# COINTEGRATION: A SURVEY OF RECENT DEVELOPMENTS

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This paper has been prepared for the 1987 volume of *Economic Perspectives*. Harwood Academic Press. New York.
We are grateful to M. Arellano, A. Banerjee, S. Burgess, A. Espasa, J. Galbraith, D. Hendry, A. Maravall, S. Nickell, N. Rau and H. Stamler for their comments. The usual caveat applies.

Banco de España. Servicio de Estudios
Documento de Trabajo n.º 8708

ISBN: 84-505-7104-9
ISSN: 0213 - 2710
Depósito legal: M. 42884 - 1987

Imprenta del Banco de España

# ABSTRACT

This paper provides an updated survey of a burgeoning literature on testing, estimation and model specification in the presence of integrated variables. Integrated variables are a specific class of non-stationary variables which seem to characterise faithfully the properties of many macroeconomic time series. Their statistical properties and implications for the interpretation of regression models are covered in a unified way.

Key words: Unit root, Cointegration, Trends, Error Correction Mechanisms.

# I. INTRODUCTION.

The majority of econometric theory is built upon the assumption of stationarity. Until recently, this assumption was rarely questioned, and econometric analysis proceeded as if all the economic time-series were stationary, at least around a deterministic trend. Stationary series should, however, at least have constant unconditional mean and variance over time; a condition which appears rarely to be satisfied in economics. The importance of the stationarity assumption had been recognised for many years, but the important papers by Granger and Newbold (1974) and Nelson and Plosser (1982) allerted many to the econometric implications of non-stationarity. *Integrated* variables are a specific class of non-stationary variables with important economic and statistical properties. These are derived from the presence of stochastic trends, as opposed to deterministic trends, with innovations to an integrated process being permanent instead of transient. For example, in terms of welfare costs, this implies that the costs of expectational errors produced by, say, policy shifts are far more serious than in the case where the shocks were purely transient.

In particular the presence of a unit root is implied in many economic models by the rational use of available information by economic agents. Standard applications include futures contracts, stock prices, yield curves, real interest rates, exchange rates, hysteresis theories of unemployment, and, perhaps the most popular, the implications of the permanent income hypothesis for real consumption. In view of this epidemic of martingales in economics a voluminous literature on testing, estimation, and model specification in the presence of integrated variables has developed in the last few years, and the purpose of this survey is to provide a guide through this increasingly technical literature.

The analysis of cointegration developed out of the work on testing for, and implications of, unit roots in economic time-series. This survey covers both literatures in a unified way. Cointegration considers the conditions under which the use of standard regression analysis, when the individual series under consideration are integrated, is valid. Some of the properties of these 'cointegrating regressions' are extremely surprising, and suggest new ways to incorporate 'long-run' information (and constraints imposed by theory) into the statistical model. In addition, the concept of cointegration is, in many ways, a statistical definition of equilibrium. As such, cointegration offers a generic route to test the validity of the equilibrium predictions of economic theories.

The analysis of non-stationary variables requires a different statistical framework from the standard stationary case, and in Section II this framework is introduced, and testing procedures for unit roots are discussed. The proper treatment of integrated processes in regression analysis is then analysed using a variety of examples. This Section includes a number of more technical parts (denoted by an asterix) which could be avoided by those readers wishing to proceed quickly to the discussion of cointegration. Section III introduces the concept of cointegration, and discusses the implications for economic modelling and estimation, and the use of cointegration to discriminate between economic theories. Section V concludes.

It should be stressed that the concept of cointegration is relatively new, and that further developments, applications and Monte Carlo studies are appearing extremely rapidly. As a result, this survey is selective, and "best-practice" methods may well change in the near future.

## II. INTEGRATION AND UNIT ROOTS.

A weakly stationary series should have a mean and variance that are time-invariant. However, many economic time-series certainly do not satisfy this condition, having first and second moments that appear to be increasing over time (see Escribano (1987) for precise definitions of integration in the $i^{th}$ moment of a stochastic process). Such series are non-stationary, and may require differencing to induce stationarity[2]. A series requiring differencing d times to induce stationarity is denoted I(d), or "integrated of order d" (see Granger (1983)). A simple example of an I(1) series is the random walk:

$$\Delta y_t = \epsilon_t \qquad y_0 = 0$$

where, for instance, $\epsilon_t$ is distributed $IN(0,\sigma_\epsilon^2)$. If, however, y were an autoregressive series such as

$$\Delta y_t = -\alpha y_{t-1} + \epsilon_t \qquad |\alpha| < 1 \qquad y_0 \sim N(0,[1 - (1 - \alpha)^2]^{-1}\sigma_\epsilon^2)$$

then y would be stationary, or I(0). In this Section some of the important and far-reaching implications of the existence of unit roots ($\alpha = 0$) in economic time series are discussed.

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[2] This condition is really too strong. In fact all that is needed is absence of trend in variance after suitable mean transformations (see, for example, Dickey *et al* (1986) and Escribano (1987)).

## 2.1 Statistical Properties of Integrated Series*

We will concentrate, in this section, on the statistical properties which stem from the presence of a single unit root, and start by considering the following data generation process (DGP) for the canonical stochastic integrated process $\{y_t\}_0^\infty$

$$\Delta y_t = -\alpha y_{t-1} + \mu + \epsilon_t \qquad \alpha = 0 \qquad y_0 = 0 \qquad \ldots .(1)$$

$$\text{or} \quad y_t = \mu t + S_t \qquad S_t = \sum_1^t \epsilon_j \qquad \ldots .(2)$$

where as a particularly interesting case we consider the driftless version of (1) with $\mu = 0$. In general, integrated series such as $y_t$ are linear functions of time (with a slope of zero if $\mu = 0$). The deviations from this function of time are non-stationary, as they are the accumulation of past random shocks, giving rise to the concept of an integrated series.

To complete the specification of the DGP we need to impose some conditions on the innovation sequence $\{\epsilon_t\}_1^\infty$. These restrictions are necessary if non-degenerate limiting distributions of the statistics discussed below are to be derived. The weakest set of conditons that achieve this aim is defined in detail in Phillips (1987a), and can be summarised as follows:

(a) $E(\epsilon_t) = 0$ for all t

(b) $\sup_t E|\epsilon_t|^{2\beta} < \infty$ for some $\beta>2$

(c) $\sigma^2 = \lim E(T^{-1}S_T^2)$ exists and $\sigma^2>0$

(d) $\epsilon_t$ is strong-mixing with mixing coefficients $\alpha_m$ such that $\Sigma\alpha_m^{(1-2/\beta)}<$ infinity

Condition (b) restrains the heterogeneity of the process, while (c) controls the normalisation at a rate which ensures non-degenerate limiting distributions. Condition (d) moderates the extent of temporal dependence in relation to the probability of outliers (see White (1984)).

The generality of the previous set of conditions implies that model (1) encapsulates a wide variety of DGP's. These include virtually any auto-regressive moving average (ARMA) model with a unit root, and even ARMAX models with unit roots and non-evolutionary exogenous processes. It is important to notice at this stage that only if we assume that the errors are iid$(0,\sigma^2)$ will $\sigma^2 = \sigma^2_\epsilon$. This restrictive case is an interesting one since most limiting distributions that have been numerically tabulated have been based on this assumption. However, this will not be the case in most empirical applications and hence in general $\sigma^2 \neq \sigma^2_\epsilon$.[3]

In order to derive the aforementioned limiting distributions, it is necessary, as in the stationary framework, to use a sequence of random variables, whose convergence is ensured by suitable transformation. More precisely, in the non-stationary framework, we need to focus on the sequence of partial sums $\{S_t\}_1^T$ which has to be transformed so that each element lies in the space D(0,1) of all real valued functions on the interval [0,1] that are right continuous and have finite left limits. This is achieved by defining the functions

$$X_T(r) = T^{-1/2}S_{j-1} \qquad \frac{j-1}{T} \leq r < \frac{j}{T} \qquad (j=1,\ldots T)$$
$$X_T(1) = T^{-1/2}S_T$$

Under the previous assumptions on the sequence $\{\epsilon_t\}$ we have that as T

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[3] As an example, if $\epsilon_t$ follows an MA(1) process then $\epsilon_t = e_t - \theta e_{t-1}$ where $e_t$ is iid$(0,\sigma^2_e)$. Then $\sigma^2/\sigma^2_\epsilon = (1+\theta)^2/(1+\theta^2)$.

tends to infinity, $X_T(r) \to W(r)$, where $\to$ denotes weak convergence in probability. That is, $X_T(r)$ converges to a Wiener process.[4] Notice that $W(r)$ behaves like a random walk in continuous time such that for fixed r it is $N(0,r)$ and has independent increments.[5]

The most striking difference between the conventional and this new asymptotic theory is that whereas in the former the sample moments converge to constants, they converge to random variables in the latter. Similarly, as a result of the absence of stationarity and ergodicity, traditional Central Limit Theorems are substituted by Functional Limit Theorems (see, for example, Billingsley (1968)).

As an example of the previous remarks, the following standardised sample moments converge to Wiener functionals:

$$\text{(i)} \quad T^{-2}\Sigma y_t^2 \quad \to \quad \sigma^2 \int_0^1 W(t)^2\, dt \qquad \ldots.(3)$$

$$\text{(ii)} \quad T^{-3/2}\Sigma y_t \quad \to \quad \sigma \int_0^1 W(t)\, dt \qquad \ldots.(4)$$

$$\text{(iii)} \quad T^{-1}\Sigma y_{t-1}\epsilon_t \quad \dot{\to} \quad \frac{\sigma^2}{2}\,[W(1)^2 - \sigma_\epsilon^2/\sigma^2] \qquad \ldots.(5)$$

Note the divergences in the orders of magnitude of these limiting distributions with the conventional stationary distributions, i.e. order in probability $T^2$, denoted $0_p(T^2)$, instead of $0_p(T)$ in (3); $0_p(T^{3/2})$ instead of $0_p(T)$ in (4); and $0_p(T)$ instead of $0_p(T^{1/2})$ in (5). These

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[4] This Wiener process will lie in the space $C[0,1]$ of all real-valued functions continuous on the interval $[0,1]$.

[5] Moreover, an extension of the Slutsky Theorem in conventional asymptotic theory also applies in this framework, in the sense that if $g(.)$ is any continuous function on $C[0,1]$ then $X_T(r) \to W(r)$ implies that $g[X_T(r)] \to g[W(r)]$.

differences shed light on the non-conventional features of the coefficient consistency and limiting distributions when testing for unit roots, and will be important in the discussion of cointegration in Section III.

If, for instance, OLS is applied to (1) , it is easy to show that, using the sample variability results summarised in (3) - (5), the slope $\hat{\alpha}$ and its t-ratio converge to the following distributions, in the case when $\mu = 0$:

$$T\hat{\alpha} \rightarrow \frac{\frac{1}{2}\left[W(1)^2 - \sigma_\epsilon^2/\sigma^2\right]}{\int_0^1 W(t)^2\, dt} \qquad \ldots.(6)$$

$$t_{\alpha=0} \rightarrow \frac{\frac{1}{2}\sigma\left[W(1)^2 - \sigma_\epsilon^2/\sigma^2\right]}{\sigma_\epsilon\left[\int_0^1 W(t)^2\, dt\right]^{1/2}} \qquad \ldots.(7)$$

From (6) we note that $\hat{\alpha}$ converges to its true value zero at a rate of $O_p(T^{-1})$ instead of the conventional $O_p(T^{-1/2})$. Similarly, from (7), the corresponding t-ratio has a non-degenerate distribution which is different from the standardised normal distribution which is used in conventional asymptotic theory.

## 2.2 Testing for Unit Roots.

The previous statistical implications of the unit root hypothesis in the time-series representation of univariate models underscore the need to have reliable procedures to test formally this hypothesis. Investigations by Dickey (1976), Dickey and Fuller (1979, 1981) and Fuller (1976) have constructed by numerical simulations the corresponding critical values of the limiting distributions expressed in (6) and (7). Table 1 collects the exact null and alternative hypotheses under which these simulations were performed. The unrestricted model contains both a constant and a trend as regressors, plus an error term subject to a first order autoregressive representation. Three interesting cases follow. Case 1 in that in which both the drift and the trend are zero, such that under the null we find a pure driftless random walk. Case 2 describes the case in which there is a drift but no trend, and consequently the model under the null is again the driftless random walk. Finally, Case 3 relates to the most general case in which both constant and trend are different from zero, and hence the model under the null hypothesis is random walk with drift. Notice that in all cases the error terms under $H_0$ are assumed to be iid$(0,\sigma^2)$ for simulation purposes.

Table 1: Use of Tables to Test for a Unit Root in Univariate Models.

$$\begin{tikzcd}
 & H_0\ (\alpha = 0) & & \begin{array}{l} H_1\ (y_t = \mu + \beta t + u_t) \\ \quad \Delta u_t = -\alpha u_{t-1} + \nu_t \end{array} \\
\underline{\text{Case 1}}\ (\mu = \beta = 0) & \Delta y_t = \epsilon_t & B_1 \arrow[l] \arrow[r] & \Delta y_t = -\alpha y_{t-1} + \epsilon_t \\
\underline{\text{Case 2}}\ (\mu \neq 0, \beta = 0) & \Delta y_t = \epsilon_t & B_2 \arrow[l] \arrow[r] & \Delta y_t = -\alpha y_{t-1} + \alpha\mu + \epsilon_t \\
 & & T_0 \arrow[ur] \arrow[dl] & \\
\underline{\text{Case 3}}\ (\mu \neq 0, \beta \neq 0) & \Delta y_t = \beta + \epsilon_t & B_3 \arrow[l] \arrow[r] & \begin{array}{l} \Delta y_t = -\alpha y_{t-1} + \beta\alpha t \\ \quad + [\mu\alpha + \beta(1-\alpha)] + \epsilon_t \end{array}
\end{tikzcd}$$

Note: $B_i$ (i=1,2,3) denote 1st, 2nd and 3rd blocks of Table 8.5.2 in Fuller (1976), $T_0$ denotes Table for critical values for the standardised Normal distribution.

From (6) and (7) two basic statistics can be derived to test the null hypothesis of a unit root. The first test refers to the scaled regression coefficient $T\hat{\alpha}$ while the second concentrates on the t-ratio $t\hat{\alpha}$. Critical values for both asymptotic distributions are found in Fuller (1976)[6]. The arrow scheme in Table 1 explains the proper use of these tables, depending on the choice of the model representing the unrestricted hypothesis. If we start with Case 1, then we should use the first block (denoted $B_1$) of critical values in Tables 8.5.1 and 8.5.2. Similarly, the choice of model with constant and constant plus trend implies the use of the second and third blocks (denoted $B_2$ and $B_3$) respectively. A very interesting case, of which some practitioners are unaware, is that, when choosing the models with a constant, if that nuisance parameter is significant under the null (checked by simply regressing $\Delta y_t$ on a constant) then the right critical value for the t-ratio will be found in the standardized normal distribution table (denoted $T_0$), rather than in the Dickey-Fuller tables (see West (1986))[7].

The same peculiar result obtains when, after using the most general model, the constant and the trend are significant under the null (checked

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[6] Chapter 8, Tables 8.5.1 and 8.5.2.

[7] In general for Case 2 it can be shown that $T^{3/2}\hat{\alpha} \sim N(0,\ 12\sigma^2/\mu^2)$ and $t_{\hat{\alpha}} \sim N(0,\ \sigma^2/\sigma_\epsilon^2)$.

by regressing $\Delta y_t$ on a constant and trend)[8]. In both instances, the interesting outcome of looking at the wrong tables is enlightened when we find $t_{\hat{\alpha}}$ ,say at the 5% level, larger than 1.96 but smaller than the corresponding critical values in the D-F tables. Upon these conditions, we should be rejecting the null hypothesis instead of accepting it. The intuition behind this peculiar result is that if there is a unit root and, say, a constant, the integrated series depends on a deterministic trend and a stochastic one. Moreover, the sample variability of the deterministic trend is of $O_p(T^3)$ which dominates the order of the sample variability of the stochastic trend which is of $O_p(T^2)$. But we know that the existence of a deterministic trend in a regression model does not affect the asymptotic normality of the standardised estimates, hence normality follows.

It is clear from the previous discussion and the derivations of the conventional and unconventional statistics shown in (6) & (7) that if the error sequence $\{\epsilon_t\}$ is correlated, the distributions will depend on the nuisance parameter $\sigma^2/\sigma_\epsilon^2$. In such a case there is a need to either change the estimation method (that is, adopt another regression model), or modify the statistics described above. Dickey and Fuller (1981) favour the first approach by enlarging the regression model by adding in a lag polynomial of $\Delta y_t$ such that these terms capture the serial correlation in any of the unrestricted models contained in Table 1. It can be shown,

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[8] The case where the DGP contains a unit root and a trend does not seem to be too realistic *a priori* since, in logarithmic form, it implies an ever increasing (or decreasing) rate of change. In general for Case 3 it can be shown that $T^{5/2}\hat{\alpha} \sim N(0,\ 180\sigma^2/\beta^2)$ and $t_{\hat{\alpha}} \sim N(0,\ \sigma^2/\sigma_\epsilon^2)$.

that under the null hypothesis, $t_{\hat{\alpha}}$ in the enlarged model has the same limiting distribution as when the errors are iid, giving rise to the so-called Augmented Dickey-Fuller tests. Note, however, that it is no longer legitimate to use $T\hat{\alpha}$ as the basis of a test in any of the variants, since they are not invariant to the true population value of the parameters of the distributed lag in $\Delta y_t$.

Nevertheless, this solution introduces the problem that we might need a large number of lags of $\Delta y_t$ in order to obtain uncorrelated residuals. Recently, Said and Dickey (1984) have shown that if $\epsilon_t$ contains moving average terms, the number of extra regressors needs to increase with the sample size at a rate $(T^{1/3})$. Given that the majority of the macroeconomic variables studied in the seminal paper by Nelson and Plosser (1982) were adequately represented by an IMA(1) process, this seems a quite likely situation. Schwert (1985), using Monte Carlo simulations, has recently shown that the exact size of the test may be far from the nominal size if the order of the autoregressive correction is not increased as the sample size increases. Accordingly, it would be desirable to have an approach for the test which takes into consideration the structure of the residuals in a non-parametric way under the assumptions (a) - (d) above. This is the approach developed by Phillips and Perron (1986) and Perron (1987), and described briefly in the Appendix.

Next, we briefly discuss a testing strategy based on the choice of the appropriate initial unrestricted model in Table 1, as well as on the choice of data sample. With respect to the first issue, we advocate estimating the most unrestricted model initially, as in Case 3. Then use the test statistic (c) in Table 2 of the Appendix to test for a unit root, using the critical values contained in $B_3$ of Table 8.5.2. If the null hypothesis of a unit root is rejected there is no need to go further. If it is not rejected, test for the significance of the trend (a rather implausible case, as discussed earlier) using the test statistic in row (d) of Table 2. If it is significant, then test for its significance under the null using the ordinary tables. Its significance under the null would imply that the ordinary tables, instead of Table 8.5.2 should have been used to test for the unit root. If the trend is not significant under the alternative, estimate the unrestricted model in Case 2 in Table 1. Test again for the unit root using the test statistic (b) in Table 2, looking at $B_2$ in Table 8.5.2. If the null hypothesis is rejected, again there is no need to go further. If it is not rejected, test for the significance of the constant under the alternative using the test statistic shown in row (e) of Table 2. If the procedure reaches the most restrictive alternative model, as in Case 1, then the unit root should be tested with the critical values contained in $B_1$ of Table 8.5.2. Failure to follow this strategy may lead to serious misinterpretations.

An alternative strand to the literature on testing for unit roots is that suggested by Sargan and Bhargava (1983). They advocate the use of the conventional Durbin-Watson (DW) statistic from the simple OLS regression of the variable under consideration on a constant, that is

$$y_t = c + u_t$$

$$\Delta u_t = -\alpha u_{t-1} + \epsilon_t \qquad \epsilon_t \text{ distributed } IN(0,\sigma_\epsilon^2)$$

Then the null hypothesis of $\alpha=0$ is tested against the alternative that the errors follow a stationary first order autoregressive process. A unit root for the error process is equivalent to the structural element following a random walk. The value of the DW statistic will obviously tend to be very low when the root in the error process tends towards unity, since DW $\simeq 2(\alpha)$. The test can be performed using the standard DW statistic generated by most statistical programs along with the table of critical values presented by Sargan and Bhargava (1983) under the unit root null hypothesis. This test can be shown to be the uniformly most powerful invariant test against the alternative of a stationary first order autoregressive error process. An important feature of the test is its invariance to whether a trend enters into the true model, unlike the other tests considered above. However, the test is only powerful in discriminating between the simple random walk and stationary first order autoregressive processes, and thus lacks generality.

Having discussed the main tests that have been proposed for unit roots, an important qualification should be noted. In practice, economic time series emerge from this testing procedure as appearing to be I(1). However, in the context of, for example, the Sargan and Bhargava approach, the estimated degree of autoregression in the residuals is often in excess of 0.95. In other words, a value of 0.1 for the DW statistic is fairly typical in the static regression (given that DW $\simeq 2\alpha$). However, as Sargan and Bhargava note, the power of the test for a unit root against such highly autoregressive alternatives is exceedingly low. This is hardly suprising, since discrimination between a 0.95 autoregressive process and a random walk is extremely difficult in the relatively short samples typically used in economics. The practical implications are, however, important when we consider the powerful cointegration results that depend upon the individual time series possessing unit roots.

It should be noted that the definitions and properties introduced up to now for scalar random variables extend to multivariate cases (see Phillips and Durlauf (1986b)) by applying the properties to each element of the vector. This extension immediately raises the question of having components with different degrees of integration, or the possibility of finding linear transformations of those components with a different order of integration to the order of the individual elements of the vectors. Both these issues are raised in the next two Sections.

Finally, as far as the choice of data sample is concerned the main result concerns the trade-off between span and sampling interval (see Shiller and Perron (1985)). For a given span, more observations lead to higher power of the previous tests. Similarly, a longer span for a given number of observations leads to higher power. Of course, this intuitive result had to be mediated by the relevant alternative. So, for example, since for macroeconomic series, the natural alternative is mean reversion over a period similar to the length of business cycles, a long span of annual data should be preferred to a shorter span with, say, quarterly or monthly data.

## 2.3 Asymptotic Theory and Monte Carlo Results*

Having examined the important statistical implications of integrated processes, we proceed to use this theory to interpret a number of results concerning the treatment of integrated series in regression analysis. An explicit analytical solution to the asymptotic behaviour of parameter estimates and regression statistics permits a unification of the disparate Monte Carlo studies that presently exist in the literature. We present a summary of results on analyses which range from inappropriate detrending of integrated series, to efficiency tests, including the familiar spurious regression results. Most of the results derive from the work of Phillips or Phillips and Durlauf in a recent long sequence of papers that are referenced at the beginning of each case.

In order to unify as much as possible the treatment of different cases, the following description procedure is adopted. Each case will be characterised by a DGP and an estimated model (denoted simply MODEL). The distributional results, which happen to be functionals of Wiener processes, will be denoted generically by f(W), whose precise expressions are given in the appropriate references. At the end of each case we offer an intuitive explanation of the analytical results, together with some remarks about the use of certain regression statistics, which prove to be useful to detect misspecifications in the estimated models.

<u>De-trending</u> (Phillips and Durlauf (1986a))

$$\text{DGP} \quad \Delta y_t = \epsilon_t$$

$$\text{MODEL} \quad y_t = \hat{\mu} + \hat{\beta} t + \hat{u}_t \qquad \text{....(8)}$$

Summary of Results:

$T^{-1/2}\hat{\mu} \rightarrow f(W)$ $\quad T^{1/2}\hat{\beta} \rightarrow f(W)$ $\quad T^{-1/2} t_{\mu=0} \rightarrow f(W)$

$T^{-1/2} t_{\beta=0} \rightarrow f(W)$ $\quad T^{-1} s^2 \rightarrow f(W)$ $\quad T.DW \rightarrow f(W)$

$R^2 \rightarrow f(W)$

This case tackles the issue of inappropriate de-trending of integrated processes, under the traditional belief that conventional asymptotic theory could be applied to detrended series. We observe that the $\hat{\beta}$ coefficient is consistent, converging to its true value of zero. However, its t-ratio diverges to infinity, confirming the Monte Carlo results of Nelson and Kang (1981). Both the drift and its t-ratio diverge. The estimated variance of the residuals ($s^2$) also diverges reflecting the fact that the residuals of the model are non-stationary around the trend. The coefficient of multiple correlation ($R^2$) converges to a non-degenerate limiting distribution. The results for the Durbin-Watson statistic (DW) appear quite promising, confirming its powerful role as a misspecification diagnostic (see Sargan and Bhargava (1983)). The intuition behind all these disparate results stems from the different orders of magnitude of the sampling variability of the regressors and regressand in the model, i.e. $O_p(T) = \hat{\mu}\, O_p(1) + \hat{\beta}\, O_p(T^2)$. The divergence of the order of magnitude highlights the fact that $\hat{\beta}$ converges while $\hat{\mu}$ diverges, according to when the sample variances of their corresponding regressors are larger or smaller than the sample variance of the regressand.

Encompassing Tests (Phillips and Durlauf (1986a))

$$\text{DGP} \quad \Delta y_t = \epsilon_t$$

$$\text{MODEL} \quad \Delta y_t = \hat{\mu} + \hat{\beta} t - \hat{\alpha} y_{t-1} + \hat{u}_t \qquad \ldots.(9)$$

Summary of Results:

$T^{3/2}\, \hat{\beta} \rightarrow f(W)$ $\quad T\, \hat{\alpha} \rightarrow f(W)$ $\quad T^{-1/2}\, t_{\alpha=0} \rightarrow f(W)$

$s^2 \rightarrow \sigma^2_\epsilon$ $\quad F_{\beta=0,\alpha=0} \rightarrow f(W)$

This case interprets the unit root test in Case 3 of Table 1, where the issue is to discriminate between trends and integrated processes. The model embodies both alternatives, and uses the F test to discriminate between the alternatives. The encompassing test works as follows: $H_A(\beta=0,\alpha=0)$ corresponds to the integrated process, whereas $H_B(\alpha=1)$ corresponds to the deterministic trend. Denoting rejection of a hypothesis by $^{\sim}H$, the following combinations of rejections and non-rejections would operate the encompassing tests: $(H_A,\ ^{\sim}H_B)$ supports the Random Walk; $(^{\sim}H_A,\ H_B)$ supports the Deterministic Trend. In view of the divergence of $t_{\alpha=0}$, we would conclude that $H_B$ is always rejected for a sufficiently large sample. The F-test for $H_A$ converges to a non-degenerate distribution, which differs from the ordinary F distribution, and hence requires the Dickey-Fuller critical values as explained above. The disparate sample variability of regressand and regressors is given by $O_p(1) = \hat{\mu}\ O_p(1) + \hat{\beta}\ O_p(T^2) - \hat{\alpha}\ O_p(T)$.

<u>Non de-trended Spurious Regression</u>. (Phillips (1987b))

$$\text{DGP} \quad \Delta y_t = \epsilon_t \ ; \ \Delta x_t = \nu_t \ ; \ E(\epsilon_t \nu_t) = \rho\sigma_\epsilon\sigma_\nu\delta_{ts} \qquad \text{....(10)}$$

$$\text{MODEL} \quad y_t = \hat{\mu} + \hat{\alpha}x_t + \hat{u}_t$$

Summary of Results:

$\hat{\alpha} \rightarrow f(W)$ $\quad T^{-1/2}\ \hat{\mu} \rightarrow f(W)$ $\quad T^{-1/2}\ t_{\alpha=0} \rightarrow f(W)$

$T^{-1}\ s^2 \rightarrow f(W)$ $\quad T.DW \rightarrow f(W)$ $\quad R^2 \rightarrow f(W)$

This case interprets the familiar Monte-Carlo results of Granger and Newbold (1974), reinforcing analytically the divergence of $t_{\alpha=0}$ despite the fact that $\hat{\alpha}$ and $R^2$ possess non-degenerate distributions. Again, as in the de-trending case, the DW statistic detects misspecification of the model, although GLS corrections fail to provide the right answer. The orders of magnitude of the sampling variability in the model are: $O_p(T) = \hat{\mu}\ O_p(1) + \hat{\beta}\ O_p(T)$. Notice that the equality of the orders of magnitude between $y_t$ and $x_t$ provides the possibility of finding certain combinations of both variables such that the residuals are stationary, despite the non-stationarity nature of the variables themselves.

<u>De-trended Spurious Regression.</u> (Phillips and Durlauf (1986a))

$$\text{DGP} \quad \Delta y_t = \epsilon_t \ ; \ \Delta x_t = \nu_t \ ; \ E(\epsilon_t \nu_s) = \rho\sigma_\epsilon \sigma_\nu \delta_{ts} \qquad \ldots.(11)$$

$$\text{MODEL} \quad y_t = \hat{\mu} + \hat{\beta} t + \hat{\alpha} x_t + \hat{u}_t$$

Summary of Results:

$\hat{\alpha} \rightarrow f(W)$ $\quad T^{-1/2}\hat{\mu} \rightarrow f(W)$ $\quad T^{1/2}\hat{\beta} \rightarrow f(W)$

$T^{-1/2} t_{\alpha=0} \rightarrow f(W)$ $\quad T^{-1} s^2 \rightarrow f(W)$ $\quad T.DW \rightarrow f(W)$

This case interprets results from spurious regression models where $y_t$ and $x_t$ are de-trended, with the aim of inducing stationarity in the variables prior to the regression. The results are similar to the previous case with the addition that $\hat{\beta}$ is consistent, since the order of magnitude of the sample variance of the trend is $O_p(T^2)$. Notice that the presence of a trend in the regression only has qualitative effects on the asymptotic distribution.

Efficiency Tests. (Banerjee and Dolado (1987))

$$\text{DGP} \quad \Delta y_t = \epsilon_t \; ; \; \Delta x_t = \nu_t \; ; \; E(\epsilon_t \nu_t) = \rho\sigma_\epsilon \sigma_\nu \delta_{ts} \qquad \text{....(12)}$$

$$\text{MODEL} \quad \epsilon_t = \hat{\mu} + \hat{\beta} t + \hat{\alpha} x_{t-1} + \hat{u}_t$$

Summary of Results:

$T\hat{\alpha} \rightarrow f(W)$ $\quad T^{1/2}\hat{\mu} \rightarrow f(W)$ $\quad T^{3/2}\hat{\beta} \rightarrow f(W)$

$t_{\alpha=0} \rightarrow f(W)$ $\quad T.R^2 \rightarrow f(W)$

This case interprets recent Monte-Carlo results by Mankiw and Shapiro (1985) on the over-rejection of the orthogonality condition which characterises rational expectations models. In this case, the three estimated parameters and $R^2$ are consistent, but $t_{\alpha=0}$, the basis of the previous test (see Flavin (1981)) does converge to a non-degenerate distribution which differs from the standardised normal. The orders of magnitude of the sample variances are $0_p(1) = \hat{\mu}\ 0_p(1) + \hat{\beta}\ 0_p(T^2) + \hat{\alpha}\ 0_p(T)$.

## III. COINTEGRATION.

Whereas the analysis and implications of unit roots in individual time series excited mainly the econometrician, far more general economic interest has developed in the concept of cointegration, which analyses groups of integrated variables. The major reason for this is the possibility of estimating, and testing the existence of, long run economic relationships suggested by theory. As was explained in the previous sections, many individual economic time series appear to be non-stationary, requiring differencing at least once to induce stationarity. Yet economic theory rarely suggests equilibria that are not stationary functions of the variables involved. This would imply that there may exist fundamental economic forces that, over time, make variables move stochastically together. In other words, whereas the individual economic variables involved in a theory may all be non-stationary, the deviations from a given equilibrium may be bounded.

For many years the problems associated with static regressions between time-series have been known (for an interesting historical account see Hendry (1986)). The problem of 'spurious' regressions discussed earlier led many economists to adopt the Box-Jenkins (1970) methodology of transforming all the variables to stationary series prior to regression, so that, for the most part, differenced variables were considered. This, of course, resulted in models that disregarded the low frequencies of the variables, and so did not allow for any of the long run relationships which economic theory normally suggested. These features made the models difficult to interpret.

One response to such problems was the use of error-correction mechanisms (ECM) in econometric models. Models including ECMs have been widely used since Sargan (1964), and have the advantage of retaining information about the levels of variables, and hence any long-run relationships between such variables, within the model (see, for example Davidson ***et al*** (1978), Currie (1981) and Salmon (1982)). In an important paper, Granger (1983) establishes the equivalence between cointegration and error-correction. That is, ECM's produce cointegrated sets of variables, and, if a cointegrated set of variables is found, it must have an ECM representation. To a great extent, cointegration provides formal statistical support for the use of error-correcting models, and suggests additional procedures to test model specification in a *static* sense, and proposes ways to parameterise the error-correcting mechanism.

A vector of variables $x_t$ is said to be cointegrated if

(i) each element of $x_t$ is I(d)

and (ii) there exists a vector $\alpha$ such that $\alpha' x_t$ is I(d-b), where $\alpha \neq 0$ and $b > 0$.

For example, in the case of d=b=1, if $x_t$ is cointegrated, each variable in $x_t$ would each be I(1), but some linear combination of them would be I(0). If such a linear combination can be found, $\alpha$ is called the cointegrating vector.

The relationship between cointegration and equilibrium now becomes clearer. One natural way to characterise equilibrium between a set of variables is to define equilibrium to occur when a linear constraint is satisfied, such as

$$\alpha' x_t = 0 \qquad \text{....(13)}$$

For example, if we believe that a proportion $\lambda$ of any increase in labour productivity is eventually passed on in the form of real wages then, in equilibrium, $w = c + \lambda Q$ where w and Q denote real wages and productivity respectively, and c is a constant. Therefore, if

$$w - c - \lambda Q = 0 \qquad \text{....(14)}$$

in any time period, then the labour market would be in equilibrium. Of course, real wages may take some time to respond to changes in productivity, and the process by which equilibrium tends to be restored may be complex, in which case the scalar $z_t = w_t - c - \lambda Q_t$ would measure the deviation from equilibrium, or disequilibrium, in period t. If w and Q are cointegrated, then, by the above definition, the deviations from equilibrium will be bounded. An obvious way of testing the theory is then to determine the order of integration of $z_t$. If it is not possible to reject the null hypothesis of a unit root for $z_t$ then there will be no tendency for the real wage to move towards the putative equilibrium, in which case the estimated equilibrium would be misleading and irrelevant.

In the case of testing for cointegration between two variables $x_1$ and $x_2$, if a cointegrating vector exists, it must be unique. To see this, suppose $x_1$ and $x_2$ are both I(1) variables and $z_t = x_{1t} + \mu x_{2t}$ is I(0). Then $\mu$ must be unique, since any other linear combination would add or subtract a term in $x_{2t}$, which would be I(1), which would result in $z_t$ also being I(1). However, when x has more than two components, if a cointegrating vector exists, it need not be unique. In general, if x has N components, there may be r linearly independent cointegrating vectors, where $r \leq N-1$.

To illustrate the possible outcomes, consider the following example, taken from Granger and Engle (1987). Suppose $y_t$ and $x_t$ are jointly distributed according to the following data generation process:

$$\begin{aligned} y_t + \beta x_t &= v_t & \qquad v_t &= \rho_1 v_{t-1} + \epsilon_{1t} \\ y_t + \alpha x_t &= u_t & \qquad u_t &= \rho_2 u_{t-1} + \epsilon_{2t} \end{aligned} \qquad \ldots.(15)$$

where $\epsilon_1$ and $\epsilon_2$ are distributed independently N(0,1). Four possible cases exist:

(i) $\rho_1 = 1$, $\rho_2 < 1$ which implies that $x_t$ and $y_t$ are I(1) and the cointegrating vector is $(1,-\alpha)$.

(ii) $\rho_1 < 1$, $\rho_2 = 1$ which implies that $x_t$ and $y_t$ are I(1) and the cointegrating vector is $(1,-\beta)$.

(iii) $\rho_1 = \rho_2 = 1$ which implies that $x_t$ and $y_t$ are I(1) but there does not exist a cointegrating vector.

(iv) $\rho_1 < 1$, $\rho_2 < 1$ which implies that $x_t$ and $y_t$ are I(0) and so any linear combination of x and y will be I(0).

The last case introduces some interesting issues. The test for cointegration is actually a conditional test: conditional on $x_t$ and $y_t$ being I(1), the discovery of an I(0) linear combination would imply that the variables are cointegrated. However, as noted above, when $\rho_1$ and $\rho_2$ are unknown, the power of tests for unit roots against alternatives of roots close to the unit circle is often exceedingly low. In such situations type II errors, that is the acceptance of a unit root rather than a root of, say, 0.95, are likely to occur. Jenkinson (1986b) presents some Monte Carlo evidence on the hazards of inference when some, or all, of the variables under consideration are, in fact, highly autoregressive rather than I(1). The intuition is that we need extremely long time-series in order to distinguish borderline from unit root cases. Banerjee et al (1987a,b) present easily computable approximations to the correct critical values in general cases.

### 3.1 Estimation

An obvious issue is the question of estimating, and testing for the existence of, cointegrating vectors. Consider again the problem of estimating $\alpha$ and testing for the stationarity of z in the model

$$\alpha' x_t = z_t \qquad \ldots.(16)$$

If all the variables in x are I(1), then in general a linear combination of these variables, and hence $z_t$ , will be I(1). Therefore, almost all the $\alpha$ vectors will produce a series z with asymptotically infinite variance. The exceptions to this will be any cointegrating vectors. Now since Ordinary Least Squares estimation minimises the residual variance of $z_t$ , the estimated $\alpha$ vector derived from an OLS regression of the simple model (16) where all variables are in levels and no dynamics are included, should yield an excellent approximation to a true cointegrating vector, if one exists.

This result is one important reason why interest in cointegration has itself exploded like a non-stationary series. It implies that to parameterise a long-run equilibrium relationship between a set of variables, all that is needed is a simple static OLS regression between those variables. This simple regression can even be performed at the first stage of a research program, as is advocated by the Engle & Granger (1987) 'two-step estimator' discussed in Section 3.2 below. In any event, such an initial check may indicate to what extent the equilibrium predictions of the economic theory are consonant with the data, and, to put the argument at its strongest, whether it is fruitful to expend resources to model the short term dynamics around the equilibrium. To make things even easier, at least one econometric software package, PC-GIVE, automatically provides basic tests to determine the order of integration of the variables in the model as an addition to such summary measures as the means and standard deviations of the variables!

Indeed, the OLS estimate of any cointegrating vector should converge to the true value extremely quickly. To see this, consider the following case characterised as in the taxonomy of Section 2.3.

$$\text{DGP:}\quad \Delta x_t = \epsilon_t$$

$$y_t = \alpha x_t + e_t \qquad e_t = (1 - \rho L)\xi_t \qquad \ldots.(17)$$

$$\text{MODEL:}\quad y_t = \hat{\mu} + \hat{\alpha} x_t + \hat{u}_t$$

Summary of Results:

$T(\hat{\alpha} - \alpha) \rightarrow f(W)$ $\qquad$ $T(1 - R^2) \rightarrow f(W)$ $\qquad$ $T^{1/2}\hat{\mu} \rightarrow f(W)$

$T.DW \rightarrow 2(1 - \rho)$ $\qquad$ $t_{\hat{\alpha}=\alpha} \rightarrow f(W)$

The interpretation of the results illustrates very clearly the previous informal discussion. The slope in the static regression converges to its true value $\alpha$ at a rate of $O_p(T^{-1})$ instead of the ordinary rate of $O_p(T^{-1/2})$. The intuition is again clear: $\hat{\alpha}$ is computed using the ratio of a covariance, which is of $O_p(T)$, by a variance, which is of $O_p(T^2)$, given that both $x_t$ and $y_t$ are I(1). Therefore, any bias in $\hat{\alpha}$ is of $O_p(T^{-1})$. However, in spite of this super-consistency of $\hat{\alpha}$, its distribution is not asymptotically normal, and therefore the computed standard errors of the coefficients lack meaning. Since both $x_t$ and $y_t$ are driftless processes, $\hat{\mu}$ converges consistently to zero, although at a slower speed than $\hat{\alpha}$. The coefficient of multiple correlation $R^2$ is also $O_p(T)$ consistent to unity, reflecting the fact that in the bivariate case, under cointegration, the product of the slope and the inverse slope is unity. This feature will be exploited in the discussion below. Finally, the DW statistic converges to the standard result under the assumption that $e_t$ follows an AR(1) process.

An important associated result relates to the existence of simultaneity biases and errors in variables. Such biases in parameter estimates normally derive from the correlation between the regressors and the errors, which is ordinarily assumed to be of $O_p(T)$. However, given the fact that in cointegrating regressions $\Sigma x_t e_t$ will be of a lower order of magnitude than $\Sigma x_t^2$, such biases are asymptotically negligible. This implies that issues of endogeneity and exogeneity are not, in general, relevant in static cointegrating regressions.

The most important result of the previous discussion relates to the super-consistency of $\hat{\alpha}$. However, biases in $\hat{\alpha}$, despite being $O_p(T^{-1})$, can still be large in small samples. In a Monte Carlo study, Banerjee et al (1986) discovered large biases in $\hat{\alpha}$ derived from bivariate cointegrating regressions. In addition, $\hat{\alpha}$ did not converge rapidly to $\alpha$. Given that the $R^2$ of the regression converges at the same rate as the bias, they propose $(1 - R^2)$ as a proxy for the latter. In fact, for the canonical model discussed previously, the linear relationship between both statistics turns out to be:

$$\hat{\alpha} - \alpha \;=\; \alpha^2(1 - R^2) + O_p(T^{-1}) \qquad \text{....(18)}$$

which suggests rather strongly that cointegrating regressions without $R^2$ very close to unity should be viewed with caution (see, for example, Campbell and Shiller (1986)). However, in the context of a multiple regression, the $R^2$ of an equation cannot fall when an additional variable is added, and this implies that a high $R^2$ is not sufficient to guarantee that each included variable is germane to the model, nor that the estimated coefficients closely approximate their true values. This issue of functional forms is discussed in more detail below.

Another important implication is that in contrast to normal regressions where multicollinearity amongst the regressors is often considered a problem, in the context of a cointegrating static regression such multicollinearity is *essential*: if variables do not follow similar trends over time then no linear combination of the (individually non-stationary) time-series will be stationary. Indeed, in terms of estimation of the cointegrating vector $\alpha$, the multicollinearity amongst the regressors will produce a nearly-singular (X'X) matrix corresponding to the cointegrating vector. In this sense multicollinearity is a positive advantage!

The effect of running the static regression (16) to estimate $\alpha$ is to push all the dynamic adjustment terms into the residual $\hat{u}_t$. These dynamic terms can all be parameterised in terms of I(0) series of the form $\Delta y_{t-i}$, $\Delta x_{t-j}$, and $(y - \beta x)_{t-k}$ where the values of i,j, and k will depend upon the nature of the ARMA processes generating x and y. To illustrate this, consider a simple model in which the true dynamic relationship is given by:

$$y_t = \alpha_1 y_{t-1} + \alpha_2 x_t + \alpha_3 x_{t-1} + u_t \qquad \text{....(19)}$$

where y and x are I(1) and CI(0). Suppose that in the long run the homogeneity restriction $\alpha_1 + \alpha_2 + \alpha_3 = 1$ holds. This is equivalent to saying that in the long run y and x move together. Now equation (9) can be rewritten as

$$y_t = \alpha_1 y_{t-1} + \alpha_2 \Delta x_t + (1-\alpha_1)\, x_{t-1} + u_t \qquad \text{....(20)}$$

or as

$$y_t = x_t + \alpha_1 (y - x)_{t-1} + (\alpha_2 - 1)\, \Delta x_t + u_t \qquad \text{....(21)}$$

Now (y - x) and $\Delta x$ must both be I(0) under our assumptions, as will $u_t$. Hence, by estimating the static regression

$$y_t = \alpha x_t + \epsilon_t \qquad \text{....(22)}$$

these dynamic terms are all contained in the residual $\epsilon_t$. The fact that the OLS estimate of $\alpha$ is super-consistent in such circumstances is truly remarkable.

## 3.2 Two-Step Estimators.

Once the cointegrating regression has been performed, this essentially parameterises the long-run relationship between the variables. Engle & Granger (1987) then suggest that the lagged value of $z_t$, the derived estimate of disequilibrium in any period, should be included in the general dynamic model. If cointegration holds, $z_t$ will be I(0), and the dynamic modelling problem is to transform the individually I(1) variables into reasonably orthogonal I(0) regressors. The lagged value of $z_t$ is completely analogous to an error-correction term in the equation. Recent applications of this methodology include Hall (1986), Jenkinson (1986a), Campbell (1986) and Campbell and Shiller (1986).

The alternative approach to estimating the cointegrating vector $\alpha$ is to include an error-correction mechanism in the dynamic model, since, as noted above, error correction and cointegration are equivalent concepts. This is clear from equation (20) above, which can be transformed into the following dynamic model

$$\Delta y_t = \alpha_2 \Delta x_t - (1 - \alpha_1)(y - x)_{t-1} + u_t \qquad \ldots.(23)$$

where the second regressor is the ECM. Unless y and x are cointegrated, the ECM will be I(1), and hence, since $\Delta y_t$ and $\Delta x_t$ are both assumed to be I(0), will have an estimated coefficient tending rapidly to zero. In other words, rather than use the static regression as a kind of pre-test of the model, the full dynamic model is formulated and estimated, with the estimate of any cointegrating vector only being derived once a satisfactory representation of the DGP has been found. Of course, the specification of dynamic adjustment processes in economic models is to a considerable extent a matter of discretion, even when broadly agreed rules are being followed. As such, the ECM approach lacks the conceptual and practical simplicity of the static regression approach, but may be considerably more robust. In fact, Banerjee et al (1986) find that the biases in the cointegrating vector are much smaller when the short-run dynamics are jointly modelled with the long-run relationship, providing some support for the dynamic ECM modelling strategy.

### 3.3 Testing.

Testing for cointegration between a set of time series is simply a test for the existence of a unit root. The analysis of Section 2 follows through entirely, except that instead of searching for unit roots in the individual time series, the tests are for the existence of a unit roots in the residuals, $z_t$, from the static cointegrating regression. Because the tests are based on constructed regressors, the critical values obtained for the previous case have to be adjusted upwards, otherwise the test will reject the null too often. If we cannot reject the null hypothesis of a unit root in the residuals, then these 'equilibrium errors' are themselves non-stationary, and cannot be relied upon to move the system systematically back towards equilibrium. In these circumstances cointegration could not be established and hence considerable statistical doubt would be cast upon the theoretical equilibrium.

In actual applications, the Cointegrating Regression Durbin-Watson (CRDW) test, suggested by Sargan and Bhargava (1983), and discussed briefly in section 2.2, has proved extremely popular with researchers. That is,

$$\text{CRDW} = \frac{\Sigma(\hat{u}_t - \hat{u}_{t-1})^2}{\Sigma\hat{u}_t^2} \qquad \ldots.(24)$$

where $\hat{u}_t$ denote the OLS residuals from the cointegrating regression. However, special problems exist with this test. Firstly, whereas in testing for integration (when there are no regressors other than a constant) the critical values of the test, as reported in Sargan and Bhargava (1983), are exact, the test statistics for cointegration depend upon the number of regressors in the cointegrating equation, and only bounds on the critical values are available. This is because, as in the case of the standard D-W test (which is based upon a null hypothesis of white noise residuals), the exact critical value for the D-W statistic is itself a function of the data generation process. The bounds on the test provide a benchmark, and can be used to accept the null hypothesis of no cointegration, but they become rather wide apart as the number of regressors is increased. Without the addition of, for example, the Imhof Routine (1961) to standard regression software, inference will be impossible whenever the value of the DW statistic falls between the bounds. It is possible to compute exact critical values using Monte Carlo methods for a given DGP, an example of which are those reported in Engle and Granger (1987) for a white noise DGP, but these values are not generally applicable to other experiments, and should be very carefully interpreted as the basis of cointegration tests.

An alternative approach suggested earlier is to test for cointegraton using the long run solution in the autoregressive distributed lag model. If the error correction term is restricted as in (23) for theoretical reasons (e.g. log consumption and log income should have a cointegrating slope of unity) then the t-ratio of the coefficient of this term is a useful statistic. Banerjee et al (1986) show that this t-test has about the correct size at the 5% level, although the results of Evans and Savin (1981) suggest that this is not true at other levels. When the level terms are left unrestricted, non-parametric tests, based on deviations of the computed long-run solution, seem a fruitful approach. Some Monte Carlo evidence in Banerjee et al (1986) suggests that the power of these tests is higher than the power of the test based on the static regression. One explanation for this may be the smaller biases obtained using the dynamic modelling approach.

All the other tests described in Section 2 can be used to test for the stationarity of the residuals from the cointegrating regression, but in practise the Dickey-Fuller and Augmented Dickey-Fuller tests have proved most popular. Of course, the choice of the lag structure in ADF tests is still to a great extent *ad hoc*, and different results can be obtained by changing the length of the autoregression, which suggests that greater use should be made of the non-parametric tests described in the Appendix.

At this stage it is important to emphasize that, given the fragility of the tests for cointegration, simple auxiliary tests may be interesting. Granger and Weiss (1983) suggest increasing (or decreasing) the coefficients of the cointegrating vector by, say, 10% and then examine whether the corresponding sum of squares is much larger than for the chosen cointegrating vector. The intuition of this additional check is clear, since only using the latter should the variance be finite, and so it should be easily distinguishable from other cases.

### 3.4 Functional Forms.

The actual functional form of the cointegrating regression, or ECM, is normally dictated by economic theory. However, what inferences are valid on completion of a set of cointegration tests? Consider first what the inability to find a cointegrating vector, or significant ECM, might imply. It may, of course, simply be that the theoretical equilibrium is without statistical foundation. On the other hand, it may be that a crucial I(1) variable has been omitted from the analysis, which if added to the model would generate I(0) residuals. The temptation of the researcher is to continue adding variables until stationarity of the residuals is achieved. Whilst such general models may be necessary to establish cointegration, the parsimony of the relationship should then be questioned. The t-ratios of the variables in the cointegrating regression will be badly biased, given the degree of autocorrelation of the residuals, but if the autocorrelation is positive, we know that t-ratios are biased upwards. On what criterion, then, should variables be included or excluded from the equilibrium, since it is perfectly possible that some subset of the variables is cointegrated? A low t-ratio will be suggestive, but standard tables cannot be used. There is in fact little alternative to testing all subsets of the variables for cointegration, and only if all these tests are rejected can the researcher be sure that each variable is germane to the relationship. Thus, the discovery of a cointegrating vector should signal the start of a further series of tests.

The choice of functional form is also, in practice, an important consideration. It can be proved that

(i) cointegration implies Granger-Causality

(ii) cointegration in levels implies cointegration in logs

and (iii) cointegration in logs does not imply cointegration in levels. If theory is used to select functional form (rather than, for example, Box-Jenkins techniques), then the use of cointegration methods to evaluate the validity of equilibrium predictions of theories can yield inconclusive results (see, for example, the debate between Jenkinson (1987) and Nickell (1987) regarding the existence of NAIRUs). Intuition suggests that there will always exist some *non-linear* combination of I(1) series that will be I(0), which, if it were true, would have important implications for the use of cointegration analysis in modelling, although much of the simplicity and strength of the original linear case would be lost (see, for example, Escribano (1986)).

## 3.5 Common Trends.

An alternative way to approach the existence of cointegration is based upon the idea of common trends (see, for example, Stock and Watson (1986), Phillips and Ouliaris (1986), King et al (1987), Aoki (1987)). Suppose that we have a DGP as considered in Section 3.1 equations (17), with $\rho = 0$ without loss of generality. If we add the following process

$$w_t = \gamma x_t + \zeta_t \qquad \ldots.(25)$$

then the vector $(1, \gamma^{-1}\alpha)$ in the regression of $y_t$ on $z_t$ is a cointegrating vector, since it eliminates the presence of $x_t$. We then say that $y_t$ and $w_t$ have a common trend of $x_t$. If a third process of the same form is added, there are at most two linearly independent cointegrating vectors. Thus, in general, with N series and r common trends, there are at most (N - r) cointegrating vectors. It is also possible to test for uniqueness of the cointegrating vector. The test basically consists of checking that no subset of regressors is cointegrated in the cointegrating relationship (see Gourieroux et al (1985)).

This approach suggests that multivariate autoregressions of the form:

$$Y_t = \sum_{i=1}^{p} A_i Y_{t-i} + \epsilon_t \qquad E(\epsilon_t) = 0, \quad E(\epsilon_t \epsilon_t') = \Omega \qquad \ldots.(26)$$

should be considered, where Y denotes an n-vector of random variables. This can be rewritten as:

$$\Delta Y_t = -BY_{t-1} - \sum_{j=1}^{p-1} C_j \Delta Y_{t-j} + \epsilon_t \qquad \ldots.(27)$$

where $B = (I - \sum_{i=1}^{p} A_i)$ and $C_j = \sum_{s=1}^{p-j} A_{j+s}$

Diagonalising B such that $P^{-1}BP = \Lambda$, the transformed system can be written

$$\Delta Y_t^* = -\Lambda Y_{t-1}^* - \sum_{j=1}^{p-1} C_j \Delta Y_{t-j}^* + \epsilon_t^* \qquad \ldots.(28)$$

where $Y_t^* = P^{-1}Y_t$ and $\epsilon_t = P^{-1}\epsilon_t$. Then we can test for the number of common trends by testing how close the largest eigenvalue of $\Lambda$ is to zero, followed by the next largest, and so forth. Dickey and Fountis (1987) show that tests of the form $T\hat{\lambda}$ can be compared with the critical values in Table 8.5.1 of Dickey and Fuller. The natural corollary to the existence of common trends is that there are linear combinations of the regression coefficients in (26) which are $O_p(\surd T)$ consistent, and are asymptotically normally distributed, a result which was first conjectured by Sims (1978) and later formally proved by Phillips and Ouliaris (1986).

The implications of this result are very interesting. Take, for instance, the case of testing whether consumption follows a random walk, when income and consumption are I(1). The test for parameter exclusion in the regression of the change in consumption on the lagged level of income and consumption will have the ordinary F distribution, if they are cointegrated, but will have a non-normal limiting distribution otherwise (see, for example, Mankiw and Shapiro (1985) and Banerjee et al (1987b)).

Finally, taking advantage of the framework used to interpret the existence of common trends, we will briefly discuss the notion of cointegration in trends and in variance (see Escribano (1987)). Consider, for example, the following DGP

$$\begin{aligned} y_t &= \mu + \varphi t + \beta x_t + e_t \\ w_t &= \mu' + \varphi' t + \gamma x_t + v_t \end{aligned} \qquad \ldots.(29)$$

where $\Delta x_t = \epsilon_t$. Then, as before, the vector $(1, \gamma^{-1}\beta)$ provides cointegration in variance, since it eliminates $x_t$ which has a trend in variance. But unless $(\gamma\mu - \beta\mu') = 0$ and $(\gamma\varphi - \beta\varphi') = 0$ there will not be cointegration in means (trends). Therefore, the relevant concept of cointegration, in the framework so far discussed, is that of variance. So in order to decide if a set of series is cointegrated in variance, we first have to detrend in mean, or include a drift and/or trend in the cointegrating relationship. Alternatively these nuisance parameters could be excluded from the cointegrating regression and test for their presence jointly with integrated variance in the corresponding residuals. If the first test, i.e. the exclusion of the constant and trend, is accepted (rejected) and the second test, i.e. the existence of a unit root, is accepted (rejected) there will be cointegration in trends but not in variance (no cointegration in trends but cointegration in variance). The use of non-parametric tests, corrected for the existence of generated regressors, seems another fruitful testing approach which needs to be developed further.

## IV. CONCLUSIONS.

The considerable gap between the economic theorist, who has much to say about ***equilibrium*** but relatively little to say about *dynamics*, and the econometrician, whose models concentrate on dynamic adjustment processes, has, to some extent, been bridged by the concept of cointegration. In addition to allowing the data to determine the dynamics of the model (in the spirit of Hendry; see, for example, Hendry (1986)) cointegration suggests that models can be significantly improved by introducing, and allowing the data to parameterise, equilibrium conditions suggested by economic theory. Furthermore, the putative existence of such long-run equilibrium relationships can, and should, be tested, using the tests for unit roots discussed in this paper.

## Appendix: Non-Parametric Tests for Unit Roots.

The basic idea of this non-parametric approach is quite appealing. The derivation of the statistics such as (6) and (7) highlights the way in which the ratio $\sigma^2/\sigma^2_\epsilon$ affects the shape of the distribution. It is then possible to find an affine transformation of the various statistics which eliminate the dependence of the limiting distribution on the nuisance parameter $\sigma^2/\sigma^2_\epsilon$ , accomplished in such a way that the transformed statistics converge to the same random variable as do the untransformed statistics when the errors are iid, i.e. when $\sigma^2/\sigma^2_\epsilon = 1$. This implies that the critical values of the transformed statistics are the same as those tabulated by Dickey and Fuller.

A simple example will help to understand the procedure. From (6), we look for a transformation such that

$$AT(\hat{\alpha}) + B = \frac{1/2[W(1)^2 - 1]}{\int_0^1 W(t)^2\, dt} \qquad \text{....(A1)}$$

that is, in this case

$$A = 1 \quad \text{and} \quad B = - \frac{1/2[\sigma^2 - \sigma^2_\epsilon]}{\sigma^2 \int_0^1 W(\,)^2\, dt} \qquad \text{....(A2)}$$

Using (A2) and consistent estimates of $\sigma^2$ and $\sigma^2_\epsilon$ , which will be discussed later, we find a consistent estimate of B, that is

$$\hat{B} = - \frac{1/2[\hat{\sigma}^2 - \hat{\sigma}^2_\epsilon]}{T^{-2}\ \Sigma y^2_{t-1}} \qquad \text{....(A3)}$$

such that $T\hat{\alpha} + \hat{B}$ has the same asymptotic critical values as those tabulated by Dickey and Fuller. Similar arguments follow for those tests which are based upon the t-ratio of $\hat{\alpha}$. Since the latter have proved to be more powerful tests than the former, we will concentrate on testing through t-statistic from now on. Rows (a),(b) and (c) in Table 2 present the corresponding transformed t-statistics for the three unrestricted models shown in Table 1. Therefore, these statistics provide a relatively easy way to implement tests of hypotheses of a unit root with possibly heterogeneously and dependently distributed data. However, an important caveat to bear in mind is that the previous equivalence is asymptotic in Tables 8.5.1 and 8.5.2 in Fuller (1976), whereas the finite sample counterparts are not the same. This implies that when dealing with relatively small samples the transformations are not adequate, and unless there is strong evidence of a moving average error term, we advise the extended regression and the Augmented Dickey-Fuller test.

The next step in the test implementation consists of discussing the choice of consistent estimates for $\sigma^2$ and $\sigma_\epsilon^2$ . The residual variance $\hat{\sigma}_\epsilon^2$ in the unrestricted models provide consistent estimates of $\sigma_\epsilon^2$ except in the case where the unrestricted model does not contain a drift, and the true DGP is a random walk with drift. To consistently estimate $\sigma^2$ , it is important to notice that it is equivalent to $2\Pi s(0)$, s(0) being the spectral density function at zero frequency. Newey and West (1987) have proposed a simple estimate which uses a triangular smoothing window. The estimate is

$$\hat{\sigma}^2 = T^{-1}\{\sum_{1}^{T} \hat{\epsilon}_t^2 + 2\sum_{k=1}^{m} [1 - \frac{k}{m+1}] \sum_{t=k+1}^{T} \hat{\epsilon}_t \hat{\epsilon}_{t-k}\}$$

The choice of the truncation lag k, i.e. the suspected number of non-zero autocorrelations, is sometimes suggested by the framework in which the test is carried out (see, for example, Corbae and Oularis (1986) for the case of unit roots in spot and forward exchange rates). In general we suggest k ranging from 1 to 8 for quarterly data, and 1 to 24 for monthly data.

Table 2: Summary of Test Statistics.

| $H_0$ | Test Statistic ($At_\alpha + B$) | |
|---|---|---|
| | A | B |
| a) $\alpha=0$ in Case 1<br>Fuller (1976) Table 8.5.2 (B1) | $\hat{\sigma}_\epsilon/\hat{\sigma}$ | $-1/2\ (\hat{\sigma}^2 - \hat{\sigma}^2_\epsilon)[\hat{\sigma}^2 T^{-2}\Sigma y^2_{t-1}]^{-1/2}$ |
| b) $\alpha=0$ in Case 2<br>Fuller (1976) Table 8.5.2 (B2) | $\hat{\sigma}'_\epsilon/\hat{\sigma}'$ | $-1/2\ (\hat{\sigma}^{2\prime} - \hat{\sigma}^{2\prime}_\epsilon)[\hat{\sigma}^{2\prime} T^{-2}\Sigma\tilde{y}^2_{t-1}]^{-1/2}$ |
| c) $\alpha=0$ in Case 3<br>Fuller (1976) Table 8.5.2 (B3) | $\hat{\sigma}''_\epsilon/\hat{\sigma}''$ | $-1/2\ (\hat{\sigma}^{2\prime\prime} - \hat{\sigma}^{2\prime\prime}_\epsilon)\{\hat{\sigma}^{2\prime\prime}[(T^{-2}\Sigma\tilde{y}^2_{t-1})(1/12) - T^{-5/2}\Sigma\tilde{y}_{t-1}\tilde{t}]\}^{-1/2}$ |
| d) $\beta=0$ in Case 3<br>Dickey and Fuller (1981) | $\hat{\sigma}''_\epsilon/\hat{\sigma}''$ | $1/2\ [1 - (\hat{\sigma}^{2\prime\prime}/\hat{\sigma}^{2\prime\prime}_\epsilon)][T^{-5/2}\Sigma\tilde{y}^2_{t-1}\tilde{t}][T^{-4}(\Sigma\tilde{y}_{t-1})(1/12) - T^{-7}(\Sigma\tilde{y}_{t-1}\tilde{t})^2(\Sigma\tilde{y}^2_{t-1})]^{-1/2}$ |
| e) $\mu=0$ in Case 2<br>Dickey and Fuller (1981) | $\hat{\sigma}'_\epsilon/\hat{\sigma}'$ | $1/2\ (\hat{\sigma}^{2\prime} - \hat{\sigma}^{2\prime}_\epsilon)[T^{-3/2}\ \Sigma y_{t-1}][T^{-4}\Sigma\tilde{y}^2_{t-1}\Sigma y^2_{t-1}]^{-1/2}$ |

Note: (^) denotes estimates based on residuals from the unrestricted model in Case 1; (^') denotes estimates based on residuals from the unrestricted model in Case 2; (^'') denotes estimates based on residuals from the unrestricted model in Case 3; (~) denotes deviations with respect to sample means.

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