A PROTOTYPICAL SEASONAL ADJUSTMENT MODEL

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Summary

The paper analyzes unobserved-components modeling and estimation for the simplest ARIMA process that accepts a full decomposition into trend, seasonal and irregular components. This prototypical model exemplifies many features of and issues arising in model-based seasonal adjustment that are less transparent in more complex seasonal time series models. In particular the analysis illuminates the major issues surrounding the specification of the component models and the identification of a unique structure for them. In so doing, the relationship between reduced- and structural-form approaches to unobserved components estimation is illustrated within an ARIMA-modeling framework. Finally, the properties of the minimum mean-squared-error estimators of the unobserved components are examined and the two main types of estimation error, revisions in the preliminary estimator and error in the final estimator, are analysed.

Key Words: Time series; ARIMA models; Unobserved-components models; Model-based seasonal adjustment; Signal extraction; Trend estimation.

1. INTRODUCTION

Model-based seasonal adjustment has been increasingly developed over the past several years, primarily as an alternative to the Census X-11 method (Shiskin, Young, and Musgrave 1967) or its ARIMA variant (Dagum 1975), which have been by far the most common procedures for producing published seasonally adjusted series. A number of approaches have involved expressing an observed seasonal series as the sum of unobserved components generated by ARIMA models, one of which is the seasonal component; see, e.g., Bell and Hillmer (1984), Box, Hillmer, and Tiao (1978), Burman (1980), Hillmer and Tiao (1982), Pierce (1978). The process of seasonal adjustment is then the estimation (by signal extraction) and removal of this component.

By far the most difficult task in developing such a procedure is the specification of the model for the seasonal component, which requires a statement (even if implicit) about what we mean by seasonality and what we want to remove in seasonally adjusting a series. In many approaches, including the use of unobserved-components ARIMA models, the embodiment of such a statement in the model specification can be a complex and unintuitive process. Thus we believe it is of value to examine the issues arising and the decisions required within the context of a very simple model, which can then serve as a prototype for more complex applications.

The model for the observable series which we have chosen for this purpose is one that we believe to be the simplest possible ARIMA model which possesses a nontrivial decomposition into trend, seasonal and irregular components – namely, the model $x_t - x_{t-2} = a_t$, for the observable series x_t in terms of white noise a_t. This model is appropriate for semiannual data or other periodic data of period 2. Section 2 develops this model and the corresponding component-model specifications which embody our usual concepts of seasonality and trend. However, there are an infinite number of such specifications consistent with the assumed model for x_t, and Section 3 characterizes the class of all admissible decompositions. Section 4 then focuses on one of these, the "canonical" decomposition, discusses some of its most relevant properties and presents a structural interpretation of the decomposition. In Section 5 the decomposition problem is analysed in the frequency domain.

Having solved the identification/specification problem, Section 6 considers estimation of the unobserved components, and discusses properties of the derived estimators. The last section analyzes the two implied types of estimation error: revision error contained in the preliminary estimator and error still present in the final estimator.

2. THE MODEL

Numerous recent applications of signal extraction consist of two-component decompositions such as into signal plus noise, or into seasonal plus nonseasonal. However, frequently there are reasons for desiring a separation of the nonseasonal component into trend and irregular components as well. Purposes given for seasonal adjustment are typically that seasonality is extraneous to and interferes with what we want to observe in a series, so that its removal facilitates interpretation of the remainder of the series. But then, insofar as the irregular component may also be extraneous, its removal should still further aid in these goals, as the remaining component (the trend) would then represent the long-term evolution of the series, which is presumably of greater interest.The estimation of trend has in fact been recommended for years as an alternative or adjunct to seasonal adjustment, a few of the more recent examples being Moore et. al. (1981), Kenny and Durbin (1981), Box, Pierce and Newbold (1986), Maravall and Pierce (1986).

It should be noted that not always is a separation into additive components desired or desirable; for some applications (an obvious example is forecasting) the overall model's relatively simple form and interpretation may suffice. We are addressing situations where a separation into components is useful (such as, for example, whenever seasonal adjustment is desired.) Thus we focus our attention on the three-component model

$$x_t = p_t + s_t + u_t \qquad (2.1)$$

where p_t, s_t, and u_t are respectively the unobserved trend, seasonal and irregular components of the observable series x_t, at time t.

For x_t, we require a seasonal ARIMA model which admits a decomposition such that the components themselves have ARIMA-model specifications and moreover reflect the essential properties ordinarily associated with them, namely, "periodicity" for the seasonal and low frequency dominance for the trend. We consider the simplest model to do so, given by

$$\nabla_2 x_t = a_t \quad , \qquad (2.2)$$

where $\nabla_2 = 1-B^2 = (1-B)(1+B)$, and where a_t is white noise.

It is illuminating to examine the frequency-domain behavior of this series. The model (2.2) is nonstationary so that the spectrum of x_t is not defined. However it is customary to define the "pseudospectrum" of this series by

$$g_x(\omega) = \left| \frac{\sigma_a}{1-e^{2i\omega}} \right|^2 = \frac{\sigma_a^2}{2(1-\cos 2\omega)} \tag{2.3}$$

for $0 \leq \omega \leq \pi$ where $|z|^2 = z\bar{z}$ for a complex number z with conjugate $\bar{z}$. This spectrum is graphed in Figure 1 where it is seen to be symmetric about $\omega = \pi/2$. The low-frequency behavior can be associated with trend, and the frequencies near π with seasonal behavior (π is the single seasonal frequency, corresponding to a seasonal cycle of period 2.)

Since: (i) the autoregressive polynomial in the model for x_t is $\nabla_2 = (1+B)(1-B)$, and (ii) the peaks for $\omega = 0$ and $\omega = \pi$ are associated with the roots $B = 1$ and $B = -1$ of ∇_2, respectively, from the additive relation (2.1) it follows that acceptable trend and seasonal component models are of the form

$$p_t = \frac{\beta(B)}{1-B} b_t \tag{2.4}$$

and

$$s_t = \frac{\gamma(B)}{1+B} c_t \quad , \tag{2.5}$$

where b_t and c_t are white noise and $\beta(B)$ and $\gamma(B)$ are polynomials in B. Multiplying both sides of (2.1) by ∇_2, it is obtained that

$$a_t = (1+B)\ \beta(B) b_t + (1-B)\ \gamma(B) c_t + \nabla_2\ u_t . \tag{2.6}$$

Since the lag-two autocorrelation of the r.h.s. of (2.6) has to be equal to zero, reasonable models for the components are of the form

$$(1-B)\ p_t = (1-\beta B) b_t \quad , \tag{2.7}$$

$$(1+B)\ s_t = (1-\gamma B)c_t \quad , \tag{2.8}$$

$$u_t = \text{white noise} \quad , \tag{2.9}$$

where, in order to avoid model multiplicity (see Box and Jenkins, 1970, p. 195-200), we assume

$$|\beta|\leq 1 \ , \ |\gamma|\leq 1 \ . \tag{2.10}$$

Recapitulating, in this model the seasonality is of period 2, appropriate for semi-annual data. The trend p_t is an integrated process, reduced to stationarity by the ordinary difference operator; and the seasonal component s_t is summed to attain stationarity. These specifications are consistent with those of several recently developed model-based seasonal adjustment procedures, including Burman (1980), Hillmer and Tiao (1982) and Bell and Hillmer (1984).

We note that the same component specifications as (2.7) through (2.9) (though with different parameter values) are consistent with a slightly more general IMA (2,2) model for x_t,

$$\nabla_2\ x_t = (1-\theta_1 B-\theta_2 B^2)a_t \ , \tag{2.11}$$

which reduces to (2.2) by taking $\theta_1 = \theta_2 = 0$. Another point is that the irregular component u_t is required to be white noise. This is not always imposed, though two of the procedures where u_t is allowed to be serially correlated (Burman 1980, Hillmer-Tiao 1982) do produce a white noise irregular for this example. In general, having specified the seasonal component, one can define the trend to have certain properties and the irregular as the residual, which could then be autocorrelated; or alternatively one could specify the irregular as white noise and the trend as the residual, which could then exhibit features in addition to the smooth, low-frequency behavior ordinarily associated with this component. Our prototypical model is sufficiently well behaved that such a choice is not necessary.

3. AUTOCOVARIANCE EQUATIONS AND ADMISSIBLE DECOMPOSITIONS

Given the difference/summation specifications for p_t and s_t and the white noise u_t, the models (2.7)-(2.9) are the most general first-order component models which imply the overall model (2.2) for x_t. As we shall see in this section, they are in fact too general, as an infinite number of models (2.7)-(2.9) are compatible with (2.2), and the components p_t, s_t, and u_t are thus unidentified.

Given $x_t = p_t + s_t + u_t$, if we multiply through by $(1-B)(1+B)$ and use the model specification (2.7) through (2.9), we obtain

$$\begin{aligned} a_t &= (1-B)(1+B)x_t \\ &= (1+B)(1-\beta B)b_t+(1-B)(1-\gamma\beta)c_t+(1-B^2)u_t \\ &= b_t+(1-\beta)b_{t-1}-\beta b_{t-2}+c_t-(1+\gamma)c_{t-1}+\gamma c_{t-2}+u_t-u_{t-2}. \qquad (3.1) \end{aligned}$$

The system of equations used to determine relationships among the parameters is obtained by equating autocovariances on the left and right sides of (3.1). In particular, for k = 0, 1, 2 we have:

$$\sigma_a^2/2 = (1-\beta+\beta^2)\sigma_b^2 + (1+\gamma+\gamma^2)\sigma_c^2 + \sigma_u^2 , \qquad (3.2)$$

$$0 = (1-\beta)^2\sigma_b^2 - (1+\gamma)^2\sigma_c^2 , \qquad (3.3)$$

$$0 = -\beta\sigma_b^2 + \gamma\sigma_c^2 - \sigma_u^2 \ . \tag{3.4}$$

Adding the first and third equations,

$$\sigma_a^2/2 = (1-\beta)^2 \ \sigma_b^2 + (1+\gamma)^2 \ \sigma_c^2 \tag{3.5}$$

which together with the second suggests expressing σ_b^2 and σ_c^2 as functions of β and γ. Adding and then substracting (3.3) and (3.5), it is obtained that

$$\sigma_b^2 = \frac{\sigma_a^2}{4(1-\beta)^2} \quad , \quad \sigma_c^2 = \frac{\sigma_a^2}{4(1+\gamma)^2} \quad , \tag{3.6}$$

and, after substitution into (3.4), that

$$\sigma_u^2 = \left[\frac{-\beta}{4(1-\beta)^2} + \frac{\gamma}{4(1+\gamma)^2} \right] \sigma_a^2 \ . \tag{3.7}$$

Equations (3.6) and (3.7) show the dependence of the component— model innovation variances on the moving average parameters β and γ, so that the specification of values for β and γ suffices to determine (identify) the system. However, in addition to (2.10) these parameters must satisfy the restriction that the variances σ_b^2, σ_c^2, and σ_u^2 be nonnegative. Equation (3.6) ensures that $\sigma_b^2 \geq 0$, $\sigma_c^2 \geq 0$; is the nonnegativity of σ_u^2 that is at issue, which from (3.7) is equivalent to the constraint

$$-\beta(1+\gamma)^2 + \gamma(1-\beta)^2 \geq 0 \quad . \tag{3.8}$$

Figure 2 shows the graph of the region (not shaded) where the constraints (3.8) and (2.10) are satisfied. All points (β,γ) in this region (and only those points) correspond to an admissible decomposition of x_t in (2.2) into components p_t, s_t, and u_t as given by (2.7) through (2.9). In the next section, we shall be interested in the decomposition which corresponds to the upper left corner of the graph.

4. IDENTIFICATION AND THE CANONICAL DECOMPOSITION

The identification of the seasonal, trend and irregular components of x_t in (2.2) is tantamount to the selection of a point in the space of admissible (β,γ) values given in Figure 2. Several approaches to the resolution of this problem are possible. One often employed is to restrict the order of the MA polynomials, which in our cases means setting $\beta = \gamma = 0$ in (2.7) and (2.8) (see Maravall, 1985). More generally, specifying in advance the model forms for p_t and s_t and a sufficient number of parameter values is what the "structural" approach would entail (see, for example, Engle, 1978, or Harvey and Todd, 1983.)

The identification problem encountered here is similar to the one that appears in standard econometric models. The model for the observed series is the reduced form, while the models for the components represent the associated structural form. For a particular reduced form, there are an infinite number of structures from which it can be generated. In order to select one, additional information has to be incorporated. The traditional approach in econometrics has been to set a priori some parameters in the structural model equal to zero (see Fisher, 1966). These zero-parameter restrictions reflect a priori economic theory information, for example that some variables that affect demand of a commodity do not affect supply and vice versa.

In the case of our unobserved-components model, such a priori information is not available. We follow instead an alternative approach, originally suggested by Box, Hillmer and Tiao (1978) and

Pierce (1978). The additional information will be the requirement that separable white noise should not be a part of either the trend or the seasonal, and should instead be regarded as irregular; thus the irregular-component variance is maximized and the resulting decomposition has been termed "canonical" by Hillmer and Tiao (1982). We now show that in the present example maximizing the variance of the irregular component u_t results in determining unique values for both β and γ and thus identifies the system (2.7) through (2.9).

By differentiation of (3.7) it is seen that the maximum of σ_u^2 occurs at $\beta = -1$, $\gamma = +1$, values which, in view of (3.6), also minimize σ_b^2 and σ_c^2, the variances of the trend and seasonal innovations. Thus in the canonical decomposition of a series generated by (2.2) into $p_t+s_t+u_t$, the models (2.7) and (2.8) for the trend and seasonal components are given by

$$(1-B)p_t = (1+B)b_t \tag{4.1}$$

and

$$(1+B)s_t = (1-B)c_t \quad , \tag{4.2}$$

where moreover from (3.6) and (3.7) the innovation variances are seen to satisfy

$$\sigma_b^2 = \sigma_c^2 = \sigma_a^2/16, \; \sigma_u^2 = \sigma_a^2/8 \quad . \tag{4.3}$$

Let h(z) denote a spectrum (or pseudospectrum) g(ω) as a function of z = cos ω. An important property of the canonical model is the following. If s_t represents any admissible seasonal component, and s_t^* denotes the canonical one, the spectra of the two are given by

$$h_s(z) = \frac{\sigma_a^2}{8(1+\gamma)^2}\left[\frac{1+\gamma^2-2\gamma z}{1+z}\right],$$

$$h_s^*(z) = \frac{\sigma_a^2}{16}\left[\frac{1-z}{1+z}\right],$$

where use has been made of (3.6). Then, it is easily seen that $h_s(z) = h_s^*(z) + k_o$, where $k_o = (1-\gamma)^2/16(1+\gamma)^2$ is a constant, and hence the two spectra are parallel. Since a similar result holds for the trend, any admissible trend or seasonal component -of the type (2.7) or (2.8)- is equal to the canonical one plus orthogonal white-noise.

The manner in which maximizing the variance of the irregular identifies the component models is easily understood by considering the following. For our simple model (2.1) and (2.2), the spectra of the trend and seasonal components are decreasing and increasing functions, respectively, of ω. Hence the minimum is obtained, in the trend case, at $\omega = \pi$ and, in the seasonal case, at $\omega = 0$. Since for the canonical decomposition these minima are zero, it follows that $g_p(\omega=\pi) = 0$ and $g_s(\omega=0) = 0$, where $g_p(\omega)$ and $g_s(\omega)$ are the trend and seasonal spectra. The first condition implies that $B=-1$ is a root of the moving average polynomial $(1-\beta B)$ in the trend model; and similarly, the second condition implies that the $B=1$ is a root of the moving average polynomial $(1-\gamma B)$ in the seasonal model. In terms of (2.4) and (2.5), these two restrictions are equivalent to $\beta(-1) = \gamma(1) = 0$, or, in our model, to the two linear constraints $1+\beta = 1-\gamma = 0$.

It follows that identification is attained by, instead of setting coefficients equal to zero, imposing linear constraints on them, reflecting the minima of zero in the trend and seasonal component spectra. There is thus a close relationship between the ARIMA-based decomposition and the structural approach. The requirement of noise-free components can be easily incorporated into the latter, and equations (4.1) to (4.3) represent the structural form associated with the reduced form (2.2).

Since, as noted before, equations (2.7)-(2.9) are consistent with the more general reduced form (2.11), equations (4.1) and (4.2) and a white-noise irregular will also represent the canonical components associated with the model (2.11). The three innovation variances, however, would not be given by (4.3) but would, instead, be functions of the parameters θ_1 and θ_2.

5. THE DECOMPOSITION IN THE FREQUENCY DOMAIN

The preceding sections have illustrated the decomposition of a time series into trend, seasonal and irregular components in the time domain. It is also of interest to examine this problem in the frequency domain, which we do with a development similar to Burman (1980). Since the components p_t, s_t and u_t are orthogonal, an admissible decomposition is characterized by a partition of the spectrum of x_t into three additive component spectra, which we write as

$$h_x(z) = h_p(z) + h_s(z) + h_u(z) \quad . \tag{5.1}$$

From (2.3), the l.h.s. of (5.1) is

$$h_x(z) = \frac{1}{4(1-z^2)} = \frac{1}{4(1-z)(1+z)} \quad ,$$

where, without loss of generality, we set $\sigma_a^2 = 1$. The factors (1-z) and (1+z) are associated with the trend and seasonal roots (B=1 and B=-1), respectively, of the AR polynomial in (2.2), and $h_u(z)$ is constant. Thus an admissible decomposition can be obtained from the identity

$$\frac{1}{4(1-z)(1+z)} = \frac{n_p(z)}{1-z} + \frac{n_s(z)}{1+z} + k \, , \qquad (5.2)$$

where the three terms of the r.h.s. represent the trend, seasonal and irregular spectra, which are nonnegative. For $|z|\leq 1$, the minimum of $h_x(z)$ is greater than zero; hence at least one of the three quantities (k, min. $h_p(z)$, min. $h_s(z)$) will be positive. Since a positive constant can be interchanged among the three component spectra without violating the admissibility constraints, it follows that the decomposition given by (5.2) will not be unique: an infinite number of combinations of non-negative $n_p(z)$, $n_s(z)$ and k exist which satisfy (5.2). This is the frequency domain equivalent of the existence of an infinite number of component models satisfying the system of covariance equations (3.2)-(3.4). It implies, as before, that, without additional assumptions, the overall model does not identify unique models for the components.

In order to derive the canonical solution, the partial fraction expansion of $h_x(z)$ provides an easy to compute two-stage procedure. First, to obtain simply an admissible decomposition, we seek values a and b such that

$$\frac{1}{(1-z)(1+z)} = \frac{a}{1-z} + \frac{b}{1+z} \, , \qquad (5.3)$$

which are obtained by noting that, from (5.3), $a(1+z)+b(1-z)=1$ so that $a=b=1/2$. Consequently, from (5.2) and (5.3)

$$h_x(z) = \frac{1}{8}\left[\frac{1}{1-z} + \frac{1}{1+z}\right]$$

hence

$$h_p(z) = \frac{1}{8(1-z)} \, , \quad h_s(z) = \frac{1}{8(1+z)} \, , \quad h_u(z) = 0 \, ,$$

with analogous component spectra

$$g_p(\omega) = \frac{\sigma_a^2}{8(1-\cos\,\omega)}\ , \quad g_s(\omega) = \frac{\sigma_a^2}{8(1+\cos\,\omega)}\ , \quad g_u(\omega) = 0\ .$$

Since the two minima

$$\min_{0\leq\omega\leq\pi} g_p(\omega) = g_p(\pi) = \sigma_a^2/16\ , \tag{5.4}$$

$$\min_{0\leq\omega\leq\pi} g_s(\omega) = g_s(0) = \sigma_a^2/16\ , \tag{5.5}$$

are both strictly positive, the decomposition obtained in the first stage is not a canonical one. Thus in the second stage the constants (5.4) and (5.5) are substracted from the trend and seasonal spectra, respectively, and added to the irregular. Consequently, for the canonical decomposition

$$g_p^*(\omega) = g_p(\omega) - \frac{\sigma_a^2}{16} = \frac{\sigma_a^2}{16}\ \frac{1+\cos\,\omega}{1-\cos\,\omega}\ , \tag{5.6}$$

$$g_s^*(\omega) = g_s(\omega) - \frac{\sigma_a^2}{16} = \frac{\sigma_a^2}{16}\ \frac{1-\cos\,\omega}{1+\cos\,\omega}\ , \tag{5.7}$$

and

$$g_u^*(\omega) = \frac{\sigma_a^2}{8}. \tag{5.8}$$

These spectra imply the same component processes and variances as previously derived, given by (4.1) through (4.3). Notice that the first stage of the procedure is equivalent to decreasing the order of the moving averages in (2.7)-(2.8), and hence the admissible decomposition obtained in the first stage is the one that results from identifying the component models by a priori setting $\beta=\gamma=0$.

The graph of $g_x(\omega)$ is given in Figure 1, which also shows the spectra of the three canonical components. Figure 3 illustrates the two stages of the decomposition. For the canonical components, the height of $g_u(\omega)$ is maximized, and the minima of $g_p(\omega)$ and of $g_s(\omega)$ are both zero.

6. ESTIMATION

The foregoing has been concerned with specification of the model forms assumed to generate the series x_t and its components. The components are unobservable and, having resolved the identification/specification problem, we proceed to obtain estimates of p_t, s_t, and u_t given a realization of $\{x_t\}$.

6.1. Signal Extraction

Consider an admissible decomposition, given by equations (2.7) to (2.9), and let

$$\psi_p(B) = (1-\beta B)/(1-B) \qquad (6.1)$$

$$\psi_s(B) = (1-\gamma B)/(1+B)$$

$$\psi_x(B) = 1/(1-B^2)$$

denote the polynomials in B of the moving average representation of p_t, s_t and x_t. Using well-known results (see Cleveland and Tiao, 1976, or Bell, 1984), the minimum mean squared error (MMSE) estimators of the three components are given by

$$\hat{p}_t = \nu_p(B)x_t \quad , \qquad (6.2)$$

$$\hat{s}_t = \nu_s(B)x_t \quad , \tag{6.3}$$

$$\hat{u}_t = [1-\nu_p(B)-\nu_s(B)]x_t \quad , \tag{6.4}$$

where the ν-polynomials represent the two-sided symmetric filters

$$\nu_p(B) = \frac{\sigma_b^2}{\sigma_a^2}\left|\frac{\psi_p(B)}{\psi_x(B)}\right|^2 \quad , \qquad \nu_s(B) = \frac{\sigma_c^2}{\sigma_a^2}\left|\frac{\psi_s(B)}{\psi_x(B)}\right|^2 \quad , \tag{6.5}$$

where the convention

$$|h(B)|^2 = h(B)h(F)$$

is employed, $F = B^{-1}$ denoting the forward shift operator.

For the trend component estimator, (6.1) and (6.5) eventually yield,

$$\nu_p(B) = \frac{\sigma_b^2}{\sigma_a^2}\,|(1+B)(1-\beta B)|^2 = \nu_{po}+\nu_{p1}(B+F)+\nu_{p2}(B^2+F^2) \quad ,$$

where $\nu_{po} = (\sigma_b^2/\sigma_a^2)(2-2\beta+2\beta^2)$, $\nu_{p1} = (\sigma_b^2/\sigma_a^2)(1-\beta)^2$, $\nu_{p2} = -(\sigma_b^2/\sigma_a^2)\beta$, and, from (3.6), $\sigma_b^2/\sigma_a^2 = 1/4(1-\beta)^2$. Similarly, for the seasonal component estimator, given by (6.3),

$$\nu_s(B) = \frac{\sigma_c^2}{\sigma_a^2}\,|1-\gamma B|^2|1-B|^2 = \nu_{so}+\nu_{s1}(B+F)+\nu_{s2}(B^2+F^2) \ ,$$

where $\nu_{so} = (\sigma_c^2/\sigma_a^2)(2+2\gamma+2\gamma^2)$, $\nu_{s1} = -(\sigma_c^2/\sigma_a^2)(1+\gamma)^2$, $\nu_{s2} = (\sigma_c^2/\sigma_a^2)\gamma$, and, from (3.6), $\sigma_c^2/\sigma_a^2 = 1/4(1+\gamma)^2$.

As is the case whenever the model for the observed series is a finite autoregression, the two filters are finite, depending in this case only on values x_{t-2} through x_{t+2}. Futhermore, they satisfy the conditions $\nu_p(1) = \nu_s(-1) = 1$ and $\nu_p(-1) = \nu_s(1) = 0$.

6.2 The models for the estimators

For the canonical decomposition, $\beta = -1$ and $\gamma = 1$, so that $\nu_{po} = \nu_{so} = 6/16$, $\nu_{p1} = -\nu_{s1} = 4/16$, $\nu_{p2} = \nu_{s2} = 1/16$; and in compact form

$$\nu_p(B) = \frac{1}{16} |1+B|^4 , \quad \nu_s(B) = \frac{1}{16}|1-B|^4 . \tag{6.6}$$

In order to analyse the estimators, it will prove helpful to obtain the models that express the three components as functions of the innovations a_t. Using (6.6) in (6.2)-(6.4), and then considering that $x_t = (1-B^2)^{-1}a_t$, the estimators of the components can be expressed as

$$(1-B)\ \nabla\ \hat{p}_t = (1+B)(1+F)^2 a_t/16 \quad , \tag{6.7}$$

$$(1+B)\ \hat{s}_t = (1-B)(1-F)^2 a_t/16 \quad , \tag{6.8}$$

$$\hat{u}_t = (1-F^2)a_t/8 \quad . \tag{6.9}$$

Comparing these three expressions with the models for the components, given by (4.1)-(4.3), it is seen that the model for the MMSE estimator of a component is different from the model for the component itself (see Grether and Nerlove (1970)). There are some similarities: First, the same stationarity-inducing transformations are required; second, since the models for $\hat{p}_t$ and $\hat{s}_t$ contain the moving average factor (1+B) and (1-B), respectively, estimation preserves the canonical properties of the trend and seasonal components.

From (6.7)-(6.9), the spectra of the three estimators are found to be

$$g_{\hat{p}}(\omega) = \sigma_a^2(1+\cos\omega)^3/(64(1-\cos\omega)) ,$$

$$g_{\hat{s}}(\omega) = \sigma_a^2(1-\cos\omega)^3/(64(1+\cos\omega)) ,$$

$$g_{\hat{u}}(\omega) = \sigma_a^2(1-\cos^2\omega)/16 ,$$

and comparing them with the true-component spectra, given by (5.6)-(5.8), it is found that, when $0\leq\omega\leq\pi$, $g^*_p(\omega) \geq g_p(\omega)$, $g^*_s(\omega) \geq g_s(\omega)$, $g^*_u(\omega) > g_u(\omega)$; hence, in each of the three cases and for all frequencies, the spectrum of the estimator is smaller than that of the component.

Figure 4 compares the two spectra for the three components. The "distorsion" induced by MMSE estimation is seen to affect mostly, the spectrum of the irregular component, which shows dips for $\omega = 0$ and $\omega = \pi$. These dips reflect the fact that, in extracting the noise from the x_t series, the MMSE ignores the frequencies dominated the trend and seasonal components (notice that, for $\omega = 0$ and $\omega = \pi$, the ratio of the irregular variance to that of the trend plus seasonal becomes zero). As a result, the spectrum of the irregular estimator displays a peak for $\omega = \pi/2$, which implies a periodic effect (in semiannual data associated with a two-year period.)

From equations (6.7)-(6.9), the autocovariance functions of the (stationary transformation of the) component estimators can also be computed. As shown in Table 1, MMSE estimation induces additional autocorrelation in the models for the three estimators. Furthermore, the variances of the theoretical components are larger than those of their estimators, particularly concerning the irregular component.

Finally, the correlations between the estimators can be computed. It is seen that estimation preserves orthogonality of the trend and seasonal component estimators, although the two are correlated with the estimator of the irregular ($\rho = -.316$ in both cases). By comparing the properties of the empirical estimates with those implied by the models for the estimators, the model-based approach provides a natural way for evaluating the results of a particular application.

7. ESTIMATION ERRORS

7.1 Revisions

The preceding has assumed that both future and past data are available, whereas this is not the case for the current time period, which is in practice often the most important application. Consider, for example, the trend estimator associated with an admissible decomposition, given by

$$\hat{p}_t = \nu_{po} x_t + \sum_{i=1,2} \nu_{pi} (x_{t-i} + x_{t+i}) \tag{7.1}$$

where the ν-coefficients were given in Section 6.1. At time t, $\hat{p}_t$ cannot be computed since x_{t+1}, and x_{t+2} are not yet known. The MMSE "concurrent" estimator of p_t, denoted $\hat{p}^o_t$, is the conditional expectation of p_t at time t. Since (for i>1)

$$E_t \, p_t = E_t \, E_{t+i} \, p_t = E_t \, \hat{p}_t \, ,$$

taking expectations in both sides of (7.1) yields

$$\hat{p}^o_t = \nu_{po} \, x_t + \sum_{i=1,2} \nu_{pi} (x_{t-1} + \hat{x}_t(i)), \tag{7.2}$$

where $\hat{x}_t(i)=E_t\ x_{t+i}$ denotes the origin-t lead-i forecast of x. Therefore, the concurrent estimator is obtained by applying the two-sided filter $\nu_p(B)$ to a forecast augmented series (see Cleveland and Tiao, 1976). This concurrent estimator will be revised in future periods, as forecasts are either updated or replaced with new observations until the historical or final estimator can be computed.

Substracting (7.2) from (7.1) shows that the total revision in the concurrent trend estimator is

$$r_{pt} = \hat{p}_t - \hat{p}_t^o = \nu_{1p}\ e_t(1) + \nu_{2p}\ e_t(2) ,$$

with $e_t(j) = x_{t+j}-\hat{x}_t(j)$ denoting the corresponding forecast error. Since $e_t(1) = a_{t+1}$ and (for this model) $e_t(2) = a_{t+2}$, we have

$$r_{pt} = [(1-\beta)^2\ a_{t+1} - \beta a_{t+2}]\ \sigma_b^2/\sigma_a^2$$

so that r_{pt} is a first order moving average. Using (3.6),

$$r_{pt} = \frac{1}{4}\ [a_{t+1} - \frac{\beta}{(1-\beta)^2}\ a_{t+2}] \quad , \tag{7.3}$$

whence

$$\mathrm{Var}(r_{pt}) = \frac{\sigma_a^2}{16}\ [1+ \frac{\beta^2}{(1-\beta)^4}\ ] \quad . \tag{7.4}$$

Similarly, the revision in a concurrent seasonal estimate $\hat{s}_t^o$ is

$$r_{st} = \frac{\sigma_c^2}{\sigma_a^2}\ [-(1+\gamma)^2\ a_{t+1} + \gamma a_{t+2}]$$

$$= \frac{1}{4}\ [-a_{t+1} + \frac{\gamma}{(1+\gamma)^2}\ a_{t+2}] \quad , \tag{7.5}$$

with variance

$$\text{Var}(r_{st}) = \frac{\sigma_a^2}{16}\left[1+\frac{\gamma^2}{(1+\gamma)^4}\right] . \qquad (7.6)$$

It is of interest to examine the revision variances in terms of the chosen decomposition. In Section 3 we derived the admissible space for the parameters β and γ. On its boundary, where (3.8) holds as an equality, at $\beta = -1$, we have $\gamma = -3+2\sqrt{2}$, and, at $\gamma = 1$, $\beta = 3-2\sqrt{2}$. Hence for all admissible decompositions (see Figure 2):

$$-1 \le \beta \le 3-2\sqrt{2} \quad \text{and} \quad -3 + 2\sqrt{2} \le \gamma \le 1 .$$

Figure 5 shows the variances of the trend and seasonal revisions as functions of the respective parameters, β and γ, over their admissible range. The figure suggests, and equations (7.4) and (7.6) show, that the revision variances are maximized at the canonical-decomposition values, $\beta = -1$, $\gamma = 1$. The occurrence of larger revisions may indicate a price paid for choosing the canonical decomposition (i.e., a trade-off between size of the revision and cleanness of signal).

For the canonical model the revision variance is:

$$\sigma_r^2 = \frac{\sigma_a^2}{16}\left[1 + \frac{1}{16}\right] \qquad (7.7)$$

for either trend or seasonal revisions, a value which is slightly above the innovation variance of the trend or seasonal models (see (4.3)), and well below the one-step-ahead forecast error variance σ_a^2. Also, from (7.3) and (7.5) the correlation between r_{pt} and r_{st} is seen to be (-15/17), so that joint confidence intervals, based on these revisions, can be built around the concurrent estimates of the trend and seasonal components.

7.2 Final estimation error

Revisions may be regarded as measurement errors in the concurrent estimate of a component, caused by limitations on the availability of data. But even if an infinite length of data is assumed (and the models known exactly), the final or historical estimate still contains an error. For the historical seasonal estimate $\hat{s}_t$ in (6.3), let the error be $\delta_t = s_t - \hat{s}_t$. Then:

$$\delta_t = -\nu_s(B)(p_t+u_t)+[1-\nu_s(B)]s_t \tag{7.8}$$

where, for the canonical decomposition,

$$\nu_s(B) = (1/16)(1-B)^2(1-F)^2 \;. \tag{7.9}$$

Using (4.1), (4.2) and (7.9) in (7.8), it is found that

$$\delta_t = (1/16)[-(1-B^2)(1-F)^2 b_t+(1/k)(1+F)(1-B)(1-kB)(1-kF)\ c_t+$$

$$+\ (1-B)^2\ (1-F)^2\ u_t] \;.$$

with $k = 3-2\sqrt{2}$. Thus

$$\mathrm{Var}(\delta_t) = (1/256)[10\ \sigma_b^2 + 74\ \sigma_c^2 + 70\ \sigma_u^2] \;,$$

and, using (4.3), we obtain

$$\mathrm{Var}(\delta_t) = \sigma_\delta^2 = \frac{\sigma_a^2}{16}\ [1-\frac{1}{8}] \;. \tag{7.10}$$

(The value (7.10) may alternatively be derived by obtaining the stochastic process followed by $n_t = p_t+u_t$ and applying results of Pierce, 1979.)

Therefore, most of the variation in the seasonal's final estimate is induced by the irregular and by the seasonal innovation. The variance of the error in the historical estimate, given by (7.10), is slightly less than the innovation variance σ_b^2 in (4.3), whereas the revision variance σ_r^2 in (7.7) is slightly larger. Roughly, the three standard deviations σ_b, σ_r and σ_δ, are of similar magnitudes and all approximately one-fourth the standard deviation σ_a of the innovation of the series.

Having obtained expressions for the errors and their associated standard deviations, the model-based procedure permits us to analyze the precision of the estimates, a major point of concern (see Moore et. al., 1981). Combining the revision and final-data error results, an approximate 95 percent confidence interval for p_t based on the concurrent measurement is given by $\hat{p}_t^o + .70\ \sigma_a$; when the final measurement is available,the interval narrows to $\hat{p}_t + .46\ \sigma_a$. The corresponding confidence intervals for the true seasonal component s_t would be of the same width.

Figure 1

Pseudospectrum of Series x_t obeying Model $\nabla_2 x_t = a_t$

and Canonical Components

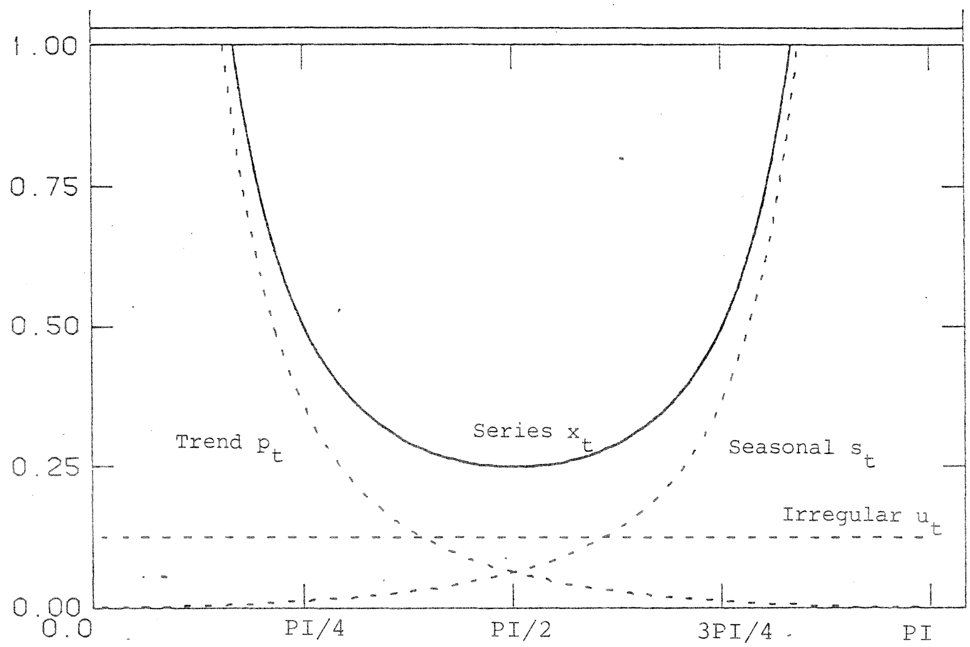


Figure 2

Admissible Parameter Region

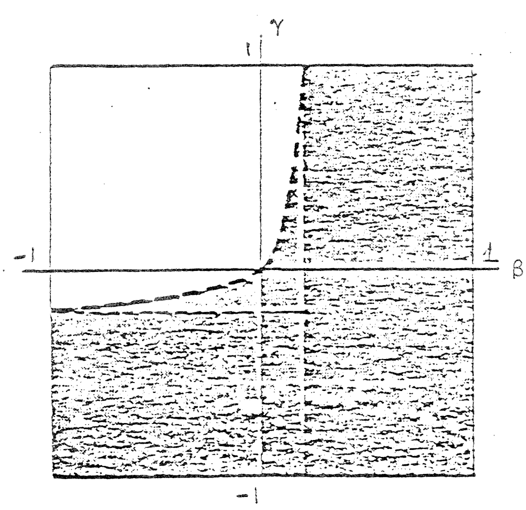


Figure 3

Decomposition of $\nabla_2 x_t = a_t$:
First and Second Stages

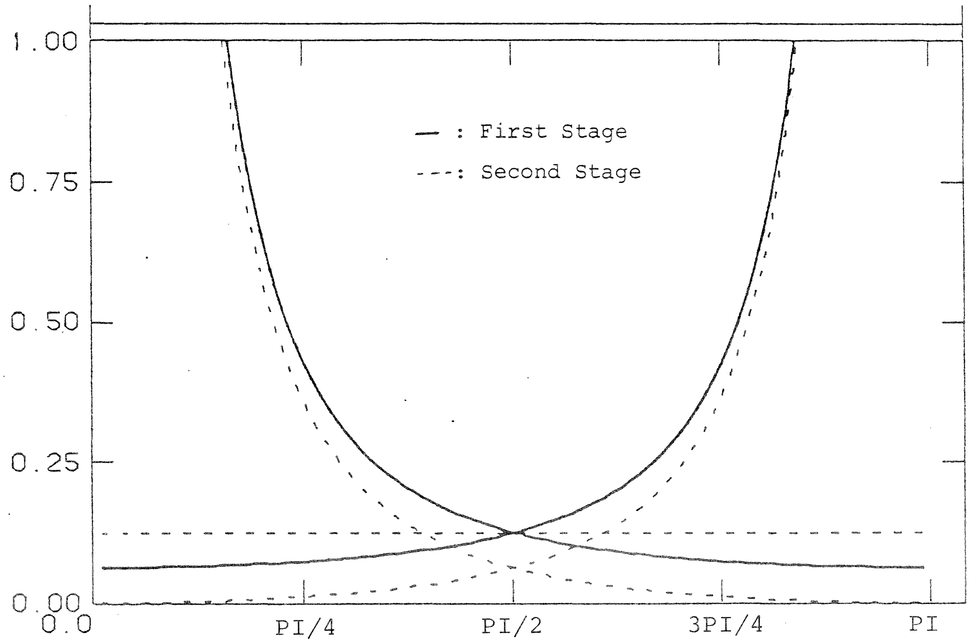


Figure 5

Variance of Revisions in
Concurrent Estimates

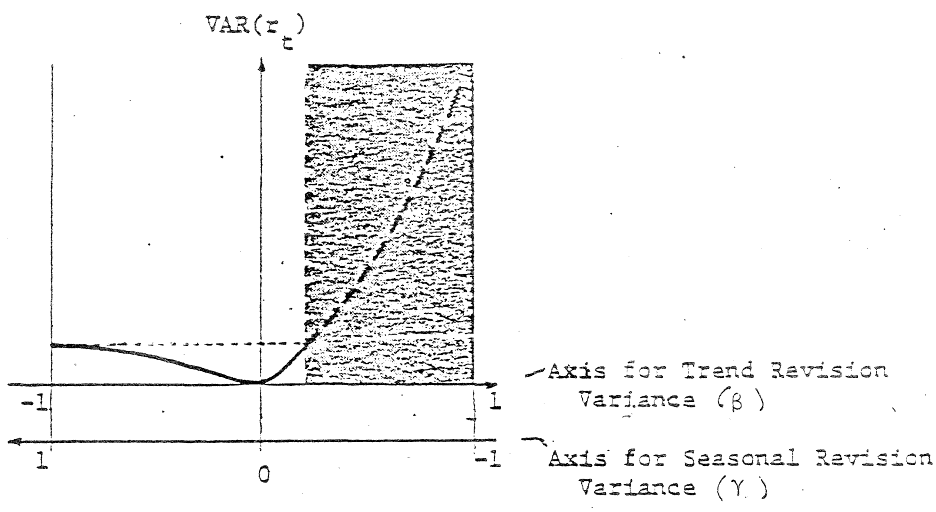
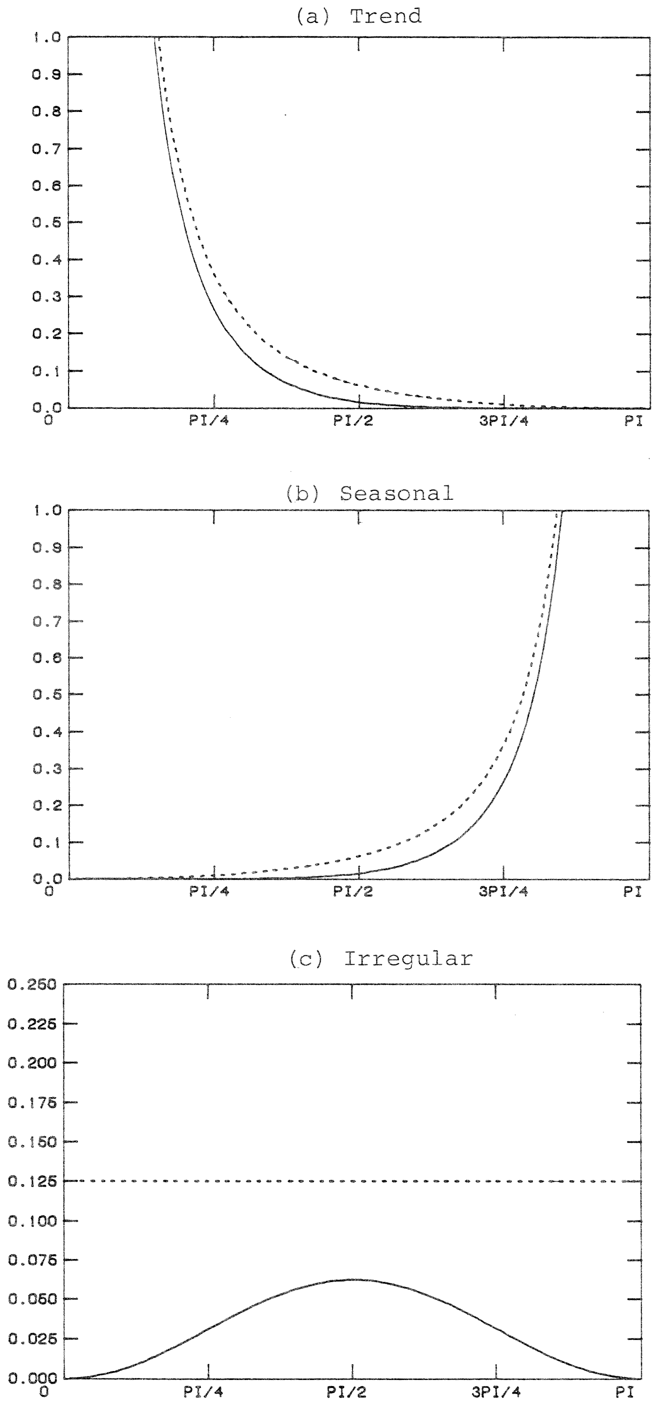


Figure 4

Spectra of Theoretical Components
and Their Estimators



... Theoretical Component
— Estimator

Table 1

Variances and non-zero autocorrelations of the stationary components and their estimators

		Var.	ρ_1	ρ_2	ρ_3
Trend	Component	.125	.50	–	–
	Estimator	.078	.45	.30	.05
Seasonal	Component	.125	–.50	–	–
	Estimator	.078	–.45	.30	–.05
Irregular	Component	.125	–	–	–
	Estimator	.031	–	–.50	–

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