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Call Out the Reserves -- Warranty, That Is

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Warranties offered on a product must be figured in its costs like any other sales cost. Yet—how much should be allowed? This article explains one simple graphical method of forecasting costs—

CALL OUT THE RESERVES—WARRANTY, THAT IS

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Almost all products, whether sold directly to the customer or to a producer for assembly into a consumer product, now carry a warranty of some kind. Articles in current periodicals, as well as increasing Congressional interest in warranties, emphasize the growing importance of this subject both to consumer and to producer.

Warranty status has changed from a sales "gimmick" to a contractual obligation with ever-increasing customer demand for fulfillment. It is therefore important that a cavalier treatment of warranty costs be replaced by an objective determination of cost. This will help manufacturers plan operations more effectively since an accurate knowledge of warranty costs allows more accurate profit expectations which may, in turn, lead to unanticipated marketing advantages.

As business men we realize that the costs of warranty claims should somehow be predictable. They are indeed, but the methods are complicated. It is hoped that this article, by use of graphical methods, will remove many of the complications. (There are other methods, but they require some sophistication in mathematics.) Even then, when the going gets rough, particularly in the symbolism, stick with it.

The payoff to you can be measured in dollars and cents.

Let us start with the premise that estimating the financial impact of customer claims against a specific warranty policy need not be a crystal-ball operation. Warranty costs can be classified and identified as business costs which are as real as inventory, materials, labor, and all other production costs. They can be calculated in several ways, but, in most of them, the mathematics tends to be complex. One simple method of calculation is a graphic method, which is demonstrated in this article. However, let us first define the nomenclature and the type of warranty problem to be considered.

Warranty reserves

Moneys designated to cover the costs of warranty claims will be called warranty reserve funds or warranty reserves. These are established as separate funds when the product is offered for sale and are used to honor warranty claims. The reserve funds are recovered as the product is sold since the product price is adjusted to prorate the expected cost of claims for all units of the product.

We will consider warranty reserve requirements for nonrepairable products where an explicit warranty is in force. It is assumed that the typical warranty guarantees the product or component to be free from failures caused by defects in material or workmanship for a specific period of time defined as the warranty time, w. If failures occur within this time, the manufacturer will replace the failed item. The customer is credited for the value of the unused portion of the warranty life. He is charged the difference between the product price and the rebate for the new replacement product.

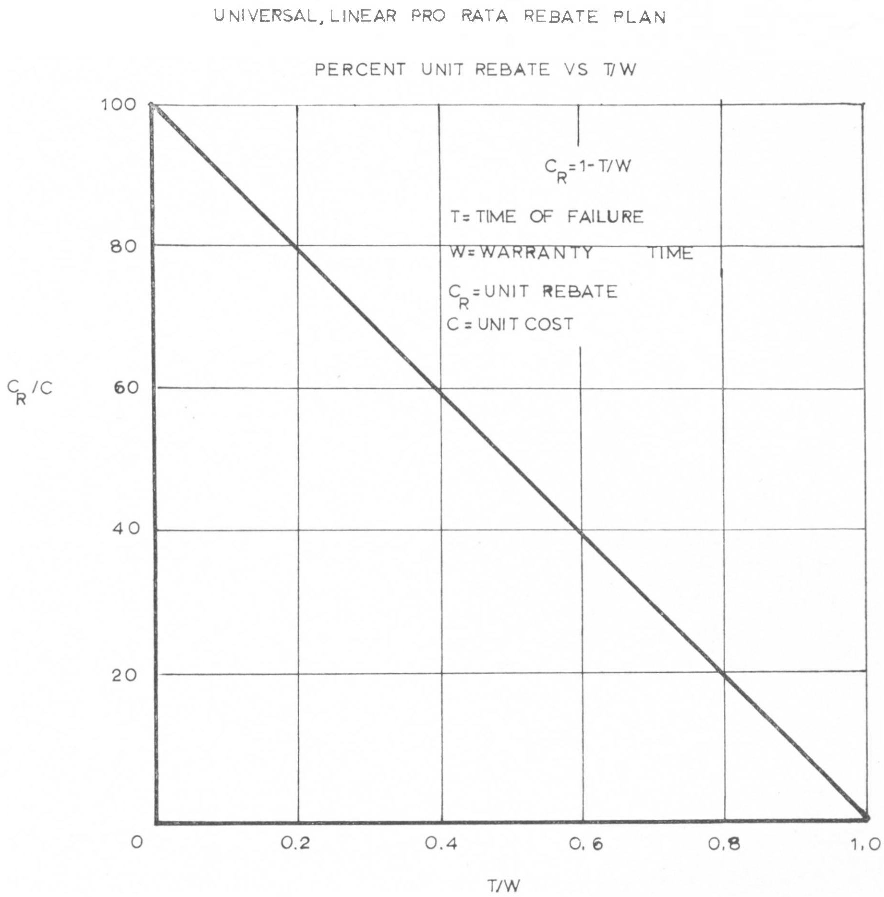

FIGURE 1

Pro rata rebate

The most common kind of warranty rebate is the pro rata rebate where the amount credited varies linearly from full product value for failure at zero life to zero rebate for failure at or after the end of the warranty period. This type of rebate policy is shown in the universal graph of Figure 1 (above), where the rebate as a percentage of the product unit cost is plotted versus the ratio of the time at failure to the warranty time.

Let us examine a graphical procedure for calculation which will allow a manufacturer to determine his product price if it should include the cost of pro rata customer rebates claimed because of product failures occurring during the warranty period. Two representative kinds of product failure patterns are considered. The first failure pattern is typical of high-reliability items where failures occur according to an exponential failure law with a mean time to failure calculated from past history. The second failure pattern is typical of items which fail because of "wear out." The failure pattern is not easily expressed mathematically but can be represented graphically.

Exactly the same approach is used for both cases. However, the first example is described in detail to emphasize the techniques that are used while the second example

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is described in abbreviated form. In both cases the approach is described by reference to the graphs accompanying this article.

Let us first consider the following case:

> A manufacturer plans to produce 4000 units, designated N.
> Unit sales price *not* including the cost of warranty claims is $50.00, designated c'.
> Product warranty period is 12 months, designated w.
> Product mean time to failure is 40 months, designated m.
> Rebate policy is pro rata as shown in Figure 1.
> The manufacturer must determine:
> The adjusted sales price, c, to include the cost of expected warranty claims.
> The total warranty reserve, R, (or total cost of rebates) which must be reserved to cover the cost of warranty claims assuming all failures are claimed.

FIGURE 2

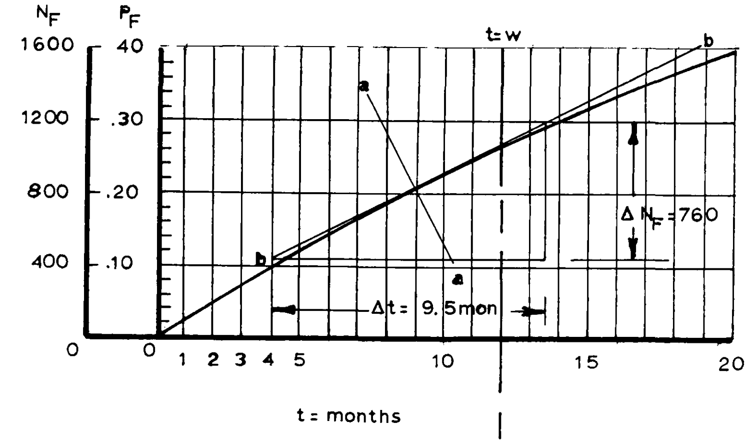
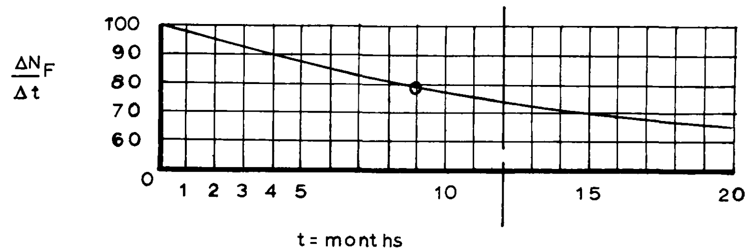


FIGURE 3



Method of calculation

In order to start the calculations for warranty reserves, the probability of failure at any time t must be known or estimated for the product. This first example represents a case commonly encountered where failures occur for reasons other than wear out of the product. This typical failure pattern can usually be described by an exponental failure law, $P_F = 1 - e^{-t/m}$, where P_F is the probability of failure at any time t, e = 2.7183, and m is the mean time to failure. Methods of calculating m from historical records or from life test data on the product are well documented.[1,2] If no previous knowledge of the failure pattern for a product exists, a good first trial is to determine if past records show the exponential failure law to be a good fit. If the exponential is not a good approximation to the actual failure pattern, then the method of estimating P_F as described in the second case to follow may be used.

The first step in the calculation is to plot P_F vs. t as shown in Figure 2 (above). Each number of the probability axis is next multiplied by N (the lot size), thereby converting the graph of Figure 2 to a plot of the expected cumulative number of failures, N_F, in the lot for any time, t. By so doing the total warranty reserve cost finally calculated will be for a production lot size, N. Note that the time scale is most convenient if it has the same units as the warranty time, that is, months in the example. However, any unit of time can be used as long as the same unit is consistently used whenever time is expressed.

Figure 3 (above) is derived from Figure 2 by determining and plotting the slope of Figure 1 at evenly spaced time intervals over the warranty period, w. For example, determine and plot the slope of Figure 1 at t = 0.1w, t = 0.2w, t = 0.3w, . . . t = w.

The slope or tangent to the graphed curve will be described by $\Delta N_F/\Delta t$ where ΔN_F is the incremental rise and Δt is the incremental run associated with the straight-line tangent to a point on a curve. This is shown in Figure 2, where the slope is constructed for t = 9 months. The slope is most conveniently determined by the graphical method described in the next paragraph.

First determine the line perpendicular to the slope line. The perpendicular to the slope can be quite accurately determined by placing a small rectangular hand mirror vertically to the plane of the graph and across the curve at the point in time being considered. Adjust the mirror by pivoting it around its intersection with the curve until the curve to the left of the mirror and its image in the mirror form a symmetrical figure. There must be no abrupt change in the curve direction at the edge of the mirror where the curve ends and its mirror image starts. At this position, use the mirror as a straight edge to draw a line (a-a in Figure 2) intersecting the curve. This straight line will be perpendicular to the slope of the curve. Next, construct the slope by drawing another line perpendicular to the first construction line (and tangent

[1] Bazovsky, Igor, *Reliability Theory and Practice*, Prentice-Hall Inc., Englewood Cliffs, N. J., 1962.

[2] Calabro, S., *Reliability Principles and Practices*, McGraw-Hill Book Co., New York, N. Y., 1962.

FIGURE 4

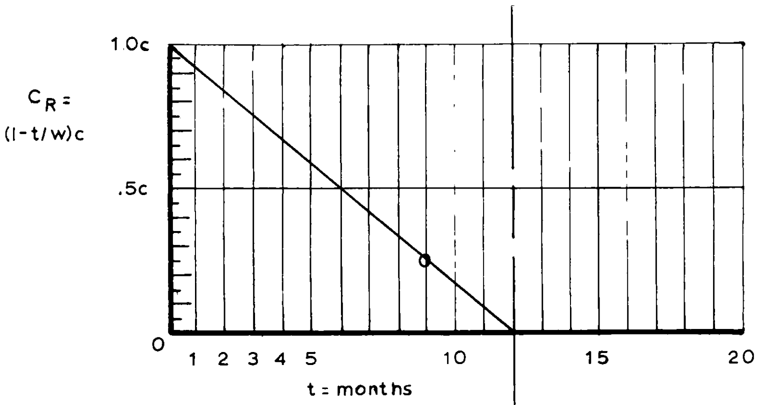
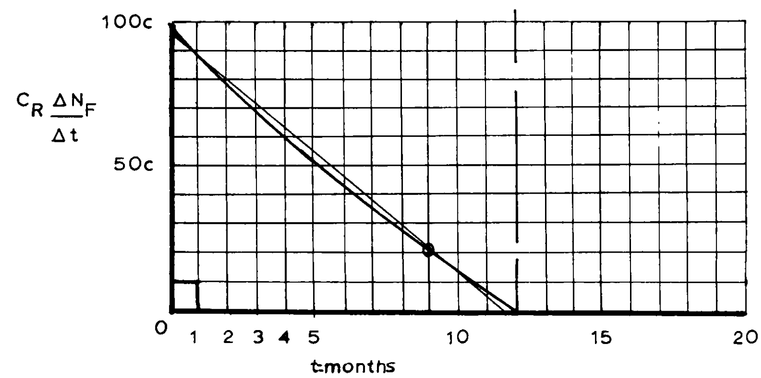


FIGURE 5



to the plot) at the point in time being considered. This is line b-b in Figure 2. A protractor or a repeat of the mirror image method may be used for this construction. The slope can be calculated as shown in Figure 2, where it is constructed at t = 9 months. The numerical value of the example is $\Delta N_F/\Delta t = 80$.

Figure 3 is then formed by plotting the slopes (as determined above) versus the time for which they were calculated. It is really a plot of the rate of change of cumulated failures versus time, where $\Delta N_F/\Delta t \approx dN_F/dt$. That is, the slope of a curve at a point on the curve is the derivative, dN_F/dt, of the function described by the curve at that point. The circled point of Figure 3 is the slope calculated from Figure 2 at t = 9 months.

Now form Figure 4 (above), which is a plot of the cost of the rebate offered for each failure at any time during the period of the warranty policy being examined. The example assumes a linear, pro rata rebate is offered. However, the method of this article is applicable to any rebate policy.

The vertical scale of Figure 4 varies from 0 to 1.00c where c is the original unit product price *adjusted for the warranty rebate policy.* Since c is to be determined, subsequent calculations will contain this yet unknown adjusted cost, c. For example, Figure 4 shows that the rebate C_R for a failure at t = 9 months is 0.25c, etc.

Figure 5 (at left) is the rate of change of the total cost of cumulated failures at time t versus time t and is next prepared by multiplying the curves of Figure 3 and 4 together ordinate by ordinate. That is, the values at each t from Figures 3 and 4 are identified, multiplied together, and the product is plotted in Figure 5 for that value of t. In our example at t = 9 months we find the value of 80 from Figure 3 and the value 0.25c from Figure 4. The product, 20c, is plotted as a point for t = 9 months. The complete series of calculations forms the curve of Figure 5.

The area under the curve of Figure 5 is equal to the total cost of failures, which then must be the warranty reserve fund, R, for the lot size N if all failures occurring during the warranty period are claimed. In most cases, a graph corresponding to Figure 5 is nearly a straight line and can be approximated as shown by the straight line plot of Figure 5. When this approximation can be made, a triangle is formed. The area of the triangle of this example is R = (½) (11.4) (96c) = 547c.

Calculating dollar reserves

Now one must calculate c in order to establish the dollar value of the total warranty reserve fund, R. Remembering that the lot size, N, is 4000, one can calculate the ratio R/Nc = 547c/4000c = 0.137. This is the estimate of the ratio of the total warranty reserve fund to the total sales value of the lot of N (4000 units in the example) articles. It is also the fraction of the unit product cost which must be allocated to the satisfaction of future warranty claims. Let us re-

TABLE I

Failure Rate

Hours of Operation to Failure	Proportion of Failures $= P_F$	Cumulative P_F	$N_F = NP_F$
100	.1	.1	10,000
200	.1	.2	20,000
300	.2	.4	40,000
400	.3	.7	70,000
500	.2	.9	90,000
600	.1	1.00	100,000
			N = 100,000 units

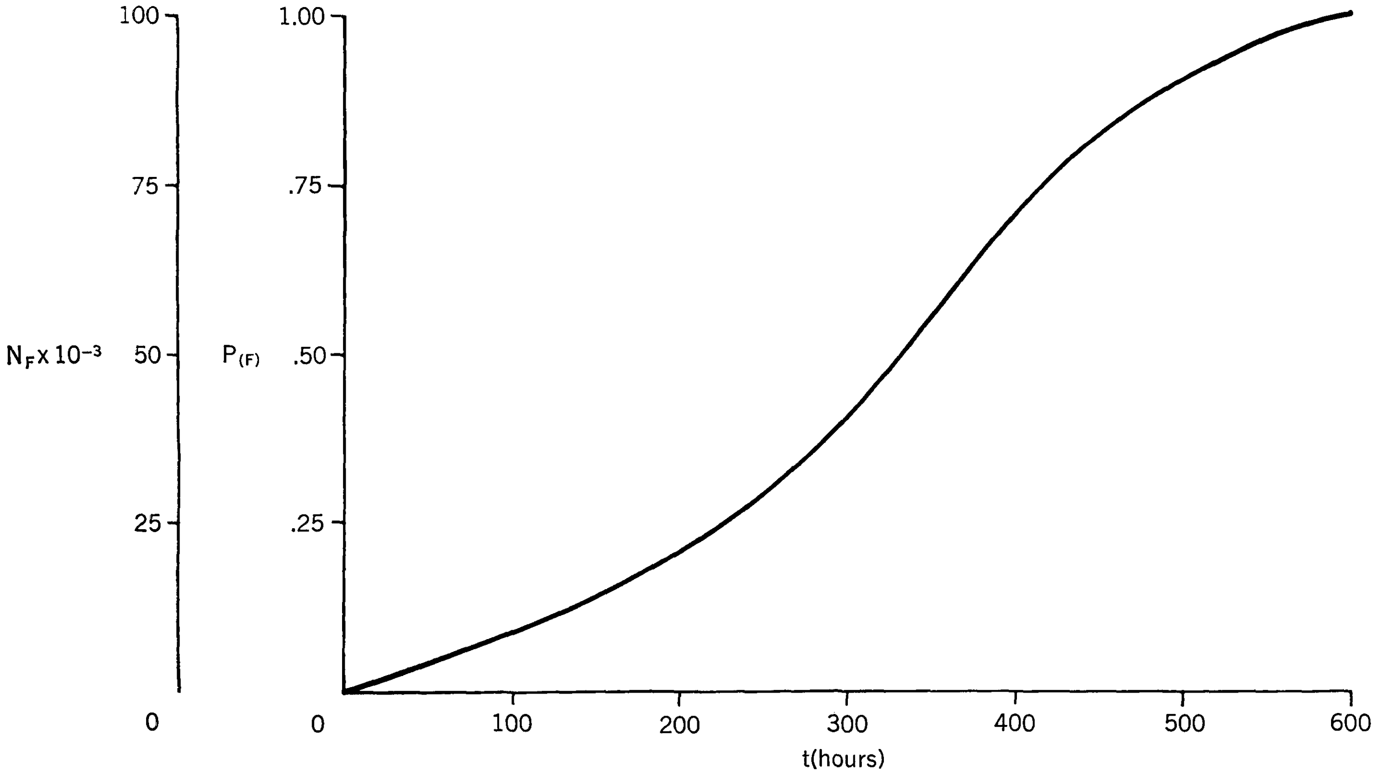


FIGURE 6

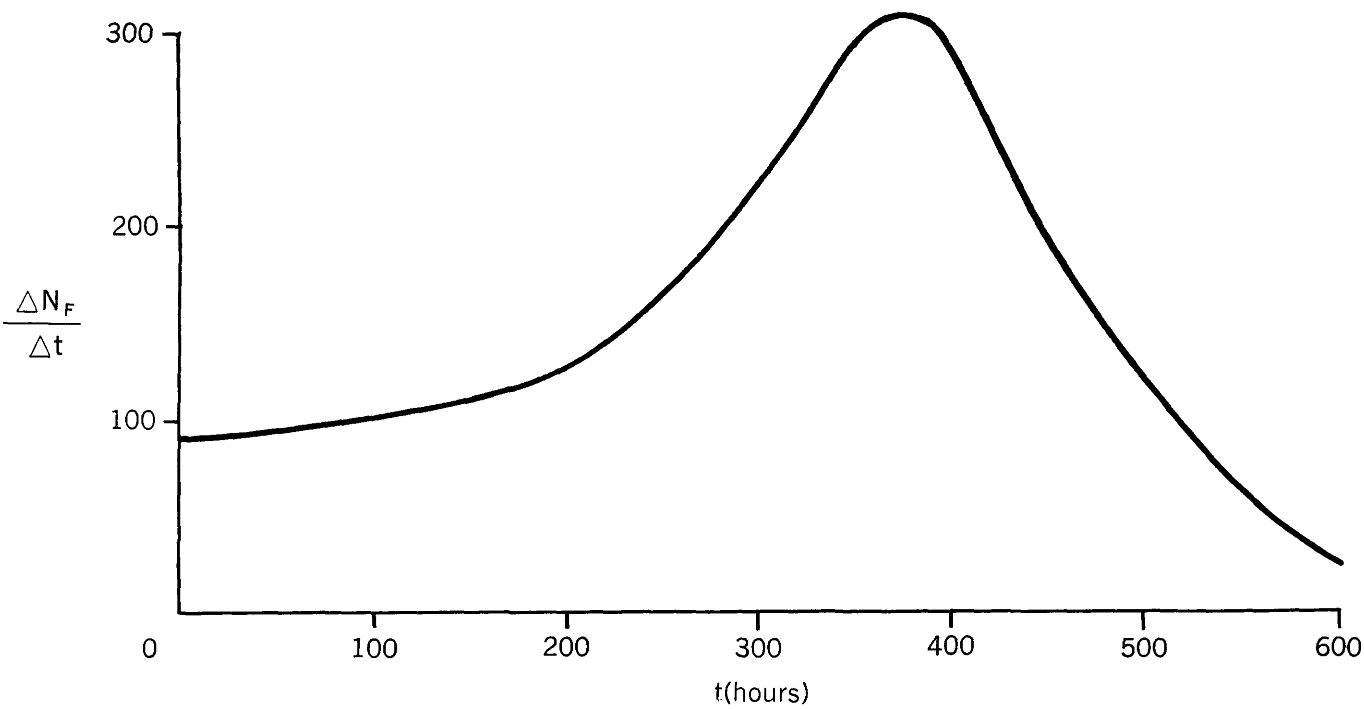


FIGURE 7

TABLE 2

Slope Calculations

Time	ΔN_F	Δt	Slope = $\Delta N_F/\Delta N_t$
hours	failures	hours	failures/hour
0	25K	274	25K/274 = 91
100	25K	230	109
200	32.5K	265	123
300	75K	335	224
360	100K	328	305
400	100K	340	294
500	50K	405	124
600	25K	370	68
			K = 1000

place the value R/Nc by the symbol, Q (Q = R/Nc).

Then 1-Q must be the fraction of the unit product cost which is *not* allocated to satisfying warranty claims [*i.e.*, 1-Q is the fraction of c which includes all other manufacturing costs (and the unit profit) *except* warranty costs]. Hence c′/c = 1-Q, where c′ is the known product unit cost *not* including warranty reserves and c is the desired adjusted unit product cost *including* cost of the warranty policy. Therefore c = c′/ (1-Q) = $50.00/(1-0.137) = $58.00; the unit product cost must be increased by $8/unit in order to prorate the cost of the warranty policy among all the units of the lot. If the market cannot accept the price increase, the manufacturer must absorb it and realize that it is the unit cost of honoring warranty claims. It is a cost factor which may reduce his unit profit, but almost certainly will enhance his reputation for product quality.

The value of the total warranty reserve fund to be allocated by the manufacturer to cover warranty claims can now be calculated and is estimated from Figure 5 to be (547) ($58.00) = $31,730, for the example. This will be recovered from the $8 price adjustment received as the product is sold. The accuracy of the preceding estimate depends primarily upon the accuracy of P_F. Experience has shown that the graphical method described will have a maximum error of about 5 per cent depending upon the accuracy of P_F (Figure 2) and the care taken with the slope constructions previously described.

If the curvature of Figure 5 is so great that it is hard to approximate by a straight line, the area can be found by using a planimeter or by counting graph squares. Then the

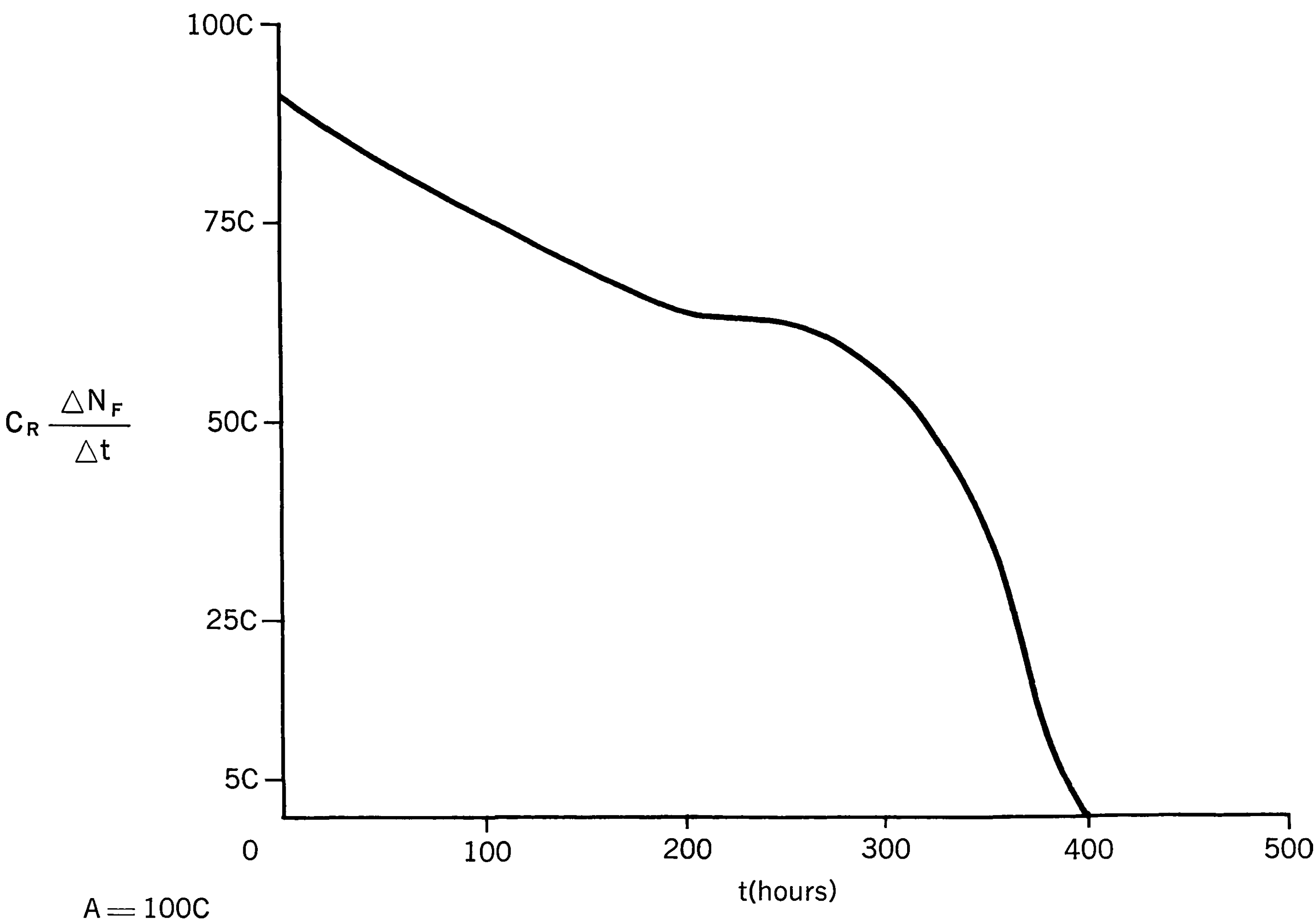


FIGURE 8

TABLE 3

Cost Calculation

t	C_R (calc.)	$\Delta N_F/\Delta t$ (from Fig. 7)	$C_R (\Delta N_F/\Delta N_t)$
0	1.00c	91	91.0c
100	.75c	102	76.5c
200	.50c	127	63.5c
250	.375c	167	62.6c
300	.25c	224	56.0c
350	.125c	295	36.9c
380	.04c	312	12.5c
400	0	294	0

cost per square unit of area must be determined from Figure 5. The measured area in square units must be multiplied by the cost per square unit of area to determine the total warranty reserve, R.

The above example considered warranty reserve calculations when product failure follows an exponential failure law because a large percentage of products fail in this way. Let us now consider the same calculation for a product which does not follow the exponential law or indeed any well known statistical distribution.

Suppose the manufacturer plans to make 100,000 units of a component which is critical to the operation of a system but because of its location in the system or because of excessive system stresses is subject to wear out failure. The component is replaced only after it fails. (This could be an electrical heating element for an oven, a critical bearing in a machine, and the like.) Past history shows that no product has more than 600 hours' life and the proportion of failures occurring during each 100 hours of operation is listed in Table 1 on page 50.

If each item carries a 400-hour warranty and a linear, pro rata rebate policy, let us calculate the adjusted sales price, c, to include the cost of expected warranty claims.

In exactly the same way as for the previous example, Figures 6, 7, and 8 (pages 51 and 52) are formed. Figure 6 plots the expected cumulated failures (N_F) versus time from the data just given. The data points are connected by a smooth curve implying that the wear out phenomenon is a continuous function of time. Construction lines for determining the slope at critical points are shown on Figure 6, and the slope calculations are listed in Table 2 on page 52.

The slope calculations, $\Delta N_F/\Delta t$ versus time, are plotted in Figure 7.

The unit rebate cost of honoring the 400-hour warranty policy for failure at any time t is $C_R = (1\text{-}t/400)c$. Table 3 (above) lists the calculations for the values of C_R $\Delta N_F/\Delta t$ which are plotted in the graph of Figure 8.

The area under the curve of Figure 8 can be easily found by using a planimeter or by approximating the curved portions of the graph by straight lines (as is shown below in Figure 9) and calculating the areas of the simple geometric figures identified schematically in the figure. The area calculations are listed below:

I	$200\,(62.5c) =$	12,500c
II	$\frac{200}{2}\,(25.5c) =$	2,550c
III	$50\,(62.5c) =$	3,125c
IV	$\frac{65}{2}\,(10c) =$	330c
V	$65\,(53.5c) =$	3,480c
VI	$\frac{95}{2}\,(53.5c) =$	2,540c
	Total	24,525c = R

Thus, following the same meth-

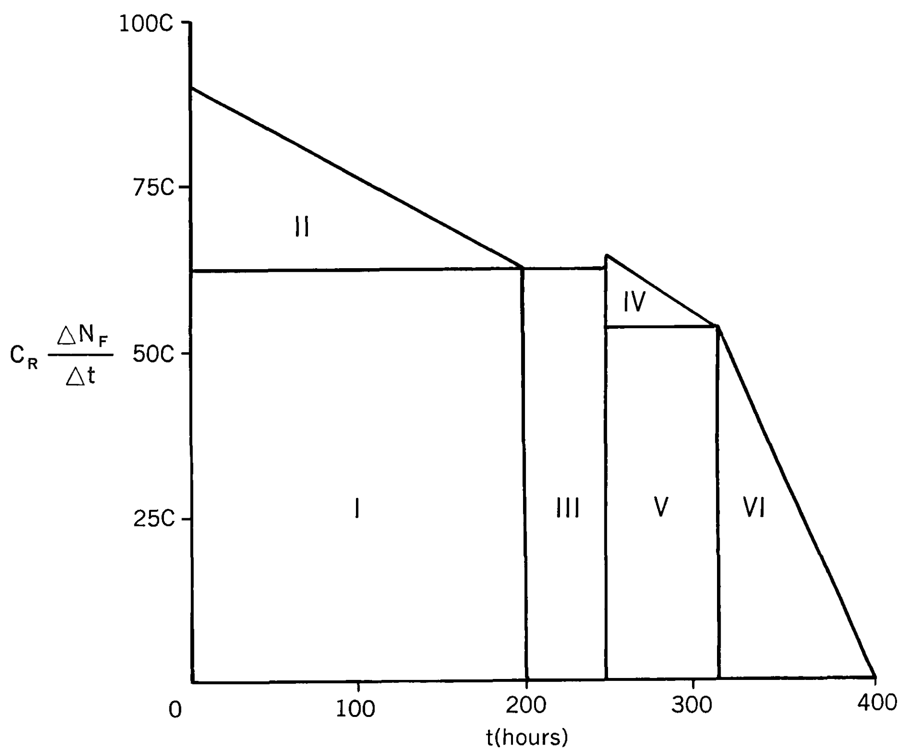


FIGURE 9

od of calculation as for the previous example for an exponential failure law:

$$R/Nc = \frac{24,525c}{100,000c} = 0.2452 = Q$$
$$1\text{-}Q \quad = 0.76$$

Therefore, $c = c'/0.76 = 1.32c'$, or the unit product price c' (without warranty reserve protection) must be increased by 32 per cent to cover the cost of honoring the warranty policy.

The warranty reserve, R, for the lot of N = 100,000 items assumed in the second example, must be equal to:

$$R = 0.245Nc = 0.245\ (100,000)c = 24,500c$$

The calculations above have been performed using a linear pro rata warranty policy. It is obvious that any other warranty policy may be handled in a similar manner though the resultant graphs may be more complicated than those of the examples shown here.

Lump sum rebate

Administration of claims using the pro rata graph of Figure 1 may be difficult. Each rebate would be different depending upon the time at failure. This could lead to errors caused by misreading the graph, miscalculation, or both. Therefore, use of this technique for administering warranty rebates should be examined for suitability in any specific case.

If errors in administering claims persist, the manufacturer should consider additional training of the rebate administrators. If this is not feasible, an alternate policy of fixed or lump sum rebate for any failure occurring within the warranty period may be considered. This kind of rebate plan can be protected by warranty reserve funds calculated *as though a pro rata warranty were being offered.*

The lump sum rebate plan may not be attractive to all customers and for all products since the usual rebate would be about 50 per cent of the original product price. This may not always be palatable to the customer when failure occurs shortly after the product is put into use.

When it is applicable

A lump sum rebate plan *should* be considered in situations where it is possible to determine that products failed before the end of the warranty period but it is *not* possible to specify the exact time of product failure for any individual item. Thus lump sum rebate plans are particularly applicable to:

1. failures in products installed in remote locations but subject to periodic inspection for failures
2. failures which are detected at a replacement time determined by a group replacement plan
3. initial or early failures of products which are stocked for long periods before use and which have a determinate shelf life.

In any of the above cases, a lump sum rebate policy could minimize arguments between manufacturer and customer about the amount of rebate due for product failure. This assumes, of course, that both parties understand and agree to the plan at the time of product sale.

How to calculate

To demonstrate the method, let us calculate the lump sum rebate for the first example of this article. Referring to Figure 2, the number of failures that can be expected by the end of the warranty period is $NP_F = 400\ (25.8) = 1032$ for t = 12 months. Then the total replacement cost of the expected number of failures is 1032c or 1032 ($58.00) = $59,860. But the reserve fund R = $31,730. Therefore k, the fraction of the replacement cost which can be given as a lump sum rebate, is $31,730/$59,860 = 0.53.

Then the unit lump sum rebate for the example can be 53 per cent of the original unit product cost. This rebate policy will still be charged to the customer by the unit price increase from $50.00 to $58.00 previously calculated and the $8(4000) = approximately $32,000 reserve fund will still be established in order to be able to honor warranty claims before all N units of the product lot are sold. Note that a more direct way of calculating k from values previously determined is given by $k = (R/Nc)(1/P_F)$.

Conclusion

A simple graphical method for performing warranty reserve calculations has been demonstrated. Costs of customer claims against a warranty can be treated quantitatively like any other production cost because adjustments in unit price can be calculated to allocate the cost of the warranty policy equally to all units sold. The total expected cost of all warranty claims can be calculated and a warranty reserve fund established to cover the costs of the expected warranty claims. Moneys in the warranty reserve fund will be recovered by the collection of the adjusted increment in unit price as the product is sold. If a lump sum rebate policy is desirable, it also can be determined by simple graphical procedures.

For those more analytically inclined, a mathematical formulation of warranty reserve calculations can, of course, also be prepared.

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