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TEMPORAL DISAGGREGATION METHODS OF ECONOMIC TIME SERIES

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I N D E X

Introduction

The current project for estimating Spanish Quarterly National Accounts, now being developed in the Research Department and, particularly, the obtaining of retrospective quarterly series pose needs of a different nature. Among these needs are those that are derived from methods that can be used to satisfactorily handle the basic problem, which can be formulated in the following general terms: given an annual magnitude, how to obtain quarterly data that are compatible with this magnitud and that incorporate exogenous information, should it be available; this information would take the form of a time series, equally quarterly, which we will call the indicator.

Choosing the indicator will be based on theoretical or other considerations. Although we will not discuss them here, we will assume that these considerations have prompted us to conclude that fluctuations in the indicator provide information on what should be the evolution of the quarterly series to be estimated.

In some cases, no available indicator will exist. Thus, the problem will be reduced to generating, under some valid criterion, four quarterly data from the corresponding annual data.

Based on what has been said up to this point, one might suggest classifying the different methods proposed in literature(*), according to whether or not they use an indicator. Nevertheless, although this outline will be followed in part in this paper, we will later see that it is possible to establish a relatively nested sequence, in the sense that some simple methods that operate without indicators can be treated as a special case of other more general and complete ones. For this reason, we will spend virtually no time at all with the first methods, limiting ourselves only to those that were used as a starting point in more elaborate alternatives. Methods using indicators from stage one of their development are examined in greater detail, and as was stated earlier, we will see that these same methods are perfectly suitable for constructing quarterly series should other information be unavailable.

Independently of the degree of information that each method attemps to incorporate, some methods tackle the problem by introducing special conditions or restrictions, whose degree of reasonableness depends on the criterion of the authors proposing them; given these conditions proposed a priori, the solution to the problem will be unique in each case. There are other methods based on fixing a unique objective criterion that the quarterly series must respect.

(*) A description of some of these methods can be found in Chapter 2 of a recent OECD report (1979). Unfortunately, in the synthesis provided therein the most important methods have been omitted.

Thus, the solution to the problem posed is reduced to a simple optimization exercise. The methods gathered in this paper are representative of these two alternative approaches.

This exposition is based on the problem that gave rise to this paper, even though what follows can also be applied to obtain, for example, monthly data from quarterly data or to any other temporal desaggregation. For the same reason, the exposition centers on obtaining quarterly data from an annual flow, that is, the aforementioned compatibility between the annual and quarterly data is understood in the sense that the sum of the quarterly data must equal the corresponding annual data, as is required to determine the quarterly series for the product, spending or income of the National Accounts. Another alternative compatibility criterion could be the fitting of the average of four quarters to annual data, which occurs when constructing quarterly data from an annual deflator or price index. The transition from one criterion to another is trivial. For this reason, without any loss of generality, henceforth we will always refer to the case of an annual flow.

In the first section, some methods that construct quarterly data without indicators are summarized. For reasons already expressed, far from trying to carry out an exhaustive compilation, the contents of this section are limited to those cases which, in turn, will be the point of departure for other more elaborate ones. Section 2 will include a brief summary of these more elaborate methods. In Section 3, which is the central part of this paper, three methods are discussed, all of which have been initially conceived to use indicators,and some important relationships between them are underlined. In Section 4 the choice of the method recommended in this paper to obtain quarterly data is justified. Last of all, in the last section where conclusions and summaries are drawn up we will take up the basic criteria to be followed when deriving the quarterly data of real economic series.

1. Methods of temporal desaggregation without indicators

Two different methods are summarized in this section. The first was developped by Lisman and Sandee (1964), the second by Boot, Feibes and Lisman (1967). Both works are representative of the two alternative approaches cited in the introduction. We will find these methods again in more elaborate ones analyzed later in this paper.

1.1. Method of Lisman and Sandee (L-S)

After posing the problem of obtaining quarterly data from an annual series with an absence of exogenous information, L-S have proposed a method in which the resulting quarterly series would conform to some previously established conditions. The first condition is that each quarterly data in year t is considered a weighted average of years t-1, t and t+1.

With $Y_t (t=1,\ldots,T)$ representing the series of known annual data and $y_{tj} (t=1,\ldots,T)(j=1,2,3,4)$ the quarterly series to be obtained, bearing in mind our starting assumption, we have:

$$(1.1) \quad \begin{bmatrix} y_{t1} \\ y_{t2} \\ y_{t3} \\ y_{t4} \end{bmatrix} = M \begin{bmatrix} Y_{t-1} \\ Y_t \\ Y_{t+1} \end{bmatrix}$$

where M is a 4x3 matrix that distributes the three annual data among the quarters of the central year.

L-S point out that, from a practical point of view, it is not worthwhile to increase the number of annual values on which y_{tj} is made to depend.

To estimate the elements of the M weights matrix, L-S have introduced four additional conditions that ensure that the solution to the problem is unique:

1. The first is a symmetry restriction: if the annual totals for years t-1, t and t+1 are X, Y, Z, the quarterly data for year t will be the same, although the order will be reversed for data obtained if the total years are Z, Y, X, respectively. This means that the M matrix would only contain six different elements, taking the following form:

$$M = \begin{bmatrix} a & e & d \\ b & f & c \\ c & f & b \\ d & e & a \end{bmatrix}$$

2. The sum of the quarters of each year must be equal to the corresponding annual data:

$$\sum_{j=1}^{4} y_{tj} = Y_t$$

3. If Y_t increases or decreases by a constant amount, k (that is, $Y_{t+1}-Y_t= Y_t-Y_{t-1}= k$), the quarterly data y_{tj} must increase or decrease by a constant amount equal to 1/4 k.

4. If Y_t is a series that alternates between constant increases and decreases (that is, $Y_{t+1}-Y_t= -(Y_t-Y_{t-1})$), the y_{tj} quarterly series will be sinusoidal.

Under these four conditions, the following weights are obtained: a=0.073, b=-0.010, c=-0.042, d=-0.021, e=0.198, f=0.302.

Given the form of (1.1), it is obvious that quarterly data cannot be obtained for the first and last years of the sample. Although the conditions imposed to estimate the weights matrix are more or less reasonable, they are, in any event, arbitrary, as would be any other criteria chosen.

1.2. Method of Boot, Feibes and Lisman (B-F-L)

Instead of starting with a set of restrictions *a priori*, B-F-L have proposed an alternative based on the minimization of some criterion and fixed an objective the series to be estimated must meet. The mathematical formulation of the criterion makes it possible to find its solution by means of a simple optimization exercise.

The first possibility consists in minimizing the sum of the quadratic differences between each pair of successive quarters, under the constraint that the sum of the four quarters of each year be equal to the corresponding known annual total.

Using the same notation as before, it involves minimizing

$$(1.2) \qquad \sum_{t=1}^{T} \sum_{j=1}^{4} (y_{tj} - y_{t(j-1)})^2$$

under the constraints:

$$\forall t: \sum_{j=1}^{4} y_{tj} = Y_t$$

In matrix notation, the above problem would be formulated as

$$(1.2b) \qquad \begin{array}{ll} \text{min.} & y'Ay \\ \text{b.c.} & B'y=Y \end{array}$$

where y, Y are 4Tx1 and Tx1 vectors, respectively, A is a 4Tx4T symmetric matrix such that A=D'D, where D is a (4T-1)x4T matrix, defined as

$$(1.3)\qquad D=\begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & 0 \;\cdot\cdot & 0 \\ & & & & \\ 0 & & \cdots\cdots\cdots & -1 & 1 \end{bmatrix}$$

and, lastly, B is a 4TxT matrix such that(*):

$$(1.4)\qquad B'=\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & \cdots\cdots & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & \cdots\cdots & 0 \\ \cdot & & & & & & & & & \cdot \\ \cdot & & & & & & & & & \cdot \\ \cdot & & & & & & & & & \cdot \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The solution to the problem could be obtained solving the Lagrangean

$$(1.5)\qquad L(y,\lambda) = y'Ay-\lambda'(Y-B'y)$$

where λ is a vector of T multipliers λ_t, whose minimization leads to the 5T equations system:

$$(1.6)\qquad \begin{bmatrix} y \\ \lambda \end{bmatrix} = \begin{bmatrix} A & B \\ B' & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ Y \end{bmatrix}$$

where the matrix 0 is TxT.

(*) To obtain quarterly data for a deflator instead of a flow, B need simply be replaced by B*, with B*=1/4B.

The solution of (1.5) requires the inversion of a 5Tx5T matrix, although B-F-L have reformulated the problem so that it can be solved by inverting a matrix of order (T+1)x(T+1). In any event, annual totals more than three years removed exercise practically no influence on the quarters of a given year. This means that by applying a seven year matrix to the central year, we can obtain the same results given by working with the complete system. Consequently, from a practical point of view, an 8x8 matrix need only be inverted. Moreover, since the matrix to be inverted does not depend on data Y -see (1.6)-, the elements of the inverted matrix may be considered as given for generating quarterly data from any series.

Another alternative minimization criterion, also proposed by B-F-L, consists in minimizing the sum of the squares of the second differences of the series, i.e.,

$$(1.7) \qquad \sum_t \sum_j (\Delta y_{tj} - \Delta y_{t(j-1)})^2$$

[where Δ is the operator of the first differences such that $\Delta y_i = y_i - y_{i-1}$, $i=1,\ldots,4T$(*)] , subject to the same sum restriction of the previous criterion. In terms of matrixes, the problem is presented in the same terms as in (1.2b), by redefining the D matrix in order to incorporate the second differences(**). For this reason, the analytical form of the solution continues to be (1.6).

(*) Strictly speaking, in this case the operator does not act only on j or t (consider the jump from one year to another).

(**) In the discussion of Denton's method in Section 3, we will return to the problem of appropriate definitions for the D matrixes associated with each criterion.

B-F-L compare the results obtained with the Lisman-Sandee method with their own method and use first or second differences -which we will call FD and SD, respectively-, applying them to two short series, with T=3. In the first, the annual data grow by a constant amount (Y_t=160, 200, 240), while in the second, the data approach a cyclical change, with increases that are equal in absolute value, though of the opposite sign (Y_t=160, 200, 160). In both exercises, the results obtained with FD are inferior to those derived from SD, in the sense that the quarterly series with FD produce figures that are less consistent with the evolution of the annual data than those with SD. The comparison with the L-S method does not make much sense since, with T=3 and the loss of the two end years implied in the method, it is reduced to the four central quarters. L-S and SD give the same results for these quarters in the case of the annual series with constant growth, and somewhat different ones for the cyclical series.

The choice between setting an objective in terms of first or second differences in methods based on quadratic minimization is important. Some aspects of the problem are discussed in detail later in section 3.

2. Towards the incorporation of indicators

This section summarizes two methods firmly based on those described in the previous section, although they take into account the information of an indicator.

2.1. Method of Vangrevelinghe

The first method, proposed by Vangrevelinghe (1966), was used in France by the INSEE to obtain the quarterly data of private consumption from the annual aggregate of the National Accounts. This method operates in two steps: in the first one, Vangrevelinghe obtains what he calls the quarterly trend of the annual series; in the second, this trend is modified, by working with the discrepancies that are observed between the quarterly series of the indicator and its own trend. More specifically, if x_{tj} represents the series indicator and, similarly to the notation already introduced, $X_t = \sum_{j=1}^{4} x_{tj}$, the method may be summarized as follows:

1st) Interpolate the annual series Y_t, X_t to obtain the "quarterly trends" $\hat{y}_{tj}$, $\hat{x}_{tj}$. This interpolation is carried out by following the L-S method.

2nd) Fit an annual regression:

$$Y_t = a+bX_t+u_t$$

to calculate the estimator $\hat{b}$.

3rd) Obtain the final quarterly series, y_{tj}, by modifying $\hat{y}_{tj}$ in the following way:

$$y_{tj} = \hat{y}_{tj}+\hat{b}(x_{tj}-\hat{x}_{tj})$$

This method has various weaknesses, such as the initial derivation of quarterly figures based on the criteria of L-S and, in particular, the regression of the second step. Vangrevelinghe states that this regression enables him to verify if "a good correlation" exists between the annual series and the indicator, and to eventually ensure the scale change between the units of measurement of the two series. As far as the first objective is concerned, it would be preferable to present other contrasts; for example, without leaving the same simple regression analysis proposed, the relationship could be estimated by using the interannual rates of change of each variable, in this way avoiding the danger of measuring correlations dominated by the strong trend usually contained in economic temporal series. Turning to the estimation of parameter b, which will play an important role in the third step, some assumptions on the distribution of u_t would have to be offered, and a suggestion would have to be ventured on what should be done, for example, when the residual estimated for a particular year is quite high.

2.2. Method of Ginsburg

The overall approach of the previous method was used by V. A. Ginsburg (1973) to propose a more interesting variant. Subject to the same limitations already mentioned in the second step, his procedure can be expressed in an alternative way that reveals its meaning. From this point of view, this method is clearly preferable to the previous one, granting that this is a special case of more general alternatives, as we will later see.

Although Ginsburg proposes following the same steps used by Vangrevelinghe, he alters the first step, in which the quarterly trend is estimated according to the B-F-L method instead of to the L-S one. That is, the three stages are:

1st) Obtain the quarterly trend series, $\hat{y}$, $\hat{x}$, from (see (1.6)):

$$(2.1)\quad \begin{bmatrix} \hat{y} \\ \lambda_y \end{bmatrix} = \begin{bmatrix} A & B \\ B' & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ Y \end{bmatrix} ; \quad \begin{bmatrix} \hat{x} \\ \lambda_x \end{bmatrix} = \begin{bmatrix} A & B \\ B' & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ X \end{bmatrix}$$

where λ_y, λ_x are the two vectors (Tx1) of Lagrange multipliers associated with the respective problems of conditional minimization.

2nd) Fit the same annual regression

$$(2.2)\quad Y_t = a+bX_t+u_t$$

3rd) Calculate the final quarterly series, y, from the first 4T equations of the system:

$$(2.3)\quad \begin{bmatrix} y \\ \mu \end{bmatrix} = \begin{bmatrix} \hat{y} \\ \lambda y \end{bmatrix} + \hat{b} \begin{bmatrix} x - \hat{x} \\ -\lambda x \end{bmatrix}$$

In the first stage, Ginsburg only considers the minimization criterion of B-F-L with first differences -see (1.2)- and not the second ones, that is, in (2.1), A=D'D with D defined in (1.3). The regression fitted in the second step suffers from the same problems commented on in the previous method. Nevertheless, with some manipulation, the final solution (2.3) can be expressed in a different, more interesting way.

By premultiplying (2.3) by $\begin{bmatrix} A & B \\ B' & 0 \end{bmatrix}$ and using (2.1):

$$\begin{bmatrix} A & B \\ B' & 0 \end{bmatrix} \begin{bmatrix} y \\ \mu \end{bmatrix} = \begin{bmatrix} 0 \\ Y \end{bmatrix} + \hat{b} \begin{bmatrix} Ax \\ B'x \end{bmatrix} - \hat{b} \begin{bmatrix} 0 \\ X \end{bmatrix}$$

and, given that $B'x = X$, we finally have:

$$(2.4) \qquad \begin{bmatrix} y \\ \mu \end{bmatrix} = \begin{bmatrix} A & B \\ B' & 0 \end{bmatrix}^{-1} \begin{bmatrix} \hat{b}\ Ax \\ Y \end{bmatrix}$$

As we will see later, this expression is the solution to the problem of minimizing:

$$(2.5) \qquad \sum_t \sum_j (\Delta y_{tj} - \hat{b} \Delta x_{tj})^2$$

under the usual sum constraint

$$\sum_j y_{tj} = Y_t$$

which, in the particular case of $\hat{b} = 1$, is reduced to the conditional minimization of:

$$(2.6) \qquad \sum_t \sum_j (\Delta y_{tj} - \Delta x_{tj})^2$$

As a result, the Ginsburg method makes it possible to obtain a series whose interquarterly increments minimize the sum of the quadratic differences with respect to the interquarterly increments of the indicator.

In these terms the interpretation of the method is totally clear. Given that this has been a particular case, for the timebeing we will not appraise this method, since its merit will become more evident when we analyze more general ones.

3. Methods of temporal disaggregation with indicators

This section provides a detailed presentation of various methods that incorporate the information of one or more indicators from the initial stages of their development. The first of these methods is due to V. L. Bassie. Among those methods based on an *a priori* set of conditions imposed on an obtainable result, V. L. Bassie's method is the most elaborate. The length of its presentation here is justified by the importance of its use in various countries -see the OECD report (1979); it was also employed in an earlier estimation of the Spanish quarterly national accounts published by the Bank of Spain (1975). The second method, that of F. T. Denton, is also the most complete among those based on a quadratic minimization of an objective criterion. Last of all, we will summarize the method of Chow and Lin, whose theoretical properties will be used to draw attention to the importance of Denton's approach and to enhance its interpretation.

3.1. Method of Bassie

In his initial presentation, Bassie (1958) proposed his method to temporally disaggregate a magnitude by using the information of an indicator. Nevertheless, in the absence of indicators, his method is perfectly usable.

In general, there will be some discrepancy between the value taken by the indicator of a given year and the corresponding annual data, which we will represent by K_t. For example, the sum of four quarterly flows contained in the indicator is less than the annual data, since the indicator represents a partial coverage of the annual data(*). In the particular case in which this discrepancy will be nule for every t, the quarterly series to be obtained will obviously be the indicator itself. Another special situation is defined by the total absence of information; here K_t would coincide with the annual magnitude from which quarterly data is to be constructed. As a result, the problem posed by Bassie of constructing quarterly data from an annual series with an indicator can be expressed as the need to construct quarterly data for the K_t discrepancy without using an indicator. And, given that no limitations have been placed on the value of K_t, the Bassie method can obviously be used to construct quarterly data from annual magnitudes, whether or not the information contained in an indicator is used.

(*) We will later give a more precise definition of K_t, since such a definition is not necessary at the moment.

One of the objectives of this method is to ensure that any new annual data added to a series for which quarterly data has previously been constructed will have a minimal effect on the data already obtained. This criterion presents undeniable practical advantages, but it may be far from optimum, for there is no reason to systematically reject that the new annual data contain relevant information that should be incorporated in the quarters of several previous years.

In order to respect this criterion of minimal reviews of the past, the method operates by making successive fittings to two consecutive years. After completing one of these fittings, or steps, let's say on years t and t+1, the process begins again starting at the beginning of t+1, with a new step fitted to t+1 and t+2. In other words, each year is dealt with twice: first of all, as the second year of a step and later, as the first year of the following step. In this way, the final quarterly profile of each year will exclusively be the result of the net effect of two steps, and the addition of new annual data to a series ending in T will not affect the quarters of year T-1 or the previous ones.

In each of these steps applied to two years, the fitting is carried out by applying four basic criteria, which can be summarized as follows:

1. For the first year of each fitting, no modification is made to correct the total annual discrepancy K_t.

2. For the second year, the total discrepancy of year K_{t+1} is distributed among four quarters.

3. To avoid breaks in the steps from one year to the next, the correction applied at the beginning of the first year will be nule.

4. Since the information contained in the data of the following years is not incorporated at the end of the second year, it will not be known how the trend of the discrepancy will evolve. The method introduces the assumption that this trend will stop at the end of the second year. In other words, the curve expressing the correction will tend to be horizontal at the end of the second year.

This fourth condition clearly reflects the price to be paid in order to maintain the criterion of minimal review of previous data. If a larger horizon of information were incorporated, in general, it would be possible to better detect the trend of the discrepancy. The operative system described, centered exclusively on two years, would be optimum only in those cases in which using more annual data, does not incorporate relevant information for an improved definition of the trend.

The four afore mentioned conditions can be formalized almost identically to the way Bassie himself presented them[*].

(*) The nature of the problem facing Bassie made him define an integral on a three-year period, thus leading him to different results than those found here. Otherwise, the method drawn up here is perfectly in keeping with his earlier-cited work.

Let K_j be the factor to be applied to discrepancy K_t in order to obtain the part of K_t imputable to quarter j, and let K_j be expressed in function of time:

$$K_j = f(t)$$

If the beginning of the first year is situated at the origin of time and the unit interval covers a year, the four conditions cited could be represented as:

(3.1) $\int_0^1 f(t)\, dt = 0$

(3.2) $\int_1^2 f(t)\, dt = K$

(3.3) $f(0) = 0$

(3.4) $\frac{df(2)}{dt} = 0$

The simplest time function enabling the unique solution of system (3.1) - (3.4) is

(3.5) $f(t) = a + bt + ct^2 + dt^3$

Given this definition of f(t), the previous system is reduced to:

(3.6) $a + \frac{b}{2} + \frac{c}{3} + \frac{d}{4} = 0$

(3.7) $a + \frac{3b}{2} + \frac{7c}{3} + \frac{15d}{4} = K$

(3.8) $a = 0$

(3.9) $b + 4c + 12d = 0$

The solution of the four equations system (3.6) - (3.9) leads to the values of the parameters of (3.5):

(3.10) $K_j = f(t) = (-1.125t+2.15625t^2-0.625t^3)K$

By dividing the previous expression by K, we obtain the correction of each quarter of the two years included in the step, which we will represent by f_j:

(3.11) $f_j = K_j/K \qquad (j = 1,...,8)$

The numerical value of each f_j is obtained by solving the defined integrals for each quarter. Given that the unit interval in (3.1) and (3.2) was represented by a year, the amplitude of each quarter will be 0,25: $t \in [0,0.25]$ in the first quarter, $t \in [0.25,0.50]$ in the second, etc. We can verify that the solution of the eight integrals takes the values included in Table 1.

Since the f_j weights are expressed in relative terms, they need only be multiplied by each K_t to obtain the values of each quarter of years t-1 and t, which will be obtained when the methods is fitted to each pair of years. The total of these weights is zero in the first year and one in the second, as a result of conditions (3.1) and (3.2). Consequently, we can see that if K_t is relatively constant in time, the resulting net effect of the two steps applied to each year will induce a seasonal behavior in the quarterly series. We will return to this subject later, at which time, for illustrative purposes, a comparison will be drawn between the results of alternative methods.

TABLE 1

Value of quarterly constants f_j

	1st year	2nd year
1st quarter...	-0.0245362	+0.1434326
2nd quarter...	-0.0360107	+0.2257080
3th quarter...	-0.0020752	+0.2947998
4th quarter...	+0.0626221	+0.3360596
Total......	0.0000000	1.0000000

Before looking at the operative system of the Bassie method in detail, the K discrepancy must be accurately defined. To this end, two alternative criteria can be followed, which we will term, respectively, "additive" and "multiplicative". In the additive case, K_t will express the difference existing in year t between the magnitude for which quarterly data is to be constructed and the sum(*) of the four quarters of the indicator, which is, of course, expressed in the same unit of measurement. In the multiplicative case, the discrepancy is the disparity existing between the interannual rate of variation of the annual magnitude and the average rate of interannual variation of the indicator(**).

(*) If the magnitud for which quarterly data is derived were not a flow, but a deflator, the average, not the sum, would be the quantity needed to calculate the difference. This means that in this case the integral (3.2) must be equal to 4K, thus, the values of the parameters b, c and d included in (3.10) are multiplied by 4. Consequently, the values of f_j presented in Table 1 would also be multiplied by 4.

(**) With the multiplicative criterion, should quarterly data be constructed for a flow as well as for a deflator, the f_j weights will be those given in Table 1 multiplied by 4.

More formally, if Y_t is the annual flow to be dissagregated, x_{tj} (j=1,...,4) the value of the indicator of quarter j of the same year t, and $\bar{x}_t$ the average of the indicator in year t, then the initial definition(*) of K_t is, in each case, the following:

a) additive fitting:

$$K_t = Y_t - \sum_j x_{tj} \tag{3.12}$$

b) multiplicative fitting:

$$K_t = \frac{Y_t/Y_{t-1}}{\bar{x}_t/\bar{x}_{t-1}} - 1 \tag{3.13}$$

The implications of each criterion will be clearly shown by examining in detail the operative system of the method. The exposition will be limited to one specific case that illustrates, without any loss of generality, the mechanism of each step. Thus, Tables 2 and 3 will play a central role, since the text that follows is fully based on them.

We will begin with Table 2, which covers the generation of quarterly data from an annual flow, Y_t. The information contained in series x_{tj} was used and the multiplicative criterion adopted. Both series, as well as their interannual rates of variation, appear in columns 1 through 4. The method begins by applying a transformation to the indicator, such that the sum of the first year coincides with the corresponding annual data. Treatment of this first year -and, as we will later see, also treatment of the last

(*) Later we will see that K_t is, in fact, always calculated on transformations of x_{tj}.

Table 2

Derivation of quarterly data of annual flow Y_t with quarterly indicator x_{tj} (t=years; j=quarters)

Multiplicative Fitting

Year	Annual Data Y_t (1)	Inter annual rate of variation (2)	Quarterly indicator x_{tj} (3)	Annual sums and rate of variation (4)	Indicator matching annual sums x^*_{tj} (5) (a)	Annual sums preceding column (6)	K_1 (7)	Bassie STEP 1 $\hat{y}^1_{tj}$ (8) (b)	Annual sums preceding column (9)	K_2 (10)	Bassie STEP 2 $\hat{y}^2_{tj}$ (11) (c)	Annual sums preceding column (12)	K_3 (13)	Bassie STEP 3 $\hat{y}^3_{tj}$ (14) (c)	Quarterly series $\hat{y}_{tj}$ (15) (e)	Quarterly series with adjusted sum (16) (f)
1			25,00		21,74			21,99							21,99	22,0
	100	-	27,50	115,0	23,91	100,00		24,32							24,32	24,3
			30,00	-	26,09			26,11							26,11	26,2
			32,50		28,26			27,42							27,42	27,5
2			33,75		29,35		$K_1 = \frac{115/100}{130,44/100} - 1 = -.1183$	27,35			27,19				27,19	27,3
	115	15,0	36,25	150,0	31,52	130,44		28,16	114,65		27,91				27,91	28,0
			37,50	30,44	32,61			28,06			28,04				28,04	28,1
			42,50		36,96			31,08			31,56				31,56	31,6
3			42,50	(d) →	(d) →	(d) →	(d) →	31,08		$K_2 = \frac{144/115}{135,28/114,65} - 1 = -.0612$	32,17			32,14	32,14	32,2
	144	25,2	45,00	185,0				32,91	135,28		34,72	143,70		34,68	34,68	34,8
			47,50	23,33				34,73			37,24			37,24	37,24	37,3
			50,00					36,56			39,57			39,66	39,66	39,7
4			52,50	(d) →	(d) →	(d) →	(d) →	(d) →	(d) →	(d) →	41,55		$K_3 = \frac{180/144}{178,07/143,70} - 1 = -.0087$	41,76	41,76	41,8
	180	25,0	55,00	225,0							43,53	178,07		43,87	43,87	44,0
			55,00	21,62							43,53			43,98	43,98	44,1
			62,50								49,46			50,04	50,04	50,1

(a) $x^*_{tj} = (Y_t / \sum_j x_{tj})\, x_{tj}$, $(j = 1,\ldots, 4;\ t = 1,\ldots, T)$.

(b) $\hat{y}^1_{tj} = (f_j K_1 + 1) x^*_{tj}$, (f_j= quarterly constants. See text).

(c) $\hat{y}^i_{tj} = (f_j K_i + 1) \hat{y}^{i-1}_{tj}$, $(i = 2,3,\ldots,T-1)$

(d) $(\hat{y}^i_{t,4} / x_{t,4})\, x_{(t+1)j}$, $(i = 1,2,\ldots,T-1)$

(e) $\hat{y}_{tj}$ = 1st year of $\hat{y}^i_{tj}$ for $t = 1,\ldots,T-1$ $(i = 1,2,\ldots T-1)$
= 2nd year of $\hat{y}^{T-1}_{tj}$ for $t = T$

(f) $(Y_t / \sum_j \hat{y}_{tj})\, \hat{y}_{tj}$

year- differs from the rest of the series, since only one Bassie step, and not two, is applied. To achieve this equality of sums, the difference between them is proportionally distributed between x_{tj}, as is indicated in note a. The calculation of first discrepancy K_1 is shown in col. 7, in which expression (3.13) is simply applied to these data. The first Bassie step, col. 8, distributes discrepancy K_1 by quarters, using the f_j weights. This quarterly distribution of the discrepancy is affected by the level of the indicator which multiplicatively weighs it in accordance with its importance at each moment. This weighed distribution of the quarterly discrepancy is added to the indicator, thus giving us "step 1" in col. 8, as is indicated in note b.

Before beginning with step 2, applied to years 2 and 3, the scale of the indicator is transformed with the criterion set forth in note a: the difference observed in the last quarter fitted -the 4th quarter of year 2- between the original indicator and the value obtained by the Bassie method is distributed in proportion to the values of the 3rd year indicator. In this way, a break is avoided between the provisional results of step 1 and the initial values of the indicator. K_2 is calculated as was K_1, and step 2 is executed with the same criterion commented on in step 1 and expressed in note c.

Here we see that the first half of the results obtained in step 2 modify the provisional estimation available for the same year and obtained from the second half of step 1.

From this point on, the process followed in each step is the same. Logically, only one step is involved in the last year, and the results obtained cannot be considered final. We should also observe that the rectification the quarterly data of year T will be subject to is a function of the discrepancy found between the annual data and the indicator in T+1.

With the exception of the last year, as is indicated in note e, the resulting quarterly series for the entire sample used, col. 15, is simply the first half of each step. The final results are those of column 16, in which the results of the preceding column have been slightly adjusted so that the sum of the quarters of each year would match the corresponding annual data, since the multiplicative criterion followed to define K_t discrepancies cannot fully respect this condition.

Table 3 presents the same numeric example and details seen in the preceding table, but here an additive fitting criterion has been adopted.

Apart from the K_t calculation, obviously, the operative differences between this case and the last one are:

a) The first fitting of the indicator designed to respect the annual sum, col. 5, is carried out linearly and not proportionally to the value of x_{tj}. See note a of both tables.

b) The distribution of K_t discrepancies only depends on the f_j constants, and is unaffected by indicator levels. See notes b and c of both tables.

c) The change in the indicator scale implemented to avoid a break with preceding steps (note d) is linear and not proportional to x_{tj}.

Table 3

Derivation of quarterly data of annual flow Y_t with quarterly indicator x_{tj} (t=years; j=quarters)

Additive fitting

Year	Annual Data Y_t (1)	Inter-annual rate of variation (2)	Quarterly indicator x_{tj} (3)	Annual sums and rate variation (4)	Indicator matching annual sums x^*_{tj} (5) (a)	Annual sums preceding column (6)	K_1 (7)	Bassie STEP 1 $\hat{Y}^1_{tj}$ (8) (b)	Annual sums preceding column (9)	K_2 (10)	Bassie STEP 2 $\hat{Y}^2_{tj}$ (11) (c)	Annual sums preceding column (12)	K_3 (13)	Bassie STEP 3 $\hat{Y}^3_{tj}$ (14) (c)	Quarterly series $\hat{Y}_{tj}$ (15) (e)	Quarterly series with adjusted sum (16) (f)
1			25,00		21,25			21,74							21,74	
			27,50		23,75			24,47							24,47	
	100	-	30,00	115,0	26,25		K_1 = 115-135 = -20	26,29							26,29	
			32,50	-	28,75			27,50							27,50	
2			33,75		30,00			27,13			27,11				27,11	
			36,25		32,50			27,99			27,01				27,01	
	115	15,0	37,50	150,0	33,75	135,0		27,85		K_2 = 144-143,12 = .88	27,85				27,85	
			42,50	30,44	38,75			32,03			32,09				32,09	
3			42,50		(d) →			32,03			32,16			32,27	32,27	
			45,00					34,53			34,73			34,89	34,89	
	144	25,2	47,50	185,0				37,03	143,12		37,29		K_3 = 180-184,31 = -4,31	37,30	37,30	
			50,00	23,33				39,53			39,83			39,56	39,56	
4			52,50			(d) →					42,33			41,71	41,71	
			55,00								44,83			43,86	43,86	
	180	25,0	55,00	225,0							44,83	184,31		43,56	43,56	
			62,50	21,62							52,33			50,88	50,88	

(a) $x^*_{tj} = x_{tj} + \frac{1}{4}(Y_t - \sum_j x_{tj})$, (j = 1,...,4; t = 1,...,T).

(b) $\hat{Y}^1_{tj} = f_j K_1 + x^*_{tj}$, (f_j = quarterly constants. See text).

(c) $\hat{Y}^i_{tj} = f_j K_i + \hat{Y}^{i-1}_{tj}$, (i = 2,3,...,T-1)

(d) $\hat{Y}^i_{t,4} - x_{t,4} + x_{(t+1)j}$, (i = 1,2,...,T-1)

(e) $\hat{Y}_{tj}$ = 1st year of $\hat{Y}^i_{tj}$ for t = 1,...,T-1 (i = 1,2,...T-1)

= 2nd year of $\hat{Y}^{T-1}_{tj}$ for t = T

(f) Unnecessary. Column 15 respects the annual sums.

Here the fitting of col. 16 is unnecessary whenever the quarterly data of col. 15 already respect the corresponding annual totals.

In short, with the multiplicative fitting the indicator is always involved in the quarterly distribution of K_t discrepancies, whereas with the additive fitting, the indicator plays no role at all.

The implication of this difference is important and can be exemplified by constructing quarterly data from an annual flow using a quarterly series as an indicator. Such a series contains a part of the aggregate included in the annual series, that is, it represents the same phenomenon, but with partial coverage. Two quarterly series are obtained from the same data: in the first, an additive criterion is followed, in the second, a multiplicative criterion.

The results obtained in each case are represented in figures 1 and 2, respectively. These figures also include the common indicator of both fittings, as well as the difference -denoted as the "remainder" in the figures- between each quarterly series and the common indicator.

We can see how the remainder of the additive fitting -figure 1- presents a profile that is adapted to the trend of the annual data -which is also quite similar to that of the indicator represented in the graph- but that is not fitted to the quarterly variation of the indicator. On the other hand, the remainder of the multiplicative fitting -figure 2- is indeed adapted to the quarterly fluctuations of the indicator. The result is that the series subject to an additive criterion shows a profile resembling that of the

FIGURE 1.

BASSIE METHOD
ADDITIVE FITTING

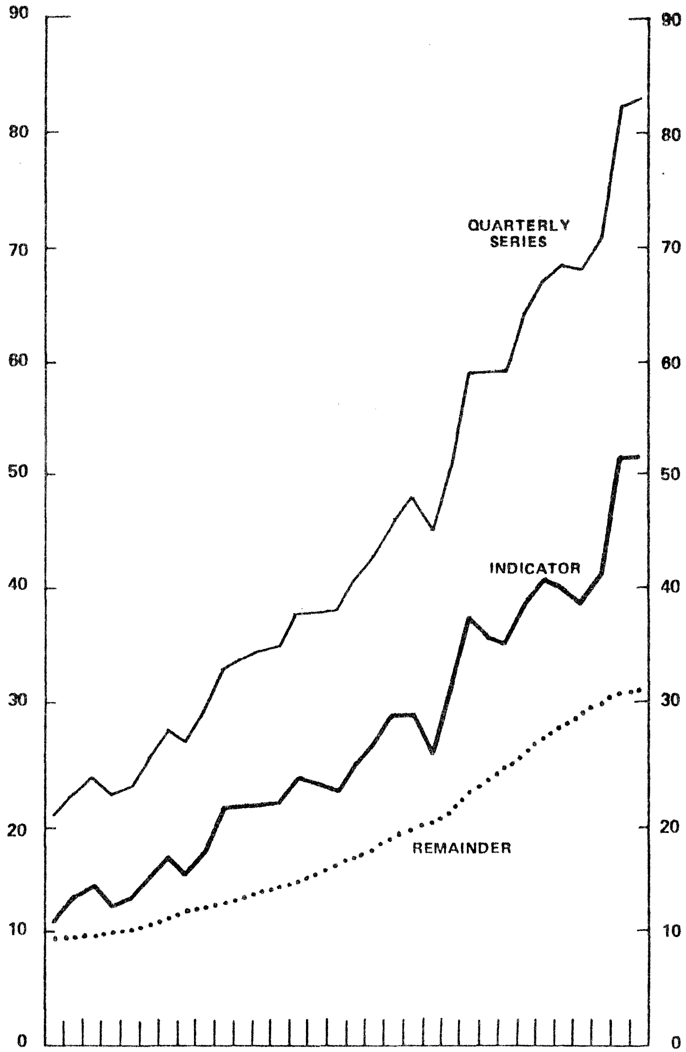
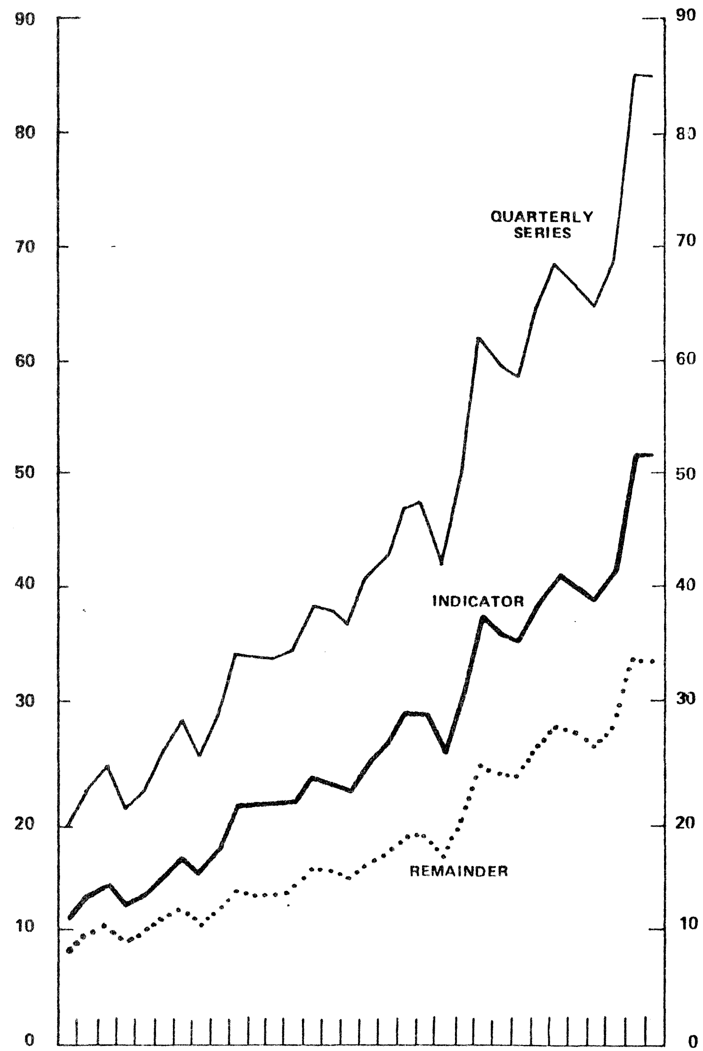


FIGURE 2.

BASSIE METHOD
MULTIPLICATIVE FITTING



indicator, while with the multiplicative criterion, the quarterly variations of the indicator are accentuated in order to better respect the rates of variation, smoothed in the previous case.

The choice between the two alternative criteria must be based on a previous decision as to whether the part of the annual data not covered by the indicator should be distributed throughout the year with the same profile as the indicator or independently of the same. Obviously, the first case will require the multiplicative criterion, the second, the additive criterion.

In support of this last statement, it is interesting to note that the results of this additive fitting are the same as those obtained when the problem is tackled by imposing a priori the independence of the remainders with respect to the indicator. That is, by deriving quarterly data without an indicator for a new annual series Y_t^*, constructed as

$$Y_t^* = Y_t - \sum_j x_{tj}$$

and by adding the series x_{tj} to the result, we obtain with precision the resulting quarterly series of the additive option of the method with the Y_t and x_{tj} series.

Lastly, we can suggest a proper way of handling the last year of the series to which, as we stated earlier, and as is the case of the first year, only one step of the two that determine the quarterly profile of any intermediate year is applied. This means that the four last quarters of a series may be modified considerably when a new annual data is added. The importance of this modification will depend on the magnitude of the discrepancy presented by the data of T+1.

So as to reduce the scope of the revisions of the last year, we can add a prediction of an annual data and of four quarterly data of the indicator to the end of the series, allow the program to operate on T+1 years, and eliminate the T+1 of the resulting quarterly series. If the prediction of the values of T+1 is "reasonable", the quarters of T will only be marginally modified when the true data of year T+1 will be included.

The prediction problem posed is relatively simple, since the interannual profile of the indicator in T+1 does not exercise any influence on the derivation of the quarterly data of T. It is only the average rate of the interannual variation of the indicator -or, simply, its absolute increment in the case of an additive fitting- that is involved in calculating the K discrepancy. As a result, the only necessary prediction, as far as the quarterly data of year T is concerned, is limited to obtaining annual data from which quarterly data is derived and the value, also annual, of the indicator, all of which is tantamount to simply predicting the K discrepancy.

In general, although the Bassie method presents some difficulties, later to be illustrated, it does lead to quarterly series that closely reflect the indicator used.

3.2. Method of Denton

The method proposed by F. T. Denton (1971) is the most general among those based on a quadratic minimization criterion, since both those of B-F-L and Ginburg, as we will see, are special cases of the former. Moreover, in some cases, Denton's results are preferable to those derived from the previous Bassie method. We will see in the following section that although the point of departure of this method is a purely mathematic criterion, it can be reinterpreted to emphasize the statistical significance of the solution, which shows quite desirable properties that cannot be established for previous methods. Thus, this method can be considered the most appropriate one for tacking the problem of generating quarterly data.

As in the Bassie case, this method takes into account the existence of an indicator, even though it must also be suitable for deriving the quarterly data from an annual series without an indicator.

The estimation of the quarterly series is based on the quadratic minimization of an objective. By using the same previous notation, the problem formally consists in minimizing.

(3.14) $(y-x)'A(y-x)$

under the T constraints

(3.15) $B'y = Y$

where B is the matrix defined in (1.4) and A is a 4T x 4T symmetric matrix that varies with the objective function. For example, if we wish to minimize the quadratic differences between the series to be estimated and the indicator, that is,

$$\sum_{t=1}^{T} \sum_{j=1}^{4} (y_{tj}-x_{tj})^2$$

A will be the unit matrix. In order to minimize the quadratic differences of the interquarterly increments of both series, i.e.,

$$\sum_{t} \sum_{j} (\Delta y_{tj}-\Delta x_{tj})^2$$

A = D'D, and Denton defines D as a 4T x 4T matrix:

$$(3.16) \qquad D=\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & \cdot & & & \cdot \\ 0 & & \cdots\cdots & & -1 & 1 \end{bmatrix}$$

which we will return to later. We will see other interesting objective criteria.

The solution of the (3.14) - (3.15) system is obtained from the Lagrangean:

$$(3.17) \qquad L(y,\lambda^{*}) = (y-x)'A(y-x) - \lambda^{*}{}'(Y-B'y)$$

where λ^{*} is the vector of the Lagrangean multipliers associated with the T restrictions.

The first-order conditions are:

$$\frac{\partial L}{\partial y} = 2A(y-x) + B\lambda^* = 0$$

$$\frac{\partial L}{\partial \lambda^*} = Y-B'y = 0$$

By representing the vector of annual discrepancies between the two series by r, with r = Y-B'x, and making $\lambda = \lambda^*/2$, the previous system can be written as:

$$(3.18) \quad \begin{bmatrix} y \\ \lambda \end{bmatrix} = \begin{bmatrix} A & B \\ B' & O \end{bmatrix}^{-1} \begin{bmatrix} A & \tilde{O} \\ B' & I \end{bmatrix} \begin{bmatrix} x \\ r \end{bmatrix}$$

where O, I are TxT, and $\tilde{O}$ is 4TxT.

Given that

$$\begin{bmatrix} A & \tilde{O} \\ B' & I \end{bmatrix} \begin{bmatrix} x \\ r \end{bmatrix} = \begin{bmatrix} Ax \\ Y \end{bmatrix}$$

(3.18) reduces to (2.4) when A = D'D, thus confirming that the Ginsburg method with $\hat{b} = 1$ is a special case of Denton.

On the other hand, in the absence of the indicator, x = 0 and (3.18) reduces to (1.6). Consequently, B-F-L is also a special case of Denton(*), which becomes obvious

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(*) Actually, in order to go from (3.18) to (1.6) we need only make x=k, where k is any constant: if the matrix elements of the partitioned inverse containing both expressions are represented by A^{ij} (ij=1,2), we can see that the quarterly series derived from (1.6) and (3.18) -say, y_1 and y_2- are, respectively, $y_1 = A^{12}Y$ and $y_2 = A^{11}Ax+A^{12}Y$. So that $y_1 = y_2$, regardless of what Y may be, $A^{11}Ax = 0$ is a necessary condition. One sufficient condition for this last equality, if x is a constant different from zero, is that the sum of the elements in the rows of $A^{11}A$ be zero. Although its analytical demonstration is complex, the $A^{11}A$ matrix satisfies this condition.

when comparing the respective objective functions (1.2b) and (3.14) and defining D similarly in both cases, since A = D'D.

The (3.18) system is the general form that adopts the solution to the problem posed, provided that the objective function is expressed in terms of quadratic differences between any degrees of differentiation of the series y, x, in other words, when we are conditionally minimizing

$$(3.19) \qquad \sum_t \sum_j \left[\Delta^h (y_{tj} - x_{tj})\right]^2 \qquad (h = 0,1,2,...)$$

where Δ^h is the operator of differences of h order, such that $\Delta^h = (1-L)^h$, with $L^h x_i = x_{i-h}$.

The differences of the h order can be obtained with successive h applications of matrix D as defined in (3.16). For example, for h = 2, DD(y-x), such that A = D'D'DD. This is possible because D, according to (3.16) is a square matrix. This fact allowed Denton to suggest a computational system for (3.18) that considerably reduces computer time, since it only requires the inversion of a TxT, and not a 4Tx4T, matrix. The cost of this simplification is, however, quite high: by defining D as the (3.16) square matrix, the initial condition $y_t = x_t$ is established for t=1, i.e., the minimization criterion imposed on the initial quarter is substantially different from the rest of the series. It is therefore preferable to adopt the definition of D given in (1.3) as B-F-L and, later, P.A. Cholette (1979) have done. Although the latter only considers the differences of the first order, while second differences are taken up in this paper, a similar criterion will be followed without placing any initial conditions. With this, for h = 2, A = D2'D2, where D2 is the (4T-2)x4T matrix, defined by:

$$(3.20) \qquad D2 = \begin{bmatrix} 1 & -2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -2 & 1 \end{bmatrix}$$

and, the matrix associated with Δ^h for $h \geqslant 3$, would have to be defined similarly. In this paper, however, we will not consider order differentiations greater than two.

The substitution of Denton's D and DD matrixes by those defined in (1.3) and (3.20), respectively, bear considerable practical importance, as will be seen later in a simple example.

Another possible family of objective criteria must be introduced, centered on relative, and not absolute, differences. Its most general formulation, carried out in a manner similar to that of (3.19), is

$$(3.21) \qquad \sum_t \sum_j \left[\Delta^h \frac{y_{tj} - x_{tj}}{x_{tj}} \right]^2$$

which, in matrix notation, translates into the Langrangean

$$(3.22) \qquad L(y, \lambda^*) = (y-x)' \; X^{-1} \; AX^{-1} (y-x)$$

where X is a diagonal matrix formed by the elements of the x indicator, and for h = 0,1,2, respectively, A = I, A = D'D with D as defined in (1.3) and A = D2'D2.

The solution of (3.22) leads to

$$(3.23)\quad \begin{bmatrix} y \\ \lambda \end{bmatrix} = \begin{bmatrix} X^{-1}AX^{-1} & B \\ B' & 0 \end{bmatrix}^{-1} \begin{bmatrix} X^{-1}AX^{-1} & \tilde{0} \\ B' & I \end{bmatrix} \begin{bmatrix} x \\ r \end{bmatrix}$$

whose parallelism with (3.18) is evident.

To underline the basic difference between minimizing absolute or relative differences -whatever the order of differentiation, h, may be-, systems (3.18) and (3.23) can be solved for y by inverting by blocks the first partitioned matrix of both solutions(*).

By representing the elements of the inverse of the complete matrix (3.18) by $A^{ij}(i,j = 1,2)$, we have(**):

$$A^{11} = A^{-1}(I-B(B'A^{-1}B)^{-1}B'A^{-1}) \quad , \text{ and}$$

$$A^{12} = A^{-1}B(B'A^{-1}B)^{-1}$$

thus, in the case of minimizing absolute differences, the estimated quarterly series takes the form:

$$(3.24)\quad y = x + A^{-1}B(B'A^{-1}B)^{-1}r$$

(*) This exercise makes it necessary to consider that D is defined as in (3.16), since, under (1.3), $A = D'D$ has rank 4T-1 and A^{-1} does not exist. As far as the solution to (3.18) or (3.23) are concerned, no problems arise, regardless of the definition of D, since the complete matrix of which A forms part can indeed be inverted. Consequently, the implicit adoption of (3.16) is only necessary to the illustration pursued at this point.

(**) See, for example, Goldberger, <u>Econometric Theory</u>, page 27.

and, operating in a similar fashion on (3.23), we can see that in the case of relative differences the solution is:

$$(3.25) \quad y = x + XA^{-1}XB(B'XA^{-1}XB)^{-1}r.$$

The interpretation of both expressions is immediate: the quarterly series is equal to the indicator plus a part of the discrepancies, r, existing each year between the annual series and the indicator. The distribution of the Tx1 vector, r, between the 4T quarters is carried out according to the weights assigned by the matrix expression that multiplies r. In the case of (3.24), these weights are independent of the indicator, since they only depend on B and on the degree of differentiation included in the objective function, that is, on A. On the contrary, in (3.25), the distribution of discrepancies is also done according to indicator X.

In this way, we can see that in order to relate this method with that of Bassie's, "absolute differences" in the objective function can be associated with the "additive criterion" in Bassie's Method, and "relative differences" with the "multiplicative criterion".

Thus, henceforth we will qualify solution (3.24) as the Denton additive and (3.25) as the Denton multiplicative. The reasons for opting between one or another alternative will be the same as those pointed out in the Bassie method.

Of course as also occurred in the Bassie additive, (3.24) guaranties that the series for which data are derived is simply the indicator plus the quarterly data obtained without an indicator from $Y - \sum_j x_{tj}$. In the case of very little real interest, for example, in which the absolute quadratic differences are minimized, we can easily see that (3.24) reduces to

$$y_{tj} = x_{tj} + \frac{1}{4} r_t \quad (3.26)$$

In the following section, with the presentation of a new method, we will return to other aspects that better clarify the properties of the Denton method. Before this, however, we should show some results that jusfify some previous statements.

We have emphasized the importance of substituting the matrixes of differentiation used by Denton based on (3.16) by those defined in (1.3) and (3.20), due to the different treatment given to the initial observations of the quarterly series. In order to illustrate this point, data was derived from the same annual series used by Denton in his work. For our purposes here, it is preferable not to use any indicator.

The annual series, with five observations, evolves smoothly, taking the values of 500, 400, 300, 400, 500. The quarterly data have been derived by minimizing the first and second differences. In each case, the Denton matrixes and those proposed here are used. The results are presented in Figures 3 and 4, which cover, respectively, the two series obtained under the first and second differences. In both cases, the series derived from the Denton matrixes -represented with a thick line- introduce a strong distortion at the beginning of the series, while the alternative series -represented with thin lines- shows a

FIGURE 3.
DERIVATION OF QUARTERLY SERIES FROM ANNUAL SERIES
WITH ALTERNATIVE DEFINITIONS OF THE DIFFERENTIATION MATRIX

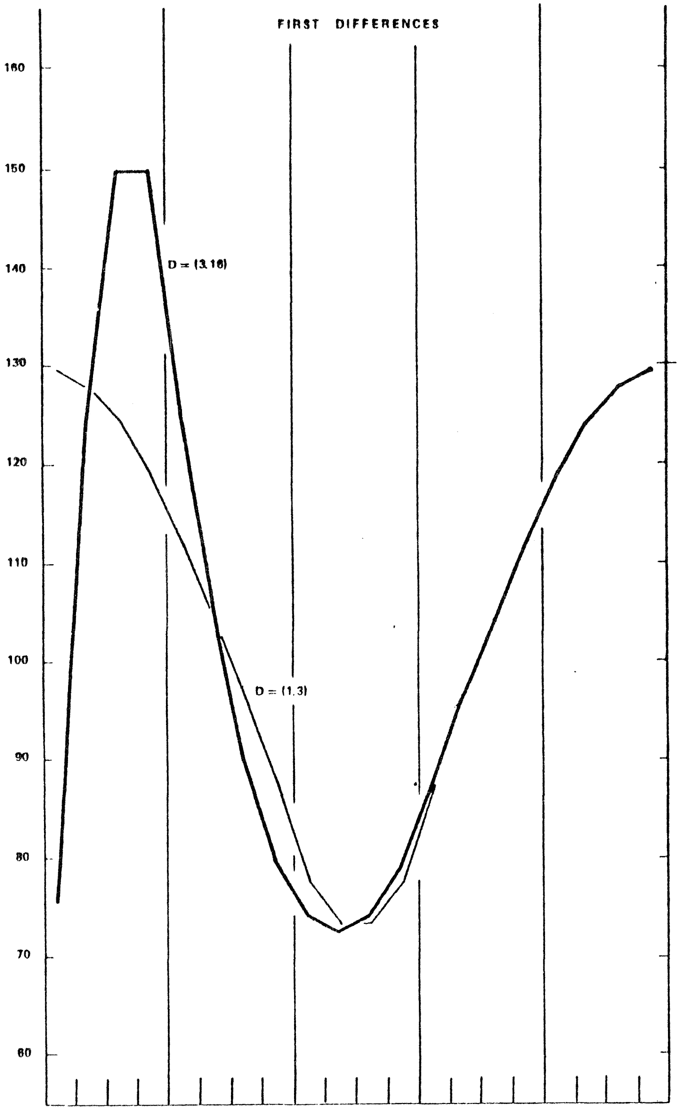
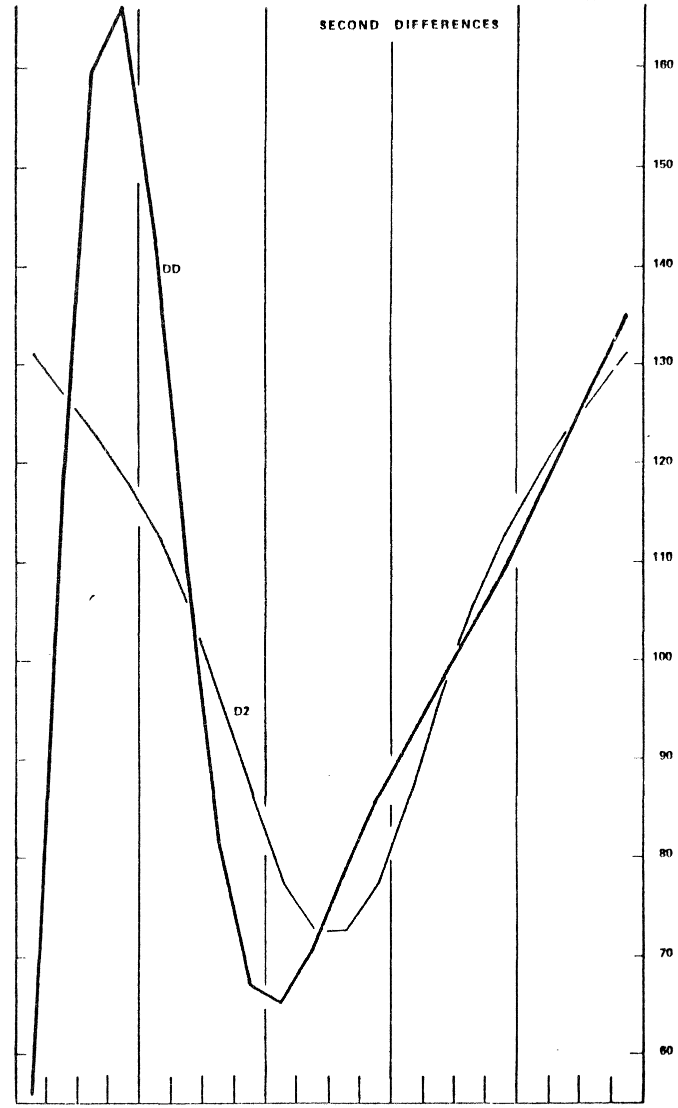


FIGURE 4.
DERIVATION OF QUARTERLY SERIES FROM ANNUAL SERIES
WITH ALTERNATIVE DEFINITIONS OF THE DIFFERENTIATION MATRIX



smooth evolution, closely reflecting that of the annual data. The discrepancies between both logically tend to reduce with time, although this occurs much more rapidly in the case of the first differences.

An important point posed by this method is the choice of the degree of differentiation of the objective function. The simple case of not taking any differences is clearly of less interest, since, when the annual series shows discontinuity in its evolution, more sudden jumps occur in the steps from one year to the next. As a result, the choice is presented between the first and second differences.

In the non-indicator case, we can explore the results obtained with and ^{2} for some series that reflect the different patterns of evolution that frequently appear in economic series. As was stated in Section 1.2, B-F-L do this by starting with two simple examples using only three annual data: in the first, they take an annual series with constant increments but of the opposite sign; in the second, they take a series that continually grows in a constant amount. To these two cases we will add a third example, with a series that grows at a constant rate. Specifically, the three series to be disaggregated are Y_t^1 = 400, 300, 400; Y_t^2 = 300, 400, 500; and Y_t^3 = 300, 360, 432.

The quarterly series obtained are presented in Figures 5, 6 and 7, respectively, and require virtually no comment. In all cases the second differences lead to quarterly series with an evolution that is more consistent with the starting annual series. Given its importance,

FIGURE 5.

DERIVATION OF QUARTERLY SERIES FROM ANNUAL SERIES
(Y_t = 400, 300, 400) WITH FIRST (FD) AND SECOND (SD) DIFFERENCES

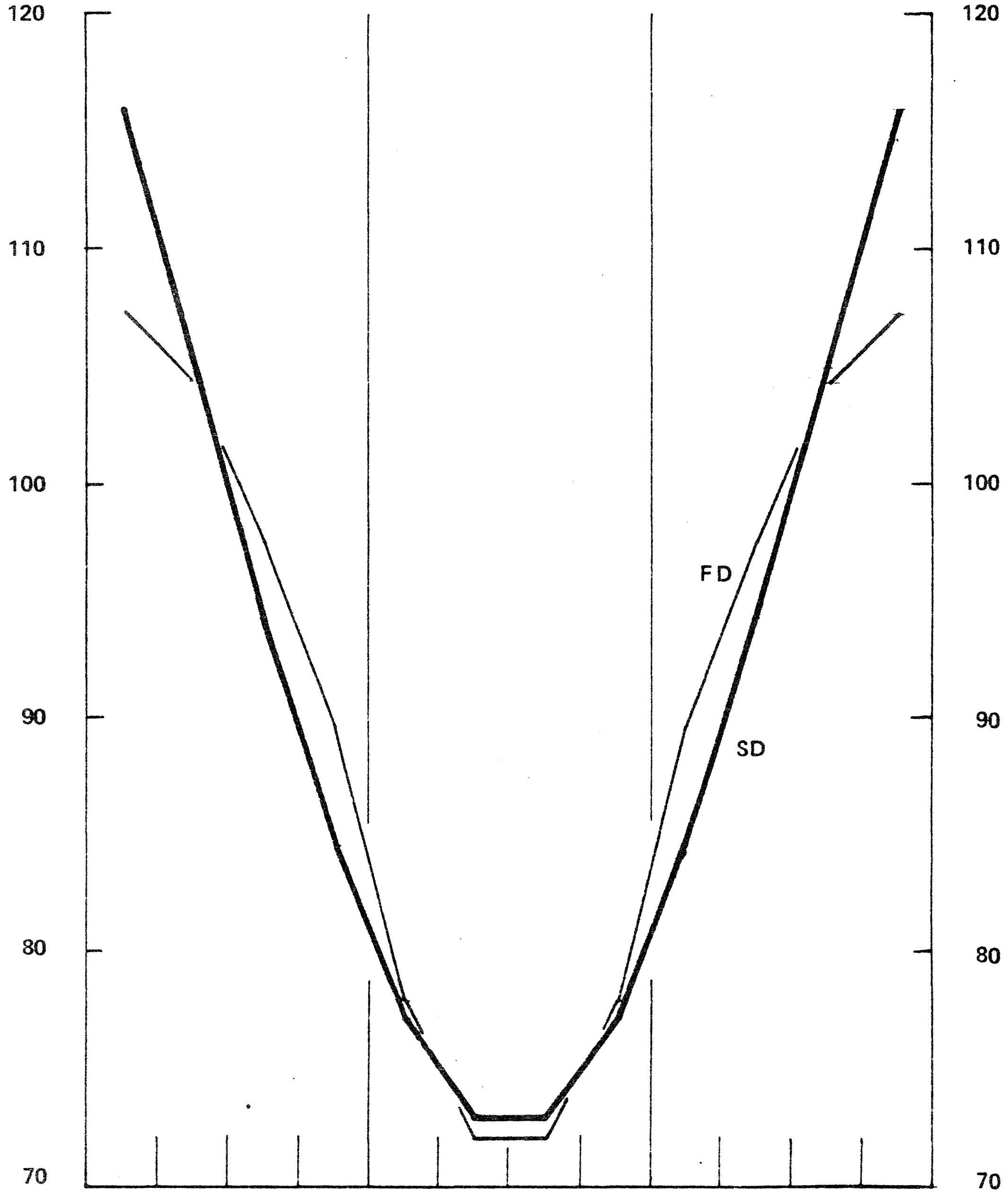


FIGURE 6.

DERIVATION OF QUARTERLY SERIES FROM ANNUAL SERIES (Y_t = 300, 400, 500) WITH FIRST (FD) AND SECOND (SD) DIFFERENCES

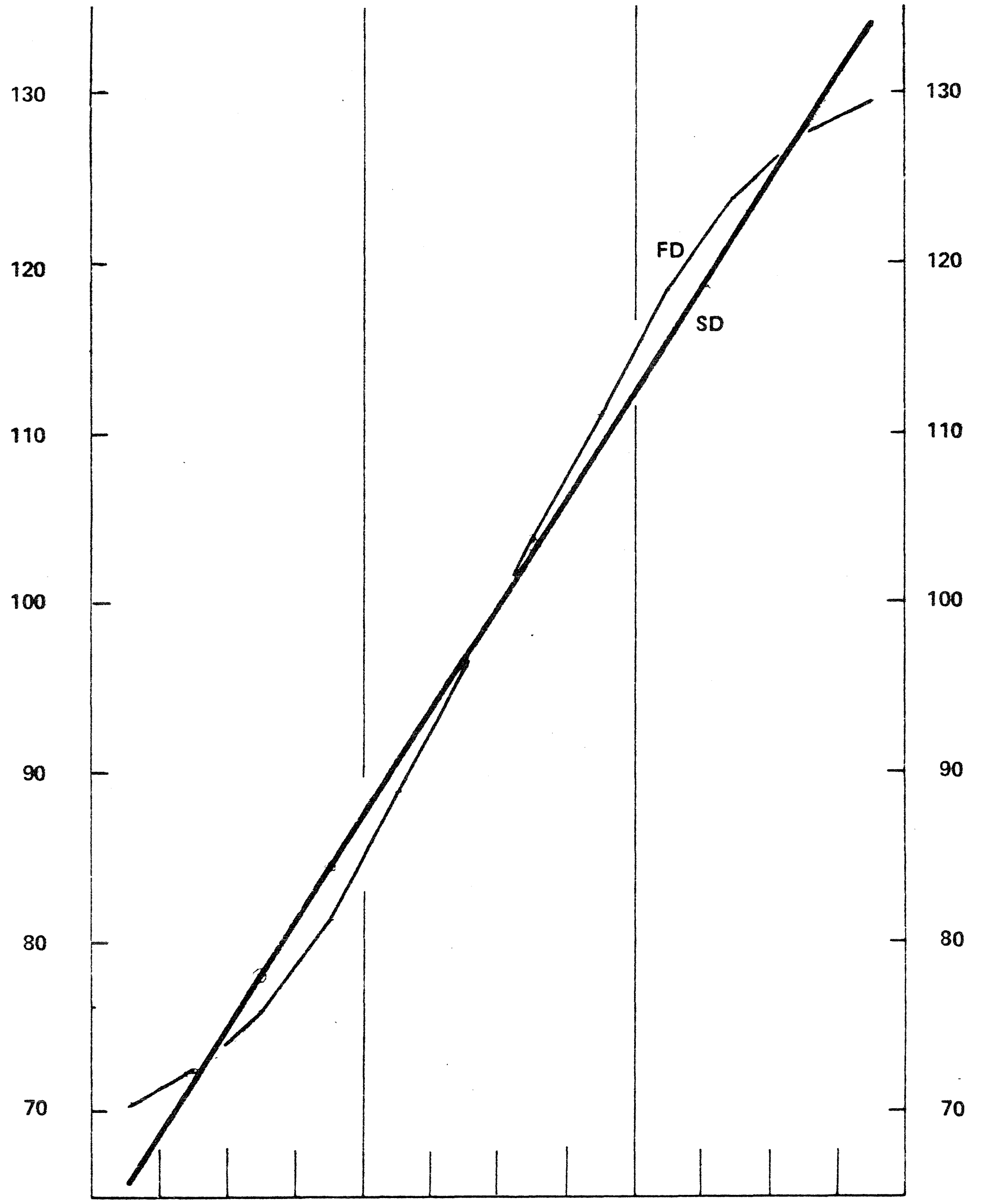
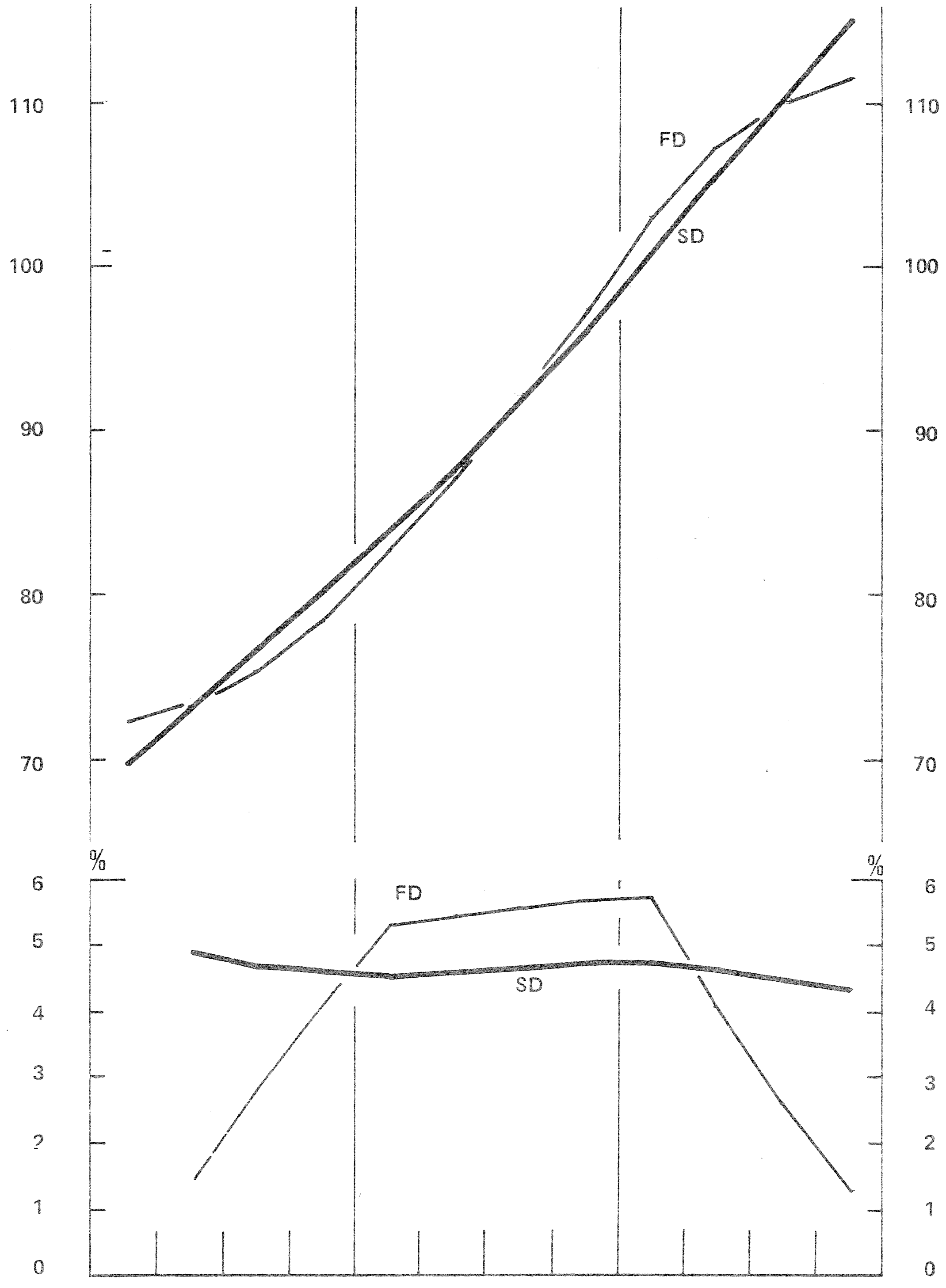


FIGURE 7.

DERIVATION OF QUARTERLY SERIES FROM ANNUAL SERIES WITH A CONSTANT RATE OF VARIATION (Y_t = 300, 360, 432), USING FIRST (FD) AND SECOND (SD) DIFFERENCES

QUARTERLY SERIES AND RATES OF VARIATION



perhaps we should mention the strong distortion introduced by the first differences in the derivation of quarterly data of a series that grows at a constant rate (Figure 7). Far from coming close to the regularity of the interannual rates of the original series, the interquarterly rates of variation show the profile represented in the lower part of the graph, sharply contrasting with the rates derived from applying second differences.

The three aforementioned figures in some way suggest an exaggerated picture of the differences existing between Δ and Δ^2. This occurs because they contain only three years, and the discrepancies arise primarily in the end years of the series. In longer series, the similarity shown by the two sets of data in the intermediate years would be more evident than it appears here. All told, the three cases under study point towards the same conclusion, which could have a general nature: if an indicator is not available, when quarterly data is derived with the Denton method it is advisable to take second differences in the objective function.

When an indicator exists, this option is not clear. The wide variety of real situations found in applied work make it very difficult to adopt a merely illustrative approach, such as the one followed in the non-indicator case. We will let this issue hang for the moment, and return to it after analyzing the method proposed by Chow and Lin.

3.3 Método de Chow-Lin

The last method to be discussed in detail is that proposed by G. C. Chow and A. Lin (1971). In some ways, this method is the most general of those analyzed up to now, although serious estimation difficulties do arise.

The approach of this method enables the unified treatment of three different problems that M. Friedman (1962) had been interested in, namely those of interpolation, distribution and extrapolation. Chow-Lin summed them up as follows: given the values that a series takes at the beginning of each year during T years, and given the values of an indicator at the beginning of each quarter, the problem of interpolation consists in estimating the observations of the remaining 3T quarters. The problem of distribution is seen in the case of the quarterly data derivations that we are concerned with. And, lastly, the problem of extrapolation, associated with the two previous problems, is that of estimating quarterly data outside of the sample of T annual data by making use of indicators for part of year T+1. All three problems can be treated jointly in the context of a linear regression model.

The presentation of the method is adapted to the general case of quarterly data. Chow and Lin describe it for the three problems simultaneously in order to obtain monthly data from quarterly ones. Here we will limit ourselves to indicating at certain points how to move from one problem to another. This will enable the notation of the original article to be simplified considerably and to be fitted, with some slight variations, to that used in previous sections.

They assume that the unknown quarterly observations, y, consistent with a known annual series, Y, are related to quarterly indicators, p, in the following linear manner:

(3.27) $y = X\beta + u$

where y is 4Tx1, X is a 4Txp matrix formed by the observations of the p indicators x_i (i = 1,...p), β is a px1 coefficients vector, and u is a random term of error with a mean of zero and covariance matrix V. In this regression model, X may include current or lagged variables, and the variables may be subject to any previous transformation, such as a logarithmic one, for example.

Using the already known B matrix -see (1.4)-, the quarterly model (3.27) could be expressed in annual terms as(*):

(3.28) $Y = B'y = B'X\beta + B'u$

where, obviously, $E(B'u) = 0$, y $E\left[(B'u)(B'u)\right] = B'VB$.

(*) Everything that follows is applicable to the problem of interpolation by substituting B for B_I, with

$$B_I = \begin{bmatrix} b & 0 & \cdots & 0 \\ 0 & b & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & b \end{bmatrix} \quad \text{where } b = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

The problem consists in estimating the vector y of 4T quarterly observations. A linear, unbiased estimator of y, let us say $\hat{y}$, will satisfy some matrix P of order 4TxT(*):

(3.29) $\hat{y} = PY = P(B'X\beta + B'u).$

Since $\hat{y}$ is unbiased:

(3.30) $E(\hat{y}-y) = E\left[P(B'X\beta + B'u) - (X\beta + u)\right] = (PB'X-X)\beta = 0$

which implies, for $\beta \neq 0$, that:

(3.31) $PB'X = X.$

Moreover, using (3.31):

(3.32) $\hat{y} - y = (PB'X\beta + PB'u) - (X\beta + u) = PB'u - u$

from which we can determine that the covariance matrix of $\hat{y}$ is:

(3.33)
$$\begin{aligned} E(\hat{y}-y)^2 &= E\left[(PB'u - u)(PB'u - u)'\right] \\ &= E\ (PB'uu'BP' - uu'BP' - PB'uu' + uu') \\ &= PB'VBP' - VBP' - PB'V + V \end{aligned}$$

Lastly, the best linear unbiased estimator $\hat{y}$ will be that which minimizes the trace of (3.33) with respect to P, subject to the 4Txp matrix (3.31).

(*) In order to incorporate the extrapolation case, the problem is presented in terms of estimating a vector z of m observations of the dependent variable, with $4T < m < 4T+4$, whose first 4T components are the vector y, while the 4T+1 to m are the extrapolations.

The previous problem can be solved by forming the Lagrangean:

$$(3.34) \qquad L(P,\Lambda) = 1/2\ \mathrm{tr}\left[PB'VBP' - VBP' - PB'V + V\right] - $$
$$- \mathrm{tr}\left[\Lambda^{*\prime}(PB'X-X)\right]$$

where Λ^* is a 4Txp matrix of Lagrange multipliers. The first -order conditions are:

$$(3.35) \qquad \frac{\partial L}{\partial P} = PB'VB - VB - \quad X'B = 0$$

$$(3.36) \qquad \frac{\partial L}{\partial \Lambda^*} = PB'X - X = 0$$

where $\Lambda = 1/2\ \Lambda^*$. From (3.35) we can determine a first definition of P:

$$P = (\Lambda X'B + VB)(B'VB)^{-1}$$

by substituting this expression in (3.36), we obtain the value of Λ which, through its introduction into the previous first definition, leads to P matrix:

$$P = X(X'B(B'VB)^{-1}B'X)^{-1}X'B(B'VB)^{-1} +$$
$$+ VB(B'VB)^{-1}\left[I-B'X(X'B(B'VB)^{-1}B'X)^{-1}X'B(B'VB)^{-1}\right].$$

As a result, the optimum (3.29) estimator is:

$$(3.37) \qquad \hat{y} = X\hat{\beta} + VB(B'VB)^{-1}\widehat{B'u}$$

where:

$$(3.38) \qquad \hat{\beta} = (X'B(B'VB)^{-1}B'X)^{-1}X'B(B'VB)^{-1}Y$$

is the GLS estimator of the model with annual data (3.28), and where $\widehat{B'u} = Y - B'X\hat{\beta}$ are the T residuals estimated for this model.

The (3.37) estimator automatically satisfies the condition that the sums of the quarters equal the annual totals:

$$B'\hat{y} = B'X\hat{\beta} + B'VB(B'VB)^{-1}\widehat{B'u} = Y$$

As we can see, the (3.37) quarterly series estimated breaks down into two parts: the first, $X\hat{\beta}$, applies the regression coefficients obtained by generalized least squares from annual data to the matrix of 4T observations on the p indicators. In the second, the t residuals of the annual regression are distributed among the quarterly data, according to the weights given by the 4TxT matrix $VB(B'VB)^{-1}$.

At this point, we should open a parenthesis to mention that Ph. Nasse (1973) proposed a method based on two steps -used in France to obtain quarterly National Accounts- that closely reflect the method we are now discussing, even though it suffers from a lack of consistency and was criticized by J. Bourney and G. Laroque (1979). In synthesis, in the first part of the method, Nasse obtains the estimator $\hat{\beta}$ of the regression with annual data by using ordinary least squares, i.e., by assuming that the covariance matrix of the annual residuals is $\sigma^2 I$. In the second stage, however the annual residuals are distributed among the quarters according to a procedure equivalent to assume that they follow a first or second order autoregressive distribution. Bournay and Laroque recommended that the method used in France be modified. Their proposal is based primarily on the contribution of Chow and Lin.

Returning to the method of Chow and Lin, it is evident that the difficulty of its application lies in the fact that the weights matrix depend on the covariance matrix of the residuals of the theoreticals model (3.27), V, which is unknown. Its estimation must be based on some assumptions on the distribution of the u_t term of error.

The simplest case consists in assuming that u_t has the classical properties of constant variance and independence, with V = I. In this case, the quarterly series (3.37) reduces to

$$(3.39) \qquad \hat{y} = X\hat{\beta} + \frac{1}{4}\widehat{Bu}$$

that is, to distribute each annual residual in four equal parts. Moving on to other more realistic and interesting specifications of V, Chow and Lin propose a method to obtain consistent estimators for generating monthly data from quarterly data when the residuals of the monthly model follow a first-order autoregressive process. Extending this method to the quarterly data case is not obvious, and there exists the basic difficulty of knowing the distribution of the residuals of the theoretical quarterly model.

In spite of the estimation difficulties, the method proposed by Chow and Lin has the great theoretical advantage of leading to an estimator of the quarterly series with extremely desirable properties, such as that of being the best linear unbiased estimator one can find. Consequently, this method can be taken as a point of reference on which to judge the relative merit of alternative methods. From this point of view, it is interesting to return to Denton's method in order to give it a broader statistical interpretation.

By comparing the Denton solution, (3.24) or (3.25), with that of Chow and Lin, (3.37), we can see that, in the particular case in which the X matrix reduces to only an indicator and $\beta = 1$, both solutions are identical if the covariance matrix, V, of Chow-Lin is made to be equal to the inverse of Denton's differenciation matrix, that is, A or XAX, according to the criterion adopted.

Actually, this similiarity surfaced earlier for the special case in which V = A = I. The solution of each method -see (3.26) and (3.39)- is the same, under $\beta = 1$ and by reducing X to x.

At a more general level, by substituting the indicator x in Denton's method with a linear combination of indicators expressed by $X\beta$, it is possible to show the relation between both methods, as R. B. Fernández does (1978)(*). This leads to the reformulation of the Lagrangean (3.17) in order to minimize it with respect to y, β, λ^*:

(3.40) $$L(y,\beta,\lambda^*) = (y-X\beta)'A(y-X\beta) - \lambda^{*\prime}(Y-B'y)$$

which gives solution:

(3.41) $$y = X\beta + A^{-1}B(B'A^{-1}B)^{-1}[Y-B'X\beta]$$

(3.42) $$\beta = (X'B(B'A^{-1}B)^{-1}B'X)^{-1}X'B(B'A^{-1}B)^{-1}Y$$

expressions that are formally identical to the Chow-Lin solution, (3.37) and (3.38), and clearly show that to take

(*) When this paper was written, we received another from E. de Alba (1979) which also emphasizes this relation in a similar manner.

A = I in Denton is equivalent to applying ordinary least squared in Chow and Lin, since (3.42) reduces to

$$\beta = (X'BB'X)^{-1}X'BY$$

and again,

$$y = X\beta + \frac{1}{4}\,(Y-X\beta).$$

As a result, the particular case in which the residuals of (3.27) satisfy classical hypothesis, the method of Denton, with A = I, is optimal.

If the residuals were random walk, that is,

$$u_t = u_{t-1} + e_t$$

where e_t is a random variable with mean zero, serially independent and with a constant variance, applying first differences to the data would lead to a white noise residual, e_t. Denton's method, with A = D'D, would be optimal in this case.

In general, when matrix A closely reflects the unknown covariance matrix of theoretical model (3.27), Denton's method leads to the best linear unbiased estimation of the unknown quarterly series. If it were possible to do so, in each case, the D matrix that transforms residual series into white noise would have to be selected. Unfortunately, the structure of the residuals cannot be observed, forcing us to limit ourselves to approximations of the optimal criterion constituted by the method of Chow-Lin. The following section is dedicated to this last point.

4. Choosing a method for deriving quarterly data

Considering what has been said up to this point, a two-fold observation can be made: 1st) from the point of view of its theoretical properties, Chow-Lin's method is superior to the rest; and 2nd) taking into account the other methods analyzed, only that of Denton can give, in some cases, the same results. This is a good reason for choosing the second method, for it has important operative advantages over the first.

Of the other methods considered in this work, that of Bassie was the only real alternative option, since, of the rest, those of interest are particular cases of Denton's method. In spite of its great operative capacity, theoretical and practical objections can be made against the Bassie method. From a theoretical point of view, the properties satisfying the results derived from this method are unknown, since they are deduced from some subjective criteria determined beforehand; whether or not this criteria are reasonable is an open question, although with some of them a degree of reservation should be shown. From a practical point of view, its wide use shows that, in general, when a good indicator is available, effective results are yielded, leading to quarterly series that are reasonably similar to the profile of the indicator. Nevertheless, some limitations already mentioned when discussing this method are presented. The most important one is the false seasonality induced in quarterly series when the K_t discrepancy factor is constant in time. Since economic series frequently grow at a relatively constant rate, this point bears considerable importance in empirical work. To illustrate this difficulty, a simple example can be cited in which quarterly data were derived from an annual flow of 14 data artificially constructed from a value of 120 and with a constant interannual rate of 20%. Quarterly data was estimated from the same series with the Bassie and Denton methods; the latter took second differences. The results obtained are presented in Table 4, which includes the annual series, the quarterly series derived from each method and their absolute differences, as well as the interquarterly rates of variation and the difference between both. The aforecited rates, which require virtually no comment, are represented in figure 8: while Denton's method gives practically constant rates for the entire sample, such as occurs with the annual data, Bassie's method, aside from showing the atypical behavior of the ends years commented on in Section 3.1, induces a strong seasonality. This exercise should be sufficient grounds for discarding the Bassie method in favor of that of Denton.

As far as the use of the latter method with an indicator is concerned, the problem of specifying the A matrix mentioned in the preceding section remains unsolved. In principle, a suggestion made by Fernández (1978) bears interest on this point: since partial information is always used to derive quarterly series, the residuals will be dominated by omitted variables. If this is so, a transformation converting annual residuals into white noise would be informative with respect to the appropriate treatment applicable to the quarterly residuals. As a result, an ARIMA model could be estimated for the annual series and used to deduce the suitable filter in quarterly data. From a real point of view, however, a large difficulty is presented by this suggestion: annual series are often too short to properly identify the ARIMA model.

Table 4

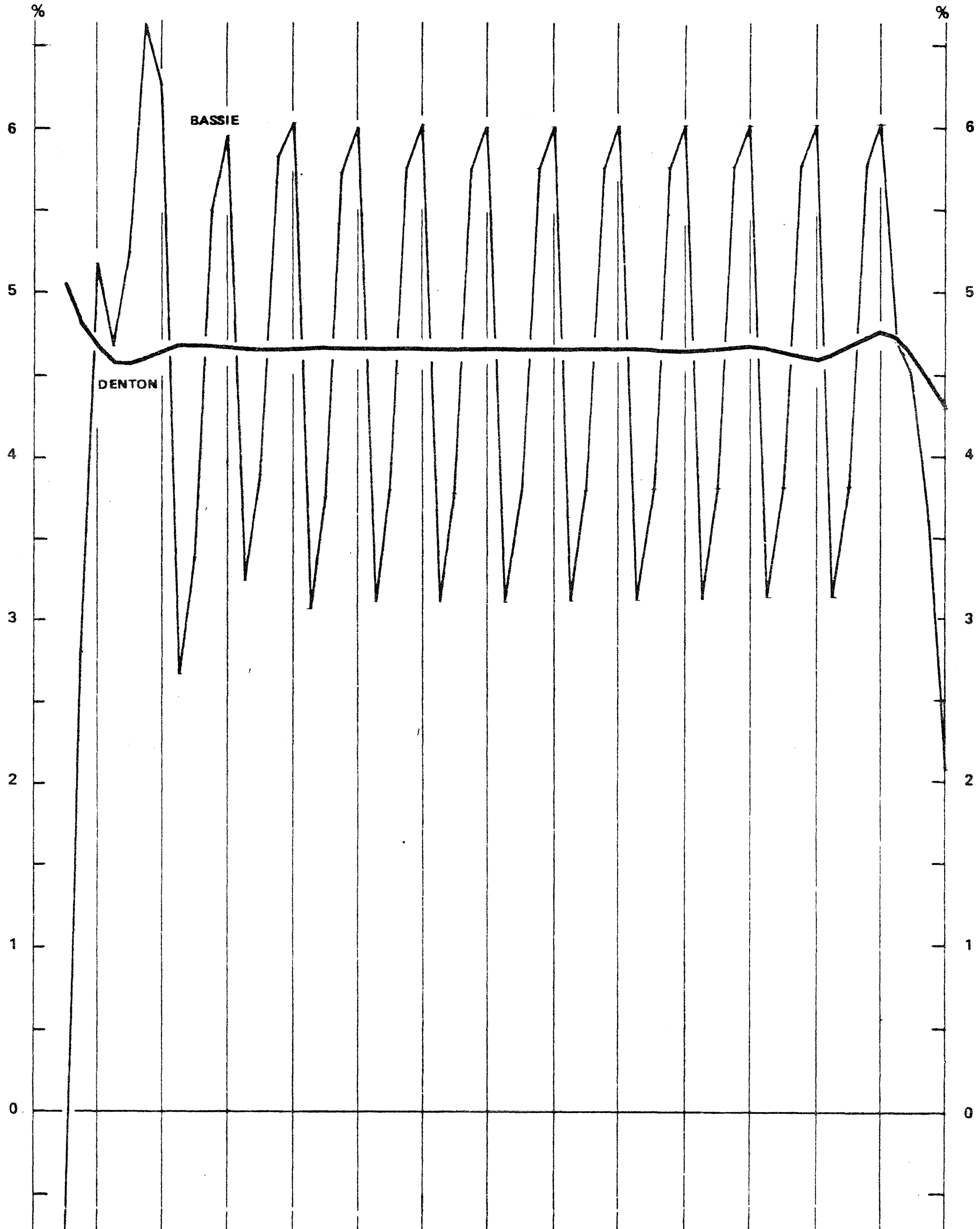
Derivation of quarterly data without an indicator from an annual flow by Bassie and Denton

	Annual Series	Quarterly Series: Bassie	Quarterly Series: Denton	Quarterly Series: Difference	Rates of Variation: Bassie	Rates of Variation: Denton	Rates of Variation: Difference
1	120	29.41	27.88	1.53			
2		29.14	29.29	-.15	-.94	5.06	-5.99
3		29.95	30.70	-.75	2.80	4.83	-2.04
4		31.50	32.14	-.63	5.18	4.67	.51
5	144	32.97	33.61	-.64	4.67	4.58	.08
6		34.70	35.15	-.45	5.24	4.57	.67
7		37.00	36.77	.24	6.64	4.61	2.03
8		39.32	38.48	.84	6.26	4.65	1.61
9	172.8	40.37	40.28	.09	2.67	4.69	-2.02
10		41.73	42.17	-.44	3.38	4.69	-1.31
11		44.04	44.14	-.11	5.51	4.68	.83
12		46.66	46.21	.45	5.96	4.67	1.29
13	207.36	48.17	48.36	-.19	3.24	4.66	-1.42
14		50.05	50.61	-.56	3.90	4.65	-.75
15		52.97	52.96	.01	5.84	4.66	1.18
16		56.17	55.43	.74	6.04	4.66	1.38
17	248.83	57.90	58.02	-.12	3.07	4.67	-1.59
18		60.07	60.73	-.66	3.75	4.67	-.91
19		63.52	63.56	-.04	5.74	4.67	1.08
20		67.34	66.53	.82	6.02	4.66	1.35
21	298.6	69.44	69.63	-.18	3.12	4.66	-1.54
22		72.08	72.87	-.79	3.80	4.66	-.87
23		76.24	76.27	-.03	5.77	4.66	1.11
24		80.83	79.83	1.01	6.03	4.66	1.36
25	358.82	83.34	83.55	-.21	3.11	4.66	-1.56
26		86.50	87.45	-.95	3.78	4.66	-.88
27		91.48	91.53	-.04	5.76	4.66	1.10
28		96.99	95.80	1.20	6.02	4.67	1.36
29	429.98	100.01	100.26	-.25	3.11	4.66	-1.55
30		103.80	104.94	-1.14	3.79	4.66	-.87
31		109.78	109.83	-.05	5.76	4.66	1.10
32		116.39	114.95	1.45	6.02	4.66	1.36
33	515.98	120.01	120.31	-.29	3.11	4.66	-1.55
34		124.56	125.92	-1.36	3.79	4.67	-.38
35		131.74	131.80	-.06	5.76	4.67	1.09
36		139.67	137.95	1.72	6.02	4.67	1.35
37	619.17	144.01	144.39	-.38	3.11	4.67	-1.56
38		149.47	151.12	-1.65	3.79	4.66	-.87
39		158.08	158.15	-.07	5.76	4.65	1.11
40		167.61	165.51	2.10	6.02	4.65	1.37
41	743.01	172.82	173.21	-.40	3.11	4.66	-1.55
42		179.36	181.30	-1.94	3.79	4.67	-.88
43		189.70	189.80	-.10	5.76	4.68	1.08
44		201.13	198.70	2.43	6.02	4.69	1.34
45	891.61	207.38	207.99	-.61	3.11	4.68	-1.57
46		215.23	217.67	-2.43	3.79	4.65	-.86
47		227.64	227.73	-.09	5.76	4.62	1.14
48		241.35	238.22	3.13	6.02	4.61	1.41
49	1069.9	248.86	249.25	-.40	3.11	4.63	-1.52
50		258.28	260.94	-2.66	3.79	4.69	-.90
51		273.17	273.34	-.17	5.76	4.75	1.01
52		289.63	286.39	3.23	6.02	4.78	1.25
53	1283.9	303.26	299.98	3.28	4.71	4.74	-.04
54		316.94	313.89	3.05	4.51	4.64	-.13
55		328.43	327.96	.47	3.62	4.48	-.86
56		335.29	342.08	-6.79	2.09	4.31	-2.22

FIGURE 8.

DERIVATION OF QUARTERLY SERIES WITHOUT INDICATOR
BY BASSIE AND DENTON

RATES OF INTERQUARTERLY VARIATION



Here we propose a more empirical criteria, based upon alternative specifications of A. Since the method is optimum when A properly reflects the structure of the differences between the quarterly series and the indicator, the solution which leads to lower and more random discrepancies with respect to the indicator will be the closest to the optimal. From a empirical point of view, this comparison can be limited to definitions of A in terms of the first and second differences of the objective function. For most series, the first differences will be suitable, since they are close to the case of a first-order autoregressive structure of the residuals with a high autocorrelation coefficient.

The analysis of the consistency between the series obtained and the indicator used may therefore be the basic criterion for accepting or rejecting the estimated quarterly data. This analysis should include the usual tools in time series analysis, such as a comparison between the structure of the indicator and that of the resulting quarterly series, the structure of the differences between them, the most important outliers, and some measures of the goodness of fit between the estimated series and the indicator.

A quarterly series obtained with Denton's method and showing consistency with an appropriate indicator can be accepted as a reasonable approximation to the best linear unbiased estimation of a real unobservable quarterly series.

5. Conclusion-Summary

Given its theoretical properties as well as its operative capacity, with a slight modification in the differentiation matrix, Denton's method can be accepted as the best alternative to solving the problem addressed in this paper.

When a series is minimized without an indicator, in general, the second differences are more suitable than the first.

In order to obtain quarterly data from a series with an indicator, an option exists between the additive and multiplicative versions of the method. The choice, which depends on the nature of the problem to be solved in each case, should be based on a prior decision as to whether the differences between the annual data, from which quarterly data is to be obtained, and the indicator measured in annual terms must be distributed throughout the year following the intraannual profile of the indicator or independently of it. In the first case, the multiplicative criterion will be used; in the second, the additive criterion. For either of these criteria, the objective function minimizing the method may be expressed in first or second differences. The choice between the two can be made by using statistical criteria, in accordance with the empirical results obtained in each case. First differences will be adequate in most economic series.

References

BANCO DE ESPAÑA (1975): Una estimación de la contabilidad nacional trimestral de España, 1962-1972, Banco de España, Madrid.

BASSIE, V. L. (1958): Economic Forecasting, McGraw-Hill, New York, 1958.

BOOT, J. C. G.; FEIBES, W., and LISMAN, J. H. C. (1967): "Further Methods of Derivation of Quarterly Figures from Annual Data", Applied Statistics, vol. 16, n° 1, pp. 65-75.

BOURNAY, J., and LAROQUE, G. (1979): "Réflexions sur la méthode d'élaboration des comptes trimestriels", Annales de l'INSEE, n° 36, pp. 3-30.

CHOLETTE, P. A. (1979): "A Comparison and Assessment of Various Adjustment Methods of Sub-Annual Series to Yearly Benchmarks", Statistics Canada, Seasonal Adjustment and Time Series Analysis Staff, Otawa, March 1978, Revised 1979.

CHOW, G. C. and LIN, An-loh (1971): "Best Linear Unbiased Interpolation Distribution and Extrapolation of Time Series by Related Series", Review of Economics and Statistics, vol. 53, n° 4, pp. 372-375.

DE ALBA, E. (1979): "Temporal Disaggregation of Time Series. A Unified Approach", New Mexico State University, Statistical Laboratory, Technical Report Series n° 32, February 1979.

DENTON, F. T. (1971): "Adjustment of Monthly or Quarterly Series to Annual Totals: An Approach Based on Quadratic Minimization", Journal of The American Statistical Association, vol. 66, n° 333, pp. 99-102.

FERNANDEZ, R.B. (1978): "Alternative Approaches of the Estimation of Short-Term Economic Indicators". International Monetary Fund, Research Department, April 5, 1978.

FRIEDMAN, M. (1962): "The Interpolation of Time Series by Related Series", Journal of the American Statistical Association, vol. 57, n° 300, pp. 729-757.

GINSBURG, V. (1973): "A Further Note on the Derivation of Quarterly Figures Consistent with Annual Data", Applied Statistics, vol. 22, n° 3, pp. 368-373.

LISMAN, J.H.C., and SANDEE, J. (1964): "Derivation of Quarterly Figures from Annual Data", Applied Statistics, vol. 13, n° 2, pp. 87-90.

NASSE, Ph. (1973): "Le système des comptes nationaux trimestriels", Annales de l'INSEE, vol. 14, pp. 119-165.

O.C.D.E. (1979) Comptes nationaux trimestriels. Un rapport sur les sources et méthodes utilisées dans les pays de l'OCDE. Paris.

VANGREVELINGHE, G. (1966): "L'évolution à court terme de la consommation des ménages", Etudes et Conjoncture, INSEE, pp. 59-102.

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