment of the class, higher self-efficacy, and greater motivation and liking for school" (p. 264-265). In order for meaningful and authentic student participation to transpire within a blended learning environment, whether the students are face-to-face with their teachers or on an online synchronous medium, students must feel as though they are personally connected with their fellow classmates within a collective community.

The current interest in the development of community in online environments has arisen from the conceptual change in thinking with regard to what constitutes learning (Shea & Pickett, 2006). McMillan and Chavis (1986) offered the following definition of sense of community, "a feeling that members have of belonging, a feeling that members matter to one another and to the group, and a shared faith that members' needs will be met through their commitment to be together" (p. 9). According to Sfaard (2008), the perception of student learning has shifted dramatically in the 21st century, and now "learning is conceived as a process of becoming a member of a certain community. This entails, above all, the ability to communicate in the language of this community and act according to its norms" (p. 6). In addition, social skills, such as lack of loneliness and social belongingness, are key factors supporting school engagement and belonging within an educational community (Kankaras & Suarez-Alvarez, 2019). Educators, at all instructional levels, can help promote these social skills and sense of community by encouraging and promoting productive student discourse within the classroom.

Research suggests that a low sense of community amongst students is related to two primary characteristics of attrition: student burnout and feelings of isolation. A number of studies examined individual differences in loneliness and mental health outcomes of teenagers during the COVID-19 pandemic (Foulkes, Blakemore, 2020; Oosterhoff, et al., 2020; Magson, et al., 2021). During the COVID-19 pandemic students' social relationships dramatically reduced, causing high stress among middle and high school students (Styck et al., 2020). Research done by Killgore et al., (2020) found a significant positive association between loneliness, suicidal ideation, and depression amongst young Americans that were forced to attend school virtually. Due to the COVID-19 pandemic, governments and school officials mandated students quarantine and isolate at home, and since then, their sense of community and belonging faded, while their feelings of loneliness increased (Lee, et al., 2020). Bu, Steptoe, and Fancourt, (2020) analyzed that these feelings are based on the fact that school is often the main environment where students' social relationships exist. However, during the school closures and social distancing measures, social relationships reduced dramatically. Schoon, (2021) discovered that socio-emotional competences, such as communication and relationships, if implemented effectively within the educational system, would give students the opportunity to thrive.

For educators to give their students the chance to excel in a blended learning environment is to create an environment where discourse can flow freely. The first step to creating mathematical discourse is to create an open classroom that allows students to express their ideas (Hufferd-Ackles et al., 2004). This development is a crucial first step in achieving a community of learners and discourse in the mathematics classroom. According to Kilic et al., for students having the opportunity to engage on a regular basis with tasks implemented with high levels of cognitive demands translated into substantial learning gains on an instrument specially designed to measure exactly the kind of student learning outcomes advocated by NCTM's professional teaching standards (2010). "For students to learn mathematics with understanding, they must have opportunities to engage on a regular basis with tasks that focus on reasoning

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and problem solving and make possible multiple entry points and varied solution strategies"
(NCTM, 2014). Students' engagement in solving tasks is more strongly connected with their
sense of identity, which is leading to an increase in engagement and motivation in mathematics.
Increased engagement is a result of teachers' efforts to incorporate these elements into
mathematical tasks (Aguirre et al., 2013; Boaler 1997; Hogan 2008; Middleton and Jansen
2011).

Tasks

Stein and Smith define a mathematical task as "a classroom activity, the purpose of
which is to focus students" attention on a particular mathematical idea" (Stein et al., 1996, p.
460). Given this ambiguous definition, it is arguable that mathematical tasks are a part of every
mathematics classroom. Tasks have the potential to influence students" learning by the
opportunities for mathematical thinking that they afford. Over two decades of research (Hiebert
& Wearne, 1993; Stein & Lane, 1996; Stigler & Hiebert, 2004) suggest that the greatest student
learning occurs in classrooms where implementation of high cognitive demand tasks originates
in a fashion that consistently maintains the cognitive demand of the task during instruction.
Hiebert and Wearne (1993) concluded that teaching and learning are related through the
instructional task and the discourse environment in the classroom. They go on to say that these
instructional factors influence the level of cognition that students will engage. Consequently,
what teachers do directly affect what students will learn. Further, Tarr (2008), and previously,
Stein and Lane (1996), have both determined that learning environments in which teachers
encourage multiple solutions, making conjectures and mathematical connections, and
explaining reasoning result in higher student performance.

In order for students to engage in purposeful discourse while teachers facilitate student-
to-student and whole-class discussions, mathematics teachers must implement tasks that
promote reasoning and problem solving (NCTM, 2014). This means that the tasks must be high
in cognitive demand in order to engage students in "solving and discussing tasks that promote

mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies" (NCTM, 2014, p. 10). When students are challenged with problem-solving tasks of this nature, teachers are placing higher-level cognitive demands on the students. Additionally, teachers should choose tasks that have the potential to strengthen student engagement by drawing on students' prior experiences and knowledge (Cross et al., 2012; Kisker et al., 2012) by understanding how contexts, culture, and language affect student engagement. These types of tasks also encourage students to be actively engaged while interacting with classmates and in their learning (NCTM, 2000).

Tasks that are both challenging and high-cognitively demanding are prime opportunities for students to delve into mathematical discussions using various instructional strategies. Students develop a more advanced understanding of mathematical concepts when confronted with appropriate learning tasks, then their understanding or thinking is challenged by their peers, and ultimately when they reflect upon their actions and learning as a result (Battista, 2001). "Mathematical ideas must be personally constructed by students as they intentionally try to make sense of situations, including, of course, communications from others" (Battista, 2001, p. 107). According to Breyfogle and Williams (2008), a useful mathematics task "allows for connections, incorporates multiple approaches and solutions, requires high-level thinking and facilitates reasoning and communication" (p. 277).

Research has shown ample benefits from using cooperative learning in mathematical problem-solving in K-12 mathematics education. It is believed that cooperative group work can promote students' creative thinking, problem-solving, mathematical reasoning, and their social relations (NTCM, 2000). However, this collaborative method may not always be the ideal way of learning and presenting core subject contents (Gillies, 2003; Manouchehri & Goodman, 2000; Whicker, et al., 1997). The design and difficulty level of a task can determine how group members interact from a student's perspective. For example, Stein et al. (2009) observed that when students worked on a high cognitive task and struggled, they persisted in pressuring the

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teacher to provide steps or procedures and eventually the teacher gave in and told the students how to solve the problem. Although the teachers' have good intensions, when they deconstructed the problem, gave hints or scaffolded the task, they reduced or eliminated the opportunity for the students to think and reason, resulting in the loss of meaningful opportunities to develop mathematical understanding.

Hiebert et al. (1997) defined high-level tasks as "problems that are intellectually challenging with opportunities for reflection and communication about important mathematics, and students walk away with valuable concepts" (p. 19). Students are not guaranteed opportunities to engage in high-level thinking and reasoning even when cognitively challenging tasks are selected for instruction. Maintaining the complexity of high-level tasks is a difficult endeavor (Stigler and Hiebert 2004; Weiss et al. 2003), often shaped by teachers' and students' beliefs about how mathematics is best taught and learned (Lloyd and Wilson 1998; Stein et al. 2007). Teachers and students accustomed to traditional, directive styles of teaching and routinized, procedural tasks experience conflict and discomfort with the struggle that often accompanies high-level tasks. In response to ambiguity or uncertainty on how to proceed, students may disengage with the task or press the teacher for step-by-step instructions, and teachers may reduce high-level demands by breaking the task into less-challenging subtasks or by shifting the focus to correct answers or procedures (Arbaugh et al. 2006; Henningsen and Stein 1997).

Recognition of high-cognitive demand tasks requires a teacher to consider how the task provides opportunities for a student to investigate mathematics content in an open way, and to assess how well the task connects with the students' background knowledge, as well as how the task is designed to push the student to think more deeply about the mathematics involved in the task. In their book, Implementing Standards-Based Mathematics Instruction, Stein, Smith, Henningsen, and Silver (2009) describe how teachers classify mathematics tasks as high-level thinking tasks. First, the teachers have to consider several factors in order to determine if the

task is a cognitively demanding task. Second, the teachers have to overlook the surface features of the task and carefully consider the kind of thinking the task requires. Examples of the surface features include tasks that require students to show or explain their thinking, incorporating manipulatives to solve the problem, having multiple solutions or multiple steps, or being set in a real-world scenario. When the instructor selects and oversees tasks with multiple pathways, students are forced not to repeat procedures but instead challenge themselves to reason and think abstractly (Smith & Stein, 2011). Clearly, teachers must know how to look at a task and accurately judge its cognitive demand before using it in their classroom in order to provide an opportunity for their students to engage in the type of learning associated with 21st century skills.

A study conducted by Boston and Smith (2009) concluded that teachers who implemented a high cognitive demand task in the classroom reduced the cognitive demand of the task. They found that teachers who implement high cognitive demand tasks in the classroom will lower the cognitive demand of the task during the instructional episode by providing too much guidance, answering student questions directly, and providing an entry point to the task. Over two decades of research (Hiebert & Wearne, 1993; Stein & Lane, 1996; Stigler & Hiebert, 2004) indicate that the greatest student learning occurs in classrooms where implementation of high cognitive demand tasks occur in a manner that consistently maintains the cognitive demand of the task throughout instruction. Hiebert and Wearne (1993) concluded that teaching and learning are related through the instructional task and the discourse environment in the classroom. Thus, what teachers do directly affect what students will learn, but no studies have been conducted to determine what happens to the cognitive demand of a task once a teacher removes herself from the learning environment in which the task is taking place.

Low-level tasks do not require as much communication because they do not demand as much thinking or processing. Smith and Stein (2011) described low-level tasks as those that

involve either memorization or application of procedures with no connection to meaning or understanding. There is usually only one pathway or one answer to share in these tasks. When attempting to implement tasks, sometimes teachers inadvertently decrease the rigor or level of the task and consequently restrict the classroom discussion. Teachers should first establish a clear and accurate goal for learning before expecting productive mathematical discussions, and then teachers should select a high-level mathematical task to match the learning target (Smith & Stein, 2011).

Recognition of high-cognitive demand tasks requires a teacher to consider how the task provides opportunities for a student to investigate mathematics content in an ambiguous way and to assess how well the task connects with the student's background knowledge, as well as how the task is intended to push the student to think more deeply about the mathematics involved in the task. Stein and Smith (1998) outlined four categories of cognitive demand: memorization, procedures without connections to concepts or meaning, procedures with connections to concepts and meaning, and doing mathematics. The first two categories (i.e., memorization and procedures without connections to concepts or meanings) are considered low-demand tasks, while the second two (i.e., procedures with connections to concepts and meaning and doing mathematics) are considered high-demand tasks. These categories served as the primary lens to analyze artifacts and tasks posed during observation sessions. Clearly, teachers who utilize tasks within their classroom must know how to look at one and accurately assess its' level of cognitive demand in order to provide an opportunity for their students to engage in appropriate discourse and learning.

The Effect of Students' Presence on Engagement

The research on student engagement has not changed significantly since the 1970s. Much of the research is still practical in nature, focusing on online pedagogical or design practice and its relationship with student engagement. Primary concerns in the research on engaging high school students in online learning contexts seem to be the quality of student-

teacher interactions and the interactive design of the learning environment. This assumes

engagement is the result of pedagogical or design choices in the online learning environment

rather than as an experience of learning inherent within the student. Crippen, Archambault, and

Kern (2013) examined the pedagogical practices and perspectives of 35 online science

teachers across 15 U.S. states who use inquiry-based methods in their online laboratory work

with students. For Crippen et al., a "psychology of presence," or the belief that one is embedded

in a learning experience within a particular 'place,' is more critical in producing learning

outcomes than the mode of technology (hands-on, virtual, or remote) used to conduct the

experiment (p. 1035). Their use of the term 'presence' is consistent with the use of the term

'social presence' in the more extensive literature on student engagement (Bangert, 2008;

Fontaine & Chun, 2010; Kanuka & Garrison, 2004; Shea & Bidjerano, 2009).

For example, Fontaine and Chun (2010) describe presence as the psychological and

subjective experience of the student in the virtual environment wherein characteristics of

"realism" and "immersion" create the illusory experience that the student is in one place, even

when situated physically in another (p. 34). Further, the student is simultaneously involved in an

immediate learning situation and is "broadly aware of a range of ecological characteristics"

associated with this learning situation (p. 32). This description of presence can be read as a

substantive description of students' experiences of engagement when learning online. However,

Fontaine and Chun (2010) note that the term presence must be distinguished from "social

presence," which instead refers to the "subjective quality of the communication medium that

reflects its capacity to transmit cues," such as facial expression, reaction of students or

teachers, or other nonverbal cues that are key to effective communication and influential in

facilitating a sense of presence (p. 35). Researchers indicate that teacher presence in face-to-

face sessions lessens psychological distance between them and the learners and leads to

greater learning. This is because there are verbal aspects like giving praise, soliciting for

viewpoints, humor, etc and non-verbal expressions like eye contact, facial expressions, and

gestures, which make teachers to be closer to learners psychologically (Kelley &

Gorham, 2009). Therefore, social presence depends upon the ability of the context of learning

to provide those social cues seen as imperative to the teaching and learning process.

Based on Gladstein's (2008) survey of 107 high school online students, of the common

design elements present: (a) online forums, (b) multimedia content, (c) simulations or interactive

elements, (d) graphics, and (e) collaboration (p. 84), most online students found multimedia

elements engaging, but not necessarily useful; interactive elements both engaging and useful;

graphics less engaging but useful; and online forums and learner to learner collaboration neither

engaging nor helpful (p. 85). Additionally, students considered student-teacher interactions to be

more critical than student-student interactions (p. 85). Gladstein (2008) attributes the teachers'

role in tailoring communication to the students' pursuit of course goals as one reason why

students might have seen their teachers' presence as crucial to their learning experience and

engagement (p. 11-12, p. 85). Overall, this study highlights pedagogical and technological

features, such as students' interactivity with online components and interaction with the teacher

as contributing to engagement within the online learning context.

Learning outcomes

Knight and Mercer (2015) studied how different groups of students engaged with one

another to complete a problem-solving task. Students were asked to seek information through a

familiar means in their study, as prior research on this topic indicates students often struggle

with this undertaking. It was hypothesized collaboration and discourse would assist students in

more efficiently completing this task. The study included three small groups of 11- and 12-year-

olds working through an information-seeking classroom activity (Knight & Mercer, 2015).

Although all students were of similar academic ability, they were not equally successful in

completing the task. The students' success appears to be related to their ability to work together

and engage in dialogue that mediates this collaboration. While the students were of similar

academic ability levels, how they collaborated and engaged in discourse impacted their learning

outcomes (Knight & Mercer, 2015).

CHAPTER III: METHODOLOGY

Purpose

The purpose of this mixed-methods study was to determine if productive mathematical discourse can occur when students are forced to learn in blended learning environments. Throughout the 2020-2021 school year, data was collected from the four AP Calculus AB classes in a Tennessee high school. Over a six-month period, data was collected from five rounds of observation and a researcher's journal. In each round, small-group conversations revolving around the completion of a task were videotaped for a total of 110 observations. The environment in which students choose to participate is essential in constructing an understanding of their engagement in mathematical discourse and how it differs by learning environment. This chapter presents the results connected to the purpose and research questions of this study.

In this chapter, data collected from observing the AP Calculus AB students completing five on-level tasks in multiple blended learning environments was used to help answer the three research questions. Using Smith and Stein's Task Analysis Guide, each task that would be administered to the AP Calculus AB class was analyzed, and it was established that all the tasks were of higher-cognitive demand. This context is essential in understanding subsequent findings because it eliminates the bias that off-level or lower cognitive level tasks provide.

With new content standards and increased accountability measures, teachers facing educational change must focus not only on what to teach, but on how students are learning. The Common Core Standards for Mathematical Practice (SMP) inform teachers that students must be equipped not only with content knowledge but also with processes for learning to prepare

them for college and ultimately the real world. To adequately prepare students for college and future careers, the SMP propose that teachers engage students in classroom discourse where they make sense of mathematics by working collaboratively with peers, and by effectively communicating their thinking. The practice of collaborative discourse is a pedagogical map for ensuring the future success of all students in mathematics. For this research, the most important SMP is the third, where students communicate by constructing viable arguments and analyzing the reasoning of others. The productive discourse that takes place in a mathematics classroom can be the primary means by which students learn and grow as advanced thinkers.

This research project might be useful for educational systems who are thinking about creating an online school within their school district. It would also be beneficial for schools and educators who are having to teach students both virtually and synchronously. It is possible that teachers and administrators could use this research to explore making changes to their programs by implementing online learning.

Research Questions

The following research questions will guide this study:

1. How does the student's learning environment impact their mathematical discourse when completing tasks in an online synchronous work environment?
2. How does the student's physical environment affect their level of productive discourse when placed in a virtual group?
3. In what ways do the dynamics of the group change because of a blended learning environment?

Research Design

While different studies reviewed and discussed in Chapter 2 employed varying means of data collection, most researchers studying discourse believe in using a mixed-methods approach. This type of research allows discourse to be analyzed within a context, as both

quantitative and qualitative data are gathered and evaluated (Mercer, 2010). "The core argument for a mixed methods design is the combination of both forms of data provides a better understanding of a research problem than either quantitative or qualitative alone" (Creswell, 2015, pp. 21-22). The qualitative portion of the study described, compared, and analyzed students' participation in discussions in a high school AP Calculus AB classroom. These perspectives are seen and heard through audio recordings of students while they collaborate in groups, then coded using a revised Mathematics Class Observation Practices Protocol (MCOP^2; Gleason, et al., 2015) instrument. A nonexperimental design was used to obtain the quantitative data (Cooper & Schindler, 2003). A characteristic of quantitative research is the statistical analysis of data using numerical representations (Creswell, 2005).

The high school classroom is dynamic, seething with human activity, and part of reality that cannot be captured or measured by a single quantitative measure. "Qualitative research assumes that reality is constructed, multi-dimensional, and ever-changing; there is no such thing as a single, immutable reality waiting to be observed and measured" (Merriam, 1998, p. 54). Furthermore, classroom discourse is best understood in real-time. Due to the researcher being a participant in the research process, while the others involved were stakeholders, this study is also considered to be action research. According to McNiff and Whitehead (2005), action research can help people to improve their practice, as well as help people see their practices as practical theorizing; in conducting action research, there is no separation of practice and theory. The applied focus of action research makes it an important means by which educators can improve their research (Creswell, 2008; National Research Council, 2000). In summation, this study employs a mixed-methods action research design that intends to explore a phenomenon and provide a detailed narrative of the participants' experiences within the phenomenon.

Researchers collecting qualitative data must be mindful of their own biases, values, and research agendas during the research process. It is imperative to maintain validity of the

research for researchers to constantly self-monitor their emotional and bias responses during the data collection process (Merriam, 2009). Understanding how discourse was affected by the different learning environments required a qualitative research approach. According to Merriam (2009), qualitative research gives organic presumption of various realities from those individuals' lives and their interaction in the world of their subjective perceptions of that experience. Carefully "chosen and applied methods of analysis allow the researcher to become an objective observer of subjective meaning" (Schwandt, 2000, p. 193).

Setting, Participants, and Materials

The traditional suburban high school in which this study took place is one of the largest secondary schools in the state of Tennessee. This particular school was selected because it was the institution at which the researcher was an educator with convenient access to mathematics students. The high school at which the study was conducted had approximately 2,900 students. Throughout the 2020-2021 school year, 739 seniors, 643 juniors, 725 sophomores, and 729 freshmen attended this particular institution. The race/ethnicity of the school consisted of 58% Caucasian, 14% Asian, 20% African American, 4% Hispanic, 1% Indian/Pacific Islander, and 2% mixed race. Of the 2,900 enrolled, 31% of the students were enrolled in one or more Advanced Placement (AP) courses, while 39% took at least one Dual-Enrollment, college credit course. According to College Board (2020), 39% of American high school students took at least one AP Exam. This high school's mathematics program offers three different calculus courses, namely AP Calculus AB, AP Calculus BC, and AP Calculus DE, which are essentially Calculus I, Calculus II and Calculus III. Each of these courses represent college-level mathematics for which most colleges and universities, nationally and internationally, grant advanced placement and credit.

During the 2020-2021 school year, due to Covid-19's stronghold on the nation, students and family were allowed to select the modality in which they received instruction. This choice was given to minimize the number of students on the campus on any given moment, which

would ultimately help decrease the spread of the Covid-19 virus. When given this decision, 38% of the students enrolled in this specific high school selected to attend school virtually, which meant they would only learn through synchronous, online learning and would never step foot onto campus. The remaining 62% of students elected to learn through a blended learning environment, having their instructional time split between synchronous, virtual learning and face-to-face, in class learning.

There were 92 students enrolled in four AP Calculus AB classes at the start of the 2020-2021 school year. Out of those students, one was a sophomore, 32 were juniors, and 59 were seniors. 24 of these 92 students opted to learn completely virtually at the start of the school year, which meant they logged into classes via Microsoft teams and received instruction synchronously. These students will be referred to as virtual students for the purpose of this research study. Virtual students had to attend all classes daily, at the same time as their hybrid counterparts. The remaining 68 students who chose to learn through a combination of learning environments, will be referred to as hybrid students throughout this study. The school schedule created for hybrid students blended the virtual, synchronous learning environment with the face-to-face, in person, learning environment.

Although initially very confusing, the blended learning schedule only permitted around 30% of students on campus on any given day. This drastic decrease in population was a valiant attempt by the school directors to keep Covid-19 out of the schools, while trying to keep students' learning as normal and consistent as possible. This newly created schedule for hybrid students allowed pupils whose last name started with A-K to come to campus and learn face-to-face with their teachers on Monday and Tuesday, while simultaneously students whose last names started with L-Z logged onto Microsoft Teams and learned synchronously alongside their peers. On Wednesday and Thursday, this schedule flipped, with hybrid students whose last names start with L-Z attending class face-to-face, but students whose last names started with A-K had to learn synchronously by logging into class via Microsoft teams. By doing this, the 30%

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of the hybrid students whose last name fell in the first half of the alphabet were never on campus with the 30% of the hybrid students whose last names started with the second half. And last, on Fridays, all students in the district remained at home, learning synchronously by logging into class through Microsoft Teams. This schedule lasted the entire Fall semester of the 2020-2021 school year.

This changed at the start of the 2021 Spring semester, as Covid-19 cases around the United States decreased, and multiple state governors across the nation declared that schools could continue normally, which resulted in the removal of the fall schedule. Based on this decision, students who opted to learn virtually during the fall semester could return to school in the Spring, and hybrid students could choose to learn virtually in the spring. Ultimately, students could change their learning modality based on their success or perceptions of the learning environment in which they were enrolled in the fall. Table 2 shows a detailed look into the daily breakdown of the instructional options throughout the two semesters.

Table 2: Student Schedules

Fall 2020 Instructional Schedule			Spring 2021 Instructional Schedule		
	In Class	Microsoft Teams		In Class	Microsoft Teams
Monday	A-K	L-Z & virtual	Monday	Hybrid students	Virtual students
Tuesday	A-K	L-Z & virtual	Tuesday	Hybrid students	Virtual students
Wednesday	L-Z	A-K & virtual	Wednesday	Hybrid students	Virtual students
Thursday	L-Z	A-K & virtual	Thursday	Hybrid students	Virtual students
Friday		All students	Friday		All students

*A-K are hybrid students whose last names begin with the letters in that alpha range
**L-Z are hybrid students whose last names begin with letters in that alpha range

Research Positionality

The researcher was also the mathematics teacher of record and the only AP Calculus AB teacher at the high school where the study was conducted. During observations of student groups throughout the study, the researcher assumed a nonparticipant status in the classroom. The researcher acknowledged that there was a degree of subjectivity and bias in this research

project in that she had an interest in promoting deeper student engagement and productive discourse. Unfortunately, researchers are susceptible to obtaining the results they want to find. These biases can be a product of personal experience, environment, and/or social and cultural conditioning. Reflexivity, self-reflection by the researcher on their biases and predispositions, is the crucial strategy for avoiding researcher bias (Johnson & Christensen, 2014). Although it was not possible to remove this potential bias altogether, the researcher was aware of the influence that these biases may cause and made every effort to avoid their influence. In order to help avoid observer bias, the observer read through each protocol item and rubric for each observation. This helped the observer to make decisions based solely on the rubric outlined by each descriptor.

Instrumentation

Based on the review of literature, it is evident that there are many ways to evaluate student discourse. It is impossible to include all the observation protocols used to evaluate discourse within a high school classroom, so an abbreviated form of the Mathematics Classroom Observation Protocol for Practices (MCOP[2]) was selected for its mathematics-specific design. This tool was also selected because it focuses on many aspects of active learning and because it has been proved to be both reliable and valid (Gleason, et al., 2017).

The MCOP[2] is a mathematics classroom observational instrument used in grades K-16 to measure the scope of alignment of a mathematics classroom with several standards set forth by multiple national organizations focused on mathematics teaching and learning such as the NCTM (1989, 2000, 2014), American Mathematical Association of Two-Year Colleges (1995, 2006), Mathematical Association of America (Barker et al., 2004), and the National Research Council (2003). This tool was specifically created to measure mathematics classroom practices for teaching lessons that are goal-oriented toward conceptual understanding emphasizing three classroom components: student engagement, lesson design and implementation, and classroom culture and discourse.

The instrument was designed with 16 descriptors focusing on mathematical classroom interactions between the teacher and students that promote conceptual understanding. The design and dialect of the MCOP[2] were created using the Standards for Mathematical Practices, and each item was created using scores on a 0-3-point scale. A user guide was created with detailed descriptions that indicated what teacher and student behaviors looked like according to each of the 16 descriptors. This tool was validated based on test content, internal structure, and response processes (Gleason et al., 2017). To validate the test content, an iterative process of expert surveys was used to clarify the test items and their descriptors and confirm if they measured the desired constructs of classroom interactions. The internal structure was validated when the test items and descriptors were modified from the test content validation process using a Horn parallel analysis and an exploratory factor analysis (Gleason et al., 2017).

The MCOP[2] measures different teacher facilitation and student engagement factors using two subscales of nine items on each subscale (Gleason et al., 2015). For this study, only the items that assessed students' discourse were used (See Appendix A). This component focused exclusively on student engagement, participation, and discourse. The authors of this instrument recommended three to six class observations to capture the students' typical interactions with one another (Gleason, et al., 2017). Items in the student engagement and discourse portion of the MCOP[2] were designed to measure multiple aspects of productive discourse as described by the CCSS.

After selecting the MCOP[2] as the instrument for this study, an extensive modification process took place. The 16 MCOP[2] items were cut in half to focus on the eight student experiences most relevant to this research. The eight chosen descriptors focused on student engagement, time devoted to the task, oral perseverance, arguments and critiques, attending to precision, environment, and questioning amongst the students. Out of the 16 original descriptors, five were used verbatim in the modified instrument, with three of the descriptors being modified by two words or less (Table 3). The other eight were removed from the modified

instrument because they focused more on the teacher and the lesson plan or task, and not the students' and their patterns of communication.

Table 3: MCOP Changes

Instrument Modifications by Descriptors

Descriptor Number	Original MCOP	Abbreviated MCOP
1	Students engaged in exploration/investigation/ problem solving	Students engaged in exploration/problem solving/ investigation
2	Students use a variety of means to represent concepts	Not used
3	Students were engaged in mathematical activities	Students were engaged in mathematical activities
4	Students critically assessed mathematical strategies	Not used
5	Students persevered in problem solving	Students persevered orally in problem solving
6	The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding	Not used
7	The lesson promoted modeling with mathematics	The lesson promoted viable arguments and critiques of the reasoning of others
8	The lesson provided opportunities to examine mathematical structure	Not used
9	The lesson included tasks that have multiple paths to a solution or multiple solutions	Not used
10	The lesson promoted precision of mathematical language	The lesson promoted precision of mathematical language

11	The teachers' talk encouraged student thinking	Not used
12	There were a high proportion of students talking related to mathematics	There were a high proportion of students talking related to mathematics
13	There was a climate of respect for what others had to say	There was a climate of respect for what others had to say
14	In general, the teacher provided wait-time	Not used
15	Students were involved in the communication of their ideas to others	Students were involved in the communication of their ideas to others
16	The teacher uses student questions/comments to enhance conceptual mathematical understanding	Not used

Selecting the MCOP[2] was not without its challenges. First, the researcher spent numerous hours analyzing the tool and reading studies that utilized it in order to fully understand how each descriptor appeared in a secondary mathematics classroom. Second, the MCOP[2] only captures snapshots of the tasks that are being observed, and it is limited to the researcher's perspective. In order to effectively capture the students' discourse as it progressed, five tasks during the 2020-2021 year were observed using the modified version of the MCOP[2] instrument.

Data Collection Procedures

The start of this study was considerably delayed as the request to conduct research in the researcher's school was not signed and returned by the appropriate personnel until months after the form was initially submitted. It is believed this was due to the chaotic nature of the start of the school year as the pandemic was taking a toll on the education system. After months of waiting and requesting the form be signed by the proper personnel in the superintendent's office, it was finally returned, signed, near the end of the fall 2020 semester.

Prior to conducting any aspect of this research, a request to the researcher's institution's Institutional Review Board (IRB) was made. Soon after the request was filed, IRB responded that this project did not meet the definition of human subjects' research and did not require IRB approval. This decision was made on the basis that the students being observed in the study would have completed these same tasks in the same random groups, regardless of if the research was taking place.

Stein and Smith conceptualized the Mathematical Tasks Framework (MTF)(Figure 1), which shows the process of choosing and executing mathematical tasks from the mathematics textbooks or other resources to their potentially revised form as teachers provide the tasks to their students and then to the finalized tasks that are implemented by the teacher to the students in the classroom. Each stage of the MTF can influence what students have an

Figure 1

Mathematical Tasks Framework
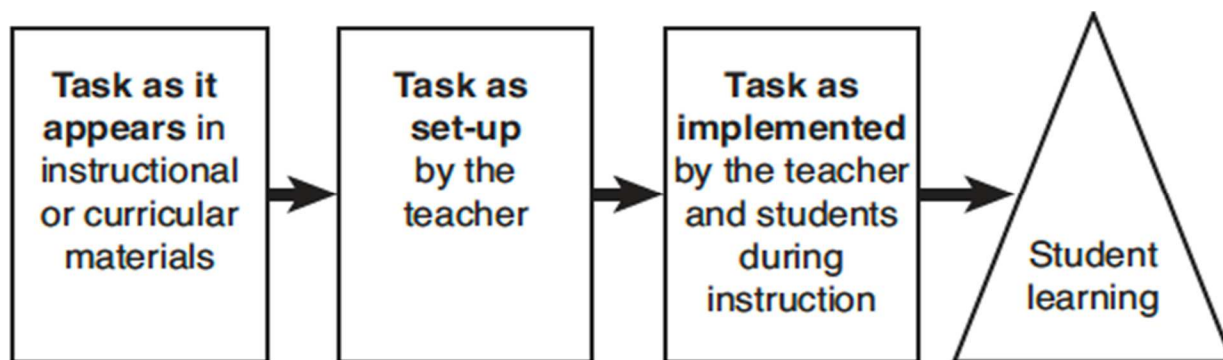
opportunity to learn. The MTF includes a four-tier rubric (Task Analysis Guide) (Figure 2) to analyze the cognitive demand of mathematical tasks. Cognitive demand is the type and level of thinking required of students in order to successfully engaged with and solve the task (Stein, Stein, Henningsen & Silver, 2000). With high-level cognitively demanding mathematical tasks, students engage in doing mathematics or make connections between concepts and procedures (Stein et al., 1996). On the other hand, with low-level cognitively demanding mathematical tasks, students engage in simple memorization and procedures without making connection. For

example, determining the derivative of $f(x) = x^2 + 4x - 8$ would be a low-level cognitively demanding task because students can use a simple procedure to find the derivative.

Figure 2

The Task Analysis Guide

Lower-Level Demands	Higher-Level Demands
Memorization • involve either reproducing previously learned facts, rules, formulae or definitions OR committing facts, rules, formulae or definitions to memory. • cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • are not ambiguous. Such tasks involve exact reproduction of previously-seen material and what is to be reproduced is clearly and directly stated. • have no connection to the concepts or meaning that underlie the facts, rules, formulae or definitions being learned or reproduced.	**Procedures With Connections** • focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections • are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • have no connection to the concepts or meaning that underlie the procedure being used. • are focused on producing correct answers rather than developing mathematical understanding. • require no explanations or explanations that focuses solely on describing the procedure that was used.	**Doing Mathematics** • require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • require students to explore and understand the nature of mathematical concepts, processes, or relationships. • demand self-monitoring or self-regulation of one's own cognitive processes. • require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Based on task analysis guide, the researcher actively sought out numerous high-level instructional tasks that aligned with the Advanced Placement Calculus curriculum to embed in her AP Calculus AB course. The implementation of tasks was engrained in curriculum from the start of the school year. The researcher was aware of and well versed in Smith and Steins' (2000) task analysis guide and utilized this tool whenever selecting tasks to use within their AP Calculus AB classroom. At the start of the school year the researcher complied numerous tasks from multiple resources to administer throughout the year to the AP Calculus AB students. Once the approval to conduct research was granted the researcher selected the five tasks that

corresponded with the rest of the years' lessons. A mathematical task is defined "as a classroom activity, the purpose of which is to focus students" attention on a particular mathematical idea" (Stein et al., 1996, p. 460). For this research, implemented tasks are defined as ones the students collaboratively worked on. Instructional tasks that are properly implemented draw students' attention to particularly important concepts, and students have opportunities to be exposed to those concepts embedded in the tasks they complete. Literature has shown that providing worthwhile tasks is a critical part of class practices and has significant impact on students' learning and the level of knowledge they attain (Boston, 2012; Munter, 2014). Mathematical tasks can either limit or broaden students' thinking on mathematics that they are engaged in (Henningsen & Stein, 1997), which is why the teachers' selection of tasks is a critical aspect of the learning process.

The next step involved observations of the AP Calculus AB students during a virtual mathematics class while they completed a task collaboratively. Afterwards the researcher observed each group using the abbreviated observational instrument MCOP[2]. Raw scores from the MCOP[2] ranged from 0-24 because the instrument was used to assess the eight subscales individually. Since this instrument was designed to compare groups or measure growth (Gleason, 2019), the results of the MCOP[2] were used for the statistical analysis to compare the changes within the groups between the tasks.

The first observed task of the year was administered on a Friday during the first semester, when all students were logging into their classes both virtually and synchronously. Within their class periods, students were randomly put into groups of three or four, depending on the class size. The teacher opted to group the students randomly because she wanted to create an environment where all students' abilities were seen as equal without social barriers within the group (Liljedahl, 2020). According to Cohen, (1992) heterogenous grouping is the preferred method of student grouping due to the benefits of low-achieving students receiving additional instruction from their higher-achieving peers. Multiple studies over the past four

decades have found that students with lower achievement benefitted from participation within heterogenous groups when compared to homogeneously low achieving groups (Swing & Peterson, 1982; Cohen, 1992; Sherman & Klein, 1995). The researcher knew this data and initially considered creating the groups for each task heterogeneously. Random grouping was ultimately selected as there are so few low achieving students within the AP Classroom AB, so the well-known benefits of heterogenous groups would not impact the participants of this course. This Calculus I course is designed for high academically achieving students and students with a lower competence would not be in this, or any Advanced Placement course.

After the random groups were assigned, they met in channels, within Microsoft Teams, for the entirety of their 30-minute class period. These channels were private, as only members of the group and the teacher had access. In each channel, the instructor uploaded a task and students were instructed to work collaboratively to complete the task and upload their results into their channel by the end of the period. They were aware that they were being recorded. By their MacBook's camera, thus, some students opted not to turn their cameras on for the entirety of the task. Once the group completed the task to the best of their ability and uploaded it, the students were allowed to leave the channel, the recording automatically stopped and was saved for the researcher to view at a later date. Throughout the time the students were in the channel, completing the task, the researcher was not present within the channel, unless her presence was requested by the group. The group contained only the students, but they were aware that everything they said or did within the channel was being recorded and would be viewed by the teacher. This process occurred once more during the Fall 2020 semester, with both hybrid and virtual students completing a task together virtually via Microsoft Teams.

At the start of the spring 2021 semester, the entire schools' learning environment changed, and hybrid students who initially chose to receive instruction face-to-face started attending school four days a week instead of two. Students who opted to remain at home during

the fall were still learning virtually and synchronously through Microsoft Teams throughout the spring; their schedule was not affected by the change.

Throughout the beginning of the Spring semester, three more tasks were administered to the AP Calculus AB students. This time not all students were virtual when this task took place, as hybrid students were physically attending school, learning face-to-face with their instructors. For these tasks, groups were randomly assigned with a mix of both hybrid and virtual students. On the day of the tasks, all students logged onto Microsoft Teams and joined their groups' channel, regardless of if they were sitting in the mathematics classroom, or at home. The tasks were completed within Microsoft Teams, just as it had been during the fall semester, the only difference was that there were students present in front of the teacher, in the classroom, while they were completing the task. For example, if a randomly assigned group for one of these tasks consisted of two hybrid students and two virtual students, the two hybrid students would be sitting next to one another with their laptops open, talking to their two virtual counterparts while they are participating at home.

At the conclusion of the research, 110 groups total were formed to complete the five tasks. The video recordings from all 110 groups were saved to the researchers' password-protected laptop that only the researcher could access. Each classroom observation was given a number (1-110) that corresponds to the sequence in which it was completed. The researcher observed each group as they completed their task using the modified observational instrument MCOP[2]. Raw scores ranging from 0-3 were assigned for each of the eight descriptors. Tables 4a – 4h are those categories that amended to the instrument used while coding the students' discourse.

Table 4a

The Eight Descriptors of the Amended MCOP[2]

	Descriptor 1			
	0	1	2	3
Student Engagement	Students did not engage in exploration, investigation, or problem solving	Students seldom engaged in exploration, investigation or problem solving	Students engaged in exploration, investigation, or problem solving more than half the task	Students regularly engaged in exploration, investigation or problem solving

Because the terms *seldom* and *regularly* are somewhat ambiguous, the researcher felt it would be useful to have distinct descriptions for each. No student in the group putting forth an effort to question or relate the task to previously learned material would ultimately earn a score of 0. If only one or two students attempted to make a connection or disagree with team members once or twice, would earn a 1. Students should determine their own solution pathway without necessarily knowing that the path will lead to a desired result throughout the entire task to earn a 3. The role of exploration, investigation, and problem solving is central in teaching mathematics as a process. Students producing these skills in a virtual setting, without a teacher present, shows that they understand how to work collaboratively, while attempting to complete a task. Student exploration may also foster a belief that mathematics is a discipline that can be explored, reasoned about, connected to other subjects, and one that makes logical sense (Barker, et al., 2004).

Table 4b

Descriptor 2				
	0	1	2	3
Student Discourse	None of the students are engaged in appropriate level mathematical discourse	Most of the students in the group spend less than one-quarter of the task engaged in appropriate level mathematical discourse	Most of the students in the group spend more than one-quarter but less than two-thirds of the task discussing appropriate level mathematical activity	The entire group spends two-thirds or more of the tasks discussing the mathematical activity at the appropriate level

Clarification might be needed to understand this category fully. To score a 0, none of the members of the group would discuss the task at an appropriate course level. Since these tasks are designed to be at an Advanced Placement (AP) Calculus level, the discourse should also be at that same mathematical level. One-quarter and two-thirds relates to the actual time devoted to the task, as thirty minutes was the amount of time each group had to discuss and complete the task. The appropriate level of discourse relates to the actual mathematics being discussed, which would be at an AP Calculus level. Keeping this discourse at a highly productive level for 20 minutes earns the group the full 3 points.

Table 4c

Descriptor 3				
	0	1	2	3
Student Oral Perseverance	Students did not orally persevere in problem solving	Students exhibited minimal oral perseverance in problem solving. Only one to two students verbally looked for multiple entry points and solution paths	Students exhibited some oral perseverance in problem solving. Not all of the students verbally looked for entry points and solution paths	Students exhibited a strong amount of oral perseverance in problem solving. All students looked for multiple entry and solution paths. When confronted with an obstacle, the group discusses together what to do next

Perseverance is more than just completion or compliance for an assignment. It should involve students overcoming a roadblock in the problem-solving process. One of the Standards for Mathematical Practices is that students will persevere in problem solving. This category was edited slightly from the original instrument by adding the word 'oral' in front of perseverance. This change was necessary since students could only hear one another in their virtual groups, as it was against district policy for teachers to require students to turn their cameras on. The tasks the students completed were at the AP level and should have provided a challenge for the students. If members of the group gave up without trying the task or attempted to solve the task without help from other members, this was coded as a 0. If the group collaborated to select an entry point into the task, while bouncing ideas off one another, then a 3 was given.

Table 4d

	Descriptor 4			
	0	1	2	3
Arguments and Reasoning	The task did not include any viable arguments or critiques of others	One or two of the students in the group use viable arguments or critiques of the reasoning of others	Viable arguments and critiques of the reasoning of is a major component, but the discourse is not productive, OR discourse is not a major component, but the students engage in a verbal activity that fits within the appropriate task	Viable arguments and critiques of the reasoning of others is an integral component of the task with students engaged in productive discourse

Following the Standards for Mathematical Practice from the Common Core State Standards (CCSS)(2010), this item describes lessons that help students to "understand and use stated assumptions, definitions, and previously established results in constructing arguments" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). This category adheres to the expectation provided by the Standards of

Mathematical Practice (2010), "Construct viable arguments and critique the reasoning of others." The CCSS does not provide an exact definition of a viable argument, but states, "Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments" (CCSS, 20111, p. 6). Given that practice, students are expected to communicate with one another as part of an effective and productive discourse community. This descriptor replaced the modeling category from the original MCOP[2] instrument. The researcher selected another CCSS to replace modeling, since arguing and critiquing is more evident verbally than modeling, which needs to be seen. If members of the group blindly followed one another without question, a code of 0 was given. If students productively argued or validated their work, that would receive a 3.

Table 4e

	Descriptor 5			
	0	1	2	3
Attend to Precision	None of the group members attend to precision in communication throughout the task	Only two of the members of the group attend to precision in communication throughout the task	Only three of the members of the group attend to precision in communication throughout the group task	All members of the group attend to precision in communication

This category, once again, follows the Standards of Mathematical Practice to 'attend to precision.' "Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning" (National Governors Association Center for Best Practices, 2010). A group received a 3 if a culture of precision of language was evident in how the students were communicating with one another. A score of 0 was earned if the members consistently used wrong terminology when referring to mathematical items or were not coherent in their communication.

Table 4f

	Descriptor 6			
	0	1	2	3
Student Discourse	No students talked related to the mathematics of the task	Less than half the group were talking related to the mathematics of the task	More than half of the students in the group were talking related to the mathematics of the task	All of the students in the group were talking related to the mathematics of the task

It should be noted that student discourse refers to student-to-student talk. This category was coded based on the number of students effectively talking while completing their task. The student talk had to be productive and directly related to an aspect of the task. If students worked independently and shared their results within their groups, they earned a 0. If all members of the group discussed entry and exit points as well as methods to complete the task, that was given a 3. Non-productive discourse and irrelevant talk between students dictated which code was given to each group.

Table 4g

	Descriptor 7			
	0	1	2	3
Climate of Respect	No students shared ideas	Only a few students in the group shared. Most students actively listen but do not share	The climate allows for students to share, question and comment throughout the entirety of the task. Most students actively listen	Majority of the group are sharing, questioning and commenting during the task. Students are active listening and clarifying the ideas of others

Effective communication means that students not only share their ideas, but listen, question and critique the ideas of their peers (Sherin, et al., 2004). This descriptor embraces the literature on equity within mathematics that believe all students have valuable ideas to share within their mathematics classroom (Boaler, 2006). The amount of task appropriate questions

and answers deemed how this category got coded. If participants of the group gave answers that were automatically accepted without critique by the rest of the group, then active listening did not take place. The expectation of this particular descriptor is that students are able to bounce ideas off one another without being ignored or blindly accepted.

Table 4h

	Descriptor 8			
	0	1	2	3
Time of Discourse	There was no productive peer to peer conversations during the task	The task was primarily led by one student A few instances occurred where discourse developed but this was not frequent	Some time was devoted to peer-to-peer discourse, and was not led by one student	Considerable time, more than half, was spent with peer-to-peer discourse related to the communication of ideas, strategies and solutions

NCTM and the Standards for Mathematical Practices expect teachers to create a mathematical community that includes discourse around the mathematics content and learning. Students are expected to talk and practice in the dialogue of the classroom (Manoucheri & St. John, 2006). This descriptor highlights the need for all students to be active participants in the classroom dialogue. This item was coded based on the amount of time students spent on their communication while completing the group task.

Data Analysis Procedures

Qualitative and quantitative data were collected by the researcher and obtained by means of observation and the researcher's journal. The data for this study was collected using the observation protocol MCOP[2]. The purpose of this mixed methods descriptive study was to explore the relationship between students' productive mathematical discourse within multiple learning environments. This study sought to understand the impact blended learning has on students' discourse as they transition from different learning modalities.

Analysis of Quantitative Data

Research Question 1: *How does the student's learning environment impact their mathematical discourse when completing tasks in an online synchronous work environment?*

- H_0: The average discourse between hybrid students in a group does is not equivalent to the average discourse between students in a blended learning group, for any of the tasks.

- H_a: The average discourse between hybrid students in a group is equivalent to the average discourse between students in a blended learning group for any of the tasks.

After all five occasions of quantitative data collection had occurred, the scores from each of the eight descriptors of the MCOP[2] were averaged together for each group, resulting in a range of scores from 0-24, for all 110 groups. Once the mean score of each group had been established, a two-sample t-test was performed to compare the discourse between groups who had all hybrid students and groups who had students with multiple learning modalities. This analysis was conducted for each of the five tasks to determine if there was a statistically significant difference between the discourse occurring in groups consisting of all hybrid students and groups consisting of both hybrid and virtual students.

Research Question 2: *How does the student's physical learning environment affect their level of productive discourse when placed in a virtual group?*

The quantitative data obtained through the eight descriptors within the MCOP[2] were analyzed using a chi-square (χ^2) statistical test. This instrument was administered in five different intervals, but only phase 1 (task 1) and phase 5 (task 5) were considered for the chi-square test. The decision to only select these two tasks was made by the teacher-researcher since the learning modality had changed for the majority of students between the first and last tasks, most members of each group was the same between the first and last tasks, and a comparison of the two would produce the most definitive results. The MCOP[2] was segregated

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into eight separate categories. Subsequently, the $MCOP^2$ is comprised of observational rankings, thus the variables are categorial in nature. There was no relationship between the subjects in each group, and a relatively large sample size (N=92) was used ensuring that the expected frequency was at least 1. As a result of all the forementioned criteria, and with the help of SPSS, a chi-square test of association was applied on all eight descriptors from task 1 and task 5.

Eight research hypotheses based on the eight categories of the $MCOP^2$ were created for research question 2:

$H_0 1$: There is no difference in the groups who regularly engaged in exploration, investigation, or problem solving over the course of the assignment from task 1 as compared to the groups who regularly engaged in exploration, investigation, or problem solving over the course of the assignment from task 5.

$H_a 1$: There is a difference in the groups who regularly engaged in exploration, investigation, or problem solving over the course of the assignment from task 1 as compared to the groups who regularly engaged in exploration, investigation, or problem solving over the course of the assignment from task 5.

$H_0 2$: There is no association of the number of groups discussing the task at the appropriate level for the entirety of the task between task 1 and task 5.

$H_a 2$: There is an association of the number of groups discussing the task at the appropriate level for the entirety of the task between task 1 and task 5.

$H_0 3$: There is no difference in the proportion of groups who exhibited a strong amount of oral perseverance in problem-solving from task 1 as compared to the proportion of students who exhibited a strong amount of oral perseverance in problem-solving from task 5.

$H_a 3$: There is a difference in the proportion of groups who exhibited a strong amount of oral perseverance in problem-solving from task 1 as compared to the proportion of students who exhibited a strong amount of oral perseverance in problem-solving from task 5.

$H_0 4$: There is no difference in the proportion of the groups who demonstrate viable arguments and critiques of the reasoning of others as an integral component of the task with students engaged in productive discourse over the course of the assignment from task 1 as compared to the proportion of groups who exhibit viable arguments and critiques of the reasoning of others as an integral component of the task with students engaged in productive discourse over the course of the assignment from task 5.

$H_a 4$: There is a difference in the proportion of the groups who demonstrate the viable arguments and critiques of the reasoning of others as an integral component of the task with students engaged in productive discourse over the course of the assignment from task 1 as compared to the proportion of groups who exhibit viable arguments and critiques of the reasoning of others as an integral component of the task with students engaged in productive discourse over the course of the assignment from task 5.

$H_0 5$: There is no difference in the proportion of the groups who "attend to precision" in communication from task 1 as compared to the proportion of students who "attend to precision" in communication from task 5.

$H_a 5$: There is a difference in the proportion of the groups who "attend to precision" in communication from task 1 as compared to the proportion of students who "attend to precision" in communication from task 5.

$H_0 6$: There is no association between the groups who maintain appropriate level mathematical conversations of the topic over the course of the assignment from task 1 as compared to the proportion of students who maintain appropriate level mathematical conversations of the task over the course of the assignment from task 5.

$H_a 6$: There is an association between the groups who maintain appropriate level mathematical conversations of the topic over the course of the assignment from task 1 as compared to the proportion of students who maintain appropriate level mathematical conversations of the task over the course of the assignment from task 5.

H$_0$7: There is no difference in the proportion of the students who were sharing, questioning, commenting, clarifying, and recognizing others' ideas over the course of the assignment from task 1 as compared to the proportion of students who were sharing, questioning, commenting, clarifying, and recognizing others' ideas over the course of the assignment from task 5.

H$_a$7: There is a difference in the proportion of the students who were sharing, questioning, commenting, clarifying, and recognizing others' ideas over the course of the assignment from task 1 as compared to the proportion of students who were sharing, questioning, commenting, clarifying, and recognizing others' ideas over the course of the assignment from task 5.

H$_0$8: There is no association between the amount of time students spent on peer-to-peer discourse from task 1 to task 5.

H$_a$8: There is an association between the amount of time students spent on peer-to-peer discourse from task 1 to task 5.

The chi-square test of independence was used to test "the association between two variables, an independent variable and a dependent variable" (Gliner & Morgan, 2000, p. 231). This test was conducted to determine the relationship of the students' discourse between task 1 and task 5. The Pearson Chi-Square test statistic, degrees of freedom, and *p*-values were calculated and reported. These results were compared to an alpha value of $\alpha = 0.10$ so as to determine if there was a statistical significance between the two tasks conducted both before and after the researcher's action research. This helped to determine the distribution of observations (frequencies) if no relationship exists. The chi-square test was also used to investigate the relationship between the students' learning environment and the change, if there was one, of productive discourse between the tasks.

Analysis of Qualitative Data

Data gathered through groups on Microsoft Teams provided a deep understanding of the phenomenon under investigation. The method for data analysis was based on Creswell's (2007) data analysis procedure. Specifically, the researcher progressed through four stages of data

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analysis: (a) data managing and transcribing; (b) reading (c) describing, classifying, and interpreting the data; and (d) comparing the data. To manage the data, relevant sections of all collaborative sessions was transcribed verbatim. Specifically, transcribed sections were when participants demonstrated their cognitive processes through verbalizations and/or observable behaviors. After transcribing the 110 sessions, the data was referenced with the researcher's journal, which helped bring possible codes and themes to light.

Coding transcripts and field notes. After each round of tasks, the field notes from the multiple groups were reviewed and any pertinent information relating to students' initial demeanor going into their groups, and behavior while in the groups was identified. By coding data this way, it made for later comparisons across sets of data more manageable. After each task was complete, the conversations captured on video were transcribed in a Word document.

After each round of tasks, the language of the transcripts was evaluated, and student contributions were categorized in ways described by the observation protocol MCOP[2]. These categories included student engagement, time devoted to the task, oral perseverance, arguments and critiques, attending to precision, environment, and questioning amongst the students. While listening and observing each group, a basic observational coding took place with the MCOP[2] instrument. Transcribing the discourse from audio, especially with its stops and starts to make sure everything was properly caught, even the mumbling, nearly incoherent parts, caused the researcher to slow down and really listen to the students' voices. Not only was the researcher forced to focus on what they said, but also how they said it. Paying attention to exactly how the conversation was spaced and articulated narrowed the researcher's focus to the speaker's areas of emphasis and meaning by paying more attention to intonation than to the exact words being spoken. Listening in this way took the researcher out of the habit of focusing more on the language and really focusing more on the interaction, on the patterns of speech, and how the individual participant presented him or herself though discourse.

Group analysis. Next, once transcription was finished, the first level of analysis was completed at the interpretive level of open coding. Cases were then analyzed by class. The researcher's journal was examined to explore summaries created of the initial groups and the overall patterns in the classroom discourse environment and student behaviors in that group. Data compilations were then compared using axial coding. Marshall (2011) reported that axial coding is a qualitative method used to group or cluster what was already sorted. In this study, the types of contributions were by learning environment.

Cross-Group analysis. Next, patterns and themes were compared between the multiple classes. The similarities and differences between groups whose members were hybrid, groups whose members were completely virtual, and groups who had members of both learning environments were explored. Not only were field notes and transcriptions analyzed from the multiple group observations, impressions and thoughts recorded in the researcher's journal were also examined.

The coding that occurred in this phase was thematic coding. This consisted of comparing codes across groups and class sessions to look for emerging themes. This phase focused on the nature of discourse to understand when and how types of discourses were used to construct understanding of each task. Parallel findings led to inferences about the influence of learning environment, and differences were interpreted.

CHAPTER IV: RESEARCH ANALYSES, FINDINGS, AND RESULTS

The design of this study was flexible in order to attend to information that emerged as the study took place. The researcher went into this study open minded and intrigued by the concept of discourse within a blended learning environment. The statistical analysis of the quantitative data explored the statistical significance of the students' discourse between tasks as measured by the MCOP^2. The collection of qualitative data created a detailed exploration into the amount and type of discourse high school AP Calculus AB students construct when in different types of learning environments. This mixed methods research study allowed the researcher to disaggregate the data by learning environment during the data analysis process to explore for any similarities and differences between tasks and amongst groups. Chapter 5 offers conclusions and implications for this study.

Quantitative Results

Research Question #1. A two-sample t-test was conducted on the category of overall student mathematical discourse in different blended learning environments in order to answer research question 1: *How does the student's learning environment impact their mathematical discourse when completing tasks in an online synchronous work environment?* A two-sample t-test was chosen to test whether the two population means (groups consisting of all hybrid learners, and groups consisting of both hybrid and virtual learners) are equal. Before implementing the two-sample t-test the researcher went through the five t-test assumptions to ensure the t-test was the correct statistical test to use with the current data set. The t-test was warranted because the test variable (the scores from the MCOP^2) follows a continuous scale, and the sample was randomly selected from the population. In this case the population was

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random students placed in an AP Calculus AB course within a large suburban high school. A

third assumption is the data, when plotted, results in a normal distribution, bell-shaped

distribution curve. When a normal distribution is assumed, but not proven, if the sample size is

at least 30, a t-test can be used. The final assumption is homogeneity of variance between

groups. There are equal variances across groups, as assessed by Levene's Test of

Homogeneity of Variance (p = .783 > .05). In order to analyze the data using the t-test, a null

and alternative hypothesis was created by the researcher.

H_0 = The average discourse between hybrid students in a group does is not equivalent

to the average discourse between students in a blended learning group, for any of the tasks.

H_a = The average discourse between hybrid students in a group is equivalent to the

average discourse between students in a blended learning group for all of the tasks.

Table 5 shows the mean and standard deviation for productive mathematical discourse

of students in groups with either all hybrid students, or groups with both hybrid and virtual

students. Five two-sample t-tests were used to evaluate the average scores the groups

acquired from the MCOP^2 for all the observed tasks. As the table shows, there was no

significant difference in the students' discourse amongst the different groups for any of the five

tasks.

Table 5

Student's mathematical discourse based on combination of members' learning environments

	Groups with all hybrid students		Groups with both hybrid and virtual students			
	M	SD	M	SD	*t*-value	*p*
Task 1	2.07	0.311	1.95	0.563	0.645	0.263
Task 2	2.09	0.302	2.01	0.503	0.508	0.309
Task 3	2.35	0.296	2.21	0.489	0.879	0.195
Task 4	2.43	0.204	2.45	0.298	-0.179	0.569
Task 5	2.55	0.112	2.6	0.205	-0.708	0.754

*Note. *p < .1. M = mean, SD = standard deviation.

Results. Five t-tests were conducted on the category of overall student productive

mathematical discourse between groups for each of the five tasks. Table 5 shows that the mean

of the all hybrid member groups in task one (mean=2.071) is greater than the groups whose

members learned both virtually and face-to-face (mean=1.955). Interestingly enough, the

average amount of productive discourse was higher for the groups with all hybrid students for

task 1 (mean=2.017), task 2 (mean=2.092) and task 3 (mean=2.35). It was not until task 4 (2.43

for all hybrid groups, and 2.45 for mixed groups), which occurred later in the 2020-2021 school

year, did groups containing both hybrid and virtual students' average discourse have higher

means. Regardless of the change in average discourse amongst the groups, according to the

results found in Table 5, there was not enough evidence to reject the null hypothesis that the

average discourse between hybrid students in a group does is not equivalent to the average

discourse between students in a blended learning group, for any of the tasks.

Research Question #2. The following section includes the results of hypothesis testing used

to answer research question 2. According to Neuman (2006), researchers reject or fail to reject

the null hypothesis when hypothesis testing. For purposes of the current study, the level of

significance for this data analysis was $p < .1$. To determine the level of significance a chi-square

test of independence is run on the independent and dependent variables in the study. The

independent variables included the two learning environments (hybrid and virtual), while the

dependent variables were data acquired from the $MCOP^2$.

The $MCOP^2$ was administered in five separate intervals throughout the school year, but

only the results from task 1 and task 5 will be considered to answer research question 2, which

states: *How does the students' physical learning environment affect their level of productive*

discourse when placed in a virtual group? Although multiple tasks were given to the AP

Calculus AB students throughout the year, the researcher decided to only use the first and last

task when analyzing the data acquired from the $MCOP^2$ in order to see the total growth, or lack

thereof, amongst the groups. For task 1, the researcher chose to randomly group the students,

and for task 5 the groups were replicated. With the members of each group in task 1, being the same as in task 5, the results from chi-square test of independence was more reliable.

Task 1 was administered towards the end of the fall semester of the 2020-2021 school year, while task 5 was administered toward the end of the spring semester, right before the AP Calculus AB exam. Subsequently, MCOP^2 is comprised of observational rankings segregated into eight separate descriptors and are therefore categorical in nature. There is no relationship between the subjects in each group, and a relatively large sample size (n=24) was used, ensuring that the expected frequency in each cell was at least 1. As a result of all the aforementioned criteria, a chi-square test of association was selected as the statistical test and was therefore applied on all eight descriptors. Each descriptor from the MCOP^2 was analyzed using SPSS$^®$ analysis software to run a chi-square test for independence. According to Creswell (2015), a chi-square test is for non-normal distribution with a category within-group comparison.

To address research question 2, the researcher aimed to determine if a relationship exists between students' physical learning environment and their level of productive discourse as observed by using the MCOP^2. The MCOP^2 instrument is the primary source of data in answering the research question; *How does the students' physical learning environment affect their level of productive discourse when placed in a virtual group?* The question was answered by testing the following eight research hypotheses.

Descriptor 1. To effectively and thoroughly discover the association between the two variables, a chi-square test of independence was run on all eight descriptors from the MCOP^2 from both task 1 and task 5. The first descriptor focused on the engagement aspect of each of the groups. The observation was to determine if each member of the group was engaged in exploration, investigation, or problem solving over the course of the task. For the chi-square test, these three components are comprised of one dependent variable.

$H_0 1$: There is no difference in the groups who regularly engaged in exploration, investigation, or problem solving over the course of the assignment from task 1 as compared to

the groups who regularly engaged in exploration, investigation, or problem solving over the course of the assignment from task 5.

H$_a$1: There is a difference in the groups who regularly engaged in exploration, investigation, or problem solving over the course of the assignment from task 1 as compared to the groups who regularly engaged in exploration, investigation, or problem solving over the course of the assignment from task 5.

A chi-square test of independence was first performed to examine the relation of students' engagement, investigation, and problem-solving between task 1 and task 5. The relationship between these two variables was significantly different, χ^2(1, N = 46) = 0.348, p = .555. This implies that there is not enough evidence to reject the claim that there is no difference in the number of groups who engaged in exploration, investigation, and problem solving from task 1 to task 5. The chi-square statistic of .348 with a 90% confidence level, as shown in Table 6, indicates that there is not a significant statistical difference between the independent and dependent variables, which means there is not enough evidence to reject the null hypothesis (H$_0$1) and accept the alternative hypothesis (H$_a$1).

Table 6

Chi-Square Results: Student engagement, investigation, and problem solving

	Value	Df	Asymp. Sig (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	0.348	1	0.555		
Continuity Correction	0.087	1	0.768		
Likelihood Ratio	0.349	1	0.555		
Fisher's Exact Test				0.768	0.384
N of Valid Cases	46				

*$p < .1$

Table 7 shows the breakdown of groups that performed at a high level (scoring only 3's for this descriptor) for task 1 and groups that scored at a low level (0, 1, or 2) for task 1 descriptor 1. Eleven of the twenty-four groups for task 1 performed at a high level 23.90% of the

time, while the remaining 13 groups scored at a low level 28% of the time. For task 5, twelve groups scored 3's for this descriptor while 10 scored either a 0, 1, or 2.

Table 7

Crosstabulation: Students engaged in exploration, investigation, and problem solving from task 1 to task 5

		Level		
		high	low	Total
Task 1	Count	11	13	24
	% of Total	23.90%	28%	52.20%
Task 5	Count	12	10	22
	% of Total	26.10%	21.70%	47.80%
Total	Count	23	23	46
	%	50.00%	50.00%	100%

Descriptor 2. The second descriptor of the MCOP[2] concentrated on the number of students discussing the task at the appropriate level for the entirety of the task. A null and alternative hypothesis was established regarding this descriptor. It is as follows:

$H_0 2$: There is no association of the number of groups discussing the task at the appropriate level for the entirety of the task between task 1 and task 5.

$H_a 2$: There is an association of the number of groups discussing the task at the appropriate level for the entirety of the task between task 1 and task 5.

A chi-square test of independence was next to run on descriptor two from the MCOP[2] to determine if there was a relationship between task 1 and task 5 concerning the proportion of students who utilized their time management techniques by staying on task discussing the mathematical activity at the appropriate level over the course of the assignment. The results of this test, as presented on table 8, showed a significant statistical difference χ^2 (1, n = 46) = 4.182, p = .041, between task 1 and task 5, which meant there is enough evidence to reject the null hypothesis and accept the alternative hypothesis. These results indicate that the discourse occurring in task 1 was not independent of the discourse transpiring in task 5. By accepting the

alternative hypothesis, the researcher can state that there is a correlation between the amount

of student discourse between the first and last tasks. The chi-square statistic of .041 with a 90%

confidence level, as shown in Table 8, indicates that there is a significant statistical difference

between the independent and dependent variables, which means there is enough evidence to

reject the null hypothesis (H_02) and accept the alternative hypothesis (H_a2).

Table 8

Chi-Square Results: Students using appropriate discourse for the entirety of the task

	Value	Df	Asymp. Sig (2-sided)	Exact Sig. (2-Sided)	Exact Sig. (1-sided)
Pearson Chi-Square	4.182	1	0.041		
Continuity Correction	3.053	1	0.081		
Likelihood Ratio	4.242	1	0.039		
Fisher's Exact Test				0.073	0.04
N of Valid Cases	46				

*$p < .1$

Table 9

Crosstabulation: Group spends more than two – thirds the task with appropriate level of mathematical discourse

		Level		
		high	low	Total
Task 1	Count	7	17	24
	% of Total	15.20%	37%	52.20%
Task 5	Count	13	9	22
	% of Total	28.30%	19.60%	47.80%
Total	Count	20	26	46
	%	43.50%	56.50%	100%

Descriptor 3. The third descriptor from the MCOP[2] focused on the oral perseverance within

each of the groups. The observation was to determine if students were looking for appropriate

entry points and solution paths and if they, as a group, displayed a strong amount of oral

perseverance while searching. A null and alternative hypothesis was established regarding this

descriptor. It is as follows:

H$_0$3: There is no difference in the proportion of groups who exhibited a strong amount of oral perseverance in problem-solving from task 1 as compared to the proportion of students who exhibited a strong amount of oral perseverance in problem-solving from task 5.

H$_a$3: There is a difference in the proportion of groups who exhibited a strong amount of oral perseverance in problem-solving from task 1 as compared to the proportion of students who exhibited a strong amount of oral perseverance in problem-solving from task 5.

After running a chi-square test of independence on the third descriptor, the results, as displayed on table 10, determined that there is a significant difference between students' oral perseverance in task 1 and task 5, χ^2(1, n = 46) =10.478, p = .001. Since the p-value is below the accepted critical value of 0.1, the researcher can state that there is enough evidence to reject the null hypothesis. This implies that there is a difference in the proportion of groups who exhibited a strong amount of oral perseverance in problem-solving over the course of the assignment from task 1 as compared to the proportion of groups who exhibited a strong amount of oral perseverance in problem-solving over the course of the assignment from task 5.

Table 10

Chi-Square Results: Students' oral perseverance

	Value	Df	Asymp. Sig (2-sided)	Exact Sig. (2-Sided)	Exact Sig. (1-sided)
Pearson Chi-Square	10.478	1	0.001		
Continuity Correction	8.653	1	0.003		
Likelihood Ratio	10.909	1	0.001		
Fisher's Exact Test				0.003	0.001
N of Valid Cases	46				

*p < .1

Table 11

Crosstabulation: Students exhibit a strong amount of oral perseverance

		Level		
		high	low	Total
Task 1	Count	6	18	24
	% of Total	13%	39%	52.20%
Task 5	Count	16	6	22
	% of Total	34.80%	13%	47.80%
Total	Count	22	24	46
	%	47.80%	52.20%	100%

Descriptor 4. The fourth descriptor from the MCOP[2] focused on the verbal reasoning aspect of each of the groups. The observation was to determine the level at which students were making viable arguments and critiques in their reasoning of the task at hand as they are engaged in productive discourse, as described in the Common Core Standards. The groups were rated on the productivity of the discourse and if it played a major component in the completion of the task. A null and alternative hypothesis was established regarding this descriptor. It is as follows:

$H_0$4: There is no difference in the proportion of the groups who demonstrate viable arguments and critiques of the reasoning of others as an integral component of the task with students engaged in productive discourse over the course of the assignment from task 1 as compared to the proportion of groups who exhibit viable arguments and critiques of the reasoning of others as an integral component of the task with students engaged in productive discourse over the course of the assignment from task 5.

H_a4: There is a difference in the proportion of the groups who demonstrate the viable arguments and critiques of the reasoning of others as an integral component of the task with students engaged in productive discourse over the course of the assignment from task 1 as compared to the proportion of groups who exhibit viable arguments and critiques of the reasoning of others as an integral component of the task with students engaged in productive discourse over the course of the assignment from task 5.

After running a chi-square test of independence on the fourth descriptor, the results, as displayed in table 12, revealed χ^2 (1, n = 46) = 6.429, *p* = .011 that there was enough evidence to reject the null hypothesis. This implies that we have enough evidence to state that there is a difference in the proportion of the students who demonstrate the viable arguments and critiques of the reasoning of others as an integral component of the task with students engaged in productive discourse over the course of the assignment from task 1 as compared to the proportion of student who exhibits viable arguments and critiques of the reasoning of others as an integral component of the task with students engaged in productive discourse over the course of the assignment from task 5.

Table 12

Chi-Square Results: Students' viable arguments and critiques of reasoning

	Value	Df	Asymp. Sig (2-sided)	Exact Sig. (2-Sided)	Exact Sig. (1-sided)
Pearson Chi-Square	6.429	1	0.011		
Continuity Correction	4.922	1	0.027		
Likelihood Ratio	6.695	1	0.01		
Fisher's Exact Test				0.025	0.013
N of Valid Cases	46				

*p < .1

Table 13

Crosstabulation: Students use viable arguments and critiques the reasoning of others

		Level		
		high	low	Total
Task 1	Count	6	18	24
	% of Total	13.2%	39%	52.20%
Task 5	Count	10	12	22
	% of Total	21.8%	26%	47.80%
Total	Count	16	30	46
		35%	65%	100.00%

Descriptor 5. The fifth descriptor focused on the attention to precision aspect of each of the groups. The observation was to determine the level at which students were "attending to precision in their communication" while completing their task. The observer rated the "attend to precision" in communication throughout the group task. A null and alternative hypothesis was established regarding this descriptor. It is as follows:

$H_0$5: There is no difference in the proportion of the groups who "attend to precision" in communication from task 1 as compared to the proportion of students who "attend to precision" in communication from task 5.

H_a5: There is a difference in the proportion of the groups who "attend to precision" in communication from task 1 as compared to the proportion of students who "attend to precision" in communication from task 5.

After running a chi-square test of association on the fifth descriptor, the test revealed $\chi^2(1, n = 46) = 7.002$, $p = .008$, that there was enough evidence to reject the null hypothesis. Since the p-value was below the accepted critical value of .1, there is enough evidence to reject null hypothesis and the alternative hypothesis can be accepted. These results, as shown in table 14, imply that there is enough evidence to state that there is enough difference in the proportion of groups who attend to precision in communication from task 1 to those who attend to precision in task 5.

Table 14

Chi-Square Results: Students attend to precision

	Value	Df	Asymp. Sig (2-sided)	Exact Sig. (2-Sided)	Exact Sig. (1-sided)
Pearson Chi-Square	7.002	1	0.008*		
Continuity Correction	5.526	1	0.019		
Likelihood Ratio	7.186	1	0.007		
Fisher's Exact Test				0.017	0.009
N of Valid Cases	46				

*$p < .1$

Table 15

Crosstabulation: All members of the group attend to precision in communication

| | | Level | | |
		high	low	Total
Task 1	Count	7	17	24
	% of Total	15.20%	37%	52.20%
Task 5	Count	15	7	22
	% of Total	32.60%	15.20%	47.80%
Total	Count	20	26	46
	%	47.80%	52.20%	100%

Descriptor 6. The sixth descriptor focused on the mathematical conversation aspect of each of the groups. The observation was to determine the level at which students were maintaining conversations related to the mathematics of the task at hand. A null and alternative hypothesis was established regarding this descriptor. It is as follows:

$H_0 6$: There is no association between the groups who maintain appropriate level mathematical conversations of the topic over the course of the assignment from task 1 as compared to the proportion of students who maintain appropriate level mathematical conversations of the task over the course of the assignment from task 5.

$H_a 6$: There is an association between the groups who maintain appropriate level mathematical conversations of the topic over the course of the assignment from task 1 as compared to the proportion of students who maintain appropriate level mathematical conversations of the task over the course of the assignment from task 5.

After running a chi-square test on the sixth descriptor from the MCOP[2], the results, as provided in table 16, $\chi^2(1, n = 45) = 5.002$, *p* = .004, indicated that there was sufficient evidence to reject the null hypothesis and accept the alternative hypothesis. Since the *p*-value is below the accepted critical value of .1, it can be determined that we have enough evidence to state that there is an association between the students who maintain an appropriate level of mathematical conversations between task 1 and task 5.

Table 16

Chi-Square Results: Students appropriate level of math talk

	Value	Df	Asymp. Sig (2-sided)	Exact Sig. (2-Sided)	Exact Sig. (1-sided)
Pearson Chi-Square	5.002	1	0.004		
Continuity Correction	6.526	1	0.025		
Likelihood Ratio	7.186	1	0.019		
Fisher's Exact Test				0.037	0.019
N of Valid Cases	46				

*$p < .1$

Table 17

Crosstabulation: Majority of the students in the group were talking related to the mathematics of the task

		Level		
		high	low	Total
Task 1	Count	7	17	24
	% of Total	15.20%	37%	52.20%
Task 5	Count	15	7	22
	% of Total	32.60%	15.20%	47.80%
Total	Count	20	26	46
	%	47.80%	52.20%	100%

Descriptor 7. The seventh descriptor focused on the students' individual reflection of the task in each of the groups. The observation was to determine the level of which students were sharing, questioning, and commenting during the task. This includes their struggles that they may have encountered during the task. The rating was given based on the attentiveness of the students as they are clarifying and recognizing the ideas of others in the group. A null and alternative hypothesis was established regarding this descriptor. It is as follows:

$H_0 7$: There is no difference in the proportion of the students who were sharing, questioning, commenting, clarifying, and recognizing others' ideas over the course of the assignment from task 1 as compared to the proportion of students who were sharing,

questioning, commenting, clarifying, and recognizing others' ideas over the course of the

assignment from task 5.

H$_a$7: There is a difference in the proportion of the students who were sharing,

questioning, commenting, clarifying, and recognizing others' ideas over the course of the

assignment from task 1 as compared to the proportion of students who were sharing,

questioning, commenting, clarifying, and recognizing others' ideas over the course of the

assignment from task 5.

After the chi-square test was run on descriptor 7, the results, as shown on table 18, χ^2(1,

n = 46) = 3.049, p = .081, showed that there was enough evidence to reject the null hypothesis.

Since the p-value was below the accepted critical value of .1, we have enough evidence to

reject the statement that there is no difference in the proportion of students who were sharing,

questioning, commenting, clarifying, and recognizing others' ideas during task 1 as compared to

task 5. Therefore, it can be implied that there is a statistically significant difference between task

1 and task 5 in regard to the students' sharing, questioning, commenting, clarifying, and

recognizing others' ideas.

Table 18

Chi-Square Results: Students share, question, and comment during the task

	Value	Df	Asymp. Sig (2-sided)	Exact Sig. (2-Sided)	Exact Sig. (1-sided)
Pearson Chi-Square	3.049	1	0.081		
Continuity Correction	2.092	1	0.148		
Likelihood Ratio	3.08	1	0.079		
Fisher's Exact Test				0.134	0.074
N of Valid Cases	46				

p < .1

Table 19

Crosstabulation: Most students in the group are sharing, questioning and commenting during the task, including their struggles

		Level		
		high	low	Total
Task 1	Count	7	17	24
	% of Total	15.20%	37%	52.20%
Task 5	Count	12	10	22
	% of Total	26.10%	21.70%	47.80%
Total	Count	19	27	46
	%	41.30%	58.70%	100%

Descriptor 8. The eighth descriptor focused on the amount of time that peer-to-peer discourse occurred in each of the groups while the students completed the assigned task. The observation was to determine the level, based on approximate time, that students spent with peer-to-peer discourse related to the communication of their ideas, strategies, and solutions during the task. A null and alternative hypothesis was established regarding this descriptor. It is as follows:

$H_0 8$: There is no association between the amount of time students spent on peer-to-peer discourse from task 1 to task 5.

$H_a 8$: There is an association between the amount of time students spent on peer-to-peer discourse from task 1 to task 5.

After running the chi-square test on the 8th descriptor of the MCOP[2] the results, as displayed on table 20, $\chi^2(1, n = 46) = 2.581$, $p = .108$, revealed a *p*-value above the accepted critical value of .1. This implies that we do not have enough evidence reject the statement that there is no association in the amount of students who were spending time with peer-to-peer discourse related to the communication of ideas, strategies, and solutions over the course of the assignment from task 1 as compared to the amount of students who were spending time with

peer-to-peer discourse related to the communication of ideas, strategies, and solutions over the course of the assignment from task 5.

Table 20

Chi-Square Results: Amount of time spent on peer-to-peer discourse

	Value	Df	Asymp. Sig (2-sided)	Exact Sig. (2-Sided)	Exact Sig. (1-sided)
Pearson Chi-Square	2.581	1	0.108		
Continuity Correction	1.714	1	0.19		
Likelihood Ratio	2.606	1	0.106		
Fisher's Exact Test				0.142	0.095
N of Valid Cases	46				

*p < .1

Table 21

Crosstabulation: Time spent on peer-to-peer discourse relate to the communication of ideas, strategies and solution.

		Level		
		high	low	Total
Task 1	Count	9	15	24
	% of Total	15.20%	37%	52.20%
Task 5	Count	10	12	22
	% of Total	26.10%	21.70%	47.80%
Total	Count	19	27	46
	%	41.30%	58.70%	100%

Summary

To fully answer research question 2; *How does the student's physical learning environment affect their level of productive discourse when placed in a virtual group?* Chi-square tests of independence were run on the eight descriptors of the MCOP². The results from these eight statistical tests revealed that there was no association between independent and dependent variables for two of the descriptors. The quantitative data analysis also revealed that there were significant differences among the relationships between task 1 and task 5 for six of the descriptors of the MCOP². The impact of this analysis, as well as students' productive discourse as related to their learning environment, will be further discussed in Chapter 5.

Qualitative Findings

Context for Mathematical Discourse. The Mathematics Class Observation Practices Protocol (MCOP[2]; Gleason, et al., 2015) is the primary source of data in answering the research question, "*In what ways do the dynamics of the group change because of a blended learning environment?"* This instrument contained eight modified descriptors that were designed to evaluate the extent and amount of time students used appropriate and productive discourse. After transcribing the 110 observations, each case was analyzed and summarized. The frequencies of individual contributions based on the MCOP[2] was tallied and organized on a spreadsheet to make various comparisons between cohorts and learning environments. Once the number and types of contributions were recorded, transcripts and field notes were analyzed for themes that emerged amongst various groups and tasks.

Qualitative studies call for thick descriptions in order to adequately convey the phenomenon in question (Merriam, 1998; Patton, 1987). Merriam (1998) defines "thick" description as a "complete and literal description of the incident entity being investigated" (p. 29). In his later work, Patton (1990) stated that qualitative studies rely heavily on direct quotations gleaned from oral or written subjects to reveal "respondent's depth of emotion, the ways they have organized their world, their thoughts about what is happening, their experiences, and their basic perceptions" (p. 24). The findings of this study are based on the quality of the evidence presented by its research, its data, so every attempt was made to fully collect answers to the research questions possible in a sound and ethical way (Creswell, 1998; Stake, 2010).

Descriptive validity. The first and most fundamental criterion of validity in qualitative research, according to Maxwell, is descriptive validity (Maxwell, 1992, p. 285). This category concerns the factual accuracy of a descriptive, narrative account. These idiosyncratic biases can influence the transcription of dialog and the descriptive narration of events, and therefore significantly impact the analysis and interpretation of these events.

In addition, the researcher, guided by theory and experience, selectively attends to certain features of interactions and ignores others when watching and analyzing the recordings. However, it is imperative to highlight the features in a representation that are relevant to the phenomenon being studied and necessary for the argument being made. In the transcript excerpts presented below, information that does not influence the outcome of the analysis (e.g., utterances that do not seem to be attended to by others in the interaction, pauses, giggles) was omitted in order to increase the readability and draw attention to the important features being analyzed.

When selecting the mathematical tasks, the intentions were for them to be "vehicles to stimulate creativity, to encourage collaboration and to study learners' untutored, emergent ideas" (Powell et al., 2009, p. 167). During the initial set up phase of this study, multiple tasks were analyzed in order to evaluate their cognitive demand. The Task Analysis Guide was used to evaluate the task during set up and implementation (Stein et al., 2009) based on the thinking expected of by the majority of the students for the length of the task. First, the task as designed was assessed with regard to whether it was a low or high level. For example, if students were being asked to only recall previous knowledge, such as identifying trigonometric identities, or practice a previously learned procedure, such as finding rates of change of a tangent line, then the task was most likely considered low level, or *memorization* or *procedures without connections*, respectively. If students were asked to discover a formula or procedure they did not already know, or were asked to engage in open-ended problem solving, then the task was most likely high level, *procedures with connections* or *doing mathematics*, respectively. During implementation, the focus was on the students' engagement with the task, as well as the discourse that resulted while the students were completing the task. Students' conversations were revealing of whether they were trying to make the required conceptual conceptions, or whether they were looking for a shortcut or just the solution. Little attention was paid to the

correctness of a student's answer than the effort they were making to understand the implications of the task.

The first group task that was observed using the MCOP[2] instrument occurred during the second quarter, in November of 2020. A majority of students in the Calculus AB class struggled on using technology to make connections between problems and real-life applications. While teaching units 1-3, the teacher did not emphasize the use of calculators, as half of the Advanced Placement (AP) exam is non-calculator. This first task, seen in Figure 3, incorporated what students learned in previous lessons, with the appropriate calculator skills that they would need moving forward in Unit 4, and ultimately on the AP exam.

Figure 3

Task 1: Tangent Lines

Given: Let function $f = \{(x, y): x \in \mathbb{R} \text{ and } 0 \le x \le 2 \text{ and } y = 4x^2 + 5\}$ and consider the Line *l* that is tangent to a graph for function **f** at the point where x = 1.
Part 1: Use a graphing calculator to view a graph of $Y2 = (4x^2+5)/(0 \le x)/(x \le 2)$ in [-1, 3.7] X [-5, 26]; draw a complete graph for function **f** on the grid provided.
Part 2: Use either nDeriv(Y2, X,1) on the MATH menu or dy/dx on the CALC menu to find an approximate value for **f**'(1); use the definition of a derivative to find the exact value.
Part 3: Use Tangent on the DRAW menu to investigate a graph for Line *l*; use an analytical method to find an exact equation for Line *l*.
Part 4: On the same axes as the graph for function **f**, draw a complete graph for Line *l*; plot and label the point of tangency.

Research Question #3

As mentioned throughout this study, at the end of each task, once group members left their teams' channel, the recording from the exercise was automatically saved to the researcher's personal laptop. Each group's dialogue was then transcribed and coded, then the findings were compared amongst each group. One of the primary objectives of this research study was to discover if the learning environment students were enrolled in impacted the dynamics of their collaborative groups. The design and modification of the MCOP[2] instrument that was used while observing the groups allowed the researcher to infer and analyze the changes within the groups' disposition. This ultimately helped answer *Research Question 3: In what ways do the dynamics of the students' group change because of the blended learning environment?*

Throughout the coding process the researcher was paying attention to themes and oddities that arose. While doing a comparison analysis between cohorts the researcher noticed a few groups whose average score across all eight descriptors of the MCOP[2] was relatively low for task 1 but showed an increase in average for task 5. Upon further analysis it was discovered that each of these groups were unique because they consisted of same gendered participants. Both table 22 and table 23 show a breakdown of the groups, their descriptor scores, and the learning environments of the students within the groups.

Table 22

Task 1 Same Sex Groups

All Females	MCOP Descriptors								Mean	Group Dynamics	
Group #	1	2	3	4	5	6	7	8			
1A	1	2	1	0	2	2	1	2	0.46	3 hybrid	1 virtual
3B	2	1	2	1	1	1	1	1	0.42	3 hybrid	1 virtual
3D	2	2	0	2	1	1	2	2	0.5	1 hybrid	3 virtual
5A	0	1	0	0	1	1	1	1	0.21	2 hybrid	2 virtual
All Males											
1C	3	2	2	2	2	3	2	3	0.79	2 hybrid	2 virtual
2B	2	2	0	1	1	2	1	2	0.45	4 hybrid	0 virtual
2D	2	2	2	2	1	2	2	2	0.63	2 hybrid	2 virtual
3C	1	0	2	2	1	2	1	2	0.46	2 hybrid	2 virtual

Table 23

Task 5 Same Sex Groups

All Females	MCOP Descriptors								Mean	Group Dynamics	
Group #	1	2	3	4	5	6	7	8			
1A	2	3	3	2	2	3	2	3	0.83	3 hybrid	1 virtual
3B	3	2	2	3	3	2	3	2	0.83	3 hybrid	1 virtual
3D	3	3	3	3	3	3	2	2	0.92	2 hybrid	2 virtual
5A	3	3	3	3	3	3	3	3	1	2 hybrid	2 virtual
All Males	**MCOP Descriptors**										
1C	3	3	3	3	3	3	3	2	0.96	0 hybrid	4 virtual
2B	2	2	3	3	3	3	1	2	0.79	2 hybrid	2 virtual
2D	3	3	2	2	3	2	3	2	0.83	1 hybrid	3 virtual
3C	3	3	2	1	3	3	3	3	0.88	0 hybrid	4 virtual

Task 1 Transcriptions

In addition to recording each group as they collaborate to complete Calculus AB task, the researcher, who was the teacher of record, took observational notes in the researcher's journal at the start and end of the class periods. If a group called the instructor into their channel to answer a question during a task, a note was made about what was observed while in the channel; the student's demeanor, cameras and microphones on or off, and if all students were

involved in the asking of the question. While there is no generally agreed upon format for writing field notes, the general purpose is to create a catalog of events that indexes the collected data corpus. Anything that, at the moment, seemed interesting or worthy of further investigation was flagged in the researcher's journal.

The goal when selecting transcripts to dissect and compare was to find instances where groups exemplified the type of discourse and collaboration the MCOP[2] labeled a 3. At the same time, groups who earned 0 or 1 scores for multiple descriptors were identified. The challenge was finding brief excerpts that encompassed one specific instance that best explained how the descriptor appears in collaborative groups. Some groups, like 2D, were consistent across the board while completing task 1, earning 2's through every descriptor besides the 5th. There were 24 groups who completed Task 1 and all of them had different scores throughout the eight descriptors, although some ended up with the same total average at the end.

When completing task 1, each student was provided with an electronic version of the task, a blank grid for drawing graphs, and plenty of space for indicating methods, explaining thinking, and justifying responses. Only one completed task was allowed to be submitted per group, so each member of the group had to come to an agreement about each parts' solution and the method best used to obtain it before finalizing. This challenge of this task lies in the connections students are forced to make between what the calculator is asking and what they already know how to do by hand. Linear approximation and drawing lines of tangency is a skill they have learned the procedures to, but this task will evaluate if they understand the reasoning behind this skill and how to analytically approach the concept without a step by step how-to.

Transcript Excerpt 1: Students regularly engaged in exploration, investigation, or problem-solving

Group	**Student**	**Discourse**
1A	Student B:	*Uhm, so does anyone know where we should start? Should we each do a part or should we start with part A together?*

Student D:	*I am going to see if I can figure out what all the parts are asking then I will get back to you guys.*	
Student B:	*Did everyone download the grid? It looks like we have to graph.*	
Everyone:	*Yes, got it*	

Approximately 8 minutes pass without anything being said

Student A:	*I have the graph for part one. I am going to send it to the group. Let me know if ya'll got the same graph*
Student B:	*Mine is the same.*
Student D:	*Mine is close enough. I might have made the window too big.*
Student C:	*How did you guys use the numbers? The -1 to 3.7 and -5 to 26? What does that mean? Is that the graphing window? How do we change that?*
Student B:	*Yes, it's the window. Literally click the window button. Top of your calculator. Anybody do part 2? I might go ahead to part 3 if someone did part 2.*
Student C:	*Oh, got it, thanks.*

From the brief conversation above it is evident the lack of community and productive discourse that takes place in the first ten minutes of the task. Student B, Student D, and Student A are all hybrid students who attended class both face to face and virtually, while Student C was strictly a virtual synchronous learner. While transcribing this groups' discourse for task 1 I was surprised about how much silence they sat in. The only noise I heard for almost half of the task was the clicking of calculator buttons and the girls' sighs. Once or twice Student B would speak up and ask a probing question, but she was never given a thorough or productive response. This group earned one point as descriptor 1, level 1, states: *Students seldom engaged in exploration, investigation, or problem solving. This tended to be limited to one or two students engaged in problem solving while other students watched but did not actively participate*.

Transcript Excerpt 2: Students discuss the task at the appropriate mathematical level

Group	**Student**	**Discourse**

3C	Student E:	*Guys, this is four parts. This is going to take forever.*
	Student F:	*We could split it four ways, but I think we need part A in order to do part B. So, let's just start at A and plug this into our calculator.*
	Student E:	*Oh, she gave us graph paper to graph on. I hate graphing.*
	Student H:	*It's just a line – better than a tangent or secant graph.*
	Student E:	*Man, I hated graphing those last year. I always got them confused with cotangent curves. Or maybe cosecant curves. I can't remember. Pre-calculus all blurred together. I just know that graphing stinks, and why do we need it when we have a graphing calculator that does all the work for us?*
	Student I:	*Part A is easy. I just drew it and uploaded my answer into the channel. Student F is this what you got?*
	Student F:	*Pretty close. Yours is neater so let's use yours to turn in.*
	Student H:	*There is a derivative button on the calculator?! Why didn't we know this?*
	Student E:	*There is? Oh wow, dy/dx. That would have been helpful last chapter. We could have just plugged it into our calculator.*
	Student F:	*No, you can't. Look at part B. It says find an approximate value of the derivative at one. But then we have to use the definition of a derivative to find the exact value.*
	Student E:	*I don't even remember the definition of the derivative.*
	Student H:	*We just had a test on it Student E… it has something to do with the limit as x approaches 0.*
	Student I:	*h approaches 0, not x. Did you guys fail that test or what?*
	Student F:	*Okay, okay, okay… I found the approximation and plugged it in. I uploaded a picture of my work into the channel. What do you think? Does that look right? It feels right.*
	Student H:	*I got the same approximation, so I am sure I would have found the same exact value. We only have about 10 minutes left to do the last two parts, so we need to hurry guys.*

The excerpt above demonstrates how discourse and productive discourse are not necessarily the same thing. During the entirety of this task someone was communicating, but it was not always about the task at hand. Both Student E and Student F were virtual students,

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while Student H and Student I were hybrid students. Student E, Student H and Student I were all seniors who had multiple courses together prior to AP Calculus AB while Student F was the only 10th grader in the class. Descriptor 2 of the MCOP² states that "Most of the students in the group spend less than one-quarter of the task engaged in appropriate level mathematical discourse. There is at least one instance of students' mathematical engagement." Student F attempted to lead his classmates into an engaging conversation, but they preferred to just accept his answers at face value, which is why this was coded as a 0 out of 3. The descriptor says that to earn zero points the group must have: *Most students are not engaged in appropriate level mathematical discourse*.

Transcript Excerpt 3: Students exhibit oral perseverance in problem-solving

Group	**Student**	**Discourse**
3D	Student G:	*I really don't think we did part 2 right. Should the approximate value and exact value be the same?*
	Student J:	*Honestly, at this point, I don't think it matters. Let's just try part 3, and if we have time we can go back to it.*
	Student L:	*What does she mean, 'find an analytical method to find an exact equation for l?' And what draw button? I don't even see a draw on this calculator.*
	Student J:	*Me neither. Not at the top by graph.*
	Student K:	*Maybe you have to press 2nd or alpha first.*
	Student J:	*But then you'd still see the word DRAW on the calculator somewhere.*
	Student G:	*There it is, on the program button, above COS.*
	Student L:	*I don't know how to use this button. There are so many options on this menu.*
	Student G:	*Student L, do you want to use google to try to help us out? Maybe there is a YouTube video I can find.*
	Student K:	*Could we figure out the exact equation for the line without using DRAW? I feel like there is definitely a way to do this by hand.*

Student L: *Maybe, but we don't have a lot of time , left, so let's just skip part 3 and go to the last part. It has to be easier.*

Student G: *Okay, you're right. We have to graph part 4, we are good at graphing, we got this.*

As written above, these four senior girls opted to skip an entire part of the task instead of attempting it. One group member volunteered to research the directions, but she was quickly redirected. The explanation for descriptor 3 in the MCOP[2] states that in order to earn just 1 of the 3 points, students must exhibit minimal oral perseverance, with only one or two students verbally looking for entry and points and solution paths. At one point, Student G offered to ask an online website for assistance, but she quickly went along with the group and opted to skip part 3 of the task. Unfortunately for group 3D, the researcher was looking to observe productive discourse, with all students exhibiting a strong amount of oral perseverance in problem solving. There was no evidence of the group looking for entry points or solution paths to this task, and it appeared the girls were so impressed with learning new buttons on their calculator they forgot the task all together. For that reason, this group earned 0 out of 3 points for descriptor 3.

Transcript Excerpt 4: Viable arguments and critiques

Group	Student	Discourse
5A	Student M:	*I just uploaded my graph [for part 1] into the channel. What do y'all think?*
	Student O:	*Uhm, Student M, did you draw line l or f?*
	Student M:	*What was the difference? Oh f is the parabola. [degrading mumbling], I drew the parabola… the initial function.*
	Student N:	*Oh my goodness I thought I was crazy. Okay, here is my graph.*
	Student M:	*I am just going to agree with what you have Student N. I'm embarrassed.*
	Student O:	*[laughter] Its okay Student M, we have all been there before.*
	Student N:	*I just uploaded my part 2. Both the approximate value and the exact. I don't think I used the definition right for the exact value.*

	Student O:	*It looks good, I am just going to trust you.*
	Student N:	*I know I wrote the definition correct, but can someone check that I plugged everything in? I've never used the dy/dx buttons.*
	Student P:	*How did you get the approximate value Student N?*
	Student N:	*I plugged it into the calculator.*
	Student P:	*Oh…..okay*
	Student O:	*So lets go to part 3. I'm sure Student N is right. I didn't even know there was a DRAW button on the calculator. This could have been helpful last chapter.*
1A	Student A:	*I didn't use the dy/dx button to find the approximate value for the at 1. I think the trace button is easier. What did y'all use?*
	Student B:	*I got exactly 9.*
	Student C:	*Same… thought it would be a decimal since its an approximate number.*
	Student D:	*Me too*
	Student A:	*I didn't get 9.*
	Student B:	*Guess that's why the directions don't say use trace and instead use Calc.*
		Silence within the group for approximately 4 minutes
	Student D:	*[laughter] well I am glad I still have the definition of a derivative memorized. That should help with the rest of part 2.*
	Student B:	*Me too, but what do we do with it?*
	Student C:	*We can plug it in… but to where?*
	Student B:	*Plug what in?*
	Student C:	*9*
	Student B:	*Why would we plug 9 in? And to where?*
	Student C:	*I don't know, do you have a better idea?*
	Student D:	*Why would the approximate and exact answers be different here? Shouldn't they both be 9?*
	Student A:	*That could be right….*

Student C: *Maybe.*

Student D: *What do you think Student B?*

Student B: *Uhm, lets plug 9 into the x in the derivative definition and see what we get.*

Student C: *Isn't 9 the y value? You're probably right Student B. Lets plug it in and see.*

Both the transcript excerpts above come from groups with all female members. After transcribing all the groups that completed task 1, one of the major themes that arose was that groups with all boys, or majority boys, scored an average or a 2 or 3 on descriptor 4. These all female groups, on the other hand, struggled to clarify their own thinking to where it made mathematical sense to their peers. In both above groups, the members were getting frustrated with one another and giving short, truncated responses. Both groups consisted of members who were hybrid learners along with members who were virtual learners. The girls were not offering up meaningful or viable arguments or critiques of one another, which is why both groups received a 0 out of 3 from descriptor 4.

Transcript Excerpt 5: Attend to precision in communication.

Group	**Student**	**Discourse**
2B	Student Q:	*Okay guys, let's look at this task. Four parts. Got it. Lets just do all four parts together, okay. This thing looks hard.*
	Student R:	*I agree, Student Q. So, we have to graph this function and find the tangent line at x = 1. And we have to use the 0 and 2 somehow. What does that matter?*
	Student S:	*Those numbers are the width of the line right? Like we zoom in the… [gets cut off].*
	Student Q:	*Four x squared is not a line Student S, it's a parabola. Can you even have a tangent line to a line?*

Student T: *Look at part 1. The 0 and 2 are included in the function we have to put into y=. But then we have more numbers for our window.*

Student Q: *Aren't those numbers the dimensions? The domain and range?*

Student Q: *I drew the function and sketched the tangent best I could. I uploaded the picture for you guys to see.*

Student S: *Looks good. Part 2 says fine an approximate value of f(1).*

Student T: *Graph looks good… but f(1)? We can just plug it into our calculator and find its value in the table…. [gets cut off]*

Student R: *It says f '(1). We have to find f prime. Why would we use the table for that? The prime value at 1.That's where that straight line touches our graph.*

Student Q: *The tangent line Student R. And isn't the tangent line the same thing as the derivative? And prime? So that's all we are looking for. The approximate derivative values. And I think Student T is right, the table could give us the approximate value but not the exact. You shouldn't use the calculator for this one.*

The transcript above shows a conversation amongst four male students which took place in less than five minutes. There was no quiet, or down time in the group, as they were all constantly talking over one another. During this observation it was notable that Student Q was the only member of the group that took charge and corrected his peers when they were wrong. He was the most assertive member of the group, and he was the only one to redirect his group into using the precise language associated with this task. The group also did not use their calculator appropriately when looking for an exact value, nor did they attempt to make sense of the problem, they just attacked it without reason or a plan of action. As an evaluator, if discourse was the topic being observed, this group would have earned maximum points, as they conversed amongst one another for the entirety of the task, but since this descriptor involved attending to precision, they only earned a 1 out of 3.

Group	**Student**	**Discourse**
3B	Student U:	*So, this thing has four parts. There are four of us. Why don't we just split it up? Take like ten minutes each to do our part then come share our answers.*
	Student V:	*Good idea. I'll do part 1, Student U you do part 2, Student X, you got three and Student W four.*

Approximately 8 minutes pass in silence

	Student V:	*I am done with my part. We only have to turn one answer sheet in. Who wants to turn theirs in?*
	Student U:	*You can Student V since you already have the graph. Upload a photo of it so we can see it.*
	Student W:	*That looks right to me Student V. How did you draw that so perfectly?*
	Student V:	*I am using my ipad! I have an app that has a ruler and different color pens. I love using it for notes.*
	Student U:	*My Ipad broke. I am so jealous, I loved using mine for notes. The colors were fun to play around with. Anyway, I don't know if I am right. I don't know if I used the definition of a derivative right. I think the approximate value is good though because its what the calculator had.*
	Group:	[chatter in agreement]
	Student X:	*I didn't know there was a DRAW button on my calculator so I played with that for awhile but then I gave up.*
	Student W:	*Why did you give up?*
	Student X:	*I don't know how to do this. What is an analytical method?*
	Student U:	*Just something you have to create by yourself by analyzing something… I think, right?. Does anyone want to attempt part 3 then? Or leave it blank? Student X I don't understand why you didn't try Google or something. You had like, what? Ten minutes to figure it out.*
	Student V:	*Student W show us what you got for part 4 and I will look at 3 real quick to see what I can do.*

Student W:	*I just had another graph to draw. I think my line l is right.*
Student U:	*It looks right to me, just combine it with Student V's part 1...* *they need to be on the same graph.*
Student V:	*Guys I got y = 8x + 1.*
Student U:	*Good, good... yes that looks right. So we are done?*
Student W:	*Do you have all the parts Student V? Can you submit it?*
Student V:	*Yep, give me a minute and I'll upload it.*

One critical feature that seemed to occur primarily in the female heavy groups during task 1 was the blind agreement they had for one another. It appeared that no one wanted to argue or ruffle their peers' feathers. Another issue this group had was communicating, in any form. They left the group after submitting their solutions only 15 minutes after entering th channel. Other groups took an average of 27 minutes to finish this task. These group members separated, did their part of the task individually, came back together and trusted one another explicitly, without bothering to check each other's work. Another theme that was coded and made note of after transcribing this groups' attempt at task 1 was that the three girls who attended school as a hybrid learner took control over the task far more than the lone virtual student. Student X, the virtual learner, only spoke a total of two times during task 1 and could not provide an answer to her portion of the task.

Transcript Excerpt 7: Sharing, questioning, and commenting

Group	**Student**	**Discourse**
5A	Student O:	*So, let's go to part 3. I'm sure Student N is right. I didn't even know there was a DRAW button on the calculator. This could have been helpful last chapter.*
	Student M:	*Wow look at all the options under the DRAW menu! I wonder if we will ever use all of these buttons. Why haven't we used this before?*
	Student O:	*Your guess is as good as mine. So I pressed Tangent and it came up saying tangent with parenthesis open. What do I put in the parenthesis? The 4 x squared equation right?*

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Student M: *We are trying to draw the tangent, wouldn't that mean we put in the equation of the tangent line we found in part 2? Student P and Student N, what do you think?*

Student P: *Hmmm, I really don't know guys, I was just trying to follow along with you guys. Doesn't give me confidence that you are all guessing though.*

[Student N is silent]

Student M: *Student N? You there?*

Student O: *Guess not…. But Student M I used Y-VARS to put in Y1 and I keep getting error. So maybe you're right Student M.*

Student M: *We can at least try the tangent equation… 8x + 1*

For this transcription I picked up where it left off from excerpt 4. This group of two hybrid students, Student P and Student M, pair with two virtual students, Student N and Student O, proved to be an interesting cohort. They started the task strong, everyone was trying to participate, but once the task proved to be challenging Student N appeared to leave the group. After the tenth minute she was not heard from again until the group submitted their solutions. Student P also got silent after she admitted confusion with part two and could not get a straight answer from her group members. This group earned one point out of a possible three because only two members of the group opted to share, while the others chose to listen.

The eighth descriptor of the MCOP[2] covers the amount of time the students were engaged in peer-to-peer discourse throughout the task. There were 23 groups that completed task 1, and the average time spent collaborating was 16.25 minutes. The code, from 0-3, that each group received was based on the time the students in the group spent discussing the task with one another, and the amount of time the group spent working on the task total. After comparing the codes, a commonality arose between the three groups who earned 0 points for this descriptor. All three groups, 1B, 2E, and 3F were comprised of two male and two female students, with the boys in all three groups learning virtually, while the girls learning hybrid.

Table 24

Time spent in peer-to-peer discourse in groups during task 1

Group	Time	Group	Time	Group	Time	Group	Time
1A	15.67	2A	16.9	3A	22.54	5A	22.03
1B	11.21	2B	19.11	3B	15.87	5B	9.22
1C	29.87	2C	27.19	3C	18.01	5C	17.84
1D	19.37	2D	12.65	3D	15.49	5D	11.98
1E	27.81	2E	3.45	3E	28.19	5E	25.19
		2F	19.12	3F	7.02	5F	18.19
		2G	20.18				

Task 5 Transcriptions

Throughout the school year the students enrolled in AP Calculus AB were given tasks to do that went along with the unit they were being taught. There were three tasks given to students while they were in groups between task 1 and task 5. A portion of this task, seen in Appendix C, was the last one administered as it aligned with the last unit of the curriculum. The final task provided was given on a Wednesday in March, less than two months before the AP Exam was to be taken. On this particular Wednesday 84 AP Calculus AP students were in attendance, and since this was the last task of the school year, the researcher attempted to replicate the same groups that were in task 1. These groups could not be duplicated perfectly as there were 92 students in attendance for the first task, and only 84 for the fifth. Out of the 84 students in attendance for task 5, 48 of them were physically in school, while 36 of them were both synchronous and virtual.

The analysis of the students' discourse using the MCOP2, and thematic coding was done to attempt to answer Research Question 3: *In what ways do the dynamics of the students' group change because of the blended learning environment?* Ultimately, the researcher compared the eight levels of discourse as described by the MCOP[2] instrument between the

groups who completed task 1 and task 5, therefore, the researcher chose to only focus on the groups that had the same participants throughout both tasks.

The instructions to this task were familiar to the students: match each graph with the corresponding definite integral, equations of revolution, and word problem, and calculators were not allowed. Students who were in physically in class joined their group in their appropriate channel via Microsoft teams and turned their cameras on so their virtual group members could see that they had each task card on different colored cardstock. Students who were completing this task at home had each individual card displayed electronically in a PowerPoint. When students in class were discussing card G2, their group members at home could find G2 on a power point slide and drag it to its matching partners. Ideally, both the hybrid learners and virtual learners could be completing the same task using different methods in multiple locations.

Below are passages from transcripts that were pulled from select groups while they completed task 5. The groups selected were comprised of the same students for both task 1 and task 5, which made comparing the qualitative results from the $MCOP^2$ less formidable. The researcher also picked selections from groups that had a significant change (2 or 3 points) from task 1 to task 5 within multiple descriptors in an attempt to show exactly how much change could possibly occur between the tasks. Each of these eight groups had only members of the same sex, and when selecting the discourse excerpts to focus on, gender was not a characteristic the researcher went into the study hoping to analyze, it just came about naturally.

Transcript Excerpt 1: Students regularly engaged in exploration, investigation, or problem-solving

Group	**Student**	**Discourse**
5A	Student M:	_Okay ladies, there are a lot of cards here. There are the problems, V cards, E cards and the graphs on G. 12 each._
	Student O:	_Would it be best to set up each problem on the first two pages? Or should we integrate the V cards first?_

Student N: *I think integrating would take too long unless we split the cards up and I do not want to do that this task. I'd rather work together because it's so easy to make one mistake when integrating and then you have to redo everything.*

Student P: *I thought starting with the E cards make the most sense because if we can identify which problems use the disk method or which used the washer, we could separate the cards to match. Does that make sense?*

Student M: *Hmmm those are both good ideas.*

Student N: *Let's try to identify disks and washer and see if that opens up an easy route to finding the integrals and equations.*

Student O: *Deal, so G1 is….*

Group: [in unison] *Washer*

Student O: *G2…*

Group: [in unison] *Disk*

[After all graphs were separated into washer or disk]

Student O: *Does it make more sense now to separate the E cards or the V cards?*

Student M: *There is no real way to split the V cards is there? We could look at the domain and compare it to the definite integrals.*

Student N: *Look at card V4… do you see how it's the cube root of y squared minus y over 4 squared? That means there is a little r and a big r.*

Student M: *So it's a washer.*

This task allows for multiple entry ways to make connections between the cards. When selecting this task, the researcher knew the primary struggle amongst most groups would be how to start, especially without the assistance of a graphing calculator. The decision was made to disallow calculators for this task because volume of revolution free responses have been on the non-calculator part of the AP Calculus AB test for the previous decade. For task 1, group

5A earned 5 out of 24 points, but for task 5 they successfully earned all 24 points. The group members and their learning environments stayed the same; both Student M and Student P stayed hybrid students and were in class during task 5, while Student O and Student N stayed virtual all year. The researcher and teacher, once the data was coded and it was revealed that this groups' average score jumped from a 21% to a 100% on the MCOP[2], was intrigued by this groups' dynamics. The researcher went back and re-listened to all five of 5A's transcripts, and the primary difference that was evident in every single task was Student N.

Student N was a senior during the 2020-2021 school year. Her family opted to keep her and her siblings home during this school year due to the threat of COVID-19. Once students elected to attend school virtually, they were no longer permitted to join any extracurriculars through the school, although they could attend games, plays, or other school events. Student N was an avid tennis player with her high school during her first three years, and her senior year was the only one where she did not play. During task 1 Student N spoke the least amount in her group, and after she was chastised for being wrong, she shut down and did not communicate for the rest of the task. Fast forward to task 5 and Student N took charge in the group, making valid arguments, speaking with precision, and engaging throughout the entire task.

Transcript Excerpt 2: Students discuss the task at the appropriate mathematical level

Group	Student	Discourse
3C	Student F:	*These three-dimensional cards really bring this unit to life. Sometimes when we are graphing, I can't see what 3D shape the graph makes. Like, I can't see if it's a cylinder or a sphere and I was afraid to say something because I thought there might be something wrong with me.*
	Student E:	*No, some of these graphs are just hard to see. How does the graph of G3 make that 3D shape? It looks like the interior of G3 is inverted or concave, but the 2D graph doesn't show that.*
	Student H:	*You're right, the 3D shape makes it look like a washer, but the 2D shape is a disk.*

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Student F and Student I: *No they're both washer.*

Student F: *Think about the 2D G3 rotating about the line x = 1. There is going to be a gap between x= 2 and x = 1 for 360 degrees. That makes it have a hole and makes it a washer method.*

Student E: *How is this so easy for y'all?*

Student H: *Student E, look at the 3D card for G3. Do you see how the pink inside creates a small circle and it looks like its coming out until it meets the green part?*

Student E: *Yah, I get that.*

Student H: *Ok, well the interior pink part is the point of the 2D graph at (1,3) while the point that looks like its getting closer to you is that point at (0,4). When we are done with this task, I'm going to find a desmos that show a figure go from two dimensional to three dimensional. It's really cool and it might help you out Student E.*

Student I: *KhanAcademy has something like that too… I will email it to you.*

Student E: *Thanks guys… I mean, I get how to integrate, and I get the procedure of revolutions I just can't visually picture them. Maybe I don't have spatial awareness…*

Although the members of this group didn't change, Student H's learning environment did, as he opted to become a virtual student during the Spring 2021 semester. That made all four members of group 3C virtual learners. As an avid hockey player, Student H did not want to risk getting sick during season, so he and his family made the decision to change his learning environment for the final semester of his senior year. When I transcribed and then coded task 5, I was especially interested in how groups faired that earned zeros in certain areas on previous tasks. The group transcribed above, 3C, earned a 0 for descriptor two: the *entire group spends two-thirds or more of the task discussing the mathematical activity at the appropriate level for the class.*

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While completing task 1 group 3C opted to follow the lead of their sophomore Student F as he, almost single handedly, completed the task. For this task every member asked questions, and their groupmates took time to answer them. Student E, in particular, seemed to be confused about the transition of a two-dimensional shape into a three-dimensional shape caused by rotating about a line. Instead of moving on with the task and telling Student E to watch while the rest of the group worked, the group stopped and tried to explain the concept to the best of their abilities, so they could finish the task together, which earned them the three full points for this descriptor.

When selecting groups' transcriptions to focus on, the researcher kept coming back to this group, 3C. As their teacher the researcher was aware of what each student is capable of, and Student F, the lone sophomore enrolled in AP Calculus AB, is a superior math student. In the fall of his sophomore year, he made a 34 on the math portion of the ACT, which frustrated him, as he aimed for a perfect 36. Student E, on the other hand, was forced to take Algebra I as an 8th grader, which propelled him down an advanced mathematics track for his entire high school career. Student E discovered in Algebra-II his tenth-grade year that math was not his strong suit, as he earned consistent C's throughout the year. Regardless, he chose to stay in Pre-Calculus honors his junior year and AP Calculus AB his senior year, despite his struggles. During task 1, the knowledge and confidence gap was evident while the researcher was transcribing the groups' discourse. Student F immediately took charge of the group, and the other three boys, including Student E, stepped back and let him. Student I and Student H contributed throughout task 1, but they looked to Student F for guidance and reassurance while Student E did the bare minimum. This dynamic shifted considerably during task 5. Student F jumped to take charge, but Student E voiced his opinion immediately. A few minutes after the task started Student E recognized where his deficit was, but instead of stepping back and watching his peers complete the task he asked for assistance.

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Group	**Student**	**Discourse**
2B	Student Q:	_Do we all have the cards in front of us? Student T and Student S, can you see them on your computer?_
	Student T:	_Yes, it's in a PowerPoint._
	Student S:	_Student T, do you see we can drag the cards and put them together? Like, make a stack of them._
	Student T:	_That is seriously cool. Does the stack tell you if you got them right or not?_
	Student S:	_I doubt we are that lucky, but we should suggest that!_
	Student Q:	_Look at these colored graphs, the three-dimensional ones. I am pumped that I can see these things in 3D now. Should we use these first and try to connect the 2D pictures? Or should we try to integrate first? We only got 45 minutes, do you think that's enough time to integrate all 12 of these?_
	Student R:	_I don't want to start this off by integrating. Let's work smarter, not harder. What is the one thing that can connect the graphs to the integral to the problems?_
	Student Q:	_We can separate the disks from the washers right? Don't they have to be one or the other?_
	Student T:	_Or we can find the line of revolution. All of the graphs have to have that. Look at G7. The line of rotation …_
	Student S:	_Axis of rotation, it's the axis_
	Student T:	_No, Student S, they can be lines… look at G7. Its line of rotation is y =-1. Now look at the E1 card. It's the only card that says 'revolve about the line y = -1.' And look again, the domain of G7 is [-1, 1] which means the definite integral has to be from -1 to 1, which means it has to be V5._
	Student Q:	_Or V8. Could be that one too._
	Student S:	_True but that really narrowed it down! Nice, Student T!_
	Student R:	_Great idea Student T. I don't know if that will work on all of the combinations because there are a lot of revolutions about the y and x axis, but it will definitely get us started._

This group earned zero out of three points for this descriptor of exhibiting oral perseverance when problem solving while they completed task 1. This score was based on their inability to get started with any of the four parts. During task 1 Student Q attempted to take charge of the group, but his peers did not follow him and kept questioning his decisions until he stopped offering them. After 15 minutes Student Q shared his answers for all four parts with the group and Student R changed a few things without discussing why, and the group submitted their work. The entirety of the task was spent with the boys conversing, but none of it was productive as they were talking over one another and disagreeing without offering solutions or valid arguments. That completely changed during task 5 where they earned top scores for their productive discourse.

During task 5 the boys came into the activity confident. It appeared they were excited about the task cards and jumped right into solving the task. Instead of debating on how to start the task they each offered a valid entry point into the task, and each group mate considered the option instead of dismissing it. Student T contributed a way to start the task that his peers liked, but when they realized his method could result in two possible answers instead of one, they brainstormed another process to complete the task.

The researcher specifically selected this group to transcribe and analyze because all four boys were hybrid during the first semester, but Student T and Student S decided to attend school virtually during the Spring of 2021. In class these boys are attentive, responsible, and motivated to do the best that they can in all that they do. They are very competitive with one another, and that was evident during task 1. This competitiveness was not witnessed in task 5, as they were incredibly cooperative and encouraging with one another. While completing task 5 Student Q and Student R were sitting right beside each other in class, while Student T and Student S were in their respective homes. It would have been easy for Student R and Student Q to take over the group, but they patiently waited on their partners to find the correct cards on the PowerPoint before moving on to the next card. This vast improvement is why group 2B earned

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the full 3 points for descriptor 3: *Students exhibited a strong amount of oral perseverance in problem solving. All students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary.*

<u>Transcript Excerpt 4: Viable arguments and critiques</u>

Group	**Student**	**Discourse**
5A	Student O:	*These 'revolve about the y-axis' all look the same to me. And there are four of them. How can we figure out which graph goes with which?*
	Student P:	*I'm looking at the 2D graphs, revolve around the y-axis has to be G11, G6, G1 and G2 right? That's four. And we can integrate to determine which function goes with which graph.*
	Student N:	*Student P I think you have it backwards, G11 revolves around the x-axis not y. I honestly don't think you can look at the 2D graphs to figure out the axis of revolution. Look at G4. That graph could revolve around y or it could revolve around x, you can't tell.*
	Student O:	*Same with G5. Is it easier to tell by looking at the 3D graphs?*
	Student P:	*I don't know guys… look at the G11 3D graph. That revolves around the x-axis. I see it looks like its rotating around the y too, but why can't it be both?*

[Student N and Student O start talking at once]

| | Student M: | *No… no stop. Wait, let me show her. She needs a visual.* |

[Student M picks up the G11 card and folds it along the x-axis to show it's a line of symmetry, not the line of reflection. Then holds a pencil horizontally and uses her other hand to create a circle by rotating around the pencil]

	Student M:	*The pencil is the y-axis, my finger the sphere, or circle, however you want to picture it. It never touches the y-axis but it does go through the x-axis, which means the x can't be a line of rotation.*
	Student P:	*You're right, you're right, you're right. So that means G8 also rotates around the y-axis.*
	Student N:	*Yes, as does G5 and G4.*

When selecting excerpts from transcripts that align with each descriptor, the researcher realized that group 5A could have been selected for all eight. In fact, this group was only one of two to earn 3's for all 8 descriptors during task 5. Student P and Student M remained hybrid students during the spring semester while Student N and Student O remained virtually synchronous. Their learning environments did not change throughout the year, but their productive mathematics discourse improved.

Although the passage above is very brief, it is evident that when a group member had a misconception about the task, the rest of the group jumped at the chance to help. Both Student M and Student P were completing the task in class, sitting next to one another, so it was easiest for Student M to use visual aides to assist Student P in making the appropriate connections. The girls in this group could have dismissed Student P and proceeded on with the task but they all stopped to ensure she had enough knowledge to move on with the group. Student M's reasoning resonated with Student P, and instead of shutting down once she realized she was wrong, Student P took her peers' critiques, learned from her mistakes, and kept productively contributing.

Transcript Excerpt 5: Attend to precision in communication.

Group	Student	Discourse
2D	Student Y:	*I am going to use deductive reasoning to figure out what integral and graph goes with card V3, but I don't think I would get this one right if it was given on a test. I don't understand the two pi on the outside of the integral when every other definite integral has just pi. What makes this different? I am looking at both the 2D and 3D cards and I don't see one that is so different from the others that would explain the two pi.*
	Student Z:	*I noticed that too…. Here is my thought. Let's pull out cards E5 and E12 because those are the only cards with only one function.*
	Student AA:	*I see what you are saying Student Z, but if you look at V3, the indefinite integral is $9 - x^2$. That's the formula for a circle. So like Student Z said, it has to match with E5, but there is no 2D circle.*

Student Z: *It has to be G1 since it revolves around the x-axis. It will make a perfect circle with diameter of 6.*

Student BB: *Integral from -3 to 3. The integral of 9 minus x^2 is 9x minus x^3 over three. Then when you integrate from 3 to 0 you get 18 minus 0. And 18 times 2 pi is 36pi.*

[Group responds in agreement with Student BB]

Student Z: *Then what graph matches with E12? I know what a sinusoidal curve looks like, and I am tempted to say G6, but depending on how zoomed in we are to the graph, it could be G7 or even G2.*

AP Calculus AB is an advanced mathematics course that generally only top math students attempt while in high school. When the researcher was listening to the groups work out the tasks, it was assumed this descriptor would have the highest scores of the eight on the MCOP[2]. Precision in communication is a skill that is focused on as early as elementary school. Proper vernacular is essential because vocabulary is strongly related to reading comprehension (Fitzgerald & Graves, 2005), vocabulary is a predictor of students' comprehension and content area learning (Espin & Foegen, 1996) and lack of vocabulary knowledge can negatively affect learning content (Fisher & Frey, 2008). In order for students to have made it to AP Calculus AB and be successful in the class, they must understand the vocabulary and be able to use it effectively. The excerpt above might appear to be a normal conversation amongst four boys completing a task together, but the researcher saw that they consistently used the right vocabulary, they articulated their thoughts using appropriate-level Calculus vernacular, and their explanations were clear and accurate. As seen throughout task 1, it is easy for high school students to get together and "forget" the proper terms for mathematics concepts. Group 2D did not fall into bad habits and kept their discourse at an appropriate AP Calculus level. For all of these reasons, group 2D earned the full 3 points for descriptor 5. During task 1, group 2D earned only 1 point for this descriptor. This was based on their hurried responses to one another, indecisiveness, and repeated use of the words 'it' and 'that.'

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Group	**Student**	**Discourse**
3D	Student K:	*Guys, I am confident that we matched our cards are matched right, so let's move on to actually solving the integrals. I don't want to split them up, that's how we make mistakes. Can we just do them together one at a time?*
	Student G:	*I'm fine with that because I am still struggling with determining which function is little r and which is the big R.*
	Student J:	*I started number one already, x^2 plus 2 squared minus negative 1 squared. We have to foil and I got x to the fourth plus four x squared plus 3 dx.*
	Student L:	*Integrate that and you get one over five x to the fifth plus four over three x to the third plus three x. I wish we had calculators.*
	Student K:	*Don't forget the pi. Times pi. When we integrate from one to negative one I got 136 pi over 15. You don't need a calculator Student L, just find the common denominator of 5 and 3, its 15. So multiply the 3 by 15, the 1 by 3 and the 4 by 5 and you get 3 over 5 plus 20 over 3 plus 45.*
	Student J:	*Then subtract the big R and its negative three over 15 minus 20 over three minus 45. That is 136 pi over 15. Like you said Student K. That wasn't bad. Let me try number two. The definite integral goes from 4 to 0…*
	Student K:	*0 to 4*
	Student J:	*Sorry, 0 to 4. That's right, because 0 is the axis. The line y = 4 is going to be the little r, the hole in the washer…*
Student K and Student L:		*Correct*
	Student J:	*So, four squared minus four minus the square root of x squared. We have to distribute that*
	Student K:	*I got 16 minus 16 minus 8 square root of x plus x dx.*
	Student J:	*I was getting there! And the 16's cross out and we need to integrate 8 square root of x minus x.*

| | Student G: | 16 over 3 x raised to the three halves minus x squared over 2. |

| Student K and Student J: | 104 pi over 3. |

When selecting an excerpt of a transcript that best describes a level three discourse for descriptor six, the researcher had a lot to choose from. Almost all the groups were engaged in this task and their talk primarily focused on the task throughout the entire period. Group 3D was specifically chosen because it was one of the few groups that had a member go from attending school virtually to attending face to face. Student K started the school year as a virtual synchronous learner, but when the district allowed students to switch learning modalities, she opted to learn in the physical classroom. Considering that, the researcher paid close attention to the groups' discourse for tasks three, four and five, all of which Student K was in class for. During tasks 1 and 2 Student K was not overtly present in her group. She offered up suggestions that usually were not valid and often ignored by her group. Student K contributed the least amount during both tasks 1 and 2 as well, even though her group members Student J and Student L were both participating virtually. In tasks 4 and 5, Student K was a different student. She was not afraid to take charge of the group, she worked ahead, and she offered valuable suggestions when her group members were stuck or confused. As you can see in the passage above, Student K moves the group from matching task cards to solving the integrals. She also works ahead of her peers to ensure she has the correct answer and does not have to rely on them for the solutions. The researcher was able to see a significant difference in Student K's level of discourse and engagement from task 1 to task 5.

Transcript Excerpt 7: Sharing, questioning, and commenting

Group	**Student**	**Discourse**
3B	Student W:	*Guys, I am sorry, I don't know why this is such a challenge for me.*
	Student V:	*It's okay Student W… do you think you're able to find the matches between the definite integrals and the equation cards?*

Student W: *I mean, I can try.*

[Student W stops talking, but is grabbing the cards in front of her]

Student U: *No, don't do it alone. We are here to help. And Student X needs to see or at least hear what you are doing.*

Student W: **[sighs]** *Okay, but I bet I am wrong. E5 with V3, E9 with V10, E10 with V12…*

Student U: *Why V12?*

Student W: *I told you guys I did not understand this! I think its V12 because it has two equations, like E10, so its washer. And the little r is 4 minus x.*

Student X: *Student W, that's all correct, but V12 has four minus the square root of x as its little r, which means the graph is a radical, or half a parabola. Look at the graphs real quick… which graph looks like a half of a sideways parabola?*

Student W: *G9, or G11, or G8. All parabolas.*

Student V: *But the 4 means something. V12 is four squared minus another four minus the square root of x. What does the four mean to the integral?*

Student W: *It's part of the integral. The end of the domain. The… I don't know.*

Student U: *You are so close Student W. Stop getting frustrated. Look at the E cards again…*

Student W: *Oh, E2.* **[mutters to herself, visibly upset]**

Student V: *Don't stop there. What cards go with what?*

Student W: *E2 with V12 with….. G9*

Student U and Student X: *Exactly!*

According to Heibert and Grouws (2007), productive struggle is using existing understandings to engage with problems that do not have apparent solutions, persevering in making sense of mathematics during problem solving, and solving problems and grappling with

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key mathematical ideas that are within reach. When evaluating the groups using the MCOP[2], while considering descriptor 7, Heibert and Grouw's definition of productive struggle was referenced. Although multiple groups struggled with task 5, the researcher selected to highlight 3B's struggle, as Student W could not grasp the connection between the definite integrals and the equations of the two bounds. Student W exemplified the definition of productive struggle during this task as her comprehension was just out of her reach, but she had the knowledge to persevere through her misconceptions. Student X, being the lone virtual student, did not contribute as much as Student V and Student U to Student W's struggle, but she still prodded her in the right direction when she was obviously stuck. Group 3B earned a 3 for descriptor 7 because all four of the girls contributed to the completion of the task and along the way they listened and clarified Student W's questions and clarified her misunderstandings with patience.

Table 25

Time spent in peer-to-peer discourse in groups during task 5

Group	Time	Group	Time	Group	Time	Group	Time
1A	37.71	2A	27.45	3A	43.48	5A	41.12
1B	30.23	2B	21.33	3B	27.14	5B	22.17
1C	39.22	2C	40.09	3C	42.56	5C	23.1
1D	24.71	2D	39.56	3D	40.15	5D	43.33
1E	44.18	2E	44.45	3E	41.23	5E	41.17
		2F	38.65	3F	30.09		

When the researcher was coding descriptor 8; *considerable time (more than half) was spent with peer-to-peer discourse related to the communication of ideas, strategies and solution,* they chose to time the meaningful and productive discourse which was taking place. Productive discourse includes students comparing and contrasting ideas and methods, constructing viable arguments, critiquing each other's reasoning, and helping each other make sense of mathematics. (Hattie, Fisher, & Frey, 2017; National Council of Teachers of Mathematics [NCTM], 2000). Meaningful discourse can precede work on a math problem, be part of the work on a problem, or follow on the heels of individual time spent working on or

thinking about a problem. The effectiveness of discourse is not based on the length of a discussion, or the amount of writing done, but in what is being communicated. For the purpose of this study, and to use this eighth descriptor effectively, the amount of time the students in each group discussed the task in a productive yet meaningful manner was coded.

The average time each group spent on task 5 was 35.79 minutes. The length of the class time for task 5 was 45 minutes, so the average group took 78% of the allocated time to complete their task. For task 1, class was 30 minutes and the average time to complete the task was 16.25 minutes, or 54% of the class time. A huge shift occurred between the first and last tasks where discourse was primarily led by one or two students during task 1 but the entire group worked together in 15 of the 23 groups during task 5.

Summary

This study focused on oral mathematical discourse in an AB Calculus AP classroom. This chapter focused on the results of the three research questions by providing multiple hypothesis, conducting a two-sample *t*-test, using a chi-square test of association to determine if the variables of the study were independent of one another, levels of student discourse, and an overview of how each descriptor appeared within the collaborative learning groups. The next chapter will further discuss these results and findings in connection with answers to the three research questions. Finally, a discussion will follow to connect the findings of chapter 4 with prior research and recommendations for future research will be explored.

CHAPTER V: IMPLICATIONS, RECOMMENDATIONS AND DISCUSSIONS

This mixed-methods action research study aimed to determine whether students' productive mathematical discourse was impacted by the blended learning environment they were placed in due to COVID-19. The study took place in all AP Calculus AB classes within a large suburban high school in West Tennessee during the 2020-2021 school year. Provasnik et al. (2016) reported that the Trends in International Mathematics and Science Study (TIMMS) indicated that the United States is lower than other First World countries in mathematics achievement. At local, state, and national levels, educators have strived to face this problem with the expectation of boosting mathematics performance. However, with the quick onslaught of COVID-19, educational systems across America had to change their instructional modalities and limit the number of students in the buildings to help prevent the spread of the virus. Chapter 2 reviewed research studies about discourse, blended learning environments, and dynamic groups. A gap was found in the literature regarding the combination of the three components of education, especially within a secondary mathematics classroom. This study investigated the relationship between productive mathematical discourse and students completing group tasks in a blended learning environment to address this educational gap. This study provided a possible solution to help get students engaged while using productive mathematical discourse in multiple learning environments. This chapter will discuss the results of the research and its findings. It will also offer implications for policy and practice and give recommendations for future research.

Interpretations of the Findings

The findings are listed in order of the research questions. Results were either acquired through statistical analysis or thematic coding. The research questions for the study were:

1. How does the student's learning environment impact their mathematical discourse when completing tasks in an online synchronous work environment?

2. How does the student's physical learning environment affect their level of productive discourse when placed in a virtual group?

3. In what ways do the dynamics of the group change because of a blended learning environment?

The research questions were addressed using two frameworks: constructivism and sociocultural theory. Mercer (2010) states, "Sociocultural researchers commonly emphasize that language is a cultural and psychological tool; classroom dialogue could have an important influence on the development of children's reasoning" (p. 2). Researchers believe knowledge and understanding are created together; talk serves as a tool in which meaning is negotiated. Education depends on students being able to develop shared knowledge through discourse (Mercer, 2010). Both constructivism and sociocultural theory are concerned with the activities that children engage in to learn. This study was completed through the lenses of both sociocultural theory and constructivism; when observing and interpreting a cooperative group, the broader social system and interpretations about the individuals' thinking based on their participation was considered and analyzed.

The data acquired from The Mathematics Classroom Observation Protocol for Practice (MCOP^2) instrument revealed that in certain situations, the amount and quality of productive discourse amongst students changed throughout the school year. Analysis of the student discourse results ushered a path toward developing an understanding of the relationship between different learning environments and productive discourse. Students in AP Calculus AB engaged in tasks and collaborative learning where they are expected to use appropriate vocabulary when talking to their teacher and peers. The population in this study consisted of advanced mathematical students who were superior in math their entire middle and high school

years. The expectancy going into this study was that these top-tier students would be able to produce highly productive discourse in a traditional classroom setting, but would they be able to produce that same quality discourse in a blended learning environment, with or without the teachers' presence?

Mathematical discourse often depends upon the potential of the mathematical task, the extent to which activity requires students to explore mathematics, multiple pathways to a problem, or make sense of the content (NCTM, 2014; Smith & Stein, 2011). According to Boston (2012), a task with high potential pushes students to use complex, nonalgorithmic thinking where they must explore concepts, which may be solving a genuine, challenging problem or identifying patterns, or making conjectures. On the contrary, tasks are of lower cognitive demand and have less potential for mathematical conversations (Boston, 2012). The five tasks selected for this study were analyzed using Smith and Stein's (1998) Task Analysis Guide. Each task was recommended from the Advanced Placement College Board website and was evaluated to be at the *Doing Mathematics* or *Mathematics without Procedures* levels.

Research Question 1: How does students' learning environment impact their mathematical discourse when completing tasks in an online synchronous work environment?

- H_0: The average discourse between hybrid students in a group is not equivalent to the average discourse between students in a blended learning group for any of the tasks.

- H_a: The average discourse between hybrid students in a group is equivalent to the average discourse between students in a blended learning group for any of the tasks.

The study found no statistical significance between the discourse of the all-hybrid groups and the hybrid, virtual student mixed groups. There were 110 groups total throughout the five tasks, with 42 of those groups consisting entirely of hybrid students, or students who learned in the physical classroom, face-to-face with their instructor. The remaining 68 groups had a combination of hybrid students and virtual students, who were students who had never stepped

foot on campus and learned strictly through synchronous online classes. The researcher administered 2-sample t-tests for each of the five tasks to compare the means of the two groups' discourses. All five t-tests resulted in a *p*-value greater than the critical value of $\alpha = 0.1$. Due to these high *p*-values, the researcher did not have enough evidence to reject the null hypothesis that there was no relationship between the discourse occurring in groups with only hybrid students and discourse occurring in groups with both hybrid and virtual students.

The researcher believes that there was no statistical significance amongst the groups because the students enrolled in AP Calculus AB are similar in academic achievement and motivation to perform. The population of these classes are 17 and 18-year-olds who have been enrolled in advanced courses their entire high school career. The expectations are there for them to be superior students in academic achievement and in every aspect of the learning environment. Students and their families did not choose the virtual learning environment over the blended learning environment due to academic concerns; their choices were primarily based on health concerns. Due to the consistent nature of the students' academic achievement and knowledge level, the researcher believed that every student would enter their random group with the same social status, regardless of if a group consisted of only students participating virtually or if a group had students who were participating in the physical school and at home, this study did not reveal a significant difference in their levels of productive discourse.

Research Question 2: How does the student's physical learning environment affect their level of productive discourse when placed in a virtual group?

$H_0 1$: There is no difference in the groups who regularly engaged in exploration, investigation, or problem solving over the course of the assignment from task 1 as compared to the groups who regularly engaged in exploration, investigation, or problem solving over the course of the assignment from task 5.

$H_a 1$: There is a difference in the groups who regularly engaged in exploration, investigation, or problem solving over the course of the assignment from task 1 as compared to the groups

who regularly engaged in exploration, investigation, or problem solving over the course of the assignment from task 5.

$H_0$2: There is no association of the number of groups discussing the task at the appropriate level for the entirety of the task between task 1 and task 5.

H_a2: There is an association of the number of groups discussing the task at the appropriate level for the entirety of the task between task 1 and task 5.

$H_0$3: There is no difference in the proportion of groups who exhibited a strong amount of oral perseverance in problem-solving from task 1 as compared to the proportion of students who exhibited a strong amount of oral perseverance in problem-solving from task 5.

H_a3: There is a difference in the proportion of groups who exhibited a strong amount of oral perseverance in problem-solving from task 1 as compared to the proportion of students who exhibited a strong amount of oral perseverance in problem-solving from task 5.

$H_0$4: There is no difference in the proportion of the groups who demonstrate viable arguments and critiques of the reasoning of others as an integral component of the task with students engaged in productive discourse over the course of the assignment from task 1 as compared to the proportion of groups who exhibit viable arguments and critiques of the reasoning of others as an integral component of the task with students engaged in productive discourse over the course of the assignment from task 5.

H_a4: There is a difference in the proportion of the groups who demonstrate the viable arguments and critiques of the reasoning of others as an integral component of the task with students engaged in productive discourse over the course of the assignment from task 1 as compared to the proportion of groups who exhibit viable arguments and critiques of the reasoning of others as an integral component of the task with students engaged in productive discourse over the course of the assignment from task 5.

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H_05: There is no difference in the proportion of the groups who "attend to precision" in communication from task 1 as compared to the proportion of students who "attend to precision" in communication from task 5.

H_a5: There is a difference in the proportion of the groups who "attend to precision" in communication from task 1 as compared to the proportion of students who "attend to precision" in communication from task 5.

H_06: There is no association between the groups who maintain appropriate level mathematical conversations of the topic over the course of the assignment from task 1 as compared to the proportion of students who maintain appropriate level mathematical conversations of the task over the course of the assignment from task 5.

H_a6: There is an association between the groups who maintain appropriate level mathematical conversations of the topic over the course of the assignment from task 1 as compared to the proportion of students who maintain appropriate level mathematical conversations of the task over the course of the assignment from task 5.

H_07: There is no difference in the proportion of the students who were sharing, questioning, commenting, clarifying, and recognizing others' ideas over the course of the assignment from task 1 as compared to the proportion of students who were sharing, questioning, commenting, clarifying, and recognizing others' ideas over the course of the assignment from task 5.

H_a7: There is a difference in the proportion of the students who were sharing, questioning, commenting, clarifying, and recognizing others' ideas over the course of the assignment from task 1 as compared to the proportion of students who were sharing, questioning, commenting, clarifying, and recognizing others' ideas over the course of the assignment from task 5.

H_08: There is no association between the amount of time students spent on peer-to-peer discourse from task 1 to task 5.

H_a8: There is an association between the amount of time students spent on peer-to-peer discourse from task 1 to task 5.

This research question was answered with the results of the chi-square test of independence which was used to analyze the data for all eight descriptors of the MCOP^2. This quantitative data analysis revealed significant differences among six of the eight group relationships. The 110 individual groups were scored using a four-scale rubric, with each of the eight descriptors from the MCOP^2 earning zero to three points in each of eight categories: engagement, appropriate level of discourse, oral perseverance, viable arguments and critiques of reasoning, attending to precision, inquiry, and time spent on productive mathematical discourse. Each portion of the MCOP^2 was worth a maximum of 3 points, which meant each group could score 24 points for each task. It is worth noting that the students were not aware that the researcher/teacher was evaluating the groups' discourse while they collaborated on the tasks, they only believed she was observing the groups to ensure they were not cheating or sitting idly by why their peers completed the work. The teacher never emphasized discourse or encouraged strategies for the students to use within their groups to promote collaboration. The changes between task 1 and task 5 happened naturally, without the teacher's input or prompting.

The Pearson Chi-square test of association established the independence of group behaviors among two behaviors; students engaged in exploration, investigation, and problem solving, (descriptor 1) and time spent engaged in appropriate peer to peer discourse (descriptor 8). Using the accepted critical value of $\alpha = 0.1$, the chi-square tests revealed that the other six descriptors show a statistically significant difference between the discourse that occurred in task one and the discourse that occurred in task 5. This result allowed the researcher to reject each null hypothesis and accept the alternative hypothesis that states there was a difference in the proportion of students who showed different aspects of discourse from task 1 to task 5 for six of the eight descriptors of the MCOP^2.

Descriptor one and descriptor eight showed no significant difference between the groups of task 1 and task 5. Descriptor eight highlights the need for all students to be active participants

in the classroom dialogue (Manoucherhri & St. John, 2006), while descriptor one focuses on student exploration that attempts to promote a stance of mathematics as a discipline that can be explored, reasoned about, and one that 'makes sense' (Barker et al., 2004). This finding was surprising as the researcher anticipated that the students' engagement would be lower when they were all participating virtually for task 1 and would be subsequently higher for task 5 when group members were participating physically in class. On the other hand, the researcher was not surprised to learn there was no association between the time the students spent on the task from task 1 to task 5. Students enrolled in this higher level, of course, want to achieve high marks, and if they perceive their grades rely on their collaboration within a group, they are going to contribute to the group discussion as much as possible.

For the remaining six items, the researcher discovered a significant statistical difference in the groups between task 1 and task 5. While completing task 1, all students were at home, virtually contributing to the task through Microsoft Teams. For descriptors two through seven, an average of 7 groups scored the highest marks for task 1, but an average of 14 groups scored highest marks for task 5. This statistic indicated that the number of groups producing productive discourse and its components doubled from task 1 to task 5. Once students started participating on the tasks in the physical classroom, their oral perseverance, viable arguments, attending to precision, and sharing and questioning increased.

In this study, the researcher drew on the work of Gee (1996), who defined discourse as "a socially accepted association among ways of using language, other symbolic expressions, and 'artifacts,' of thinking, feeling, believing, valuing, and acting" (p. 131). When evaluating the 110 groups using the MCOP^2, the researcher had to observe and determine if the mathematics discourse within each group was at an appropriate level. The first finding of this study was that the appropriate level of productive mathematics discourse in task 1 was dependent of the appropriate level of productive discourse in task 5. The students within the groups were enrolled in in an AP Calculus AB course, and the discourse they produced was initially at a low average,

with only 15% of groups earning a three from descriptor 2. During task 1, all students participated while at home, learning virtually synchronously. During task 5, 28% of groups scored a three from descriptor 2, which is a statistically significant difference. While completing task 5, 48 students were physically in the classroom, while 36 of the students were at home. These students were working collaboratively through Microsoft Teams, yet their level of discourse rose. According to Liegel (2014) and Hampton and Gupta (2008), when learners move from one physical setting to the next, they are exposed to many environmental cues, and changes in environmental stimuli can disrupt the engagement of the learner. The results of this study showed that moving from a virtual learning environment to a face-to-face learning environment produced a greater number of groups collaborating at an appropriate AP Calculus AB level.

The second finding of this study emerged after the researcher observed the groups' oral perseverance in problem-solving while they completed the different tasks. During the first task, only 6 of the 24 groups scored a 3 with this descriptor, but for the fifth task, 16 of the groups scored a maximum score of 3. The groups' oral perseverance in problem-solving significantly increased when students returned to the classroom. Problem-solving has long been a focus of research and curriculum reform (NCTM, 1989, 2000; CCSSI, 2010). The importance of problem-solving is not new, but the Common Core introduced the idea of making sense of problems and persevering in solving them as an aspect of problem-solving (CCSSI, 2010). Perseverance is imperative for solving meaningful mathematical problems and tasks, and a lack of perseverance is a real obstacle to the development of interesting and significant mathematics. The tasks chosen for this study were analyzed using Smith and Stein's (2011) Task Analysis Guide and determined to be cognitively demanding. In a group consisting of members together in a classroom and members at their individual homes, oral perseverance is necessary to tackle a challenging mathematical task effectively. The results of this study showed that oral

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perseverance increased amongst groups when students left the virtual learning environment and returned to the physical classroom.

The fourth descriptor of the MCOP[2] focused on the students' viable arguments and critiques of the reasoning of others. The results of this study found that for the first task, while students were at home participating virtually, only four of the 24 groups scored a 3 for descriptor 4. The researcher believes this is because it is hard to dispute a peers' opinions when they are not in the same physical environment. On the other hand, the drastic increase in groups' viable arguments and critiques from the first task to the last might also have to do with the classroom environment and norms. For students to critique the reasoning of others and feel comfortable enough to argue with their peers, they must be in a classroom where reasoning is made public and open to review and comment. "A classroom culture that values critique rather than the one right way to get the one right answer is a culture in which students are far more actively engaged in their learning" (Max & Welder, 2020).

Another significant finding of this study involved descriptor five of the MCOP[2], "students attend to precision in their communication." While completing task 1, only 7 of the 24 groups earned a maximum score of 3 on the MCOP[2] observational tool, but during task 5, 15 of the groups scored a 3. Within mathematics courses, precision is highly valued because imprecision can lead to holes in arguments and miscommunication can lead to misunderstanding. While observing the 110 groups, the researcher acknowledged that dialogic instances of students attending to precision were rare, and generally involved constructing definitions, refining student conjectures, or building upon another students' idea. The number of groups who attended to precision while completing their tasks increased significantly from task 1 to task 5, as students went from participating virtually to 60% of the population participating while physically in the classroom.

Finally, the researcher found statistically significant results after analyzing the data acquired while observing the groups' sharing, questioning, commenting, clarifying, and

recognizing of others' ideas. During task 1, 7 of the 24 groups earned a score of 3 on the

$MCOP^2$ observational instrument, but for the final task, 12 groups earned the maximum score.

Students are expected to communicate with each other as part of an effective classroom

community. Effective communication means that students will listen, question, and critique; this

is part of the discourse expected in a mathematics classroom (Sherin, Mendez, & Louis, 2004).

This descriptor also embodies the literature on equity and mathematics in that all students have

valuable ideas, strategies and thinking to share within the mathematics classroom (Boaler,

2006). By creating equitable spaces, interactions amongst students within a mathematical

community increases participation and engagement while working to remove potential barriers

(Yackel & Cobb, 1996; Sherin, Mendez, & Louis, 2004; Hiebert & Grouws, 2007; NCTM, 2000).

Research Question 3: In what ways do the dynamics of the group change because of a

blended learning environment?

The findings from the Mathematics Classroom Observational Practice Protocol ($MCOP^2$)

instrument revealed patterns and themes amongst numerous groups. The qualitative portion of

this mixed-methods study involved observing 110 groups with a revised version of the $MCOP^2$

instrument, as the students collaboratively tackled five on-level, high cognitive demand calculus

tasks. Data collected by transcribing the multiple groups' conversations was coded using

descriptive coding. Descriptive coding summarizes a passage of qualitative data in word or

phrase (Saldana, 2016), and in this study, a passage of transcripts described a behavior or

characteristic seen within specific learning environments.

Saldaña and Omasta (2016) described a code as an essence-capturing word or short

phrase for a portion of language-based or visual data. A category organizes similar codes with

shared characteristics in a group, and unlike a code or a category, a theme summarizes a piece

of data in a word or phrase (Belotto, 2018). Categories were used to group similar codes into

groups based on commonalities and shared characteristics. Categories established a basis for

identifying patterns and revealing themes in the data. Themes provided an insightful explanation

of the experiences of AP Calculus students in blended learning environments and led to interesting findings within the research.

Although a few minor themes arose from the observations and subsequent coding, the most predominant theme was that of same-gender collaboration. In groups that had both male and female members, girls were hesitant to answer, prefacing answers with statements of uncertainty, whereas boys aimed to be the first to answer and then reacted to being wrong or right. Boys, in general, acted in this manner in all groups, regardless of if there were females present or not. Throughout all five tasks, a common theme arose in all the groups that had a male member; the boys attempted to be the first to voice their thoughts and opinions as experts. On the other hand, the researcher was called into the groups to be asked a question by the girls, who constantly sought direction and affirmation. According to the researcher's journal, she was called into eleven out of 24 groups for task 1 to answer questions about directions, or the potential of answers. Out of these eleven times, ten requests were made by females. At the end of the course, for task 5, the teacher was only called into two groups, and both times were for technical issues. This drastic transformation implies a change in the girls' confidence and mindset.

After analyzing the groups that consisted of all boys throughout all five tasks, it was apparent that the same themes kept arising: competition. In the all-male groups, regardless of social status or knowledge, the boys wanted to be heard. In some regards, their confidence was misplaced, but there appeared to be a minor disregard for the correct answer – as long as they were first to contribute. The boys also did not want to show their work for any of the tasks, and they tried to find the pattern or method that allowed them to complete the task while putting in minimal effort. In some cases, it seemed the boys were competing to see who could find the quickest and easiest route to the solution.

Once the researcher realized there were gender-based themes arising from the codes and transcript analysis, she chose to look more carefully at groups with mixed genders. In

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groups of four students with one or two girls, the girls took on the role of helper or teacher, as the actual teacher for the course was not present. Although generally, the boys in the group would eagerly contribute their thoughts or answers to the group, if incorrect, the girls were observed patiently probing for justification or a more specific thought process. The boys would state answers succinctly, to the point, and want to argue their case instead of listening and explaining their thought process thoroughly. The girls, in turn, would want to show their answers visually by writing on the virtual whiteboard or uploading their work into the group's channel. They were quick to shy away from their answers if a group member pointed out a mistake or confronted them about their work.

In groups that consisted of only females, the girls still took on the role of teacher, but their collaboration contained more back and forth discourse than those groups that had mixed genders. In fact, a few statements said by female students though out the tasks display their perception of the group's make-up. Brief snippets like the one below indicated to the researcher that there was an underlying issue that the study had uncovered, and that issue appears to be females not feeling like equals next to their male counterparts.

From Task 4:

Student V:	I missed you, girls! My last group had Student R and I knew how to do those Reimann Sums, but he wouldn't listen to me and we failed the task! I should have just submitted my answers as the group's answers, but I was worried I was wrong.
Student W:	Your group sounds just like mine did! I couldn't get word in with those guys. They just don't listen at all and it's frustrating. Student H drew those rectangles for the left-hand sum above the graph and then forgot to divide for the Trapezoid rule and Student E said he was right! I almost emailed [the researcher] and asked her if I could do the task on my own but Student O was in the group and she pointed out his mistakes, and we convinced them they were wrong.
Student V:	You had to convince them? They did not believe the two of you? You are both A students!

Regardless of the genders within the groups, discourse patterns and the group dynamics did ultimately change from task 1 to task 5. Although the boys' competitiveness and girls' quiet nature did not appear to vary too much, the actions and behaviors of the groups did. For the first two tasks, when all students were at home, majority with their cameras off, there was a sense of independence and segregation within the groups. There was considerably more 'quiet time' amongst the group, as most students worked alone on the tasks. Once the scheduling changed and students could come to school four days a week and tasks were administered on those days, the groups became more collaborative in nature and the quiet time almost diminished completely. The students were able to see one another in the classroom, and even though they had one or two group members at home working virtually, the students sitting in the class were consistently engaged in the task.

While transcribing the 110 groups, the researcher made special notes of student opinions about the tasks and groups. Students were randomly assigned to a group for task 1, then were randomly put into new groups for tasks 2, 3, and 4 before ultimately returning to the same group they were in for task 1. The researcher noted that students who were in class face-to-face with the instructor for tasks 4 and 5 led their groups in productive mathematical discourse. For those two tasks, the researcher was present in the room with the hybrid learners, walking around, monitoring their progress, but saying nothing without volunteering help. As a result, the hybrid students contributed more actual minutes of discourse within the task and kept their groups on task throughout the period. Multiple studies have reported that achievement increases with the teachers' presence (Rodgers & Raider-Roth, 2006). Still, few studies have shown that the teachers' presence has impacted students' productive discourse.

Implications

The United States of America is underachieving in the area of mathematics (Provasnik et al., 2016). "The TIMSS video study compared teaching in the U.S. to teaching in other countries and concluded that other countries had a curriculum that was conceptual and deeper,

and involved more student discussion" (Boaler, 2016, p. 192). There are many areas for improvement in mathematics acknowledged by stakeholders. "Teachers, principals, superintendents, and policymakers on a local and national level recognized the need for educational reform and attempted to address deficiencies in mathematics" (Arbini, 2016, p. 86). The statistically significant results found in this study show productive mathematical discourse can occur in multiple learning environments, including asynchronous, virtual environment.

The results of this study provide information that can be used to supplement or improve undergraduate-level teacher education programs and the teaching strategies of current and experienced teachers. In a world full of uncertainty caused by the onslaught of a vicious, life-altering Covid-19 virus, students are feeling more isolated, lonely, and suicidal than any generation before them (Grubic et al., 2020). These feelings are prominent in students regardless of whether they are attending school on a regular basis or learning virtually (Unger & Meiran, 2020) from the comfort of their homes. The results of this study could help educators and educational systems realize that discourse is possible in any learning environment, and in the right group, students can become engaged and produce productive mathematical discourse. Improving a students' engagement and discourse does not necessarily mean their academic achievement will change, but it does indicate that the student will feel like they are part of a community, and that should decrease their negative personal feelings.

The practical implications of this research could have a great impact on the allocation of resources and the decisions made by educational leaders and policymakers. The amount of money that schools are spending on technology is growing each year (Schaffhauser, 2016). Through the lens of the present study, these leaders can promote the technology-enhanced instruction found in an effective blended classroom environment. Teacher training is a key factor in creating an effective blended classroom environment, and my future research interests may include supporting teachers with effective strategies that provide strategies for promoting engagement and productive discourse in collaborative groups.

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Students also need access to technology inside and outside of the classroom. Students need to be able to use technology in their homes to participate in a virtual learning environment. If they do not have access to the necessary technology or wi-fi, they will not be able to access this learning environment, which might be necessary. Teachers, principals, and district technology leaders must make certain each student has technology available at home (Celano & Neuman, 2010). Districts should implement a program that allows students the necessary computer equipment along with an appropriate wi-fi connection via hot spot or other types of devices, as necessary, to support their blended learning experience at home.

Recommendations

Based on the results of this research study, there are several recommendations for future research. One recommendation for future research is to administer a survey or questionnaire to the students before the first task, and after the last, about their opinions of groups. A better understanding of their perceptions of groups might lead to a better understanding of their discourse patterns. Another recommendation would be to conduct this study in a less advanced math course with average knowledge students. This would make the study more generalizable for other schools.

Since there is plenty of research regarding the separation of genders in math courses, future studies can incorporate that factor into the strategies for grouping. Based on the results of this study, following same-gendered groups throughout the year as they tackle tasks through different learning environments could provide further insight into literature that condones same sex classes at the secondary level. Following same-sex groups in a blended learning environment could fill a gap in the considerable literature on the benefits of same-sex classes.

Carpenter et al. (2003) maintain that the very nature of mathematics presupposes that students cannot learn mathematics with understanding without engaging in discussion and argumentation. Future research could use the premise of this study but focus on the outcomes

of the tasks to determine if the discourse had any effect on the academic achievement of groups.

There is a need to understand the complex relationships between learners, their ways of learning and studying, and the environments within which they study, both physical and virtual. With a few exceptions (Alphonse et al., 2019), research in online learning has not focused on where exactly learners do their learning and studying and how a physical place (e.g., the home) supports and constrains learning activities. Such an understanding would have implications for environmental designers, educators in pedagogical design, and online distance learners.

And last, based on the new insight gained through this study, future scholars can narrow down and take specific research avenues or peer discourse among students to explore why the discourse occurs instead of how. This research could ultimately help pre-service teachers master strategies they can incorporate into their lessons and instructional techniques. Once researchers further expand on the creation of productive mathematical discourse and subsequently locate specific strategies and practices, mathematics teachers at all levels may use the benefits of productive mathematical discourse within their classes.

Conclusion

This study explored the relationship between students' mathematical discourse and their untraditional learning environments caused by the onslaught of the COVID-19 virus. This study explored the differences between different blended learning environments in a large suburban high school in West Tennessee during the 2020-2021 school year.

This study revealed a difference in multiple aspects and areas of discourse amongst students in collaborative blended learning groups throughout the school year. These areas include discussing the task an appropriate mathematical level, exhibiting strong oral perseverance, viable arguments and critiques amongst group members, attending to precision, time spent devoted to the task, and sharing, questioning and commenting on one another's

ideas and struggles. This study also concluded that discourse was consistently high in groups where members had the same gender, regardless of their learning environment. It is clear from the analyses above that students contribute in substantive ways to the creation of productive mathematical discourse. In fact, they often find ways to resist and transform discourse norms without any major conflict. This study found that students were more apt to create and ultimately be involved in mathematical disagreements when they were in the physical classroom. While in the comfort and privacy of their own home, virtual students preferred to fly under the radar in discourse negotiation and critiques. Because of their perceived roles within the groups, students who stayed virtual throughout the 2020-2021 school year were often less verbally explicit with their contributions to their groups.

This study is beneficial to schools that have implemented a virtual component to their schools or school districts. The virtual instruction context for teaching secondary education is relatively new. Educational systems throughout the world have created virtual academies for students who have grown accustomed to learning at home and might not wish to return to a traditional school setting, and this research will help directors and leaders of those schools promote engagement through appropriate level mathematical discourse. This study is also beneficial to mathematics teachers who are struggling to teach virtually, as they are struggling to get their virtual students to communicate with them and their peers. Mathematics teachers at all levels could challenge themselves to explore effective methods to promote and effectively manage productive student discourse. Students need social interaction opportunities using online services so that they can continue to explore and discuss mathematics as if they were in a face-to-face classroom. Administrators and educational stakeholders should do their best to support teachers' virtual instructional practices by providing mathematics engagement-specific professional development and opportunities to collaborate with other mathematics instructors.

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LIST OF APPENDICES

MATHEMATICS CLASSROOM OBSERVATION PROTOCOL FOR PRACTICES -

ABBREVIATED

SE	**Descriptor 1**
3	Students regularly engaged in exploration, investigation, or problem solving. Over the course of the task, the majority of the group engaged in exploration/investigation/problem solving.
2	Students, more than half the time, engaged in exploration, investigation, or problem solving. Several students engaged in problem solving, but not the majority of the group.
1	Students seldom engaged in exploration, investigation, or problem solving. This tended to be limited to one or two students engaged in problem solving while other students watched but did not actively participate.
0	Students did not engage in exploration, investigation, or problem solving. There were either no instances of investigation or problem solving.
SE	**Descriptor 2**
3	The entire group spends two-thirds or more of the task discussing the mathematical activity at the appropriate level for the class.
2	Most of the students in the group spend more than one-quarter but less than two-thirds of the task discussing appropriate level mathematical activity. It does not matter if it is one prolonged activity or several shorter activities.
1	Most of the students in the group spend less than one-quarter of the task engaged in appropriate level mathematical discourse. There is at least one instance of students' mathematical engagement.
0	Most of the students are not engaged in appropriate level mathematical discourse.
SE	**Descriptor 3**
3	Students exhibited a strong amount of oral perseverance in problem solving. All students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), the group discusses together what to do next.

2	Students exhibited some oral perseverance in problem solving. Not all of the students verbally looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary.
1	Students exhibited minimal oral perseverance in problem solving. Only one to two students verbally looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary.
0	Students did not orally persevere in problem solving.
SE	**Descriptor 4**
3	Viable arguments and critiques is an integral component of the lesson with students engaged continuously in the task (as described in the Common Core State Standards).
2	Viable arguments and critiques of the reasoning of is a major component, but the discourse is not productive, or discourse is not a major component, but the students engage in a verbal activity that fits within the corresponding standard of mathematical practice.
1	Only one or two of the students in the group use viable arguments or critiques of the reasoning of others as a productive discourse
0	The lesson does not include any viable arguments or critiques of the reasoning of others or productive discourse
SE	**Descriptor 5**
3	All members of the group "attend to precision" in communication, or the teacher guides students to modify or adapt no precise communication to improve precision
2	Only 3 of the members of the group "attend to precision" in communication throughout the group task
1	Only 2 of the members of the group "attend to precision" in communication throughout the group task
0	None or one of the members of the group "attend to precision" in communication throughout the group task
SE	**Descriptor 6**
3	More than three of the students in the group were talking related to the mathematics of the task
2	More than two the students in the group were talking related to the mathematics of the task

1	Less than two of the students in the group were talking related to the mathematics of the task
0	No students talked related to the mathematics of the task.
SE	**Descriptor 7**
3	Many students are sharing, questioning, and commenting during the task, including their struggles. Students are also listening (active), clarifying, and recognizing the ideas of others
2	The environment is such that some students are sharing, questioning, and commenting during the task, including their struggles. Most students actively listen
1	Only a few students in the group share. The climate supports those who understand or who
SE	**Descriptor 8**
3	Considerable time (more than half) was spent with peer to peer discourse related to the communication of ideas, strategies and solution.
2	Some group time (less than half, but more than just a few minutes) was devoted to peer to peer discourse related to the mathematics of the task
1	The task was primarily led by one student. A few instances occurred where discourse developed but this happened infrequently
0	No productive peer to peer conversations occurred during the lesson.

TASK 1

| |
|---|
| <u>Given</u>: Let function
f = {(x, y): x ∈ ℝ and $0 \le x \le 2$ and y = $4x^2$+ 5}
and consider the Line l that is tangent to a graph for function f at the
point where x = 1. |
| Part 1: Use a graphing calculator to view a graph of
Y2 = ($4x^2$ + 5)/(0 $\le$ x)/(x $\le$ 2) in [-1, 3.7] X [-5, 26]; draw a complete graph
for function f on the grid provided. |
| Part 2: Use either nDeriv(Y2, X,1) on the MATH menu or dy/dx on the
CALC menu to find an approximate value for f'(1); use the definition of a
derivative to find the exact value.

 |
| Part 3: Use Tangent on the DRAW menu to investigate a graph for Line l;
use an analytical method to find an exact equation for Line l.

 |
| Part 4: On the same axes as the graph for function f, draw a complete
graph for Line l; plot and label the point of tangency.

 |

It's A Solids of Revolution Match-Up   Name:_____

1. Revolve $y = x^2 + 1$, $x = -1$, $x = 1$, and $y = 0$, about the line $y = -1$.	**2.** Revolve $y = \sqrt{x}$ and $x = 4$ about the line $y = 4$.
3. Revolve $x = y^{\frac{2}{3}}$, and $y = 1$, about the y-axis.	**4.** Revolve $x^2 - y^2 = 16$ and $x = 5$ about the y-axis.
5. Revolve $y = \sqrt{9 - x^2}$ about the x-axis.	**6.** Revolve $x = y^2$ and $x = 4$ about the line $x = 6$.

7. Revolve $x = \sqrt{y-1}$ and $y = 2$ about the y-axis.	**8.** Revolve $y = 2x$, $y = x$, and $x = 2$ about the x-axis.
9. Revolve $y = x^2 + 1$ and $x = 2$ about the x-axis.	**10.** Revolve $f(x) = x^2 + 2$ and $g(x) = 4 - x^2$ about the line $y = 1$.
11. Revolve $f(x) = x^3$ and $g(x) = 4x$ about the y-axis.	**12.** Revolve $y = \sin x + 1.5$ about the x-axis using a graphing calculator.

G1

G2

G3

G4

G5

G6

G7

G8

G9
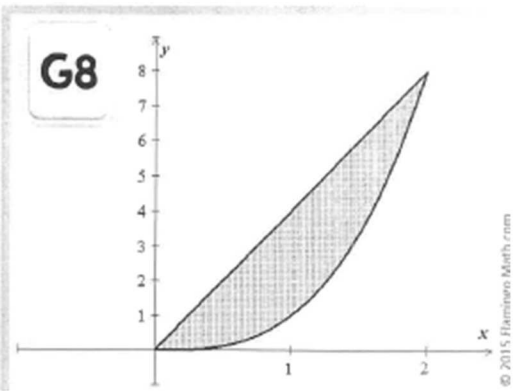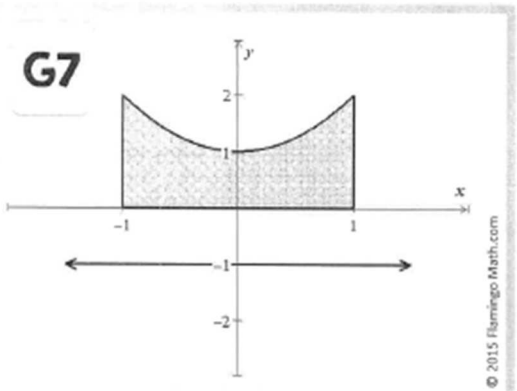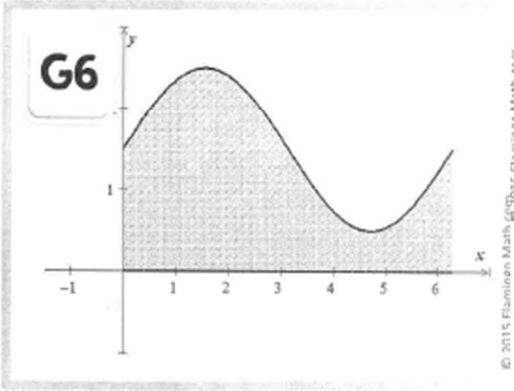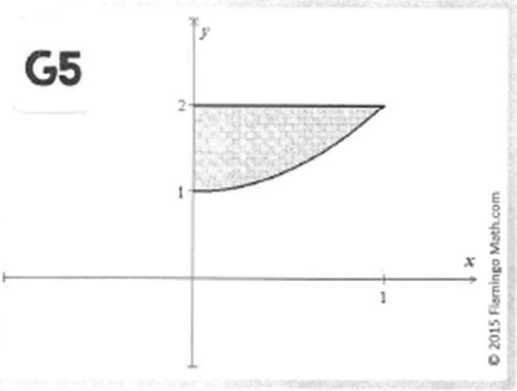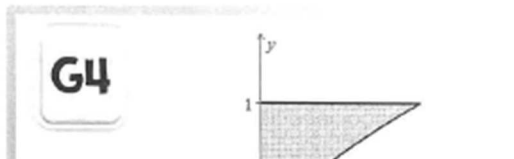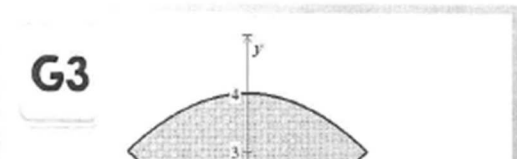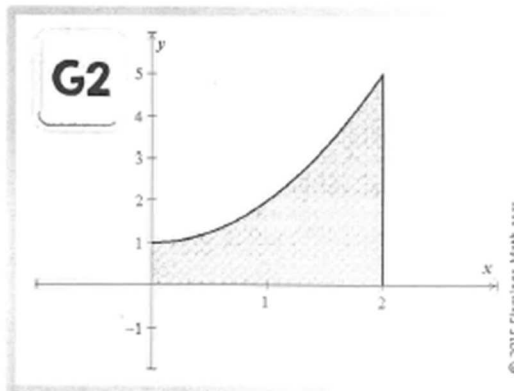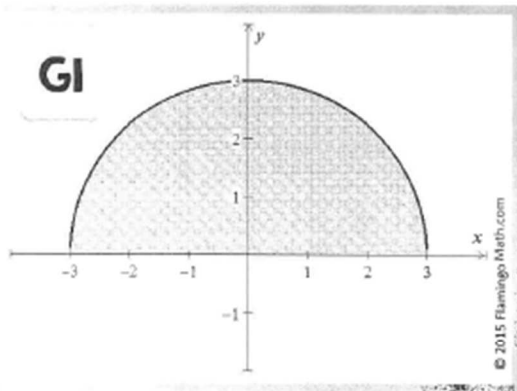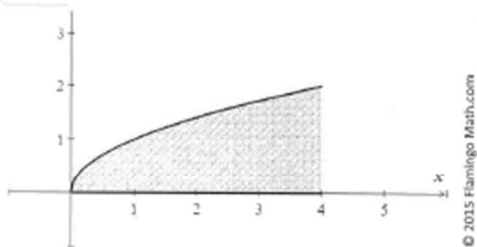
G10
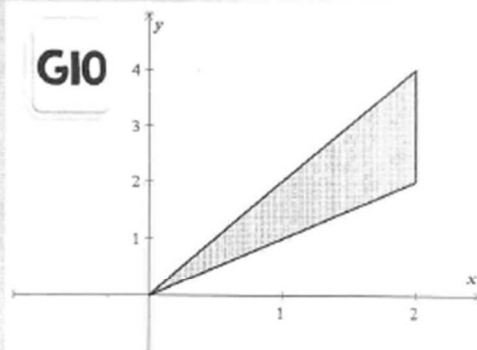
G11

G12

E5

Revolve about x-axis:

$$y = \sqrt{9 - x^2}$$

E6

Revolve about line x=6

$$x = y^2$$
$$x = 4$$

E7

Revolve about y-axis:

$$x = \sqrt{y - 1}$$
$$y = 2$$

E8

Revolve about x-axis:

$$y = 2x$$
$$y = x$$
$$x = 2$$

E9

Revolve about x-axis:

$$y = x^2 + 1$$

$$x = 2$$

E10

Revolve about line y=1:

$$f(x) = x^2 + 2$$

$$g(x) = 4 - x^2$$

E11

Revolve about y-axis:

$$f(x) = x^3$$

$$g(x) = 4x$$

E12

Revolve about x-axis:

$$y = \sin x + 1.5$$

V1

$$V = \pi \int_0^{2\pi} (\sin x + 1.5)^2 \, dx$$

V2

$$V = \pi \int_0^1 \left(y^{\frac{3}{2}}\right)^2 \, dy$$

V3

$$V = 2\pi \int_0^3 (9 - x^2) \, dx$$

V4

$$V = \pi \int_0^8 \left[(\sqrt[3]{y})^2 - \left(\frac{y}{4}\right)^2 \right] dy$$

V5

$$V = \pi \int_{-1}^{1} \left[(3 - x^2)^2 - (x^2 + 1)^2 \right] dx$$

V6

$$V = \pi \int_{0}^{2} \left[(2x)^2 - x^2 \right] dx$$

V7

$$V = \pi \int_{-3}^{3} \left[5^2 - \left(\sqrt{y^2 + 16} \right)^2 \right] dy$$

V8

$$V = \pi \int_{-1}^{1} \left[(x^2 + 2)^2 - (-1)^2 \right] dx$$

V9

$$V = \pi \int_{1}^{2} (y - 1) \, dy$$

V10

$$V = \pi \int_{0}^{2} \left(x^4 + 2x^2 + 1 \right) dx$$

V11

$$V = \pi \int_{-2}^{2} \left[(6 - y^2)^2 - 2^2 \right] dy$$

V12

$$V = \pi \int_{0}^{4} \left[4^2 - (4 - \sqrt{x})^2 \right] dx$$

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CURRICULUM VITA

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EDUCATION

Ph.D. in Education. Division of Secondary Education. The University of Mississippi, Oxford, MS. Major Field: Secondary Education with an emphasis on Mathematics.
Dissertation: *The Effects of Blended Learning on Students' Productive Discourse in a Secondary Mathematics Classroom.* Chair: Professor Allan Bellman. April 2022.

Master of Educational Theory and Practice. Arkansas State University, Jonesboro, AR. Professional Specialization: Secondary Mathematics. May 2011.

Master of Education. Secondary Education, University of Arkansas. Little Rock, AR. Professional Specialization: Secondary Mathematics. May 2008.

Bachelor of Science. Business Administration. Mississippi Valley State University. Itta Bena, MS. May 2006.

PUBLICATIONS

Cheng, Q., Brice, B., & Broadway, N. (2016). Palette of problems (No. 9-15). Mathematics Teaching in the Middle School, 21(9), 524-525.

Cheng, Q., Brice, B., & Broadway, N. (2017). Palette of problems (No. 14-16). Mathematics Teaching in the Middle School, 22(8), 460-461.

PRESENTATIONS

Brice, B. "Cooperating Teachers' Effect on Elementary PreService Teachers" National Council of Teachers of Mathematics Regional Conference, Philadelphia, PA. November 10-11, 2016.

Brice, B. "Geometrical Tasks for the Middle and High School Classrooms" Mississippi Council of Teachers of Mathematics State Conference, Oxford, MS. October 6-7, 2015.

ADDITIONAL RESEARCH EXPERIENCE/CERTIFICATES

Certificate of Program Evaluation
  Internship: *Diversity Certificate*
- Developed curriculum for graduate level diversity courses
- Analyzed the diversity within all 300 and 400 level educational courses within the School of Education
- Ran reports on the data acquired then ran statistical analysis of the data

- Created and presented a report to the Dean of Education and other stakeholders
- Developed online and written curriculum for history teachers within MS school districts regarding the Rosenwald schools

  Internship: *World Class Teaching Program*
- Through the Kellogg Initiative on Transforming Literacy through Board Certification
- Created an interview and Likert Scale questionnaire for each of the 6 directors of the World Class Teaching Programs throughout the state of Mississippi.
- Interviewed each of the 6 directors
- Analyzed and coded data from the interview and questionnaire
- Created a report that helped determine which university's program was most effective and the strategies they used
- Presented the report to each of the 6 schools' School of Education Dean

Interdisciplinary Certificate of Applied Statistics
> *Courses***:**
> - Data Management and Statistical Software
> - Mathematical Statistics I
> - Mathematical Statistics II
> - Educational Statistics I
> - Educational Statistics II
> - Advanced Quantitative Research Methods
> - Applied Multivariate Analysis

LEADERSHIP AND PROFESSIONAL DEVELOPMENT

Service
> Supervision of Pre-Service Teachers, The University of Mississippi, 2017-2019
> Collaborated on graduate students' Master Thesis, The University of Mississippi, 2018
> TODOS: Mathematics for ALL Reader, 2016-2018
> NCTM Mathematics Teacher referee, 2016-2018

Leadership
> Member of Responsibility Centered Discipline Team, Collierville High School
> Instructional Coach, Collierville High School
> New Teacher Mentor, Collierville High School & Southwind High School
> Head Softball Coach, Southwind High School & Central High School
> Created Algebra II Remediation Curriculum, Collierville High School

Affiliations
> National Council of Mathematics (NCTM)
> Mississippi Council of Mathematics (MCTM)
> Tennessee Council of Mathematics (TCTM)
> American Educational Research Association (AERA)

Professional Development
- Reviewer of conference proposals, MCTM State Conference, 2015.
- Presented *The Importance of Discourse in a Secondary Mathematics Classroom* at Collierville High School, 2022.
- Presented *Algebra II Activities and Tasks for Academic Achievement* at Collierville High School, 2021.
- Presented *Projects and Inquire Based Learning* at Southwind High School, 2015.
- Hiring Committee at Fuller Middle School, 2010-2012.

Software Programs
- SPSS
- SAS
- Microsoft Word
- Microsoft PowerPoint
- Microsoft Excel
- Canvas
- Keynote
- Goodnotes

COLLEGE TEACHING & SUPERVISION EXPERIENCE

Fall 2015   Education, Society & the K-12 Learner, EDCI 352 - TA
Spring 2016   Classroom Mgt & Behavioral Interventions, EDSP 327 - GA
Fall 2016   EDCI 352 & Math Elementary Strategies, EDEL 403 - TA
Spring 2017   Classroom Mgt & Behavioral Interventions, EDSP 327 - TA
Fall 2017   Education, Society & the K-12 Learner, EDCI 352 – TA
Spring 2018   Student Teaching, EDEL 464 – Mentored and Evaluated 9 pre-service students in science and mathematics. Implemented cooperative individualized improvement program for each individual.

PREVOUS TEACHING EXPERIENCE

Collierville High School – Collierville, TN. Algebra II, Algebra II Co-Teach, Calculus CLEP, AP Calculus AB Teacher. PLC lead, Instructional Coach, Member of Principals' Advisory Committee and Member of Responsibility Centered Discipline Leadership Team.   **2018-Present**

Southwind High School – Memphis, TN. Algebra II, Geometry, Pre-Calculus Teacher. Head of math tutoring program. Head Softball Coach. Teacher of the Year nominee.   **2012-2015**

Fuller Middle School – Little Rock, AR. Algebra I, Geometry, 7th grade math Teacher. Assistant Director of gifted and talented program.   **2009-2012**

Mayflower High School – Conway, AR. AP Calculus AB, Algebra II Teacher. Assistant Volleyball Coach. Created and Sponsored Math Olympiad Program.   **2008-2009**

Central High School – Little Rock, AR. Algebra I Teacher. Assistant Softball Coach. Assistant Volleyball Coach.   **2006-2008**

REFERENCES

Dr. Allan Bellman Professor, Teacher of Education, The University of Mississippi, Oxford, MS. Tel: (662) 915-5309; email: abellman@olemiss.edu

Dr. Andy Cheng Associate Professor, Elementary Education, The University of Mississippi, Oxford, MS. Tel: (662) 342-4765; email: qcheng@olemiss.edu

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Dr. Lauren Balentine Assistant Principal, Math Department Head, Collierville High School, Collierville, TN. Email: LBalentine@colliervilleschools.org