GOVERNMENT PROCUREMENT AND ACCESS TO CREDIT: FIRM DYNAMICS AND AGGREGATE IMPLICATIONS

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Abstract

We provide a framework to study how different public procurement allocation systems affect firm dynamics and long-run macroeconomic outcomes. We build a new panel dataset of administrative data for Spain that merges credit-register loan data, quasi-census firm-level data and public procurement project data. We find evidence consistent with the hypothesis that procurement contracts provide valuable collateral for firms, and that they do so to a greater extent than private-sector contracts. We then build a model of firm dynamics with both asset-based and earnings-based borrowing constraints and a government that buys goods and services from private-sector firms, and use it to quantify the long-run macroeconomic consequences of alternative procurement allocation systems. We find that policies that promote the participation of small firms have sizeable macroeconomic effects, but their net impact on aggregate output is ambiguous. These policies help small firms grow and overcome financial constraints, which increases output in the long run. However, they also reduce saving incentives for large firms, decreasing output. The relative strength of these two forces and hence which of them dominates crucially depends on the type of financial frictions firms face and the specific way the policy is implemented.

Keywords: government procurement, financial frictions, capital accumulation, aggregate productivity.

JEL classification: E22, E23, E62, G32.

Resumen

Este trabajo analiza cómo los diferentes sistemas de asignación de licitaciones públicas entre empresas afectan a la dinámica empresarial y al crecimiento económico a largo plazo. Por un lado, construimos una base de datos administrativos que combina información sobre las características de las empresas, el acceso al crédito bancario y la participación en licitaciones públicas. La evidencia empírica sugiere que las licitaciones públicas (ventas al sector público) proporcionan a las empresas un valor como colateral que facilita su acceso a la financiación bancaria en mayor medida que las ventas al sector privado. Por otro lado, construimos un modelo de dinámica empresarial con restricciones financieras —basadas tanto en activos físicos como en flujos de caja— y con la presencia de un sector público que compra bienes y servicios a las empresas del sector privado. A continuación, calibramos los parámetros del modelo con la evidencia empírica disponible a nivel micro, y se utiliza para cuantificar las consecuencias macroeconómicas a largo plazo de diferentes sistemas de asignación de licitaciones públicas entre empresas. En concreto, encontramos que los sistemas que promueven la participación de pequeñas empresas tienen efectos macroeconómicos significativos, y su impacto neto sobre el crecimiento económico a largo plazo es ambiguo. Estas políticas ayudan a las pequeñas empresas a crecer, al relajar sus restricciones financieras, lo que aumenta la producción y el crecimiento a largo plazo. Sin embargo, estas políticas también reducen los incentivos de ahorro para las grandes empresas, lo que implica un efecto negativo sobre la acumulación de capital y el crecimiento económico. El impacto neto de ambas fuerzas depende del tipo de restricciones financieras que afrontan las empresas y de la forma específica en que se implementa la política.

Palabras clave: licitaciones públicas, fricciones financieras, inversión, productividad.

Códigos JEL: E22, E23, E62, G32.

1 Introduction

Governments play a key role in economic activity. They set taxes and transfers, they are large employers, and they purchase goods and services from the private sector. The purchases of goods and services are done by awarding *public procurement contracts* to private firms. The size of public procurement varies over time and across countries, but it consistently represents a large fraction of GDP —12.8% in the OECD countries, 14% in EU countries, and 9.3% in the United States.[1] Because of its large size and high level of discretion, governments use the public procurement process to allocate resources to specific sectors or firms. In the U.S., for example, the Small Business Act aims to "ensure that a fair proportion of federal contracts is awarded to small business".[2] Similarly, in the EU, promoting the participation of small firms is at the core of the European Commission's agenda for public procurement regulation.[3] Given these different possible designs of procurement programs, it is surprising that we have little understanding of how the procedure of awarding public procurement contracts to private firms may affect the macroeconomy.

In this paper we study the effects of public procurement on firm outcomes and the macroeconomy. We argue that the long-run macroeconomic impact of public procurement depends crucially on the severity as well as on the type of firm-level financial frictions. As a consequence, procedural differences in the awarding of contracts to firms can have first-order effects on macroeconomic outcomes. In particular, we show that granting procurement contracts to small firms —either by directly targeting smaller firms or by slicing large contracts into smaller ones— helps these firms grow and overcome financial constraints in the long run, but the aggregate effects can reduce GDP.

Framework. We carry out an analysis that integrates a novel firm-level dataset with a macroeconomic model of firm dynamics. Our dataset merges administrative data on public procurement, credit allocation at the bank-firm level, and firm outcomes for the Spanish economy over the 2000-2013 period. Our model builds on the canonical framework of firm dynamics with financial frictions (e.g., Midrigan and Xu 2014) and incorporates two novel elements. First, there is a government that purchases goods and services from private sector firms. Firms that are willing to sell to the government must make a risky investment in advance, which reflects the costs of preparing a good proposal and increasing the chances to win the contract. Second,

[1]See EU Commission's web and OECD for details.

[2]See Report from Congressional Research Service for details.

[3]See the Public Sector Directive 2014/24/EU for details. There has been strong support from the European Parliament for explicit regulation that discriminates in favor of small firms: "From this House, we must insist that...administrative bodies incorporate terms into their tender specifications that facilitate positive discrimination in favor of SMEs and remove contractual provisions that hinder their participation." See EU Parliament Debate.

we allow for both asset- and earnings-based borrowing constraints. That is, firms not only borrow against their assets but also against their earnings.[4]

Motivating facts. We start by documenting four empirical facts that motivate our paper. First, we show a positive relationship between obtaining a procurement contract and firms' credit growth at impact. Second, we show that this credit growth is mostly explained by an increase in loans for which no tangible collateral is posted. Given that we observe firms' credit at the quarterly level, we can run these sets of regressions including *firm*×*year* fixed effects, which allows us to control for time-varying firms' unobservables at the annual level. Third, we exploit loan applications data to show that this association arises, at least partly, due to an easing of financial constraints as opposed to an increase in firms' demand for credit: obtaining a procurement contract is associated with an increase in the probability of getting a loan conditional on applying for it. These facts together point towards procurement contracts serving as collateral that relaxes borrowing constraints. Finally, we provide evidence consistent with the fact that winning a procurement contract eases firms' financial constraints more than selling to the private sector. In particular, we find that the composition of sales, i.e., private vs. procurement, matters in explaining the evolution of firms' credit: we estimate the effect of procurement to be positive and significant even when we control for the change in firms' total sales.

What we do. The core of our analysis consists in using our model to study the interplay between procurement and the macroeconomy. We calibrate the model to reproduce several micro-moments related to firm selection into procurement and to firm dynamics after winning a procurement contract, as well as to macro-moments.

Selection. In terms of selection, we show a strong pattern based on firm size. In particular, we document that firms that end up participating in procurement are 72% bigger in terms of value-added before they do so. We refer to this difference as the "procurement size premium." Our model generates a procurement size premium through two state variables of firms: productivity (TFPQ) and net worth. As is standard in models with firm heterogeneity, the value of participating in a given market —the procurement market in this case— depends on firms' ability to deliver large projects (e.g., Melitz (2003) in the context of international trade). In our model, this ability uniquely depends on firms' TFPQ in the case of financially unconstrained firms. However, for constrained firms, that ability also depends on their financing capacity, which itself depends on firms' net worth (e.g., Chaney (2016) also in the context of international trade). In our baseline calibration, where we match the 72% value-added procurement size premium, our model implies a procurement premium of 36% in terms of TFPQ and 53% in terms of net worth.

[4]Recent papers by Lian and Ma (2020), Aguirre et al. (2021), Caglio et al. (2021), Drechsel (2021), Gupta et al. (2021) and Li (2022) find empirical evidence of earnings-based borrowing constraints.

Treatment. Regarding the treatment effect of procurement on credit growth, the model is calibrated to reproduce a structural regression in which the change in constrained firms' leverage, i.e., total credit divided by fixed assets, depends on two variables: the change in total earnings divided by fixed assets and the change in total earnings from procurement divided by fixed assets. The model structure implies that the coefficient associated with the former pins down the parameter that governs the pledgeability of firms' earnings from selling to the private sector, whereas the coefficient associated with the latter pins down the difference in the pledgeability of earnings from procurement relative to the pledgeability of private earnings.[5] We run this regression for firms that are likely to be financially constrained in our data, i.e., young firms, and find that firms can pledge 42% of their annual earnings from selling to the private sector and 110% of their annual earnings from procurement.

There are several possible explanations for why government contracts are more pledgeable than sales to the private sector. The government may be less likely to default than private firms, winning a government contract may reduce the uncertainty about a firm's total demand in the future, or getting a contract might be a signal of a firm's higher productivity. We introduce this featured in reduced form in our framework, and hence we do not take a stand on its causes.

In terms of the dynamics of real variables, the model is able to reproduce the (un-targeted) fact that procurement generates a positive long-run effect on firms' performance. For example, both in the model and in the data, we find that procurement is associated with a cumulative increase of firms' sales to the private sector of around 5% in the fourth year after obtaining the contract. The high pledgeability of procurement contracts together with the extra profits generated by them reinforces the self-financing channel previously emphasized in the literature (Moll, 2014). In this respect, public procurement is a powerful policy tool to help small firms overcome financial frictions and achieve closer to optimal size in the long run. Importantly, we find that this positive long-run effect takes place despite the fact that procurement temporarily crowds out constrained firms' sales to the private sector. In our model, this within-firm negative spillover occurs because financially constrained firms have to split their scarce collateral to serve both procurement and private sector operations. The fact that government sales can be collateralized partly alleviates but does not eliminate this problem.

Policy counterfactuals. To assess the interplay between procurement and the macroeconomy, we use our calibrated model to perform some *expenditure-neutral* counterfactual experiments that consist of reallocating procurement contracts across firms while keeping government expenditure unchanged. In particular, we compare

[5]This structural identification of the earnings-based constraints is similar to the one used by Li (2022) for the case of private sector earnings only.

our benchmark economy with counterfactual economies in which a higher share of procurement contracts is allocated to small firms. Our preferred counterfactual, which consists of promoting small firms' participation by directly targeting them in the procurement allocation system, aims to reproduce the "set-aside" policies for small businesses implemented by the U.S Small Business Administration. In practice, we reproduce this policy by targeting a procurement size premium of 50% —as opposed to the 72% in the baseline calibration— while keeping the fraction of firms from which the government buys constant.

Macroeconomic implications. As a result of the policy, we find that aggregate GDP would go up by 2.07%, of which around 1/6 is explained by an increase in TFP and the rest is explained by an increase in aggregate capital. The increase in TFP is the result of an increase in TFP in the private sector —which is explained by a reduction in misallocation across firms due to the reinforcement of the self-financing channel— and a reduction of TFP in the procurement sector— which is explained by the fact that the selection pattern based on firms' TFPQ weakens. That is, reaching out to small firms implies buying goods from less efficient firms.

Three different channels. To understand the mechanisms behind the evolution of capital accumulation and GDP, we conduct a decomposition of the policy experiment's effects that allows us to isolate three different channels. The first channel is a negative *short-run partial equilibrium effect* on directly affected firms, which is the result of aggregating the crowding-out effects at impact mentioned above. We find that this channel would reduce GDP by 0.10%. The second channel is a positive *long-run partial equilibrium effect* also on directly affected firms, which is the aggregate consequence of the strengthening of the self-financing channel. We find that this channel would increase GDP by 2.19%. The third channel is a negative *long-run general equilibrium effect* coming from the change in capital accumulation incentives of all firms (not only those that ex-post obtain procurement projects) and their responses to general equilibrium price changes. One of the main reasons why firms accumulate financial wealth in our model is the fact that they expect to obtain a public procurement contract at some point. That is, obtaining a procurement contract acts as a large demand shock in response to which firms want to expand their capital stock, causing even relatively big firms to accumulate precautionary savings. Intuitively, productive firms want to have enough net worth so that they minimize the probability of being constrained in case an opportunity for a big procurement contract is realized. A procurement policy that targets smaller firms very aggressively will remove savings incentives for middle-size and large firms. We find that this channel shrinks the output gains associated to the reform.

An alternative counterfactual. We also conduct a policy experiment that consists in promoting small firms' participation by reducing the average size of contracts. This reform aims to mimic the European Commission's strategy to increase the presence of small firms in public procurement in Europe.[6] In practice, we implement this reform by solving for an economy in which the fraction of firms to which the government allocates contracts increases from 3.8% (which is our target in the baseline economy) to 13.8%, and the average contract size decreases accordingly so that government expenditure remains unchanged. Our main finding is that this alternative policy counterfactual would reach out to a higher number of small firms than in our preferred counterfactual but would actually generate a fall in aggregate GDP of 2.68%. The reason is that the reduction in incentives of relatively big firms to accumulate capital – the *long-run general equilibrium effect* – is particularly high as a result of the fall in the average size of procurement contracts. Our results hence imply that the particular way in which the promotion of small firms is implemented is crucial to understand its aggregate effects.

Importance of high pledgeability of government contracts. Finally, we find that the aggregate effects of these counterfactuals would be significantly less expansionary in a world in which earnings from government contracts exhibit the same pledgeability as earnings from selling to the private sector. For example, for the case of our preferred counterfactual, we find that GDP gains would be around 80% lower than when running the same counterfactual under our baseline calibration (0.38% vs. 2.07%). By reducing the extent to which borrowing capacity increases when participating in procurement, the above-mentioned positive *long-run partial equilibrium effect* weakens. This result points towards the importance of the extra collateral provided by government contracts when evaluating the aggregate effects of changes in the procurement allocation system.

1.1 Related literature

There is practically no literature that analyzes how the microeconomic aspects of public procurement can affect the macroeconomy. One recent exception is Cox et al. (2021), who document several new facts using micro-level data on public procurement contracts awarded by the U.S Federal Government, and investigate how accounting for these facts— in particular that government spending is concentrated in sectors where prices are more sticky— can affect the short run fiscal transmission mechanism in a New Keynesian model. Our interest instead is in quantifying the long-run macroeconomic effects of different procurement allocation systems.

Governments have been pointed to as directly responsible for the long-run economic performance of countries through the implementation of policies that distort the allocation of resources across firms. Some examples are credit subsidies to state-owned-enterprises (Song et al., 2011), the reservation of goods for small firms (García-Santana and Pijoan-Mas, 2014), labor market regulations (Garicano et al., 2016),

[6]See Trybus (2014) for details.

tariffs (Berthou et al., 2019), or capital markets regulation (Bau and Matray, 2021). However, one of the most important roles that governments play in modern economies, i.e., their role as buyers of goods and services from private sector firms, has been overlooked. We focus on this by analzying specific size-dependent procurement policies aimed at helping small firms. In this respect, our work is related to Guner et al. (2008), Restuccia and Rogerson (2008), and Hsieh and Klenow (2009), who show the importance of firms' idiosyncratic distortions in affecting misallocation across firms and aggregate productivity.

Our focus on firm-level financial frictions as a channel through which public procurement can affect the macroeconomy builds on the literature that quantifies the effects of financial constraints on aggregate output and productivity (Buera et al., 2011; Midrigan and Xu, 2014; David and Venkateswaran, 2019; Gopinath et al., 2017; Catherine et al., Forthcoming).[7] A few papers in this literature have studied the interplay of financial frictions with different forms of taxation (Erosa and González, 2019; Itskhoki and Moll, 2019; Guvenen et al., 2019; Blanco and Baley, 2022) but none has focused on the expenditure side of government policies. Our finding that the type of financial frictions matters in understanding the effects of procurement on the macroeconomy is also related to recent papers that show that the type of financial frictions, i.e., earnings- vs. asset-based, and not only their severity, plays a crucial role in explaining important economic outcomes: the gains from trade liberalization (Brooks and Dovis, 2020), aggregate productivity (Li, 2022), macroeconomic fluctuations (Drechsel, 2021), and the transmission of monetary policy (Caglio et al., 2021).

Our results on the treatment effects of winning procurement contracts on firms are related to the recent literature analyzing the relationship between public procurement and firm dynamics. Ferraz et al. (2016) and Lee (2021) use quasi-experimental designs for Brazil and South Korea, respectively to show that firms winning procurement contracts have a positive and permanent effect on firms' performance. Hebous and Zimmermann (2021) document for the U.S. a positive relationship between winning a procurement contract and firm investment, and show that the effect disappears when looking at firms that are less likely to be financially constrained. Our results are consistent with all this body of research. We provide novel evidence on loan acceptances and on the fact that only non-collateralized credit increases, which along with the other empirical facts that we document, can be taken as direct evidence of earnings-based financial constraints that are alleviated with procurement projects. Additionally, our results on the short-run crowding out of sales to the private sector by procurement sales are related to recent papers that investigate within-firm spillover effects across markets, like Almunia et al. (2021) with domestic versus foreign markets and Alfaro-Ureña et al. (Forthcoming) with multinational corporations versus other

[7]See Buera et al. (2015) for a survey of this literature.

buyers. Finally, Cappelletti and Giuffrida (2021) use data for Italy to show that firms that receive public procurement contracts survive longer, a dimension of the data that we do not explore.

The rest of the paper is structured as follows. Section 2 describes the construction of the dataset and provides summary statistics. Section 3 provides our empirical evidence organized in five stylized facts. Section 4 presents the model of firm dynamics with procurement. Section 5 discusses how we parameterize the model. Section 6 describes our benchmark economy. Section 7 provides the main quantitative results. Section 8 concludes.

2 Data and Summary Statistics

Our empirical work is based on merging three large datasets at the firm level. First, we construct a novel dataset on Spanish public procurement contracts published by the official bulletin of the Spanish Central Government (*Boletín Oficial del Estado*, BOE) over the 2000-2013 period. We have information on the type of good or service provided, the institution awarding the contract, the initial bidding and final price of the contract, the type of procedure used to allocate the contract, and the firm(s) that won the contract. Second, we use more standard firm-level data on balance sheets and income statements of the quasi-universe of Spanish companies between 2000 and 2016, a dataset that is maintained by the Banco de España and taken from the Spanish Commercial Registry. And third, we use credit registry for Spain, which contains detailed information (e.g., whether or not a non-personal collateral was posted on a particular loan) on all outstanding loans over 6,000 euros to non-financial firms granted by all banks operating in Spain. Additionally, it contains rich information on loan applications. Online Appendix A provides details about the different data sources and samples that we use.

Types and size of procurement contracts in BOE. For many people, procurement is associated with large infrastructure projects. However, only 20% of the contracts in our BOE data are in the construction sector and the median size of procurement projects in construction (0.74 million euros) is of the same magnitude as the median size in the other categories reported by BOE: services (0.42), consulting (0.37), supplies (0.37), and other sectors (0.35) respectively. The major differences in project size across sectors appear in the right tail of the distribution, with the top 1% of projects in construction being much larger than in other sectors. We also note that there is a large number of relatively small projects in all sectors: 25% of projects have a value of less or equal to 230,000 euro in construction, 200,000 euro in services, and 170,000 in consulting and in supplies. See Table A.I in the Online Appendix for details. Although we do not have direct information about the duration of the contracts in our sample, we were able to collect information about the duration of the contracts awarded in Spain in the year 2015. Around 71% of the contracts have a duration which is one year or less, and 91% have a duration which is two years or less.[8]

Presence of procurement firms. Looking at the firm-level data, we find that procurement firms are present in most industries of the economy: firms with at least one procurement contract in a given year operate in 71 out of the 91 industries based on NACE 2-digit classification. The share of procurement firms in our data is 0.5% percent, but it varies a lot across industries, with the highest fraction —around 15%— in industries like "Manufacture of coke and refined petroleum products" and "Manufacturing of Pharmaceutical Products." Because procurement firms tend to be larger, the share of employment, sales, assets, or credit of procurement firms tend to be larger than the share of firms, see Table A.II in the Online Appendix for details.

Procurement vs. non-procurement firms. We find that firms participating in procurement are significantly larger and older on average, but there is considerable overlap in the support of the size and age distribution for procurement and non-procurement firms (see Table A.III in the Online Appendix). For example, the average number of employees of a procurement firm is around 6 times larger than for the rest of the firms (73.56 vs. 12.75), total sales are 7 times larger (8.9 millions of euro vs. around 1.2 million), and procurement firms are 9 years older (20 vs 11 years). Yet, around 25% of procurement firms have less than 16 employees, have revenues that are lower than 1.14 million euro, and are 12 or fewer years old. We also find that conditional on having at least one procurement project, there is a lot of variation on the importance of these projects as a fraction of firms' total revenue. The average ratio of all the procurement value to total revenue is 0.20, with 25th, 50th, and 75th percentiles of 0.01, 0.03 and 0.10 respectively. Finally, we observe large differences between procurement and non-procurement firms in terms of their composition of credit. In particular, procurement firms seem to rely more on non-collateralized credit (86% vs 71% on average) despite holding higher levels of assets.

3 Motivating Empirical Evidence

We begin by documenting several facts related to the effects of procurement on firms' outcomes. First, we show a positive relationship between obtaining a procurement contract and firms' credit growth. Second, we show that this credit growth is mostly explained by an increase in credit for which no tangible collateral is posted. Crucially, given our empirical specification and our data at the quarterly frequency, these reduced-form regressions allow for fixed effects that control for *time-varying* firms' unobservables at the *annual level*, such as demand or productivity. Third, we exploit

[8] As a reference for a different country, Cox et al. (2021) find that the median contract in the U.S. has a duration of 31 days and 90% of contracts last less than one year.

loan-level data to show that this association arises, at least partly, due to an easing of financial constraints, in contrast to an increase in firms' demand for credit. We interpret these facts together as evidence for procurement contracts serving as collateral in earnings-based borrowing constraints. Finally, we provide evidence consistent with the fact that winning a procurement contract eases firms' financial constraints more than selling to the private sector. For facts one and two we use two different samples: a sample of firms that obtain at least one procurement project between 2000 and 2013 (the main sample) and the sample of firms for which information and ranking of the other bidders is available (bidders sample).[9] For facts 3 and 4, we can only use the main sample. For fact 4, we work with annual frequency because firms' sales to the private sector come from firm-level balance sheet information, which is only available at the annual frequency.

3.1 Procurement and credit growth

We start by regressing firms' credit growth on a dummy variable for procurement as follows:

$$\Delta \log l_{it} = \alpha_{iy} + \alpha_{st} + \beta_1 \text{PROC}_{it} + \beta_2 \log l_{it-1} + \varepsilon_{it} \tag{1}$$

where the dependent variable $\Delta \log l_{it}$ is the annualized quarterly growth of credit (loans) of firm i between quarter $t-1$ and quarter t defined as $\Delta \log l_{it} \equiv \log l_{it} - \log l_{it-1}$, winsorized between −1 (−100%) and +2 (+200%). The regressor PROC_{it} is a dummy variable that takes value one if the firm obtained a procurement contract in quarter t. We include the firm's lagged credit at $t-1$ to control for the fact that firms with large outstanding loan volumes may mechanically have less room for credit growth than firms with smaller outstanding loan levels.[10] We further include a stringent set of fixed effects. In particular, we use firm×year fixed effects, α_{iy}, in order to capture firm-level characteristics that vary over time at the yearly (y) level. Importantly, these fixed effects help control for several factors that may otherwise bias the estimation. First, as they vary at the year level, they pick up the overall firm-level trend of credit growth and thus helps assuage the concern of any potential bias arising from differences in trends pre/post "treatment" by procurement events. Second, these fixed effects control for firm-level unobserved variables that may change at annual level such as productivity or demand. We further include 4-digit sector×quarter effects, α_{st}, which control for both sector and macroeconomic conditions that vary over time. Therefore, identification of the key parameter of interest, β_1, comes from the variation of a firm's credit growth across quarters within a year conditional on obtaining a procurement contract.

[9] For the main sample, one could alternatively use a sample with all firms, but results would be very similar. This is because all our specifications use firm fixed effects, and hence the identification of the effects of procurement comes from the panel variation and not from comparing firms that participate in procurement on a regular basis with firms that never compete for procurement. Results are available upon request.

Table 1, column (1), presents the results of this regression for the main sample. The estimate of β_1 is positive and significant at the one-percent level.[11] The estimated coefficient implies that winning a procurement contract in a quarter translates into an increase of credit growth of 5.5 percentage points annually.

We next use the sample of procurement projects where we have information on all bidders as well as the final ranking. Doing so allows us to run regressions analogous to (1), except that we can identify the association between a firm's ranking in a given auction and its ensuing credit growth. To be more precise, we run two regressions similar to specification (1) at the auction level. In the first regression, we include all bidders and the PROC variable indicates which firm wins the auction ('First' place). Table 1, column (2), shows the results. We find that the winner of a procurement contract has higher credit growth relative to the firms it competes against in a given auction. Note that identification of the coefficient is exploiting the full time series of bidders, so the comparison is based on the within-auction group of firms but also with respect to each firm's annual credit growth given the inclusion of firm×year effects. The coefficient on the winner is 0.073, which indicates that winning the auction is associated to a 7.3 percentage points higher credit growth annually.

Table 1. Credit Growth and Procurement

	All firms	**Bidders only**	
		First	Second
	(1)	(2)	(3)
PROC_{it}	0.055^a	0.073^a	-0.061
	(0.004)	(0.028)	(0.049)
$\log(\text{Credit}_{it-1})$	-0.410^a	-0.175^a	-0.229^a
	(0.001)	(0.043)	(0.044)
Observations	700,780	8,310	3,683
R-squared	0.786	0.360	0.458
Sector×quarter FE	Yes	No	No
Firm×year FE	Yes	Yes	Yes
Quarter FE	No	Yes	Yes
Auction FE	No	Yes	Yes

Notes: Results from estimating the relationship between total credit growth and procurement participation (PROC) by regression (1): with firms obtaining at least one procurement project over 2000-13 in column (1), and with firms who participated in procurement contests over 2013–15 in columns (2) and (3), where the PROC dummy indicates the winning firm ('First') in column (2) and the runner-up firm ('Second') in column (3). All regressions use quarterly data. Standard errors clustered at the firm level; [a] indicates significance at the 1% level, [b] at the 5% level, and [c] at the 10% level.

While the bidder firms' sample is more restrictive than the full sample, we are reassured that we are picking up an unbiased "procurement effect" for a few reasons. First, the point estimates of PROC in columns (1) and (2) are remarkably similar, even though the sample and variation exploited are slightly different. Second, we

[10] The estimation results without lagged credit are similar and are available upon request.

[11] We cluster standard errors at the firm-level in all regressions unless otherwise noted.

are able to control for firm×year effects in both regressions, thus helping dilute productivity or demand firm-specific effects at the annual level. Third, an identification threat would be that productivity or demand shocks at the quarterly level are correlated with the concession of procurement projects such that winning the contract may be a proxy for these shocks. Therefore, our estimate may capture the effect of being ranked above other firms as opposed to the effect of obtaining the procurement contract. In Column (3) of Table 1 we drop the winner of the procurement contest and the PROC dummy now indicates which firm was runner-up ('Second' place). We run this second regression to make sure that winning the contract, as opposed to the relative ranking, is what is really associated with differences in credit growth across auction participants. The estimated coefficient on PROC implies that there is no statistical difference in quarterly credit growth for the firm that placed second relative to other losers of the auction. Fourth, as in any diff-in-diffs type of environment, it could be the case that winner firms are in different credit trajectories than non-winner firms. In principle, this should be captured by the firm×year effects. Still, in Online Appendix A.8, we provide evidence which shows that this is not the case. In particular, we show that the evolution of credit growth for winners and non-winners was similar before the auction and that it diverged afterwards. Finally, we show in the Online Appendix (Table A.IV), the procurement dummy is positive, significant, and relatively stable when running regression (1) across sub-samples based on quartiles of firms' (i) total assets, (ii) employment, (iii) net worth, or (iv) age.

3.2 Procurement and the composition of credit

We next decompose the increase in credit associated with winning a procurement contract into that coming from collateralized vs. non-collateralized credit, which will help us motivate the type of financial constraint we will use in Section 4. To this end, we use the information on the composition of firms' loans, which indicates whether these loans require collateral or not to be posted by a firm to receive financing from a bank. We therefore run a similar regression as (1), constructing the dependent variable at the firm×credit-type×quarter level, and split the estimation between collateralized and non-collateralized credit growth.

Table 2 presents the main results, where c denotes the additional collateral/non-collateral dimension that we exploit in the data. Looking at the main sample, we see that a procurement contract is not significantly correlated with the growth rate of collateralized credit in column (1). However, when turning to column (2) we see a positive and significant association with a firm obtaining a procurement contract and non-collateralized credit growth. The results with the bidders sample, in columns (3) and (4), mimic the findings for the main sample. That is, a firm winning a contract experiences significantly larger growth in non-collateralized loans relative to losing firms, but there is no differential for collateralized loan growth. Regressions for the second vs. the rest samples in columns (5) and (6) do not yield any significant estimates. Overall, these findings point to the growth rate in overall credit associated with obtaining a procurement contract observed in Table 1 being driven by the growth in loans that do not require tangible-assets backing.

Table 2. Composition of Credit Growth and Procurement

	All firms		**Bidders only**			
			First		Second	
	Collat.	NoCollat.	Collat.	NoCollat.	Collat.	NoCollat.
	(1)	(2)	(3)	(4)	(5)	(6)
PROC_{it}	0.001	0.070[a]	-0.011	0.080[b]	-0.019	-0.058
	(0.006)	(0.005)	(0.029)	(0.031)	(0.044)	(0.057)
$\log(\text{Credit}_{ict-1})$	-0.474[a]	-0.421[a]	-0.449[a]	-0.192[a]	-0.461[a]	-0.254[a]
	(0.003)	(0.001)	(0.073)	(0.040)	(0.064)	(0.044)
Observations	224,011	557,873	2,690	8,110	1,423	3,606
R-squared	0.791	0.764	0.357	0.368	0.435	0.435
Sector×quarter FE	Yes	Yes	No	No	No	No
Firm×year FE	Yes	Yes	Yes	Yes	Yes	Yes
Quarter FE	No	No	Yes	Yes	Yes	Yes
Auction FE	No	No	Yes	Yes	Yes	Yes

Notes: Results from estimating the relationship between collateralized (Collat.) and non-collateralized (NonCollat.) credit growth and procurement participation (PROC) by regression (1) with firms obtaining at least one procurement project over 2000-13 in columns (1) and (2), and with firms who participated in procurement contests over 2013-15 in columns (3)-(6) respectively, where the PROC dummy indicates the winning firm ('First') in columns (3)-(4) and the runner-up firm ('Second') in columns (5)-(6). All regressions use quarterly data. Standard errors clustered at the firm level; [a] indicates significance at the 1% level, [b] at the 5% level, and [c] at the 10% level.

3.3 Procurement and firms' borrowing capacity

The previous evidence shows a positive relationship between procurement participation and credit growth. However, it is silent about whether the observed increase in credit actually comes from a relaxation of firms' financial constraints or simply from the fact that firms demand more credit when they expand their operations. We next ask whether firms are able to use their procurement contracts to access credit more easily at the extensive margin. A unique piece of information contained in the Banco de España's credit registry allows us answer this question: the information on the loan application process for firms and banks. In particular, we can see whether a firm has applied to a given bank and whether the loan application has been accepted or rejected throughout our sample period. We use this information to help identify an increase in firms' borrowing capacity. To do so, we run regressions at the firm-bank level and relate the probability of firms obtaining a loan to whether they have received a procurement contract using the following linear probability specification:

$$\text{Loan granted}_{ibt} = \alpha_{ib} + \alpha_{bt} + \alpha_{st} + \beta \text{PROC}_{it} + \varepsilon_{ibt} \quad (2)$$

where the variable 'Loan granted' is a 0/1 dummy variable that is turned on when the firm i receives a loan from bank b in quarter t conditional on the firm applying for it during that same quarter. We include firm×bank fixed effects, α_{ib}, which implies that we are identifying the coefficient β on the procurement variable via the variation within a firm-bank relationship over time. We further control for overall bank credit supply in a given period with bank×quarter fixed effect α_{bt}, and for macroeconomic events with sector×quarter fixed effects α_{st}.

Table 3. Probability of a New Loan and Procurement

	All firms	
	(1)	(2)
$PROC_{it}$	0.024^{a}	0.023^{b}
	(0.008)	(0.011)
Observations	36,857	26,924
R-squares	0.395	0.628
Firm×bank FE	Yes	Yes
Bank×quarter FE	No	Yes
Sector×quarter FE	No	Yes

Notes: Results from estimating the relationship between loan participation and procurement participation (PROC) by regression (2) with firms obtaining at least one procurement project over 2000-13 using quarterly data. Standard errors clustered at the firm level; ^{a} indicates significance at the 1% level, ^{b} at the 5% level, and ^{c} at the 10% level.

Table 3 shows the results from running this regression. We include only firm×bank fixed effects in column (1), and add the time-varying bank and sector fixed effects in column (2). Overall, regardless of the specification, the probability of receiving a bank loan conditional on having applied for it increases by approximately 2 percent in the quarter that a firm wins a procurement project.

3.4 Differential impact of earnings from procurement on firms' credit

Our fourth piece of evidence is that public revenues can be pledged to a larger extent than revenues from the private sector. To show this, we use the main sample at the annual frequency to run regression (1) with total revenue growth as an extra control on the right hand side. Notice that we introduce the growth rate of revenue between t and $t+1$. The reason is that the sales carried out by the firm in t will manifest in the data in $t+1$.[12] Revenue growth captures revenues from both private and public sector.

We show the results of running this type of regressions in Table 4, whose columns reflect specifications using different sets of fixed effects. In all cases, we find that the coefficient associated to the change in total sales is positive and significant, which is

[12] This timing is also consistent with the one we will use in our model.

Table 4. Differential impact of earnings from procurement on firms' credit

	All firms		
	(1)	(2)	(3)
PROC_{it}	0.053^{a}	0.043^{a}	0.041^{a}
	(0.006)	(0.006)	(0.006)
Sales growth_{it+1}	0.107^{a}	0.027^{b}	0.024^{c}
	(0.020)	(0.011)	(0.011)
$\log(\text{Credit}_{it-1})$	-0.047^{a}	-0.267^{a}	-0.274^{a}
	(0.003)	(0.023)	(0.023)
Observations	86,537	86,096	83,652
R-squared	0.051	0.282	0.330
Year FE	Yes	Yes	No
Firm FE	No	Yes	Yes
Sector×Year FE	No	No	Yes

Notes: Results from estimating an equation similar to (1) but using the annual sample and controlling for the change in firms' total sales. Standard errors double clustered at the firm and year levels; ^{a} indicates significance at the 1% level, ^{b} at the 5% level, and ^{c} at the 10% level.

consistent with the idea that firms can borrow against their earnings. Importantly, we also find that the procurement dummy remains positive and significant. If earnings-based constraints were similar for both types of revenues, the extra effect of a dummy for procurement should be null. If the effect of the procurement dummy is positive (negative), this reflects public revenues being more (less) pledgeable than private revenues. In Section 5, we will exploit a similar type of variation to identify some of the model parameters.

4 The Model

We set up a model of privately held heterogeneous firms. We build on standard models of firm dynamics with collateral constraints —as Midrigan and Xu (2014), Moll (2014), or Buera and Moll (2015)— and extend this setting to allow for (a) earnings-based borrowing constraints, (b) a public sector demanding goods from private firms, (c) downward-sloping demands in both the private and public sectors, and (d) a choice to compete for procurement projects.

4.1 Technology

Time is discrete and we omit the subscript t unless it is strictly needed. The economy is populated by a continuum of size 1 of heterogeneous infinitely-lived households indexed by i. Each household is also an entrepreneur running a firm that produces a differentiated intermediate good y_i. There are two final goods in the economy: the "private sector" good, Y_p, used by households to consume, invest in productive capital, or prepare applications for procurement projects, and the "public sector" good Y_a, purchased by the government to produce (useless) public consumption.

Final goods. The two final goods are assembled by two final good producers combining the differentiated intermediate goods y_i through the following CES aggregators:

$$Y_p = \left(\int_{[0,1]} y_{ip}^{\frac{\sigma_p - 1}{\sigma_p}} \, di \right)^{\frac{\sigma_p}{\sigma_p - 1}} \quad \text{and} \quad Y_g = m_g^{\frac{1}{1-\sigma_g}} \left(\int_{I_g} y_{ig}^{\frac{\sigma_g - 1}{\sigma_g}} \, di \right)^{\frac{\sigma_g}{\sigma_g - 1}} \quad \text{with } \sigma_p, \sigma_g > 1 \quad (3)$$

where I_g is the subset of goods purchased by the public sector and m_g is the measure of this set. Note that Y_g is corrected by m_g to prevent love for variety.[13] We also note that I_g (and the implied m_g) is a policy variable and the identity of firms in this set is discussed below. The final goods producers are perfectly competitive and choose the optimal demand of intermediate goods y_{ip} and y_{ig}, respectively, to maximize profits taking intermediate good prices p_{ip} and p_{ig}, final good prices P_p and P_g, and the set I_g as given. We assume that firms compete independently in each sector and face the following downward-sloping demands,

$$p_{ip} = B_p y_{ip}^{-1/\sigma_p} \quad \text{and} \quad p_{ig} = B_g y_{ig}^{-1/\sigma_g} \quad (4)$$

where for convenience we define $B_p \equiv P_p Y_p^{1/\sigma_p}$ and $B_g \equiv m_g^{-1/\sigma_g} P_g Y_g^{1/\sigma_g}$. The prices p_{ip} and p_{ig} faced by the private and public sector producers in the purchase of the same intermediate good i may differ because intermediate good i producer has monopoly power over its variety and may be selling different quantities to each market. Y_g is the demand of the public good from the government and is a policy variable in the model, while Y_p is the demand of the private good from the households and it is determined in equilibrium. The aggregate prices P_p and P_g of the private and public goods are given by the usual aggregators:

$$P_p = \left(\int_{[0,1]} p_{ip}^{1-\sigma_p} di \right)^{\frac{1}{1-\sigma_p}} \quad \text{and} \quad P_g = \left(\int_{I_g} \frac{1}{m_g} p_{ig}^{1-\sigma_g} di \right)^{\frac{1}{1-\sigma_g}} \quad (5)$$

We will use the final private good as the numeraire, so we set $P_p = 1$ in what is to follow.

Intermediate inputs. The intermediate inputs are produced by heterogeneous firms. At any period in time, these firms are characterized by their idiosyncratic stochastic productivity s_i, their capital stock k_i (which depreciates at rate δ), their debt level l_i (when $l_i > 0$ the firm is a net borrower), and whether they currently hold a procurement project $d_i = 1$ or not $d_i = 0$. Output y_i is given by a simple CRS production function, $y_i = f(s_i, k_i) = s_i k_i$, that depends on capital k_i and managerial productivity s_i. The firm-specific s_i follows a first order Markov process, specified in more detail below. If a procurement project is active ($d_i = 1$) a fraction of output u_i,

[13] Governments purchase only a fraction of goods and services provided by the private economy mainly because their needs are different than the needs of private households and firms. By removing 'love-for-variety' we want to eliminate this trivial effect from the analysis of the effects of the number of contracts offered.

chosen by the firm, is sold to the private sector and a fraction $1 - u_i$ is sold to the public sector, otherwise all output is sold to the private sector. Our simple production function implies that u_i is also the fraction of capital used for the production of the private sector variety, that is, $k_{ip} = u_i k_i$.

4.2 Participation in public procurement

The government has control over the subset I_g of goods purchased by the public sector, and a choice of the subset I_g naturally implies its measure m_g. In order to introduce structure in this choice, we consider that the government follows a simple stochastic rule for the allocation of procurement contracts based on the quality of the proposals. In particular, we assume that firms who wish to sell to the government next period ($d_{it+1} = 1$) must invest an amount of private sector good $b_{it} > 0$ today. This quantity may reflect the costs of learning how the process works, the actual costs of preparing a proposal, or the costs of establishing connections with government officials. There is always uncertainty in the outcome of the application, which reflects the fact that "equally capable" firms usually compete in the same auction with only one winner.[14] The probability of being able to sell to the government next period depends on the amount invested, $Pr\left(d_{it+1} = 1 \mid b_{it}\right) = g\left(b_{it}\right) = 1 - e^{-\eta_0 b_{it}^{\eta_1}}$ with $\eta_0 > 0$ and $1 > \eta_1 > 0$ to ensure positive and diminishing returns. Also notice that $\lim_{b \to 0} \frac{\partial g(b)}{\partial b} \to \infty$, so there will always be an interior solution in the optimal choice of b_{it}. This probability function captures in reduced form the competition for procurement projects. As such, we think of η_0 as an equilibrium object that ensures that the fraction of firms obtaining a procurement project equals the measure m_g of goods purchased by the public sector. Hence, the probability of procurement depends on firms' own actions through b_{it} as well as on the actions of all other firms through the equilibrium object η_0.[15] The winners of the competition for procurement form the set I_g in that period.

4.3 Entry and exit

A fraction $1-\theta$ of households die every period and are replaced by the same number of new households running new firms. To avoid changing the composition of the goods produced in the economy, the entrant households produce the varieties left vacant by the exiting households. Dying households leave accidental bequests that for simplicity are taken by the government. Entrant households start with a joint distribution of financial wealth and productivity Γ_0 and with no procurement project. The wealth

[14] In practice, the final ranking of firms is decided based on a number of attributes as the price, quality, and technical requirements. Therefore, firms always face uncertainty about how the public entity awarding the contract will perceive them and their competitors fulfilling these attributes.

[15] Alternatively, we could have followed a more structural approach in modelling the competition for public contracts. For instance, in a different setting, Michelacci and Pijoan-Mas (2012) model competition for jobs with a job finding probability depending on individual human capital relative to the average human capital of the economy. Yet, our formulation is flexible and does not require taking a stand on the complex procurement competition process.

of the entrants is provided by the government. Alternatively, we could have assumed that all accidental bequests go the newborns, but we want to break this link in order to have the flexibility to choose the amount of financial wealth for entrants.

4.4 Preferences and constraints

Firms are owned by entrepreneurs. Entrepreneurs have CRRA preferences over consumption with curvature μ, and their objective is to maximize the discounted sum of utilities.[16] They obtain income only from their firm so their budget constraint is given by:

$$c_{it} + b_{it} + k_{it+1} - l_{it+1} \leq p_{ipt}y_{ipt} + p_{igt}y_{igt} + (1-\delta)k_{it} - (1+r_t)l_{it} - \text{tax}_{it} \tag{6}$$

where $\text{tax}_{it} = \tau\left[p_{ipt}y_{ipt} + p_{igt}y_{igt} - (r_t + \delta)\, k_{it}\right]$ denotes the proportional taxes on profits paid by entrepreneur i at time t. The tax function is purposely simple because we focus on revenue neutral counterfactuals.[17] As it is standard in the literature, we only allow for one-period debt contracts l_t that pay a risk-free interest rate r_t. The amount of debt is limited by the repayment capacity of the firm through a combination of earnings-based and asset-based collateral constraint. In particular, the amount of debt of a firm coming into $t+1$ is limited by,

$$l_{it+1} \leq \varphi_a\, k_{it+1} + \varphi_p\, p_{ipt+1}y_{ipt+1} + \varphi_g\, p_{igt+1}y_{igt+1} \tag{7}$$

If $\varphi_a = 0$, $\varphi_p = 0$, and $\varphi_g = 0$ no external finance is available and all production needs to be self-financed. With $\varphi_a > 0$ the firm can lever up. With $\varphi_p > 0$ and $\varphi_g > 0$ firms can borrow against the revenues generated in the private and the public sector respectively.[18]

4.5 Timing and state space

Regarding the non-procurement part of the model, we follow the timing convention commonly used in the firm dynamics literature. First, we assume that resources

[16]Modeling firms as being run by entrepreneurs with curvature in preferences over consumption could also be justified by firms' dividend-smoothing motives, as empirically documented, e.g. Leary and Michaely (2011).

[17]Note that this simple tax function is allowing for tax deductibility of depreciation δk_{it} and of the (interest) opportunity cost of capital $r_t k_{it}$. Since $l_{it} \leq k_{it}$ whenever the firm's net worth is non-negative, we are implicitly allowing the firm to deduct more than just the interest payments on debt $r_t l_{it}$.

[18]An alternative and more structural borrowing constraint would limit repayment $(1+r_{t+1})l_{it+1}$ explicitly by a fraction of undepreciated capital $(1-\delta)k_{it+1}$ plus revenues,

$$(1+r_{t+1})l_{it+1} \leq \tilde{\varphi}_a\,(1-\delta)k_{it+1} + \tilde{\varphi}_p\, p_{ipt+1}y_{ipt+1} + \tilde{\varphi}_g\, p_{igt+1}y_{igt+1}$$

In steady state with constant r this specification would be equal to (7) with the redefinitions: $\varphi_a \equiv \frac{(1-\delta)\tilde{\varphi}_a}{1+r}$, $\varphi_p \equiv \frac{\tilde{\varphi}_p}{1+r}$, and $\varphi_g \equiv \frac{\tilde{\varphi}_g}{1+r}$. In counterfactual exercises, increases (decreases) in the equilibrium r would tighten (loosen) the borrowing constraints. Our formulation ignores this effect, but this is quantitatively second order, as seen by the results from our counterfactuals below.

devoted to consumption are spent at the beginning of each period t. Second, we assume that production in $t+1$ is carried out using capital installed at the end of period t. Third, we assume that the household survival shock and the firms' productivity in $t+1$ are revealed (in this order) before firms decide how much capital to install for next period, k_{it+1}, and how much debt to issue for next period, l_{it+1}. Regarding the variables related to procurement, we follow a similar logic. The amount of resources devoted to increase the probability of being active in procurement in $t+1$, i.e., b_{it}, is spent at the beginning of each period t. Whether or not the firm is successful and becomes active in procurement in $t+1$, i.e, $d_{it+1} = 1$, is revealed at the same time as productivity in $t+1$ and right after the survival shock. This means that procurement applications of dying households are ignored by the government and hence dying households are not awarded a procurement project that cannot be delivered.

Alternative specification. These assumptions on timing simplify the state-space dimensionality of the problem. In particular, let $a_{it+1} \equiv k_{it+1} - l_{it+1}$ be the firm's net worth to be carried to next period in units of private good today. Then we can redefine the budget constraint as

$$c_{it} + b_{it} + a_{it+1} \leq (1-\tau)\left[p_{ipt}y_{ipt} + p_{igt}y_{igt} - (r_t + \delta)k_{it}\right] + (1 + r_t)a_{it} \tag{8}$$

The collateral constraint becomes

$$k_{it} \leq \phi_a a_{it} + \phi_p p_{ipt} y_{ipt} + \phi_g p_{igt} y_{igt} \tag{9}$$

where the parameters in the borrowing constraint are re-defined as:

$$\phi_a \equiv \frac{1}{1-\varphi_a} \in [1, \infty), \quad \phi_p \equiv \frac{\varphi_p}{1-\varphi_a} \in [0, \infty), \quad \phi_g \equiv \frac{\varphi_g}{1-\varphi_a} \in [0, \infty) \tag{10}$$

Hence, the production decisions (capital and sales composition) are intratemporal, while the accumulation of net worth and the investment in procurement are intertemporal. This allows to split the firm's problem in two: a *static production* problem and a *dynamic consumption-saving* problem. Next, we describe them in turn.

4.6 The static production problem

The intratemporal production problem is characterized by firm productivity s, firm net worth a, and the availability of a procurement project d. For simplicity we drop the firm subindex i. Firms with $d = 0$ only have to choose their optimal size k subject to the borrowing constraint, while firms with $d = 1$ also decide on the fraction of output $u \in [0,1]$ sold to the private sector. We can write the formal maximization problem for the firm of type $(s, a, d = 1)$ as,

$$\pi(s, a, 1) = \max_{k,u} \{p_p y_p + p_g y_g - (r + \delta) k\}$$

subject to:

$$p_p y_p = B_p \left[\quad u \quad sk \right]^{\frac{\sigma_p - 1}{\sigma_p}}; \quad p_g y_g = B_g \left[(1-u)\, sk \right]^{\frac{\sigma_g - 1}{\sigma_g}}$$
$$k \in [0, \phi_a a + \phi_p p_p y_p + \phi_g p_g y_g]; \; u \in [0,1]$$

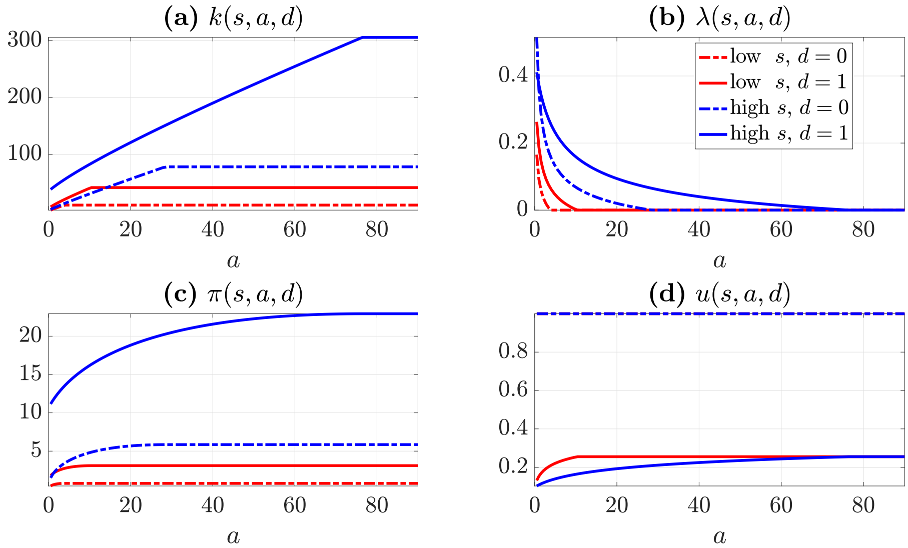
while for the firm of type $(s, a, d = 0)$ all the terms $p_g y_g$ trivially disappear and u becomes equal to 1. Let λ be the multiplier of the intratemporal borrowing constraint and let's consider the general case with $d = 1$. The optimal choices are described by the following FOC:

$$(1 + \lambda \phi_p) \frac{\partial p_p y_p}{\partial u} + (1 + \lambda \phi_g) \frac{\partial p_g y_g}{\partial u} = 0 \tag{11}$$
$$(1 + \lambda \phi_p) \frac{\partial p_p y_p}{\partial k} + (1 + \lambda \phi_g) \frac{\partial p_g y_g}{\partial k} = r + \delta + \lambda \tag{12}$$
$$\lambda \geq 0, \; \phi_a a + \phi_p p_p y_p + \phi_g p_g y_g - k \geq 0, \; \lambda \left[\phi_a a + \phi_p p_p y_p + \phi_g p_g y_g - k \right] = 0 \tag{13}$$

These optimality conditions show how financial frictions distort the two decisions faced by firms: production composition and firm size. Equation (11) characterizes the composition of sales. With $\lambda = 0$, the optimal choice requires the equalization of the marginal revenues obtained from each sector. Because of the concave revenue functions in both sectors, there is always an interior solution to this problem. With binding financial constraints ($\lambda > 0$), production is shifted towards the sector whose output can be better collateralized. For instance, if procurement contracts offer better collateral value than sales to the private sector ($\phi_g > \phi_p$) the optimal choice requires lower marginal revenues from public procurement relative to the marginal revenues from the private sector, which happens when production is shifted towards the public sector and away from the private sector.

Equation (12) determines optimal firm size. With $\lambda = 0$ the optimal choice requires to equalize the marginal revenue product of capital to its cost, which is just $r + \delta$. With binding financial constraints ($\lambda > 0$), the effective cost of capital is $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ for sales to the private sector and $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$ for sales to the public sector. The multiplier of the financial constraint λ has two opposite effects on the cost of capital: on the one hand it increases the cost of capital as in standard asset-based financial constraints, but on the other hand it decreases the cost of capital because a fraction of the generated output can also be collateralized. We will restrict ϕ_p and ϕ_g as indicated in Assumption 1 below to ensure that the earnings-based constraints cannot self-finance the optimal capital of the unconstrained problem, that is, to ensure that the financial constraints are binding for at least the entrepreneurs with zero net worth. Otherwise all firms would be unconstrained, see Lemma 2 and Proposition 2 in Online Appendix B. An implication of Assumption 1 is also that the values of ϕ_p and ϕ_g are below $(r + \delta)^{-1}$. This implies that the effective costs of capital for the private and public sector, $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ and $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$, are monotonically increasing in λ, which in turn means that financially constrained firms operate with less capital, see Lemma 1 and Proposition 1 in Online Appendix B.

Figure 1. Solution of the static profit maximization problem



Notes: This figure shows the solution to the firm's problem. Panel (a) shows the size of the firm represented by the amount of capital $k(s,a,d)$; Panel (b) shows the multiplier of the financial constraint $\lambda(s,a,d)$; Panel (c) shows the profits $\pi(s,a,d)$. All of them are plotted against firm's productivity s, for two different levels of net worth, and for the cases $d=0$ and $d=1$.

Assumption 1 *The model parameters satisfy the following boundary constraints:* $\phi_p < \frac{\sigma_p-1}{\sigma_p}(r+\delta)^{-1}$ *and* $\phi_g < \frac{\sigma_g-1}{\sigma_g}(r+\delta)^{-1}$.

where $\frac{\sigma_p-1}{\sigma_p}(r+\delta)^{-1}$ and $\frac{\sigma_g-1}{\sigma_g}(r+\delta)^{-1}$ are the capital to revenues ratios for the unconstrained problem in the private and public sector respectively.

Static policy functions. The solution of this problem yields optimal choices $k(s,a,d)$ and $u(s,a,d)$, an associated shadow value of the financial constraint $\lambda(s,a,d)$, and a profit function $\pi(s,a,d)$. In Online Appendix B we characterize analytically these objects for both non-procurement ($d=0$) and procurement firms ($d=1$) whenever $\sigma_g = \sigma_p$. In Figure 1, we illustrate the numerical solution for both cases with the parameterization discussed in Section 5. First, as it is common in standard models of firm dynamics with collateral constraints, constrained firms with no procurement see their capital and profits increase with net worth (while the shadow value of the borrowing constraints declines) until the point in which the financial constraints stop binding and net worth plays no role. Second, different from models with only asset-based collateral constraints, financially constrained firms without procurement increase capital and profits when productivity increases. This happens through the earnings-based constraint, which allows more productive firms to generate more revenues at the same level of net worth and hence expand production. Note also that more productive firms are more financially constrained at any level of net worth (their shadow value of the borrowing constraint is larger) because the expansion of borrowing possibilities with s is lower than the increase in the optimal size. Third, looking at firms with procurement, the fraction of output sold by constrained firms to the private sector is decreasing in productivity s and increasing in net worth a, which simply says that more financially constrained firms, conditional on participating, have a higher fraction of their capital allocated to the production of goods sold to the government. This last result is true under $\phi_g > \phi_p$ and it would be the opposite if $\phi_g < \phi_p$. Finally, note also that capital, profits, and the shadow value of the borrowing constraint for firms with procurement evolve with s and a as in the case without procurement.

A procurement shock. We can also analyze the static effect of a procurement shock by comparing the solutions of the $d = 1$ and $d = 0$ cases at any value of the state variables s and a. For unconstrained firms, a procurement shock leaves operations in the private sector unchanged and increases firm size (and profits) to serve the public demand. This is due to the constant returns to scale production assumption and the absence of adjustment costs. For constrained firms, a procurement shock tightens the financial constraint whenever $\phi_g \leq \phi_p$. With $\phi_g = \phi_p$ this is because the firm with $d = 1$ has two demands to serve, which are equally pledgeable, and has the same net worth to finance capital in the two different markets. As a result the firm scales down the operations in the private sector to free up collateral for the production in the public sector, which generates a negative within-firm private sector spillover of the procurement contract, that is, $k_p(s, a, 1) \equiv u(s, a, 1)\, k(s, a, 1) < k(s, a, 0)$. When $\phi_g < \phi_p$ the financial situation is aggravated because the public sector demand can be self-financed to a lesser extent than the private sector one and the negative private sector spillover is larger. When $\phi_g > \phi_p$, instead, public procurement may alleviate the firm financial situation because the public sector demand can be self-financed to a larger extent. This will only be relevant for firms with little or no wealth, which will be less constrained when obtaining a procurement project and will use the extra financing capacity coming from the public sector to scale up operations in the private sector. This is precisely stated in Proposition 13 in Online Appendix B. In our numerical exercises with $\phi_g > \phi_p$, with a realistic calibration, and endogenously accumulated net worth distributions, however, a procurement shock always increases firm size, makes the firm more constrained, and almost always generates a *negative* spillover on the private sector sales for constrained firms that obtain procurement. Finally, a procurement shock always increases profits. Among unconstrained firms, this is more so for the more productive ones because more productive firms can deliver larger projects. Among constrained firms, and for the empirically relevant case $\phi_g > \phi_p$, this is more so for the more productive and the richer firms, because these two variables determine the capacity to deliver large projects. The only exception is for the firms with little or no wealth discussed above, in which case the increase of profits with procurement actually falls with net worth, see Proposition 15 in Online Appendix B.

4.7 The dynamic problem

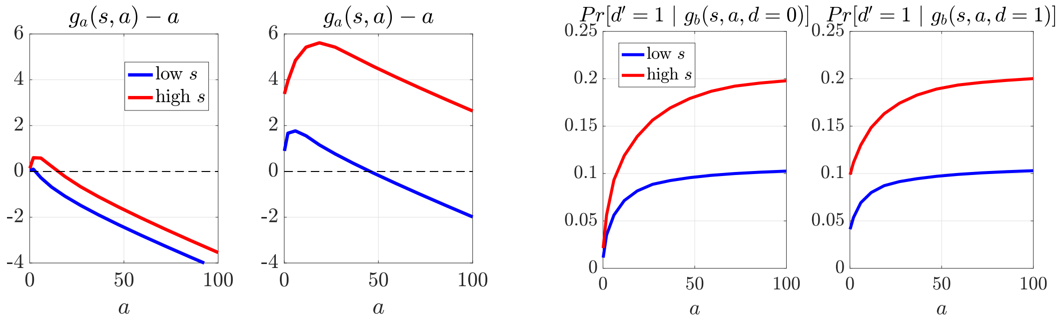
The *dynamic consumption-saving* problem can be written in recursive form,

$$V(s, a, d) = \max_{c,b,a'} \left\{ u(c) + \beta\theta\, \mathbb{E}_{s',d'|s,b} \left[V(s', a', d') \right] \right\}$$
subject to:

$$\mathbb{E}_{s',d'|s,b}\left[V\left(s',a',d'\right)\right] = g\left(b\right)\mathbb{E}_{s'|s}V\left(s',a',1\right) + \left(1-g\left(b\right)\right)\mathbb{E}_{s'|s}V\left(s',a',0\right)$$
$$c + b + a' = \left(1+r\right)a + \left(1-\tau\right)\pi\left(s,a,d\right) \quad \text{and} \quad a' \geq 0$$

The first constraint says that the expected firm's value for tomorrow is an average of the firm's value under procurement, i.e., $d' = 1$, and no procurement, i.e., $d' = 0$, weighted by the endogenous probability of procurement $g(b)$. This is why the expectations operator $\mathbb{E}$ depends on b in addition to s. The FOC for the choices of

Figure 2. Decision rules



Notes: The first and second panels show the net saving rules $g_a(s,a,d))$ for firms of different levels of productivity and net worth, both for non-procurement and procurement firms respectively. The third and fourth panels show the endogenous probability of obtaining procurement contracts evaluated at the optimal rules $g_b(s,a,d)$ for firms with different levels of productivity and net worth, both for non-procurement and procurement firms respectively.

a' and b are:

$$u_c\left(c\right) \geq \beta\theta\,\mathbb{E}_{s',d'|s,b}\left[\left(1+r+\left(1-\tau\right)\frac{\partial\pi\left(s',a',d'\right)}{\partial a'}\right)u_c\left(c'\right)\right] \tag{14}$$

$$u_c\left(c\right) = \beta\theta\frac{\partial g\left(b\right)}{\partial b}\mathbb{E}_{s'|s}\left[V\left(s',a',1\right) - V\left(s',a',0\right)\right] \tag{15}$$

The first equation is the standard Euler equation that emerges in models of heterogeneous firms with financial constraints. If a firm is expected to be financially constrained next period in the *static profit maximization* problem, that is $\partial\pi\left(s,a,d\right)/\partial a = \phi_a\lambda\left(s,a,d\right) > 0$, then there is an extra return above r to accumulating net worth that is given by the increase in (after tax) profits due to relaxing the firm's collateral constraint, see Online Appendix B. The second equation determines the optimal spending in b: the entrepreneur will equalize its marginal utility of consumption to the marginal return of b, which is given by the expected increase of the firm's value coming from the possibility of selling to the government. Because of the properties of $g\left(b\right)$ and because $E_{s'|s}\left[V\left(s',a',1\right) - V\left(s',a',0\right)\right] > 0)$ the right hand side declines with b.[19]

Decision rules. Figure 2 illustrates the net saving decision $a' - a$ of firms without and with procurement (first and second panel respectively). At low levels of net worth there is a hump-shaped relationship between net savings and net worth that is driven by the tradeoff between smoothing consumption vs. relaxing future borrowing

[19]Proposition 15 in Online Appendix B shows that $\pi\left(s,a,1\right) - \pi\left(s,a,0\right) > 0$, and $V\left(s,a,1\right) - V\left(s,a,0\right)$ inherits this property as d plays no other role than increasing profits π in the value function.

constraints, a feature present in similar models like Midrigan and Xu (2014). At larger levels of wealth, the saving behavior follows the logic in Aiyagari (1994): net savings decrease monotonically with net worth and there is a target level of wealth that is larger for larger productivity s. This figure also shows big differences between procurement and non-procurement firms in terms of saving decisions. In particular, procurement firms save more conditional on their current net worth a and productivity shock s. This difference is driven by the fact that profits are higher for firms that are active in procurement, which relaxes their budget constraint and hence allow them to save more without sacrificing too much consumption.

Figure 2 also shows the function $g(b)$ evaluated at the actual choice of b for firms with different levels of net worth and productivity, both for non-procurement (third panel) and procurement firms (fourth panel). The first thing to notice is that high-net worth firms invest more resources in increasing their probability of being able to sell to the government. This emerges as a result of an interesting trade off. In the dynamic problem there are two competing mechanisms to lessen borrowing constraints. On the one hand, households can accumulate wealth to relax the asset-based constraint and increase profits next period (right hand side of equation (14)). On the other hand, they can alternatively invest in applications for procurement projects that will relax the earnings-based constraint if $\phi_g > \phi_b$ and allow to increase revenues and accumulate net worth in any case (right hand side of (15)). Online Appendix B shows that $\frac{\partial^2 \pi(s,a,d)}{\partial a^2} < 0$, which means that the return of accumulating net worth is lower for firms with more net worth. The profit premium of a procurement project $\pi(s,a,1) - \pi(s,a,0)$ increases with net worth for constrained firms, see Proposition 15 in Online Appendix B, and so does $V(s,a,1) - V(s,a,0)$, which means that the return of investment in procurement is larger for firms with more net worth. This happens because selling to the government does not relax the borrowing constraints completely, which means that firms still rely on their own assets for determining the size of their procurement contracts. This reflects a "size effect": the bigger the procurement projects the firm expects to be able to deliver, the higher the expected profits that participating in procurement generates. Therefore, we obtain the result that the investment in procurement projects increases with firm net worth.

The second thing to notice is that there are almost no differences between procurement and non-procurement firms. The reason is that, conditional on b, the probability of obtaining contracts tomorrow is independent from whether the firm is active in procurement today. Procurement firms spend a bit more on b though for low levels of a, which simply reflects the fact that these firms have more available resources at hand.

4.8 Steady state equilibrium

A steady state equilibrium requires that (a) entrepreneurs solve their optimization problem; (b) the probability measure over the state space of households is stationary;

(c) the market for the private good clears; (d) the market for the public good clears; (e) the probability of obtaining procurement projects is consistent with the measure of goods bought by the public sector; (f) the budget constraint of the government holds; (g) by Walras law, the credit market clears. A more detailed definition of the equilibrium is provided in Online Appendix C.1. Several comments are in order. First, the parameter η_0 driving the average probability of a procurement project is an equilibrium object that ensures meeting equilibrium condition (e). It summarizes in reduced form the competition for projects. Second, the government can accumulate financial wealth D, which serves as an aggregate counterpart for the loans of entrepreneurs such that loans do not need to be in zero net supply in condition (g). Indeed, D will be calibrated to match the total amount of debt relative to capital held by firms in the data at a targeted interest rate r. Third, condition (f) establishes that the government budget constraint in steady state is such that procurement is financed by taxes, plus interest revenues from the stationary amount of government wealth D, plus accidental bequests left by dying entrepreneurs, minus the initial net worth provided by the government to newly born entrepreneurs (which is dictated by the exogenously fixed distribution of entrants Γ_0). Finally, we note that the aggregate objects determined in general equilibrium that are relevant for the optimization problem of households are Y_p, r, τ, and P_g.

4.9 Two types of misallocation

Our model generates two types of misallocation. First, the presence of financial frictions generates misallocation of capital *across firms*. This is a type of misallocation that is well understood by the literature that studies the effects of financial frictions on aggregate productivity, see for instance Midrigan and Xu (2014) or Moll (2014). After some manipulations of the firm's FOCs (equations (11) and (12)), defining $k_p = uk$ and $k_g = (1-u)k$, we obtain the following expressions:

$$\text{MRPK}_{ip} \equiv \frac{\partial p_p y_p}{\partial k_p} = \frac{r+\delta+\lambda}{1+\lambda\phi_p} \quad \text{and} \quad \text{MRPK}_{ig} \equiv \frac{\partial p_g y_g}{\partial k_g} = \frac{r+\delta+\lambda}{1+\lambda\phi_g}$$

Unconstrained firms ($\lambda = 0$) equalize their marginal products of capital to $r+\delta$ and hence operate at their optimal size. In contrast, constrained firms ($\lambda > 0$) face an effectively higher cost of capital in the private and public sectors $(r+\delta+\lambda)/(1+\lambda\phi_p)$ and $(r+\delta+\lambda)/(1+\lambda\phi_g)$ respectively and hence operate at a suboptimal scale, that is, they operate at inefficiently high MRPK_{ip} and MRPK_{ig} compared to unconstrained firms.

Misallocation across firms. In an economy with binding financial frictions misallocation has two consequences: first, the average marginal revenue products in the private and public sectors, $\overline{\text{MRPK}}_p$ and $\overline{\text{MRPK}}_g$ defined in Online Appendix C, will be inefficiently high and the average capital in each sector, K_p and K_g, inefficiently low. Second, because λ depends on the firm state variables, s and a, there will be heterogeneity in the MRPK_{ip} and MRPK_{ig} across constrained firms, which lowers TFP_p and TFP_g (defined in Online Appendix C). This type of misallocation across firms within a sector is similar to the one emphasized by Hsieh and Klenow (2009), with the key difference that in our model the same firm may produce in two sectors at the same time, and hence the marginal products of capital are firm-sector specific.

Misallocation within firms. Second, the model also generates misallocation of capital *within firms*. As it is apparent from the two equations above, unconstrained firms ($\lambda = 0$) equalize their marginal products across the two sectors. Constrained firms ($\lambda > 0$), instead, shift their production towards the sector that gives higher collateral value. In particular, whenever $\phi_g > \phi_p$ the marginal product of capital from selling to the private sector, MRPK_{ip}, will be inefficiently large relative to the one from selling to the government, MRPK_{ig}. This has again two consequences. First, the average marginal revenue product in the private sector, $\overline{\text{MRPK}}_p$, will be inefficiently higher than the one in the public sector, $\overline{\text{MRPK}}_g$. Second, the dispersion in λ across firms generates a larger dispersion in MRPK_{ig} than in MRPK_{ip}, which lowers TFP_g more than TFP_p. Therefore, and for both reasons, there would be efficiency gains from reallocating capital from the public to the private sector within the firm.

5 Calibration

The model period is one year. We classify the model parameters into four different blocks. The first block contains parameters related to preferences, technology, and productivity that we set to predetermined values. We calibrate the parameters in the other three blocks such that in equilibrium the model matches several moments measured in the data for the year 2006.

Block #1: preferences and technology. The first subset of parameters in this block are the relative risk aversion coefficient μ, which we set equal to 2; the CES elasticities σ_p and σ_g, which we set both to 3; the discount factor β, which we set to 0.94; and the annual depreciation rate δ, which we set to 0.10. These are within the range of standard values in the literature. In this block, we also include the parameters governing firms' idiosyncratic productivity. We assume that the log of a firm's productivity process s evolves over time according to an AR(1) process with Gaussian shocks and unconditional mean $\bar{s} \equiv \mathbb{E}[\log(s)]$. Because we already calibrate a large number of parameters internally, we exogenously set the autocorrelation coefficient ρ_s to 0.80 and the standard deviation of the innovations σ_s to 0.30, as estimated by Ruiz-García (2020) using the same dataset of firms. Since ρ_s and σ_s have been found to be critical for assessing the aggregate effects of financial frictions in this class of models (e.g., Midrigan and Xu (2014); Moll (2014)), we will check ex-post how well our model matches moments that are usually used to pin down these two parameters. We discretize the process following the Rouwenhorst method, allowing for $N_s = 5$ different states.

Block #2: financial constraints. Our model contains three parameters governing firms' financial constraints: ϕ_a, ϕ_p, and ϕ_g. We choose a value of ϕ_a so that the model matches the credit-to-capital ratio observed in our micro-level data, 0.55. Regarding ϕ_p and ϕ_g, we proceed as follows. Given the credit constraint in equation (7), and after dividing by k and taking first differences, changes in firms' leverage for constrained firms are given by:

$$\Delta\left(\frac{l_{it}}{k_{it}}\right) = \varphi_p \, \Delta\left(\frac{p_{it}y_{it}}{k_{it}}\right) + (\varphi_g - \varphi_p)\Delta\left(\frac{p_{igt}y_{igt}}{k_{it}}\right) \tag{16}$$

where l_{it}/k_{it} is the firms' leverage, i.e., total credit divided by fixed assets; $p_{it}y_{it}/k_{it}$ is the firms' total average product of capital, measured as total value added (minus wages) divided by total fixed assets; $p_{igt}y_{igt}/k_{it}$ is the firms' value added (minus wages) coming from selling to the government divided by the firm's total stock of capital. Therefore, for constrained firms, the coefficients from an OLS regression directly pin down φ_p and $(\varphi_g - \varphi_p)$, which together with ϕ_a allow to recover ϕ_p and ϕ_g (see equation (10)).

The construction of "output", i.e., $p_{it}y_{it}$ in the data requires some explanation. Following the recent literature on earnings-based constraints (e.g., Drechsel (2021)), we assume that the flow variable that firms can collateralize is EBITDA: sales net of overhead and labor costs, without subtracting investment, interest payments or taxes. Because we do not have labor in our model, that variable is equal to the firm's value added $p_{ipt}y_{ipt} + p_{igt}y_{igt}$. However, in the data, we compute the counterpart of that variable as:

$$p_{ipt}y_{ipt} + p_{igt}y_{igt} = \text{VA}_i - \text{wage bill}_i \tag{17}$$

Because we do not observe the use of factors separately for what is used to deliver sales to the private vs. the government sector. To compute value added generated by selling to the government, we assume that the intermediate goods and the labor share in total expenditure is constant within the firm, i.e., it does not change depending on whether the firm sells to the private sector or the government.

Block #3: participation and size of procurement. There are four parameters driving the size and participation in procurement. The parameters Y_g and m_g are policy parameters governing the relative size of procurement in the economy and the fraction of goods bought by the government. We set Y_g to match the share of procurement in GDP equal to 12.1%, which is the value we measure in the Spanish national accounts in the year 2006. We set m_g equal to the share of firms that participate in procurement, which we calculate to be 3.8%.[20] Regarding the probability function

[20] In our sample only 0.5% percent of firms participate in procurement in a given year. However, our procurement data only captures 13% of the total procurement value measured in national accounts (see Online Appendix A). Assuming that the number of procurement firms relative to the value of procurement is the same in our sample as in the whole population, we can scale up the share of procurement firms to 3.8% (0.005 / 0.13 = 0.038). This is probably a lower bound for the number of firms active in procurement because our procurement dataset is biased towards contracts awarded

of winning a contract we proceed as follows. We calibrate the level parameter η_0 to ensure that the the fraction of firms doing procurement equals the fraction of goods bought by the government, m_g, which is the equilibrium condition (e). Regarding the curvature parameter η_1, we identify it by matching the selection pattern of firms into procurement observed in the data. We proceed as follows. In the data, we select firms with no procurement contracts between 1999 and 2005. Then, we classify as procurement firms those firms that obtain at least one contract in 2006. We define the "procurement premium" as the relative difference in size (measured by value added) between procurement and non-procurement firms in 2005 (only exploiting variation across firms within the same 4-digit industry). That is, we want the model to match the ex-ante difference in size between procurement and non-procurement firms. We measure this procurement premium to be 72%. The intuition why the parameter η_1 affects the selection of firms into procurement is as follows. When η_1 approaches zero, the probability function $g(b)$ exhibits strong diminishing marginal returns in b: the marginal increase in probability falls quickly as firms invest more. This makes differences in b across firms inconsequential for their probability of selling to the government, and hence generates very little selection, with complete randomness in allocation when $\eta_1 = 0$. Conversely, when η_1 approaches 1, the diminishing marginal returns are small: the marginal increase in probability falls slowly with b as firms invest more. This implies that differences in b translate into big differences in the probability of participating in procurement, which generates a strong selection pattern.

Block #4: rest of the parameters. We use firms' average productivity level $\bar{s}$ to match the capital-to-output ratio observed in our firm-level data. The reason why this moment is informative of the average productivity in the economy has to do with our AK assumption on firms' technology. Using our measure of output and firms' fixed-capital stock, we compute an aggregate capital-output ratio of 3.88. To discipline government's wealth D, we target an equilibrium interest rate equal to 5%. Finally, we calibrate the survival probability $\theta = 0.95$ to the firms' exit rate in Spain of 0.05.

5.1 Calibration results

Our model matches all the targeted moments. Panel A in Table 5 shows the definition of the parameters as well as their inferred values. Panel B shows the description of moments and their value in the data and in the model.

by the central government, which are bigger than the contracts awarded by other governments layers (e.g., local governments), and hence are probably more concentrated in a few firms. To the best of our knowledge, there are no official statistics on the number of firms selling to governments in Spain. As a reference from a different country, Lee (2021) calculates that 5.3% participate in the procurement market in South Korea.

Block # 1. Although we exogenously set the parameters governing the AR(1) productivity process to $\rho_s = 0.80$ and $\sigma_s = 0.30$, we check that the model matches well two non-targeted moments that are informative about these two parameters: the one-year autocorrelation of firms' output and the standard deviation of firms' output growth. In our Spanish data, we find the one-year autocorrelation of firms' log sales to be 0.89 (which compares to 0.82 implied by our model) and the standard deviation

Table 5. Calibration

Panel A: parameters		(1) Baseline	(2) $\phi_p = \phi_g$	Panel B: Moments	Data = Model
Block 1					
μ	CRRA coefficient	2.00	2.00		
σ_p	CES private sector	3.00	3.00		
σ_p	CES government	3.00	3.00	PREDETERMINED	
β	Discount factor	0.94	0.94		
δ	Depreciation rate	0.10	0.10		
ρ_s	AR(1) correlation	0.80	0.80		
σ_s	AR(1) variance	0.30	0.30		
Block 2					
ϕ_a	borrowing const. (a)	2.17	2.34	Credit/K	0.55
ϕ_p	borrowing const. ($p_p y_p$)	0.92	0.99	reg. coefficient (φ_p)	0.42
ϕ_g	borrowing const. ($p_g y_g$)	2.40	0.99	reg. coefficient ($\varphi_g - \varphi_p$)	0.68
Block 3					
η_0	probability function (level)	0.21	0.21	Consistency of $g(b)$ with m_g	–
η_1	probability function (slope)	0.53	0.55	Procurement premium	0.72
Y_g	demand shifter	0.83	0.63	Share of procurement in GDP	0.12
m_g	measure of procurement goods	0.038	0.038	Percentage of procurement firms	3.8%
Block 4					
D	Government lending	0.86	0.84	Interest rate	5%
$\bar{s}$	Productivity shifter	-6.51	-6.53	K/Y (aggregate)	3.88
θ	Survival probability	0.95	0.95	Exit rate	5%

Notes: This table summarizes our baseline calibration. All moments, with the exception of the regression coefficients, have been computed for the year 2006. Government lending D is expressed as a fraction of total credit in the model economy. In column (1), we show the parameter values in our baseline calibration. In column (2), we show the parameter values in our alternative calibration where we set $\phi_p = \phi_g$ (see Section 7.4) for details. Notice that we do not report data and model's moments separately because the model matches the data moments perfectly in the two calibrations.

of firms' sales growth to be 0.57 (which compares to 0.49 implied by our model).

Block # 2. With respect to financial frictions, we find $\phi_a = 2.17$, which implies a $\varphi_a = 1 - 1/\phi_a = 0.54$. Therefore, our calibration implies that firms can collateralize 54% of their capital stock. Regarding the earnings-based constraints, Table 6 presents the results from running the empirical counterpart of equation (16) for firms that receive procurement contracts in at least two consecutive years.[21] Because that equation should hold with equality only for firms whose financial constraint is binding, we further restrict the sample to firms that are likely to be financially constrained

[21] This is what we need in order to exploit intensive margin variation in $\Delta p_{igt} y_{igt}$.

Table 6. Change in Leverage and Procurement

	(1)	(2)	(3)
$\Delta p_{it}y_{it}/k_{it}$	0.425^c	0.543^b	0.419^c
	(0.227)	(0.257)	(0.229)
$\Delta p_{igt}y_{igt}/k_{it}$	0.682^c	0.797^c	1.047^c
	(0.391)	(0.478)	(0.588)
Observations	579	403	282
R-squared	0.391	0.437	0.421
Sector×year FE	Yes	Yes	Yes
Sample by age	≤ 10 yrs	≤ 9 yrs	≤ 8 yrs

Notes: This table presents results from estimating the relationship between the change in firm's leverage and the change in its average product of capital and change in its earnings coming from selling to the government divided by the firm's total stock of capital. Regression (16) is estimated with firms obtaining at least one procurement project over 2000-13 using annual data. Standard errors are clustered at the firm level; ^{a} indicates significance at the 1% level, ^{b} at the 5% level, and ^{c} at the 10% level.

according to our model, i.e., young firms. In particular, column (1) restricts the sample to firms that are ten years or younger (the median of the age distribution), while columns (2) and (3) further cut the sample to nine or eight years and less, respectively. Both the coefficients on total and government earnings are positive and statistically significant, indicating that $\varphi_p > 0$ and that $\varphi_g > \varphi_p$. For example, in our preferred specification, i.e., column (1), our estimates imply values for φ_p and φ_g of 0.42 and 1.10 respectively. Hence, we find that firms can pledge 42% of the annual earnings from selling to the private sector and 110% of their annual earnings from selling to the government. Together with $\varphi_a = 0.54$, these numbers translate into $\phi_p = 0.92$ and $\phi_g = 2.40$.[22] These numbers mean that firms can increase their capital by 92% of their annual earnings in the private sector and by 240% of their annual earnings from selling to the government. These two last numbers are the result of a multiplier effect: firms can borrow against their revenues, allowing them to buy more capital, which can be partly collateralized to obtain further credit. This is an important interaction: how earnings-based constraints affect a firm's ability to grow also depends on the value of φ_a.

Blocks # 3 and 4. Regarding the probability function of winning procurement contracts, we find the level η_0 to be equal to 0.21 and the slope η_1 to be equal to 0.53. To match the aggregate capital-output ratio of 3.88 the model needs an average log productivity $\bar{s} = -6.51$. Finally, the model needs a high level of government lending to match an interest rate r of 5%. In particular, the amount of government lending represents around 86% of the total amount of credit in the economy.

[22]We note that both ϕ_p and ϕ_g satisfy Assumption 1, which means that capital cannot be self-financed through the earnings-based constraints, see Lemma 2 in Online Appendix B. This is true despite $\varphi_g > 1$ because the optimal unconstrained capital to output ratio in procurement is $\frac{\sigma_g - 1}{\sigma_g}(r+\delta)^{-1} = 4.44$, which means that ϕ_g should equal 4.44 and φ_g should equal 2.04 for procurement to be self-financed.

6 The Benchmark Economy

In this Section we describe three dimensions of our benchmark economy: the selection pattern of firms into procurement, the treatment effect of a procurement shock on firm dynamics, and the macroeconomic consequences of procurement.

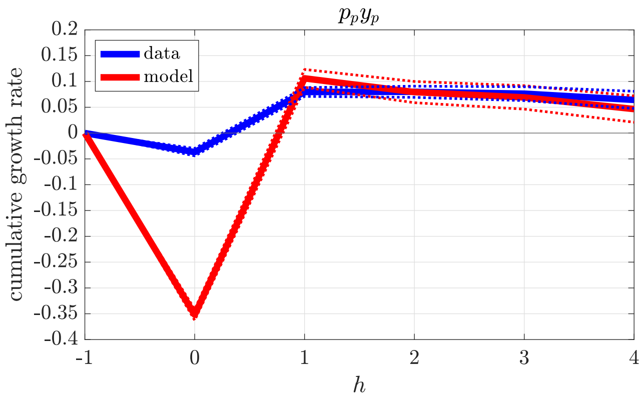
6.1 Selection

We have calibrated our model economy to match a "procurement premium" in value added of 72% (by measuring the relative size of procurement vs. non-procurement firms before they obtain a procurement project). As discussed in Section 4.6, the value of procurement, $V(s, a, d = 1) - V(s, a, d = 0)$, is increasing in firms' ability to deliver large projects, which is determined by productivity s for unconstrained firms and also by net worth a for constrained firms. Hence, in the model firms self-select into procurement based on their productivity s and their net worth a. When we compute the "procurement premium" for s and a we find that procurement firms are ex-ante 36% more productive and hold ex ante 53% more net wealth.

6.2 Treatment

We next describe the treatment effects of procurement on firm dynamics implied by our calibrated model. First, we use model-simulated data to estimate a local projection regression of a procurement shock on private sales, which we show in the red line in Figure 3 (see Online Appendix D for details). We find that procurement generates a crowding out of sales to the private sector on impact (a 35% decline in sales to the private sector), and a crowding in during the subsequent years. As discussed in Section 4.6, a constrained firm has to split resources between the two sectors, despite the extra credit generated by selling to the government, which explains the fall in sales to the private sector on impact. In fact, the firm obtaining procurement becomes more constrained at impact, i.e. higher λ, as a result of the increase in demand (see Figure A.III in the Online Appendix). However, the new profits generated from procurement allow the firm to accumulate more net worth over time. This higher level of net worth will ease the firm's financial constraint (lowering λ) and hence allow it to increase output in the private sector in the subsequent periods. We observe a similar pattern in the data. Using the main sample of Section 3, we estimate the same local projection regression and find that a procurement contract is associated with a fall in sales to the private sector of around 4% at impact and a subsequent increase over time, see blue line in Figure 3. Hence, both in the data and in the model, the crowding out effect disappears and actually gets reversed over time: four years after obtaining a procurement contract, firms increase their sales to the private sector by around 5% in the model and 6% in the data.

Figure 3. Crowding out/in of procurement



Notes: This figure shows the cumulative estimated impact of obtaining a procurement contract on a firm's private sales for different time horizons $h = 0, 1, 2, 3, 4$. Dotted lines are the associated confidence intervals at 5% confidence. See Online Appendix D for details.

Second, we find that the procurement contract has a permanent positive effect on b, which is the result of an increase in the firm's cash on hand. In particular, we find an increase of around 10% estimated four years after the firm obtains a contract (see Figure A.III). This is a channel through which the model endogenously generates some persistence in firms' participation in procurement. To have a sense of how strong this channel is as compared to the data, we compute the probability of obtaining a contract over the next three years, i.e., $t + 1$, $t + 2$, or $t + 3$, conditional on a firm having a contract in t. In the data, this number is quite high, 75%, which compares to its model counterpart of 16%. That is, our model generates around 1/5 of the persistence observed in the data.

6.3 The macroeconomy

We report the most relevant aggregate numbers of the benchmark economy in column (1) of Table 7. We find significant differences in aggregate TFP across the two sectors. In particular, our model implies that TFP in the procurement sector is 21% higher than its counterpart in the private sector (0.308 vs. 0.255), see Online Appendix C for derivations. This difference is mainly due to selection on s. To see why, we note that absent financial frictions but keeping the same selection on s into procurement, the *first-best* level of TFP in the procurement sector would be 19% higher than its equivalent in the public sector. The calibrated economy displays modest levels of misallocation in both sectors, but significantly more in the private one: the aggregate TFP gains of equalizing MRPK across firms are 4.7% in the private sector and 3.3% in procurement. There are two reasons for the difference of misallocation between the two sectors. The first one is $\phi_g > \phi_p$, which increases procurement firms' borrowing capacity relative to firms producing only in the private sector. The second reason is that firms with higher net worth self select into procurement, which reduces the dispersion in net worth across procurement firms and hence the dispersion in λ. These two reasons together imply a variance in the log MRPK across firms in the procurement sector which is around 18% lower than that in the private sector: 0.023 vs. 0.026.

Relative prices. Finally, we look at the price of public goods relative to private goods, which will be important to understand some of the results from the policy counterfactuals. The relative price can be written as:

$$\frac{P_g}{P_p} = \frac{\overline{\text{MRPK}}_g}{\overline{\text{MRPK}}_p} \frac{\text{TFP}_p}{\text{TFP}_g} \tag{18}$$

where $\overline{\text{MRPK}}_g$, $\overline{\text{MRPK}}_p$, TFP_g, and TFP_p are the weighted average marginal revenue products and sectorial TFP's (see Online Appendix C for derivations). As in standard multi-sector models, the ratio of relative prices is inversely related to sectorial TFPs. But equation (18) also implies that the relative price is positively related to the ratio of average marginal revenue products in each sector. That is, a relatively high sectorial "wedge" is associated with a higher relative price. Because firms active in procurement are on average more financially constrained, i.e., have a higher λ, the average wedge in the procurement sector is higher. In particular, in our benchmark economy, $\overline{\text{MRPK}}_g$ is around 8% higher than $\overline{\text{MRPK}}_p$. However, as mentioned above, TFP_g is 21% higher than TFP_p. All together the relative price of public goods is $P_g/P_p = 0.899$.

7 Policy Experiments

Our empirical evidence in Section 3 and model results in Section 6 show that procurement contracts help firms grow out of their financial constraints. At the same time, in Section 5 we have seen that smaller firms, typically the most constrained, participate less in procurement. This suggest that making procurement contracts available to smaller firms may lead to aggregate output gains. For this reason, in this Section we quantify the aggregate effects of reforming the public procurement allocation system through *expenditure-neutral* changes that favor small firms.

7.1 Counterfactual 1: Targeting the selection pattern

We first run an experiment that consists of encouraging the participation of smaller firms and hence change the selection pattern of firms into procurement. This counterfactual aims to reproduce the type of "set-aside" policies for small businesses implemented by the U.S. Small Business Administration. Policies of this type could be those that facilitate access to the competition for procurement contracts —like better publicity or direct assistance to prepare the process— or measures to provide more transparency of the whole process —which should diminish the importance of political connections. To model this type of policies, we reduce the parameter η_1 so that the model generates procurement size premium of 50%, as opposed to the

72% in the baseline calibration. That is, we solve for a new economy in which the procurement system gives relatively lower weight to firms' investment in b, making it easier for small firms to participate. We also change η_0 and Y_g so that the fraction of procurement firms m_g and total government expenditure $P_g Y_g$ remain unchanged.

We present the main results from this exercise in column (1) of Table 7, which shows the relative change of some relevant variables compared to their counterparts in the benchmark economy.[23] We use aggregate GDP in private good units as the main measure to assess the macroeconomic impact of the policy. We report two different measures of GDP: "nominal" GDP, which uses the relative price of procurement P_g in the reformed economy, and "real" GDP, which keeps the price P_g of the benchmark economy. We also report changes in the levels of capital in the two sectors as well as in the aggregate, together with changes in variables related to misallocation and aggregate TFP.

Aggregate output. We find that the reform increases nominal GDP by around 2.07%.[24] Since we are keeping $P_g Y_g$ constant in our experiment, this increase comes entirely from a 2.36% increase in Y_p. We find an increase in "real" GDP of 1.18%, which is lower than in "nominal" GDP. The reason is that the relative price of procurement P_g increases as a result of the policy experiment, making the provision of public goods by the government more costly than in the benchmark economy. We will come back to the increase in P_g below.

TFP vs. K. We can decompose the increase in "nominal" GDP into that coming from capital accumulation vs. TFP. We find that most of the increase in GDP (around 72%) is accounted for by an increase in the aggregate stock of capital K, which increases by 1.88%. In Section 7.3 below, we will provide more details that will help understand the evolution of the stock of capital. The rest is explained by an increase of 0.29% in aggregate "nominal" TFP, which is the result of a slight increase in TFP_p (0.02%), a large reduction in TFP_g (5.72%), and the above-mentioned increase in P_g. When keeping constant P_g to its value in the benchmark economy, our model predicts a decrease in "real" TFP (0.69%) due to the fall in TFP_g.

Why do TFP_p and TFP_g change? The increase in TFP_p is the result of the beneficial effects of procurement on wealth accumulation of small firms. In the new steady state, firms that had a relatively high MRPK_{ip} in the benchmark economy are more likely to be active in procurement, which allows them to accumulate more assets and hence operate with a higher level of capital in the private sector (see Section 6). This reallocation of procurement contracts towards relatively high MRPK_{ip} firms implies a reduction in the dispersion of MRPK_{ip} and hence in misallocation. The

[23]We report the difference not the relative change for variables that are already in shares, i.e., the percentage of procurement firms and the share of procurement in GDP, as well as for the interest rate r, the tax τ, and the parameters η_0 and η_1.

[24]This compares to a 12% increase in GDP of eliminating financial frictions in our economy (setting $\phi_a \to \infty$).

decrease in TFP_g is explained by a change in the selection of firms into procurement and hence by the change in the composition of procurement firms. In particular, procurement firms in this counterfactual economy have lower productivity s (6.47% less) and lower net worth a (13.90% less). As a result, procurement firms are less productive and more constrained (as reflected by a larger λ). This leads to the decline in TFP_g due to both lower s (lower first-best productivity) and lower a (more misallocation).

Why does P_g increase? As explained above, the relative price of public goods depends on the ratio of sectorial wedges times the inverse of relative sectorial TFPs, see equation (18). We have just seen that the ratio $\text{TFP}_p/\text{TFP}_g$ increases substantially, which raises P_g. In addition, because procurement firms are now more constrained on average, their average wedge $\overline{\text{MRPK}}_g$ also increases, which pushes P_g further up.

7.2 Counterfactual 2: Decreasing contract size

We perform a second experiment that consists in reducing the size of contracts to reach out to more firms, while keeping the same level of expenditure P_gY_g. This experiment is motivated by the fact that decreasing the size of procurement contracts as a tool to promote the participation of small firms is at the core of the European Commission's agenda for public procurement regulation. In practice, we solve for a counterfactual economy in which the fraction of firms from which the government buys, m_g, increases by 10 percentage points, i.e., from 3.8% to 13.8%.[25] We do so by increasing η_0 and adjusting Y_g so that P_gY_g remains unchanged. We also keep η_1 unchanged.

We show the results from running this policy experiment in column 3 of Table 7. In contrast to the previous counterfactual, we find that the reform reduces nominal GDP by around 2.68%. Out of this decline, around 85% is explained by a fall in aggregate capital and the rest by a decline in aggregate TFP. As in the previous counterfactual, we will explain the behavior of capital accumulation below (see Section 7.3).

Also different from the first counterfactual, the model predicts a decline in P_g, which means that the provision of public goods by the government becomes cheaper. As in the other counterfactual, $\text{TFP}_p/\text{TFP}_g$ goes up, which should raise P_g. However, P_g decreases in this counterfactual because the average wedge in the procurement sector $\overline{\text{MRPK}}_g$ decreases by 6.09% instead of increasing. As in the previous counterfactual, the policy allocates procurement contracts to smaller firms, which leads to a pool of procurement firms that are both less productive and have less net worth.

[25] This represents a large change in the average size of contracts. In the counterfactual economy, the average size of the contract is 27% of that in the benchmark economy. The European Commission is not explicit about by how much governments should decrease the size of the contracts: "[...] Such division could be done on a quantitative basis, making the size of the individual contracts better correspond to the capacity of SMEs [...]." (see the Public Sector Directive 2014/24/EU for details.)

Table 7. Counterfactuals

	Panel A: $\phi_g > \phi_p$			Panel B: $\phi_g = \phi_p$		
	(1) Benchmark	(2) Count 1	(3) Count 2	(4) Benchmark	(5) Count 1	(6) Count 2
Output						
Y_p	5.462	2.36%	-3.05%	5.365	-0.04%	-8.00%
Y_g	0.835	-7.42%	4.03%	0.636	-12.19%	18.78%
GDP	6.214	2.07%	-2.68%	6.092	0.38%	-7.04%
real GDP	6.214	1.18%	-2.19%	6.092	-1.07%	-4.80%
Capital						
K_p	21.385	2.34%	-3.37%	21.422	0.61%	-8.16%
K_g	2.710	-1.81%	6.50%	2.230	-5.70%	19.63%
$K_p + K_g$	24.094	1.88%	-2.26%	23.653	0.01%	-5.54%
Productivity						
TFP_p	0.255	0.02%	0.34%	0.250	-0.18%	0.17%
TFP_g	0.308	-5.72%	-2.31%	0.285	-7.70%	-0.71%
TFP	0.258	0.29%	-0.43%	0.254	1.00%	0.17%
real TFP	0.258	-0.69%	0.07%	0.260	-1.08%	0.78%
$\overline{\text{MRPK}}_p$	0.256	-0.01%	0.32%	0.251	-0.21%	0.17%
$\overline{\text{MRPK}}_g$	0.277	1.84%	-6.09%	0.327	6.27%	-16.28%
TFP_p gain	0.047	-0.37%	-7.39%	0.042	4.14%	-4.71%
TFP_g gain	0.033	2.10%	-21.45%	0.090	9.49%	-36.60%
Prices/tax						
P_g/P_p	0.899	8.01%	-3.87%	1.144	13.80%	-15.78%
r	0.050	0.000	0.001	0.049	0.000	-0.001
τ	-0.07	-0.008	0.005	-0.071	-0.002	0.026
Procurement						
% firms	0.038	0.000	0.100	0.038	0.000	0.100
Share GDP	0.121	-0.002	0.003	0.121	-0.002	0.008
η_0	0.209	-0.047	0.555	0.213	-0.057	0.602
η_1	0.527	-0.11	0.000	0.550	-0.128	0.000
ratio mean s	1.246	-6.47%	-0.96%	1.259	-7.97%	-2.60%
ratio mean a	1.727	-13.90%	-17.47%	1.902	-16.14%	-20.75%
ratio mean λ	2.973	8.74%	-36.72%	6.221	14.79%	-56.68%

Notes: Panel A of shows the results from running the two policy experiments under our baseline calibration, i.e., $\phi_g > \phi_p$. Panel B shows the results from running the experiments for the alternative calibration in which we impose that $\phi_g = \phi_p$. Columns (1) and (4) show the variables from the respective benchmark economies. Columns (2) and (5) show the results from running counterfactual 1, which consists in changing η_0 and η_1 so that the procurement premium decreases from 72% to 50% (while keeping the % of procurement firms equal to 3.8%) and changing the average size of contracts accordingly so that P_gY_g remains constant. Columns (3) and (6) show the results from running counterfactual 2, which consists in increasing η_0 so that the model generates a % of procurement firms of 13.8% and decreasing the average size of contracts accordingly so that P_gY_g remains constant, while keeping η_1 constant.

However, different from the previous counterfactual this policy also reduces the average contract size. This makes procurement firms less constrained (instead of more constrained as in the previous counterfactual) because they have a smaller demand

Table 8. Channels

	Panel A: Count. 1			
	(0)	(1)	(2)	(3)
	Benchmark	Step 1	Step 2	Full
Y_p	5.462	-0.12%	2.50%	2.36%
K	24.094	-0.14%	2.09%	1.88%
TFP	0.258	0.05%	0.10%	0.29%
GDP	6.214	-0.10%	2.19%	2.07%
r	0.050	0.000	0.000	0.000
	Panel B: Count. 2			
Y_p	5.462	-0.93%	5.06%	-3.05%
K	24.094	0.21%	5.22%	-2.26%
TFP	0.258	-1.01%	-0.73%	-0.43%
GDP	6.214	-0.81%	4.45%	-2.68%
r	0.050	0.000	0.000	0.001

Notes: This table shows the results from running different versions of our model. Columns (0) and (4) show the valuex of the variables both in our benchmark economy and in the new steady state. Column (1) refers to the "Short-run partial equilibrium effect." Column (2) refers to the "Long-run partial equilibrium effect." Importantly, in columns (1) and (2), we solve the model by using the η_0 and η_1 that we use to compute the new steady states, and adjust Y_g so that $P_g Y_g$ remains unchanged.

to serve (the average λ of procurement firms declines by 36%) and hence the wedge in the procurement sector is smaller.

7.3 Channels

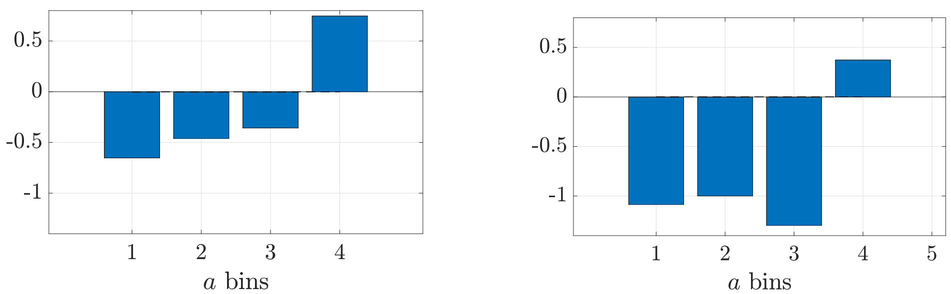
The main factor driving changes in aggregate GDP between the two steady states is capital accumulation. In the first counterfactual, a higher K explains around 72% of the rise in GDP. In the second counterfactual, a lower K explains around 85% of the fall in GDP. In this section, we provide more details about the specific mechanisms driving changes in K, as well as some other interesting effects that our model generates.

To do so, in Table 8 we solve for two different intermediate versions of the two counterfactual economies that aim to isolate the different channels at play. In column (0) we report again statistics of our benchmark economy, in columns (1) and (2) statistics of these intermediate exercises, and in column (3) we report again statistics of the complete counterfactual economy. We refer to this last column as "Full," capturing the idea that it contains all the different mechanisms we want to isolate in the previous columns.[26]

Short-run partial equilibrium effect. As we discussed in previous sections, a procurement shock ($d = 0 \rightarrow d = 1$) makes constrained firms decrease their output sold in the private sector at impact. This within-firm spillover manifests with the opposite

[26] Importantly, in columns (1) and (2), we solve the model by using the η_0 and η_1 that we use to compute the complete counterfactual, and adjust Y_g so that $P_g Y_g$ remains unchanged.

Figure 4. Static spillover effects



(a) % in $p_p y_p$ by bins of a (counterfactual 1)

(b) % in $p_p y_p$ by bins of a (counterfactual 2)

Notes: This figure shows the relative change (in %) in total $p_p y_p$ across equally-sized (in terms of total $p_p y_p$) groups of firms split according to the distribution of a as a result of a change in the procurement allocation system (step 1). Panel A and B refer to the first and second counterfactual exercises respectively.

sign when a firm becomes inactive in procurement ($d = 1 \rightarrow d = 0$). Our policy reforms reallocate procurement contracts across firms, and hence generate crowding out effects for some firms (constrained firms that start selling to the government), crowding in effects for others (constrained firms that stop selling or sell smaller contracts to the government), and no change for the rest (constrained firms unaffected by the policy or unconstrained firms).

We want to measure the aggregate effects of this short run crowding out/in effects. To do so, we solve the static firm's problem using the parameters that characterize the new procurement allocation system, i.e., the new η_0 and η_1, without taking any general equilibrium or dynamic effects into account. This means that (a) we keep Y_p, r, and τ as in the benchmark economy (and hence markets do not clear and the government's budget constraint is not satisfied), and (b) we keep the distribution of a and b unchanged.

In Figure 4, we provide some evidence on how these crowding out/in effects operate for different types of firms. In particular, we plot the relative change in total $p_p y_p$ across groups of firms with different levels of net worth a. We do so by ordering firms based on the benchmark distribution of a, splitting them in four groups so that each group accounts for 1/4 of the production of aggregate Y_p, and calculating the change in total $p_p y_p$ produced by each group, as caused by the procurement reform at impact. We find that the crowding out effect dominates within the first, second, and third bins of the distribution of a, and that the opposite is true for the fourth bin. This result reflects the fact that our policy reforms consists of reallocating procurement contracts from relatively big to relatively small firms. That is, firms with relatively lower a are more likely to be "new procurement firms" as a result of the policy change, whereas firms with relatively higher a are less likely to do procurement.

Looking at column (1) of Table 8, we find that the policy reform generates a short-run partial equilibrium fall in the private sector output Y_p and hence in GDP of the two counterfactuals. In particular if prices and the distributions of a and b were fixed, the policy reform would generate a fall in GDP of 0.10% in the first counterfactual of 0.81% in the second one.

Long-run partial equilibrium effect. Financially constrained firms that get procurement projects increase their revenues and can accumulate net worth at a faster pace and hence increase their private sector activity in the long run. In step 2, we quantify the aggregate effects of this strengthening of the self-financing mechanism. To do that, we solve for a new steady state distribution of a and s of our model under the new η_0 and η_1, but imposing that the policy functions of the dynamic problem $c(s, a, d)$, $a'(s, a, d)$ and $b(s, a, d)$, as a ratio of the entrepreneurs' cash on hand $(1+r)a+(1-\tau)\pi(s, a, d)$, remain unchanged. Our goal is to isolate the mechanical accumulation effect that income from procurement generates on directly affected firms, without taking into account adjustments in dynamic decisions. We also abstract from general equilibrium effects by using the same r and τ as in our benchmark economy. Column 2 of Table 8 shows a significant positive effect of this channel on the macroeconomy. In counterfactual 1, the implied capital stock and GDP are 2.09% and 2.19% higher than in the benchmark. In counterfactual 2, their counterparts are 5.22% and 4.45% higher. Hence, if we keep the policy functions of the dynamic problem and the interest rate unchanged, reforming the procurement allocation system in a way that favors small firms would generate a positive aggregate effect because it allows more constrained firms to accumulate more net worth, grow out of their financial constraints, and hence produce more in the long run. The enhancement of the self-financing channel is more effective in counterfactual 2, that is, in the policy experiment that reduces contract sizes to reach out to a higher number of firms.

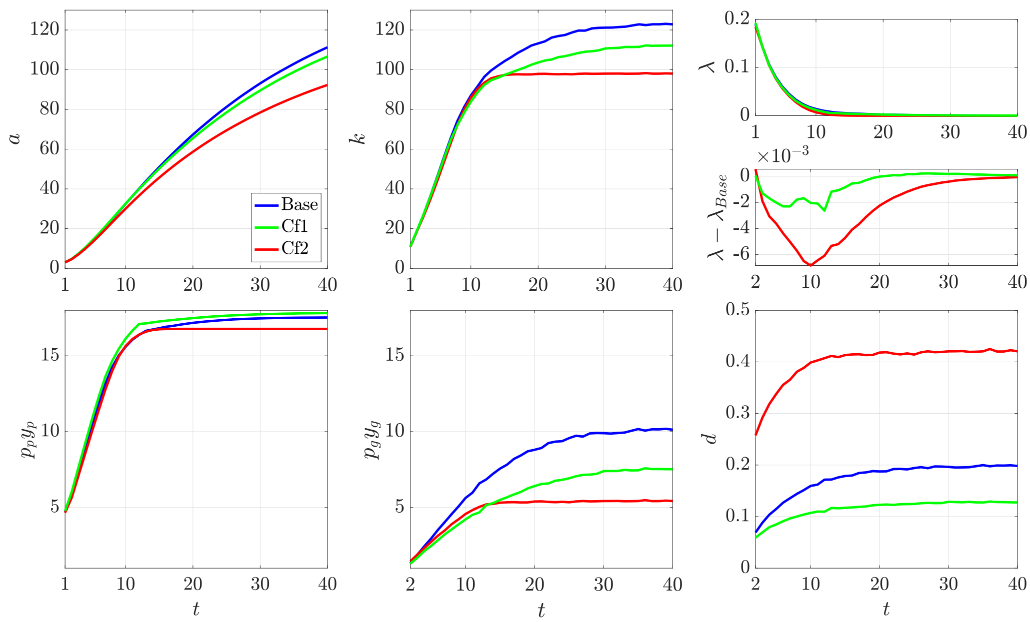
Full effect. Finally, we study the aggregate effect of the changes in the policy functions of the dynamic problem and prices in general equilibrium. We find that the reforms reduce the incentives for big firms to accumulate assets over time, which shrinks (in counterfactual 1) or even turns negative (in counterfactual 2) the output gains associated to the reforms.

One of the main reasons why firms accumulate assets in our model is the fact that they expect to obtain a public procurement contract at some point. That is, obtaining a procurement contract is a big demand shock in response to which firms want to expand their invested capital stock, causing even relatively big firms to accumulate precautionary savings. Intuitively, productive firms want to have enough net worth so that they minimize the probability of being constrained in case the procurement shock is realized. In a context in which the average contract size is lower or in which obtaining a contract is less likely, this precautionary savings motive becomes weaker.

In Figure 5, we show how this reduction in firms' incentives manifests in our model. This figure simulates the average life cycle profile of a cohort of firms in the benchmark economy (solid blue line), in the counterfactual 1 (dashed green line), and in the counterfactual 2 (dashed red line). In particular, we take a large number of newborn firms that draw the highest productivity state in every period and stochastically obtain procurement projects according to the probability $g(b)$ and their choices of b. We focus on firms that draw the highest productivity state in every period, to capture the fact that changes in saving incentives across the different economies will be specially apparent in firms that expect to operate at large scales.

We find that the three economies exhibit common patterns. As they age, firms become larger —accumulate more net worth, operate with more capital, and sell more both in the private and procurement sector— and less financially constrained. The panels for $p_g y_g$ and d show the differences in the procurement allocation systems across the three economies. The probability of participating in procurement, given

Figure 5. Firms' life cycle profiles



Notes: This figure shows the "average" life cycle profile of a large number of firms, all drawing the highest productivity level s in every period, simulated in our model under three different scenarios: the benchmark economy (blue line), the counterfactual 1 (green line), and the counterfactual 2 (red line). This particular figure uses firms with a productivity shock which is the highest among the five productivity shocks that we use to solve our model.

by d, is the highest in counterfactual 2 (there is a higher number of contracts) and the lowest in counterfactual 1 (the number of contracts is the same but high s firms are less likely to get them). In terms of the revenues from procurement, $p_g y_g$, the highest ones are under the benchmark economy, despite the fact that the probability of getting a contract is significantly higher in the counterfactual 1. This is driven by the fact that contracts are considerably bigger in the benchmark.

The most important finding from this figure has to do with the evolution of a and k over the firms' life cycle. We find that net worth accumulation by high s firms is the highest in the benchmark economy. In counterfactual 1, these firms prefer to accumulate slightly less because, although the size of contracts is still big, it is less likely for high s firms to obtain them.[27] The big difference becomes visible in counterfactual 2, where firms' net worth accumulation is significantly lower. This also becomes apparent when looking at the evolution of k. In the counterfactual 2 economy, firms reach their optimal size at an age that is considerably earlier than in the other two economies. To provide some intuition on this result, let's go back to the Euler equation given by equation (14). The strength of the precautionary savings motive (or self financing channel) is given by the term $\frac{\partial \pi(s',a',d')}{\partial a'}$, which is equal to $\phi_a \lambda'$ (see Online Appendix B.2 and B.3). That is, the expected value of the financial constraint multiplier represents an extra return to asset accumulation. In other words, firms that expect to be financially constrained next period will accumulate more assets today.

The panel $\lambda - \lambda_{Base}$ in Figure 5 compares the λ's in the two counterfactual economies with the one in the benchmark economy. We find that the λ's tend to be smaller in the two counterfactuals for high s firms, and particularly so in counterfactual 2. Importantly, these differences become bigger as firms approach their optimal size in the counterfactual from below, which points towards the fall in incentives to accumulate assets being particularly high for relatively bigger firms. This is driven by the fact that, in counterfactual 2 the procurement contracts are smaller so firms do not need much financial capacity to expand in order to service them, and in counterfactual 1 the old high productivity firms become less likely to win procurement contracts as these are being reallocated to firms with lower a and s.

7.4 The importance of $\phi_g > \phi_p$

In our final exercise we want to show the quantitative importance of the fact that revenues from public procurement help obtain credit to a larger extent than revenues from the private sector ($\phi_g > \phi_p$). To do so, in this section we compare the macroeconomic effects of the policy reforms in a world where $\phi_g = \phi_p$. We apply the same calibration strategy presented in Section 5, but imposing that $\varphi_p = \varphi_g$ both equal to the value of $\varphi_p = 0.42$ in the baseline, and ignore the targets associated with φ_p and φ_g in the calibration. In column 2 of Table 5, we show that most of the parameters are similar to those found in our baseline calibration. Because the model has to generate the same credit-to-capital ratio as before and ϕ_g is lower by construction, ϕ_a must be higher, mechanically increasing ϕ_p.

In columns 4, 5, and 6 of Table 7, we show the benchmark economy and its associated counterfactual exercises for this new calibration. Comparing the benchmark economy in both cases we can understand the role of ϕ_g in the aggregate economy. There are three main results. First, as discussed in Section 4.9, there is no within-firm misallocation when $\phi_g = \phi_p$, that is, the fraction of output sold to the private

[27] Instead, in counterfactual 1, all firms with s lower than the highest state, accumulate *more* net worth compared to the baseline (this is not shown in the paper, but it is available upon request).

and public sectors does not depend on the financial situation of the firm. Second, whenever $\phi_g = \phi_p$ the pattern of selection into procurement in terms of net worth a is more acute. This is because it is harder to finance procurement with lower ϕ_g, and hence procurement becomes relatively more attractive for firm with more financing capacity. And third, with $\phi_g = \phi_p$ the public good become more expensive relatively to private good. This is the result of an increase in the two components of P_g/P_p. The ratio $\text{TFP}_p/\text{TFP}_g$ increase slightly because there is more misallocation and hence lower TFP in the government sector due to firms operating in procurement being more financially constrained. The ratio $\overline{\text{MRPK}}_g/\overline{\text{MRPK}}_p$ increases mainly due to the loss of within-firm misallocation, which decreases capital in the procurement sector for all firms.

Regarding the procurement reforms, our main finding is that if government contracts were equally pledegable as revenues from selling to the private sector, changes in the procurement system that facilitate the presence of small firms would be associated with worse macroeconomic outcomes. In the case of keeping the average size of contracts but increasing the strength of diminishing returns to b, i.e., counterfactual 1, the increase in nominal GDP would be around 1.69 percentage points smaller. In the case of reducing the average size of contracts, i.e., counterfactual 2, we find that the fall in nominal GDP would be more than twice as big as in the baseline calibration. In fact, we find that GDP would also fall as a result of counterfactual 1 when we measure the change in GDP in real terms. The reason for these results is as follows. When $\phi_g = \phi_p$, the private sector negative spillover of procurement in the short run is larger because there is no extra financing through public revenues to alleviate the problem of scarce collateral; see Proposition 13 in Online Appendix B. In addition, by reducing the extent to which borrowing capacity increases when participating in procurement, the long-run positive effects also weaken. Overall, procurement is less effective in helping constrained firms increase their production.

8 Conclusion

In this paper, we quantify the macroeconomic impact of changes in the public procurement allocation system. To do so, we use a comprehensive framework that builds on three steps: selection, treatment, and the interplay between procurement and the macroeconomy. We use our framework to evaluate some of the policy reforms that are currently implemented in the US or are in the industrial policy agenda of the European Commission. In particular, we quantify the long-run macroeconomic effects of a size-dependent expenditure-neutral policy reform that consists of facilitating small firms' participation by either targeting them in the allocation process or by breaking down big projects into smaller ones.

Our results point towards the presence of long-run positive effects for directly affected firms, but also suggest the existence of important changes in big firms' dynamic behaviors that could shrink the expansionary effects or even make them negative. Our findings show that both the sign and size of these effects and hence the overall macroeconomic impact of this type of policies crucially depends on the severity and type of financial frictions in the economy. But they also depend on the type of reform, which determines how the change in procurement harms larger firms. These findings suggest that the optimal procurement allocation system in a country would depend on the specific institutional characteristics of the economy.

We view our contribution as part of a broader research agenda on the macroeconomic effects of government procurement, a policy that is surprisingly understudied. In our work, we only investigate the long-run consequences of expenditure-neutral changes in the procurement allocation system. Issues like the short-term consequences of reforms, or the potential implications for the effectiveness of fiscal policy are still unexplored. We emphasize that pursuing this research agenda will deliver important policy implications.

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A Details on the data

A.1 Public procurement in National Accounts

According to the System of National Accounts (SNA), "Government consumption expenditures and gross investment", i.e., G, measures the fraction of GDP, or final expenditures, that is accounted for by the government sector. In that respect, the government is treated as a consumer/investor. In addition, the SNA treats the government as a producer that uses labor, capital, and intermediate goods to provide its own consumption and investment. The total value of this output, which equals G, is measured by the total cost incurred:

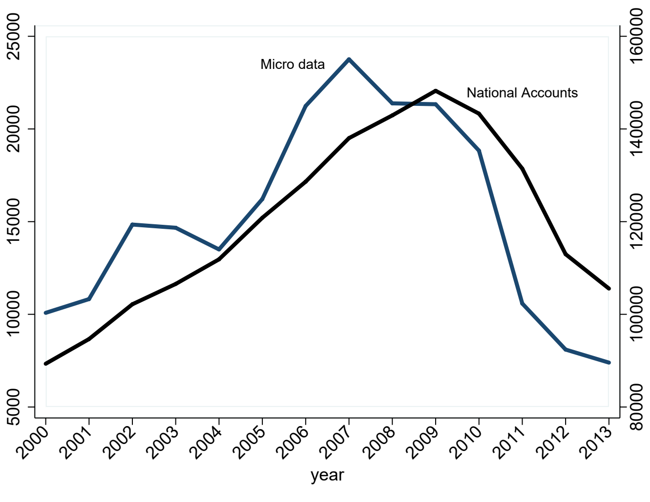
$$\begin{aligned} G &= \text{Gross Output of General Government} + \text{Gross Investment} \\ &= \text{Value Added} + \underbrace{\text{Intermediate goods and services} + \text{Structures} + \text{Equipment}}_{\text{public procurement}} + \text{IPP} \end{aligned}$$

Figure A.I shows the evolution of procurement value as measured with our micro data and compare it to the counterpart from national accounts. On average, our micro data accounts for around 13% of total government procurement as measured in Spanish national accounts. As apparent in the figure, our micro data reproduces well the cyclical aspect of pubic procurement expenditure, increasing during the boom and decreasing during the recession.

A.2 Public procurement data

Public procurement is defined in the *System of National Accounts (SNA)* as the sum of intermediate consumption (e.g., purchases of goods like medical consumables and services like accounting services), gross capital formation (e.g., building new roads), and social transfers in kind via market producers (e.g., medicines). Roughly speaking, one can think of public procurement as "government consumption expenditures and

Figure A.I. Evolution of Public Procurement in Spain, 2000-13



Notes: This figure shows the evolution of public procurement in Spain over 2000-13. The blue line ("Micro data", left y-axis) is computed by aggregating the individual projects scraped from the BOE, https://www.boe.es/. The black line ("National accounts", right y-axis) is measured from Spanish national accounts.

gross investment" (the G part of GDP) minus "compensation of employees" and "consumption of fixed capital." The size of public procurement varies across countries and over time. For the case of OECD countries, public procurement represented approximately 12% of GDP and 30% of G averaged over the 2007-2017 period.

Main sample of projects published in BOE. According to Spanish law, all procurement contracts above a certain threshold awarded by public institutions must be published in official bulletins.[1] If the contract is awarded by the central government, the information on this contract must be published in the *Agencia Estatal Boletín Oficial del Estado* (BOE), which is the official bulletin of the central government of Spain. In contrast, if the entity that awards the contract is a regional government or a municipality, the information about this contract can alternatively be published at their respective regional or local bulletin.

We construct a novel dataset on Spanish public procurement contracts by scraping the BOE website over the 2000-2013 period. Each contract provides considerable information on each awarded project. In particular, we collect information on the type of contract (kind of good or service provided), the institution awarding the contract, the initial bidding and final price of the contract, the type of procedure used to allocate the contract, and the firm(s) that won the contract. In total, we scraped more than 150,000 projects over 2000-2013, which we assign to the month that the project was awarded. Of these, 130,633 projects have a value assigned to them that we were able to recover. The sum of all these projects totals around 220 billion euros. On average, our micro data account for around 13% of total public procurement as measured in Spanish National Accounts. Despite the level differences, our micro data are able to capture the overall evolution of public procurement over time, which increased from 9.9 to 13.8 percent between 2000 and 2009 and decreased from 13.8 to 10.0 percent between 2010 and 2013; see Figure A.I.

Small sample of projects with information on bidders. Although the BOE website provides detailed information about the characteristics of the contracts, it does not provide the identity of the firms that competed for the project but did not win. This is a limitation of our dataset because it does not allow us to construct a well-defined control group. To overcome this limitation, we construct a sample of procurement projects for which we have detailed information about the awarding process. Although we did not find any government agency that provided information about the awarding process during our main sample period (2000-2013), we could identify around 50 agencies that started providing detailed information about their projects starting in 2013. Putting all these agencies together, we were able to uncover the identity of the firms competing for the same projects as well as their final rankings for around 1,000 contracts over the 2013-2016 period.

A.3 Balance sheet data

We use the balance sheets and income statements of the quasi-universe of Spanish companies between 2000 and 2016, a dataset that is maintained by the Banco de España and taken from the Spanish Commercial Registry. For each firm and year, this dataset includes information on the firm's name, fiscal identifier, sector of activity (4-

[1] The thresholds above which the contract must be advertised in official bulletins depend on the type of contract. In the case of supplies and services, for example, the threshold is 60,000 euros.

digit NACE Rev. 2 code), age, net operating revenue, material expenditures, number of employees, labor expenditures, total fixed assets, total assets, and net worth. The final sample covers a total of 1,801,955 firms with an average of 993,876 firms per year. This represents around 85-90% of the firms in the non-financial market economy for all size categories in terms of both turnover and number of employees. This database is used by García-Santana et al. (2020) among others and is described in detail by Almunia et al. (2018).

A.4 Credit data

The *Central de Información de Riesgos* (CIR) is maintained by the Banco de España in its role as primary banking supervisory agency, and contains detailed monthly information on all outstanding loans over 6,000 euros to non-financial firms granted by all banks operating in Spain since 1984. Given the low reporting threshold, virtually all firms with outstanding bank debt appear in the CIR. In addition to the total amount of credit, CIR also contains information on whether or not a non-personal collateral ("Garantía real") was posted for a particular loan. These collaterals include assets like real estate, land, machinery, securities, deposits, and merchandise (i.e., hard collateral).[2] With this information, we can hence assess whether a particular loan for a bank-firm pair was granted on the basis of tangible collateral. We use data from 2000 to 2016.

Loan applications. Besides the information on outstanding loans, we also have information about loan applications at the firm-bank level. The construction of this dataset is as follows. Spanish banks can request information about a firm whenever this firm "seriously" approaches them to obtain credit.[3] Because banks already have information about the firms with which they have a credit relationship, banks only request information on firms that have never received a loan from them or that ended the credit relationship before the current request. By matching the loan applications with the information on outstanding loans from CIR, we can infer whether the loan was granted or not.[4]

[2] See Ivashina et al. (Forthcoming) for more details.

[3] The Law stipulates that a bank can not request information about the firm without its consent, which indicates the seriousness of the approach

[4] Both the CIR and loan application data provide the identity, i.e., fiscal identifier, of the firm involved in every loan, allowing us to easily match the loan data with the balance sheet and income statements of firms.

A.5 Types and size of procurement contracts

Table A.I provides descriptive evidence for the pool of projects and years between 2000 and 2013. In particular, we report statistics on the number of projects and distribution of projects' values, separately for the five broad sector categories reported by the BOE: construction, consulting, services, supplies, and others.

Table A.I. Value of Procurement projects (budget value in millions of euro), pool of years 2000-13

Sector	mean	10th	25th	50th	75th	99th	obs.
Construction	5.28	0.13	0.23	0.74	4.00	70.84	22,549
Consulting	0.66	0.10	0.17	0.37	0.84	3.91	12,427
Services	1.22	0.11	0.20	0.42	1.05	13.47	44,581
Supplies	0.95	0.10	0.17	0.37	0.86	10.20	45,552
Others	1.99	0.09	0.15	0.35	0.99	38.18	5,524

Notes: This table presents summary statistics on the size of procurement projects in our sample as measured by the budget value. All the numbers have been computed after trimming the top 1% of the projects in terms of value, which mostly correspond to typos in the numbers, e.g., displaced comma, reported in BOE.

A.6 Procurement firms across different industries

In Table A.II, we present summary statistics for the top 20 NACE 2-digit industries in terms of the fraction of firms in that industry that sell to the government. In column (1), we show the share of firms active in procurement in that industry. In columns (2-5), we show the share of employment, sales, fixed assets and credit accounted for by procurement firms in that industry. We provide the numbers for the year 2006, but the ranking and shares are similar for the rest of the years.

Table A.II. Importance of procurement firms, 2006

Sector	Description	Firms (1)	Emp. (2)	Sales (3)	Assets (4)	Credit (5)
19	Manufacture of coke & refined petroleum prod.	0.150	0.332	0.315	0.310	0.243
21	Manufacturing of Pharmaceutical Products	0.149	0.240	0.225	0.231	0.288
42	Civil Engineering	0.093	0.260	0.324	0.366	0.386
80	Security and investigation activities	0.064	0.198	0.299	0.269	0.312
30	Manufacturing of Transport Equipment	0.052	0.176	0.177	0.205	0.180
94	Activities of membership organisations	0.051	0.069	0.127	0.037	0.018
36	Collection, purification and distribution of water	0.040	0.116	0.117	0.088	0.121
61	Telecommunications	0.038	0.217	0.192	0.189	0.207
51	Air transportation	0.033	0.054	0.049	0.078	0.142
81	Services of Buildings Maintenance	0.031	0.137	0.232	0.151	0.211
63	Information services	0.026	0.127	0.100	0.080	0.087
62	Programming, consultancy, other IT activities	0.025	0.151	0.193	0.157	0.214
26	Manufacturing of IT, electronic, & optical prod.	0.025	0.087	0.095	0.125	0.165
71	Technical services of architecture & engineering	0.024	0.152	0.159	0.084	0.103
2	Forestry and logging	0.019	0.069	0.068	0.033	0.080
6	Extraction of crude petroleum and natural gas	0.017	0.021	0.036	0.016	0.026
91	Libraries, archives, museums and cultural activities	0.016	0.061	0.051	0.021	0.017
29	Manufacture of motor vehicles and trailers	0.015	0.030	0.036	0.030	0.086
72	R&D activities	0.014	0.017	0.014	0.003	0.003
17	Paper industry	0.014	0.033	0.032	0.038	0.067

Notes: This table presents summary statistics on the number of firms, employment sales, and credit for the year 2006 at the 2-digit sectors. 'Firms' refers to the share of procurement firms, 'Emp.', 'Sales', 'Assets' , and 'Credit' are the share of employment, sales, assets and credit accounted for by procurement firms.

A.7 Procurement vs. non-procurement firms

Table A.III. Descriptive evidence from the final merged dataset, year 2006

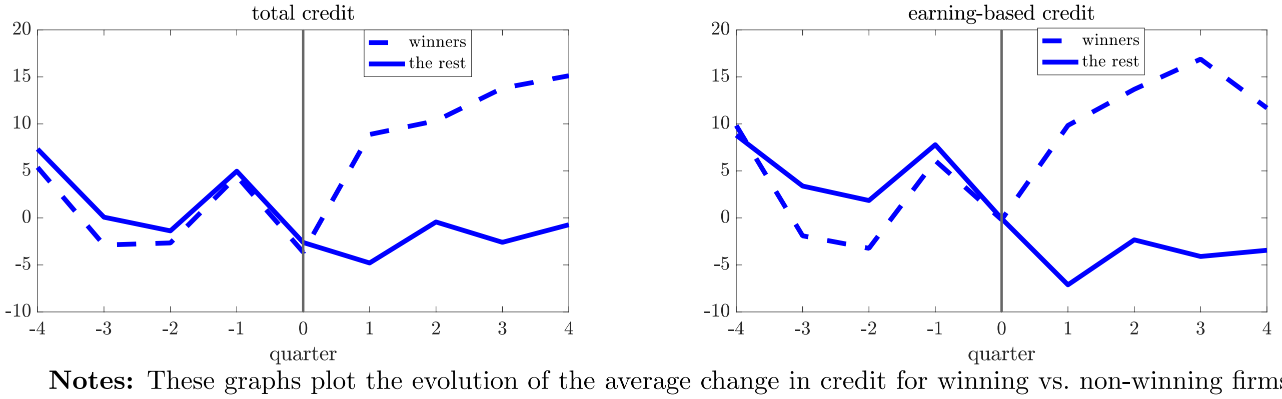
	mean		**25th pctile**		**50th pctile**		**75th pctile**	
	Proc	NoProc	Proc	NoProc	Proc	NoProc	Proc	NoProc
Age	20.42	10.95	12.00	5.00	17.00	10.00	24.00	15.00
Employment	73.56	12.75	16.00	3.00	45.00	6.00	155.0	12.00
Sales	8.96	1.19	1.14	0.10	4.22	0.28	16.89	0.86
Procurement/Sales	0.20	0.00	0.01	0.00	0.03	0.00	0.10	0.00
Fixed Assets	3.80	0.85	0.21	0.03	0.82	0.14	3.58	0.50
Net worth	3.92	0.43	0.36	0.01	1.27	0.07	6.12	0.30
Credit	2.51	0.57	0.11	0.03	0.48	0.08	2.32	0.30
Coll. Credit (share)	0.14	0.29	0.00	0.00	0.00	0.00	0.14	0.74

Notes: This table presents summary statistics from our merged dataset for the year 2006, separately for firms with at least one procurement contract ($n = 2,411$) vs. the rest of the firms ($n = 406,261$). The variable *Employment* measures the number of full-time workers employed by the firm; the variable *Sales* is just firm's revenue measured in millions of euro; *Procurement/Sales* measures the value of all the procurement projects awarded to a firm in a given year divided by total revenue in that year; *Assets* measures the value of fixed assets; *net worth* measures total assets minus total debt; *Credit* measures the value of all firm's outstanding loans in millions of euro; *Coll. Credit (share)* is the share of *Credit* collateralized against firm's assets; *Def. Credit (share)* is the share of defaulted credit over total *Credit*; *age* measures the age of the firm. We winsorize the 1% tails of all variables to make numbers result to outliers.

A.8 Pre-trends for winners vs. the rest

Graphically, the right panel in Figure A.II shows the average growth of credit without collateral of firms that win a procurement project in quarter 0 before and after winning the project, and compares it to the rest of firms. Again, there is a similar evolution of credit growth before procurement (parallel trends) and a clear (and persistent) divergence after that.

Figure A.II. Credit Growth: bidders sample



Notes: These graphs plot the evolution of the average change in credit for winning vs. non-winning firms, before and after the quarter in which the auction takes place (Quarter=0). The left panel is for all credit. The right panel is for non-collateral credit only.

A.9 Robustness: Procurement and Credit across the Firm Size Distribution

Table A.IV. Credit Growth and Procurement: Estimation across the Firm Distribution

	Assets (1)	Employment (2)	Net Worth (3)	Age (4)
Quartile 1	0.057	0.028	0.040	0.029
	(0.009)	(0.008)	(0.008)	(0.008)
Quartile 2	0.040	0.065	0.054	0.039
	(0.009)	(0.009)	(0.009)	(0.009)
Quartile 3	0.051	0.058	0.049	0.068
	(0.009)	(0.009)	(0.009)	(0.009)
Quartile 4	0.070	0.070	0.071	0.065
	(0.009)	(0.009)	(0.009)	(0.009)

Notes: This table presents results from estimating the relationship between total credit growth and procurement participation (PROC) by regression (1) with firms obtaining at least one procurement project over 2000-13. Only the coefficient of the PROC dummy is present for each quartiels of the firm distribution based on: (1) total assets, (2) employment, (3) net worth, and (4) age. All regressions use quarterly data. Standard errors are clustered at the firm level and all coefficients are significant at the 1% level.

B Details on the static production problem

In this Appendix we characterize the solution of the static production problem. First, in Section B.1 we derive the results that serve to restrict the parameters ϕ_p and ϕ_g such that the problem is well-behaved. Then, in Section B.2 we characterize analytically the solution to the production problem for firms without procurement ($d = 0$), which is useful to understand the interaction of asset based and earnings based financial constraints. Next, in Section B.3 we characterize analytically some of the solutions to the production problem for firms with procurement ($d = 1$) for the case $\sigma_p = \sigma_g$. Finally, in Section B.4 we show analytically the effect of a procurement shock for the case $\sigma_p = \sigma_g$, that is, the differences in allocations and profits between a firm with $(s, a, d = 1)$ and a firm with $(s, a, d = 0)$.

Before going to all these results, we start the Appendix by rewriting the FOC of the static production problem as follows. First, note that because the FOC for u, equation (11), states that the marginal revenue per unit of output sold —including its value as collateral— has to be equalized across the two sectors, and using the fact that $\frac{\partial p_p y_p}{\partial k} / \frac{\partial p_p y_p}{\partial u} = u/k$ and $\frac{\partial p_g y_g}{\partial k} / \frac{\partial p_g y_g}{\partial (1-u)} = (1-u)/k$ we can write the FOC for k, equation (12), as,

$$\frac{\partial p_p y_p}{\partial k}\frac{1}{u} = \frac{\partial p_p y_p}{\partial k_p} = \frac{r + \delta + \lambda}{1 + \lambda \phi_p} \tag{B.1}$$

or as

$$\frac{\partial p_g y_g}{\partial k}\frac{1}{1-u} = \frac{\partial p_g y_g}{\partial k_g} = \frac{r + \delta + \lambda}{1 + \lambda \phi_g} \tag{B.2}$$

or combining them both,

$$\frac{\partial\left[p_p y_p + p_g y_g\right]}{\partial k} = u\left(\frac{r+\delta+\lambda}{1+\lambda\phi_p}\right) + (1-u)\left(\frac{r+\delta+\lambda}{1+\lambda\phi_g}\right)$$

That is, the revenue marginal product of capital in each sector is equal to the capital cost of each sector and the revenue marginal product of capital for the whole firm is a weighted average of the capital costs in the two sectors, with the weights given but the cost shares of each sector.

It will be useful later on to use the actual revenue functions and substitute in equations (B.1) and (B.2) to obtain,

$$\left(\frac{\sigma_p - 1}{\sigma_p}\right)\frac{p_p y_p}{k}\frac{1}{u} = \frac{r+\delta+\lambda}{1+\phi_p\lambda} \tag{B.3}$$

$$\left(\frac{\sigma_g - 1}{\sigma_g}\right)\frac{p_g y_g}{k}\frac{1}{1-u} = \frac{r+\delta+\lambda}{1+\phi_g\lambda} \tag{B.4}$$

and using the production function one can write them as

$$\left(\frac{\sigma_p - 1}{\sigma_p}\right)p_p\, s = \frac{r+\delta+\lambda}{1+\lambda\phi_p} \tag{B.5}$$

$$\left(\frac{\sigma_g - 1}{\sigma_g}\right)p_g\, s = \frac{r+\delta+\lambda}{1+\lambda\phi_g} \tag{B.6}$$

Finally, dividing these two equations we get an expression for the optimal relative prices,

$$\frac{p_p}{p_g} = \frac{1+\lambda\phi_g}{1+\lambda\phi_p}\frac{(\sigma_g-1)/\sigma_g}{(\sigma_p-1)/\sigma_p} \tag{B.7}$$

Note that whenever $\sigma_p = \sigma_g$, $p_g/p_p = 1$ for firms without binding financial frictions ($\lambda = 0$). For firms with binding financial frictions ($\lambda > 0$) $p_g/p_p < 1$ ($p_g/p_p > 1$) whenever $\phi_g > \phi_p$ ($\phi_g < \phi_p$) because production is shifted towards the sector that provides better collateral, and $p_g/p_p = 1$ whenever $\phi_g = \phi_p$.

B.1 Some preliminary results

Lemma 1 *The terms $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ and $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$ describing the cost of capital for the production of the private sector and the public sector goods respectively, are (a) strictly below $1/\phi_p$ and $1/\phi_g$ respectively, (b) increasing in λ, and (c) strictly above $r+\delta$ when $\lambda > 0$, if and only if $\phi_p < (\delta + r)^{-1}$ and $\phi_g < (\delta+r)^{-1}$ respectively.*

Proof: Part (a) is straightforward:

$$\frac{r+\delta+\lambda}{1+\lambda\phi_p} < \frac{1}{\phi_p} \Leftrightarrow \phi_p\,(r+\delta+\lambda) < (1+\lambda\phi_p) \Leftrightarrow \phi_p\,(r+\delta) < 1 \Leftrightarrow \phi_p < (r+\delta)^{-1}$$

For part (b) note that

$$\frac{d}{d\lambda}\left(\frac{r+\delta+\lambda}{1+\lambda\phi_p}\right) \propto (1+\lambda\phi_p) - \phi_p\,(r+\delta+\lambda) > 0 \Leftrightarrow \phi_p\,(r+\delta) < 1 \Leftrightarrow \phi_p < (r+\delta)^{-1}$$

Finally, part (c) is proved by noting that $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ equals $r+\delta$ whenever $\lambda = 0$ and its derivative w.r.t. λ is positive, see part (b). The same arguments apply for $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$. ■

Proposition 1 *Holding s constant, more constrained firms sell less to the private sector, sell less to the public sector, and demand less capital if both* $\phi_p < (\delta + r)^{-1}$ *and* $\phi_g < (\delta + r)^{-1}$.

Proof: Let's combine the FOC (B.3) with the demand equation in (4) to produce the expression,

$$y_p = \left(\frac{\sigma_p - 1}{\sigma_p} B_p s \frac{1 + \lambda \phi_p}{r + \delta + \lambda} \right)^{\sigma_p}$$

Then, by virtue of Lemma 1 y_p falls with λ whenever $\phi_p < (\delta + r)^{-1}$. The case for y_g is analogous. Finally, note that total output is split between private sector and public sector sales, that is, $y_p + y_g = f(s, k) = sk$, so the derivative of capital with respect to λ is just,

$$\frac{dk}{d\lambda} = \frac{1}{s} \left(\frac{dy_p}{d\lambda} + \frac{dy_g}{d\lambda} \right)$$

which is negative given the previous results in this Proposition. ■

Lemma 2 *The optimal unconstrained capital for the private and the public sector respectively cannot be self-financed through its own revenues if and only if* $\phi_p \frac{\sigma_p}{\sigma_p - 1} (r + \delta) < 1$ *and* $\phi_g \frac{\sigma_g}{\sigma_g - 1} (r + \delta) < 1$ *respectively.*

Proof: The optimal unconstrained solution for the private sector capital is given by equation (B.3) when $\lambda = 0$, which implies $\frac{p_p y_p}{k} \frac{1}{u} = \frac{\sigma_p}{\sigma_p - 1} (r + \delta)$. When $\phi_p \frac{\sigma_p}{\sigma_p - 1} (r + \delta) < 1 \Leftrightarrow \frac{\sigma_p}{\sigma_p - 1} (r + \delta) < \phi_p^{-1}$ this leads to $\frac{p_p y_p}{k} \frac{1}{u} < \phi_p^{-1} \Leftrightarrow \phi_p p_p y_p < uk$, that is, the optimal unconstrained capital for the private sector, uk, cannot be self-financed through its own revenues. The proof for the public sector capital is analogous by use of the FOC (B.4) ■

Proposition 2 *Entrepreneurs with zero net worth are financially constrained if both* $\phi_p \frac{\sigma_p}{\sigma_p - 1} (r + \delta) < 1$ *and* $\phi_g \frac{\sigma_g}{\sigma_g - 1} (r + \delta) < 1$.

Proof: Note that if both $\phi_p \frac{\sigma_p}{\sigma_p - 1} (r + \delta) < 1$ and $\phi_g \frac{\sigma_g}{\sigma_g - 1} (r + \delta) < 1$, then following Lemma 2 both $\phi_p p_p y_p < uk$ and $\phi_g p_g y_g < (1 - u)k$. Adding them up leads to $\phi_p p_p y_p + \phi_g p_g y_g < k$, which implies that the capital of the unconstrained solution cannot be financed through revenue based constraints and hence entrepreneurs with zero net worth are constrained. ■

Lemma 3 *The term* $\phi_p \frac{\partial p_p y_p}{\partial k} + \phi_g \frac{\partial p_g y_g}{\partial k}$ *describing the share of capital that can be self-financed through revenues is positive and strictly smaller than one for constrained firms.*

Proof: That this term is positive is straightforward. To show that it is lower than one, note that for constrained firms the borrowing constrain in (9) holds with equality. Hence, for $a \geq 0$ it must be that $k \geq \phi_p p_p y_p + \phi_g p_g y_g$ or $\phi_p \frac{p_p y_p}{k} + \phi_g \frac{p_g y_g}{k} \leq 1$ (with strict equality for $a = 0$). Given our revenue function, the marginal products are proportional to the average products $\frac{\partial p_p y_p}{\partial k} = \left(\frac{\sigma_p - 1}{\sigma_p} \right) \frac{y_p p_p}{k}$ and $\frac{\partial p_g y_g}{\partial k} = \left(\frac{\sigma_g - 1}{\sigma_g} \right) \frac{y_g p_g}{k}$, so we can rewrite

$$\phi_p \frac{\partial p_p y_p}{\partial k} + \phi_g \frac{\partial p_g y_g}{\partial k} = \phi_p \left(\frac{\sigma_p - 1}{\sigma_p} \right) \frac{p_p y_p}{k} + \phi_g \left(\frac{\sigma_g - 1}{\sigma_g} \right) \frac{p_g y_g}{k}$$

Note that $\sigma_p > 1$ and $\sigma_g > 1$ implies that $\frac{\sigma_p - 1}{\sigma_p} < 1$ and $\frac{\sigma_g - 1}{\sigma_g} < 1$ (the marginal products are below the average products), and hence it is the case that $\phi_p \frac{\partial p_p y_p}{\partial k} + \phi_g \frac{\partial p_g y_g}{\partial k} < \phi_p \frac{p_p y_p}{k} + \phi_g \frac{p_g y_g}{k} \leq 1$ ■

Lemma 4 *The term $\phi_p \frac{\partial p_p y_p}{\partial u} + \phi_g \frac{\partial p_g y_g}{\partial u}$ describing the increase in credit that can be achieved by reallocation output to the private sector has the sign of $(\phi_p - \phi_g)$ for constrained firms.*

Proof: Using equation (11) we can write:

$$\phi_p \frac{\partial p_p y_p}{\partial u} + \phi_g \frac{\partial p_g y_g}{\partial u} = \frac{\partial p_p y_p}{\partial u} \left[\phi_p - \phi_g \frac{1 + \lambda \phi_p}{1 + \lambda \phi_g} \right] = \frac{\partial p_p y_p}{\partial u} \phi_g \left[\frac{\phi_p}{\phi_g} - \frac{\lambda^{-1} + \phi_p}{\lambda^{-1} + \phi_g} \right]$$

Note that with $\phi_p > \phi_g$ ($\phi_p < \phi_g$), this expression is positive (negative) when λ tends to zero, it decreases (increases) monotonically with λ, and tends to zero when λ tends to infinity. ■

B.2 Firms without procurement

We start analyzing the production problem for firms without procurement, that is, firms with $d = 0$.

B.2.1 Unconstrained firms

With $\lambda = 0$ the FOC for k in (12) becomes,

$$\frac{\partial p_p y_p}{\partial k} = r + \delta$$

which states that firms must equalize the marginal revenue product of capital to the cost of capital. This equation defines the optimal demand of capital $k^* (s, a, 0)$ for every entrepreneur of type $(s, a, d = 0)$. In particular, one gets $\frac{\sigma - 1}{\sigma} \frac{p_p y_p}{k} = r + \delta$ and substituting for the revenue function yields the optimal demand for capital

$$k^* (s, a, 0) = \left[\left(\frac{\sigma_p - 1}{\sigma_p} \right) \frac{B_p}{r + \delta} \right]^{\sigma} s^{\sigma - 1} \tag{B.8}$$

Next, note that profits are given by $\pi = p_p y_p - (r + \delta) k$, which given the optimal choice of capital can be written as $\pi = \frac{1}{\sigma_p} p_p y_p$ or $\pi = \frac{1}{\sigma_p - 1} (r + \delta) k$. Substituting optimal capital demand to the revenue function gives $p_p y_p = B_p \left[\left(\frac{\sigma_p - 1}{\sigma_p} \right) \frac{B_p}{r + \delta} \right]^{\sigma_p - 1} s^{\sigma_p - 1}$, which can be substituted back to the profit function to obtain:

$$\pi^* (s, a, 0) = \frac{1}{\sigma_p} \left[\left(\frac{\sigma_p - 1}{\sigma_p} \right) \frac{1}{r + \delta} \right]^{\sigma_p - 1} B_p^{\sigma} s^{\sigma_p - 1} \tag{B.9}$$

Hence, capital demand and profits increase monotonically with the shock s and are independent from net worth a.

B.2.2 Constrained firms

If the firm is constrained, then $\lambda > 0$ and the FOC of the problem are:

$$(1 + \lambda \phi_p) \frac{\partial p_p y_p}{\partial k} = r + \delta + \lambda \tag{B.10}$$

$$k = \phi_a a + \phi_p p_p y_p \tag{B.11}$$

which determine k and λ. In particular, the borrowing constraint, equation (B.11), defines the capital demand $k(s,a,0)$, the FOC, equation (B.10), delivers the shadow value of the constrain $\lambda(s,a,0)$, and the objective function delivers the profit function $\pi(s,a,0)$. The next propositions chacarcterize the derivatives of these three functions with respect to the state variables a and s.

Let's start by totally differentiating equation (B.11) in turns with respect to a and s to obtain,

$$\frac{\partial k(s,a,0)}{\partial a} = \phi_a \left(1-\phi_p \frac{\partial p_p y_p}{\partial k}\right)^{-1} \tag{B.12}$$

$$\frac{\partial k(s,a,0)}{\partial s} = \phi_p \frac{\partial p_p y_p}{\partial s}\left(1-\phi_p \frac{\partial p_p y_p}{\partial k}\right)^{-1} \tag{B.13}$$

With $\phi_p = 0$ we are in the case without earnings-based collateral constraints and these derivatives are just equal to ϕ_a and 0 respectively: higher net worth allows to operate with more capital but higher productivity does not. With $\phi_p > 0$ both derivatives are positive, that is, constrained firms with more net worth or higher productivity operate with more capital. Indeed, in this case $\frac{\partial k(s,a,0)}{\partial a} > \phi_a$ because an increase in net worth has a multiplier effect through the increase in revenues and the easing of the earnings-based financial constraint (see Lemma 3). This is stated in the next proposition:

Proposition 3 *The derivative of $k(s,a,0)$ with respect to a is positive, while the derivative of $k(s,a,0)$ with respect to s is positive as long as $\phi_p > 0$ (and zero othwewise).*

Proof: The derivatives of $k(s,a,0)$ with respect to a and s are given by equation (B.12) and (B.13). $\phi_a \geq 1$ and Lemma 3 states that $\phi_p \frac{\partial p_p y_p}{\partial k} < 1$, so the derivative with respect to a is strictly positive. For the derivative with respect to s, note additionally that $\frac{\partial p_p y_p}{\partial s} > 0$. Hence, this derivative is strictly positive (zero) if $\phi_p > 0$ ($\phi_p = 0$). ■

Note also that the derivatives of capital with respect to a and s are higher for more constrained firms (higher λ) because the multiplier effect of the earnings-based constraints is larger for firms with higher marginal product of capital, that is, the increase in capital demand with net worth a or productivity s is larger for more financially constrained firms. This is stated in the next corollary:

Corollary 1 *The derivatives of $k(s,a,0)$ with respect to a and with respect to s increase with λ*

Proof: The derivatives are characterized by equations (B.12) and (B.13). Using the FOC (B.10) and the fact that $\frac{\partial p_p y_p}{\partial s} = \frac{k}{s}\frac{\partial p_p y_p}{\partial k}$ we can further rewrite them as

$$\frac{\partial k(s,a,0)}{\partial a} = \phi_a \left(1-\phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}\right)^{-1} \tag{B.14}$$

$$\frac{\partial k(s,a,0)}{\partial s} = \phi_p \frac{k}{s}\frac{r+\delta+\lambda}{1+\lambda\phi_p}\left(1-\phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}\right)^{-1} \tag{B.15}$$

To prove this corollary it is enough to show that the term $(r+\delta+\lambda)/(1+\lambda\phi_p)$ in equations (B.32) and (B.33) increases with λ, which is proved in Lemma 1. ■

Next, equation (B.10) allows to recover $\lambda(s,a,0)$. It can be shown that $\lambda(s,a,0)$ declines with a —wealthier entrepreneurs can finance larger amounts of capital and are hence less constrained— and increases with s —s increases optimal capital by more than it increases the amount of capital that can be self-financed through revenues. This is stated formally in Proposition 4.

Proposition 4 *The derivative of $\lambda(s,a,0)$ with respect to a is always negative, while the derivative of $\lambda(s,a,0)$ with respect to s is always positive as long as $a > 0$ (and zero otherwise).*

Proof: Equation (B.10) can be rewritten as

$$\frac{\partial p_p y_p}{\partial k} = \frac{r+\delta+\lambda}{1+\lambda\phi_p}$$

The r.h.s, the cost of capital, increases with λ, see Lemma 1. Hence, the sign of the derivative of $\lambda(s,a,0)$ with respect to a or s is equal to the sign of the derivative of $\frac{\partial p_p y_p}{\partial k}$ with respect to a or s. We start by obtaining an expression of the marginal revenue product of capital by use of the revenue function:

$$\frac{\partial p_p y_p}{\partial k} = \frac{\sigma_p-1}{\sigma_p}\frac{p_p y_p}{k} = \frac{\sigma_p-1}{\sigma_p}B_p s^{\frac{\sigma_p-1}{\sigma_p}} k^{-\frac{1}{\sigma_p}}$$

where $\frac{\partial p_p y_p}{\partial k}$ declines with $k(s,a,0)$. For net worth a it is straightforward to see that $\lambda(s,a,0)$ declines with a because $k(s,a,0)$ increases with a, see Proposition 3. For the shock s we take the derivative of the marginal revenue product of capital w.r.t. s, and asking it to be non-negative delivers:

$$\frac{\partial^2 p_p y_p}{\partial k \partial s} \propto \left[(\sigma_p-1) - \frac{\partial k}{\partial s}\frac{s}{k}\right] \geq 0$$

where the first term reflects the positive direct effect of s on the marginal revenue product of capital for fixed capital, while the second term reflects the negative indirect effect of s on the marginal revenue product of capital through its induced increase in the choice of capital. Using $\frac{\partial p_p y_p}{\partial s} = \frac{k}{s}\frac{\partial p_p y_p}{\partial k}$, equation (B.13) shows that

$$\frac{\partial k}{\partial s}\frac{s}{k} = \phi_p \frac{\partial p_p y_p}{\partial k}\left(1-\phi_p\frac{\partial p_p y_p}{\partial k}\right)^{-1}$$

Then, we can rewrite

$$(\sigma_p-1) - \frac{\partial k}{\partial s}\frac{s}{k} \geq 0 \quad \Leftrightarrow \quad \phi_p\frac{\partial p_p y_p}{\partial k} \leq \frac{\sigma_p-1}{\sigma_p} \quad \Leftrightarrow \quad k \geq \phi_p p_p y_p$$

where the last step uses the fact that $\frac{\partial p_p y_p}{\partial k} = \frac{\sigma_p-1}{\sigma_p}\frac{p_p y_p}{k}$. Note that whenever a firm has zero net worth it will be able to self-finance capital up to the point $k = \phi_p p_p y_p$. In this case the derivative of $\lambda(s,a,0)$ with respect to s will be zero. Whenever a firm owns $a > 0$ then capital k is going to be above $\phi_p p_p y_p$ and the derivative of $\lambda(s,a,0)$ with respect to s will be positive.

■

Next, with Corollary 1 and Proposition 4, one can also show that $\frac{\partial^2 k(s,a,0)}{\partial a^2} < 0$ (the increase in capital due to an increase in net worth is larger for firms with less net worth) and that $\frac{\partial^2 k(s,a,0)}{\partial a \partial s} > 0$ (the increase in capital due to an increase in net worth is larger for firms with higher productivity), see Corollary 2.

Corollary 2 *The derivative of $\partial k(s,a,0)/\partial a$ with respect to a is always negative, while the derivative of $\partial k(s,a,0)/\partial a$ with respect to s is positive as long as $a > 0$ (and zero otherwise).*

Proof: By the chain rule we can write

$$\frac{\partial^2 k(s,a,0)}{\partial a^2} = \frac{\partial^2 k(s,a,0)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,0)}{\partial a}$$
$$\frac{\partial^2 k(s,a,0)}{\partial a \partial s} = \frac{\partial^2 k(s,a,0)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,0)}{\partial s}$$

The first derivative in the r.h.s. of these expressions is positive by Corollary 1. Hence, the sign of the derivatives $\frac{\partial^2 k(s,a,0)}{\partial a^2}$ and $\frac{\partial^2 k(s,a,0)}{\partial a \partial s}$ is the same as the sign of the derivatives $\frac{\partial \lambda(s,a,0)}{\partial a}$ and $\frac{\partial \lambda(s,a,0)}{\partial s}$ described in Proposition 4. ■

Finally, we can also characterize the derivatives of the profit function $\pi(s,a,0)$, which are given by

$$\frac{\partial \pi(s,a,0)}{\partial a} = \left[\frac{\partial p_p y_p}{\partial k} - (r+\delta)\right]\frac{\partial k(s,a,0)}{\partial a} \quad \text{(B.16)}$$
$$\frac{\partial \pi(s,a,0)}{\partial s} = \left[\frac{\partial p_p y_p}{\partial k} - (r+\delta)\right]\frac{\partial k(s,a,0)}{\partial s} + \frac{\partial p_p y_p}{\partial s} \quad \text{(B.17)}$$

We can substitute the partial derivatives of capital w.r.t. a and s described by (B.12) and (B.13) into equations (B.16) and (B.17) respectively. Then, using the FOC in (B.11) we obtain

$$\frac{\partial \pi(s,a,0)}{\partial a} = \phi_a \lambda(s,a,0) \quad \text{(B.18)}$$
$$\frac{\partial \pi(s,a,0)}{\partial s} = (1 + \phi_p \lambda(s,a,0))\frac{\partial p_p y_p}{\partial s} \quad \text{(B.19)}$$

Profits increase with a because more net worth allows to increase capital and hence profits. Profits increase with s because two reasons. First, there is the direct increase of revenues with s for given capital.

Second, if $\phi_p > 0$ a larger s implies higher revenues and hence more capital can be borrowed. Second, the increase in revenues with s allows to increase capital, which in turn increases profits. This is proved in the next Proposition:

Proposition 5 *The derivatives of $\pi(s,a,0)$ with respect to a and s are always positive.*

Proof: The derivatives of the profit function with respect to a and s are given by (B.18) and (B.19). These derivatives are positive because $\lambda(s,a,0) > 0$ for constrained agents and $\frac{\partial p_p y_p}{\partial s} > 0$ (see the revenue function). ■

Finally, we can also characterize the second derivatives of the profit function:

Corollary 3 *The derivative of $\partial \pi(s,a,0)/\partial a$ with respect to a is always negative, while the derivative of $\partial \pi(s,a,0)/\partial s$ with respect to s is always positive as long as $a > 0$ (and zero otherwise).*

Proof: Using equation (B.18) we can write the second derivatives as,

$$\frac{\partial^2 \pi(s,a,0)}{\partial a^2} = \phi_a \frac{\partial \lambda(s,a,0)}{\partial a}$$
$$\frac{\partial^2 \pi(s,a,0)}{\partial a \partial s} = \phi_a \frac{\partial \lambda(s,a,0)}{\partial s}$$

Then, one only needs to check the signs of the derivatives of λ in Proposition 4. ■

B.2.3 Binding constraints

Finally, we need to characterize the set of entrepreneurs that are financially constrained. Under Assumption 1, Proposition 2 says that $k(s,0,0) < k^*(s,0,0)$, and we have shown that $\frac{\partial k(s,a,0)}{\partial a} > 0$ and that $k^*(s,a,0)$ is invariant in a. Hence, for every s there will be a unique threshold $\underline{a}(s,0)$ satisfying $k(s,a,0) = k^*(s,a,0)$ such that for every s entrepreneurs with $a \geq \underline{a}(s,0)$ are unconstrained while entrepreneurs with $a < \underline{a}(s,0)$ are constrained.

B.3 Firms with procurement

We now analyze the production problem for firms with procurement, that is, firms with $d = 1$ for the case $\sigma_p = \sigma_g = \sigma$.

B.3.1 Unconstrained firms

With $\lambda = 0$ the FOC for k and u in (12) and (11) become

$$\frac{\partial p_p y_p}{\partial u} + \frac{\partial p_g y_g}{\partial u} = 0$$
$$\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} = r + \delta$$

which states that unconstrained firms allocate output between the two sectors to equalize the marginal revenues and choose capital such that the marginal revenue product of capital equals the capital costs. These two equations determine the optimal capital demand $k^*(s,a,1)$ and allocation of output in the private sector $u^*(s,a,1)$ for entrepreneurs of type $(s,a,d=1)$. In particular, the FOC for k can be written as $\frac{\sigma-1}{\sigma}\frac{p_p y_p + p_g y_g}{k} = r + \delta$. Substituting for the revenue functions yields the optimal demand for capital:

$$k^*(s,a,1) = \left[\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{r+\delta}\right]^{\sigma}\left(B_p^{\sigma} + B_g^{\sigma}\right)s^{\sigma-1} \quad \text{(B.20)}$$

Using the FOC for u one gets $\frac{p_p y_p}{ku} = \frac{p_g y_g}{k(1-u)}$ where again we can substitute the revenue functions to obtain:

$$u^*(s,a,1) = \left(1 + \left(\frac{B_g}{B_p}\right)^{\sigma}\right)^{-1} \quad \text{(B.21)}$$

Clearly $k^*(s,a,1)$ increases monotonically with the shock s and is invariant with the net worth a, while $u^*(s,a,1)$ is independent from both s and a and is only determined by the relative demands B_p/B_g. Next, note that profits are given by $\pi = p_p y_p + p_g y_g - (r+\delta)k$, which given the condition for the optimal choice of capital can be written as $\pi = \frac{1}{\sigma}(p_p y_p + p_g y_g)$ or $\pi = \frac{1}{\sigma-1}(r+\delta)k$. Substituting the optimal capital demand into the revenue function gives total revenues as $p_p y_p + p_g y_g = \left[\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{r+\delta}\right]^{\sigma-1}\left(B_p^\sigma + B_g^\sigma\right)s^{\sigma-1}$, which can be substituted back into the profit function to obtain

$$\pi^*(s,a,1) = \frac{1}{\sigma}\left[\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{r+\delta}\right]^{\sigma-1}\left(B_p^\sigma + B_g^\sigma\right)s^{\sigma-1} \tag{B.22}$$

The profit function increases with productivity s and is invariant with assets a.

B.3.2 Constrained firms.

For constrained firms with procurement, equations (11)-(13) jointly determine $k(s,a,1)$, $u(s,a,1)$, and $\lambda(s,a,1)$. The characerization of these functions is simple whenever $\phi_g = \phi_p$ and more involved when not. To characterize $u(s,a,1)$ let's start by noting that the FOC for u, given by equation (11), can be rewritten as in (B.7) and that after substituting prices we obtain,

$$\frac{u}{1-u} = \left(\frac{B_p}{B_g}\right)^\sigma\left(\frac{1+\lambda\phi_p}{1+\lambda\phi_g}\right)^\sigma \tag{B.23}$$

To characterize $k(s,a,1)$ we totally differentiate equation (13) with respect to a and s in turn, which gives,

$$\frac{\partial k}{\partial a} = \left[\phi_a + \left(\phi_p\frac{\partial p_p y_p}{\partial u} + \phi_g\frac{\partial p_g y_g}{\partial u}\right)\frac{du}{da}\right]\left[1-\left(\phi_p\frac{\partial p_p y_p}{\partial k} + \phi_g\frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \tag{B.24}$$

$$\frac{\partial k}{\partial s} = \left[\left(\phi_p\frac{\partial p_p y_p}{\partial s} + \phi_g\frac{\partial p_g y_g}{\partial s}\right) + \left(\phi_p\frac{\partial p_p y_p}{\partial u} + \phi_g\frac{\partial p_g y_g}{\partial u}\right)\frac{du}{ds}\right]\left[1-\left(\phi_p\frac{\partial p_p y_p}{\partial k} + \phi_g\frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \tag{B.25}$$

Finally, the derivatives of the profit function $\pi(s,a,1)$ are given by

$$\begin{aligned}\frac{\partial\pi(s,a,1)}{\partial a} &= \left[\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} - (r+\delta)\right]\frac{\partial k(s,a,1)}{\partial a} \\ &+ \left[\frac{\partial p_p y_p}{\partial u} + \frac{\partial p_g y_g}{\partial u}\right]\frac{\partial u(s,a,1)}{\partial a}\end{aligned} \tag{B.26}$$

$$\begin{aligned}\frac{\partial\pi(s,a,1)}{\partial s} &= \left[\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} - (r+\delta)\right]\frac{\partial k(s,a,1)}{\partial s} \\ &+ \left[\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k}\right]\frac{\partial u(s,a,1)}{\partial s} + \frac{\partial p_p y_p}{\partial s}\end{aligned} \tag{B.27}$$

Now, substituting (B.1), (B.2), and (B.24) into (B.26) and using (11) we obtain

$$\frac{\partial\pi(s,a,1)}{\partial a} = \phi_a\lambda(s,a,1) \tag{B.28}$$

while substituting (B.1), (B.2), and (B.25) into (B.27) and using (11) we obtain

$$\frac{\partial\pi(s,a,1)}{\partial s} = (1+\phi_p\lambda(s,a,1))\frac{\partial p_p y_p}{\partial s} + (1+\phi_g\lambda(s,a,1))\frac{\partial p_g y_g}{\partial s} \tag{B.29}$$

Profits increase with a because more net worth allows to increase capital and hence profits. Profits increase with s because two reasons. First, there is the direct increase of revenues with s for given capital. Second, if $\phi_p > 0$ and/or $\phi_g > 0$ the increase in revenues with s allows to increase capital, which in turn increases profits.

For the case $\phi_g = \phi_p$ it can be shown that $u(s,a,1) = u^*(s,a,1)$ —as revenues from both sectors are equally pledgeable— and hence $u(s,a,1)$ is invariant in a and

s. This makes the problem analogous to the case without procurement ($d = 0$), and hence the derivatives of $k(s,a,1)$, $\lambda(s,a,1)$, and $\pi(s,a,1)$ with respect to a and s are as in the $d = 0$ case. This can be seen in the next propositions.

Proposition 6 *When $\phi_g = \phi_p$, the optimal choice of $u(s,a,1)$ is as in the unconstrained case and it is hence independent from a and s*

Proof: Equation (B.23) clearly shows that whenever $\phi_g = \phi_p$ the optimal solution for u for constrained firms is equal to the one for unconstrained firms, see equation (B.21). This means that $u(s,a,1)$ is independent from s and a and only determined by the relative demands B_p/B_g of each sector. ■

Proposition 7 *When $\phi_g = \phi_p$, the derivative of $k(s,a,1)$ with respect to a is positive, while the derivative of $k(s,a,1)$ with respect to s is positive as long as $\phi_p > 0$ (and zero othwewise).*

Proof: Note that with $\phi_g = \phi_p$ the optimality condition (11) implies that $\frac{\partial p_g y_g}{\partial u} = -\frac{\partial p_p y_p}{\partial u}$ and hence we can rewrite equations (B.24) and (B.25) as follows,

$$\frac{\partial k}{\partial a} = \phi_a \left[1 - \phi_p\left(\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \tag{B.30}$$

$$\frac{\partial k}{\partial s} = \phi_p\left(\frac{\partial p_p y_p}{\partial s} + \frac{\partial p_g y_g}{\partial s}\right)\left[1 - \phi_p\left(\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \tag{B.31}$$

Given $\phi_a \geq 1$ and $\phi_p > 0$ both $\partial k/\partial a$ and $\partial k/\partial s$ are positive because of Lemma 3. If $\phi_p = 0$ then $\partial k/\partial s = 0$. ■

Corollary 4 *When $\phi_g = \phi_p$, the derivatives of $k(s,a,1)$ with respect to a and with respect to s increase with λ*

Proof: Equation (12) can be written as,

$$\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} = \frac{r + \delta + \lambda}{1 + \lambda\phi_p}$$

Then, using the fact that $\frac{\partial p_p y_p}{\partial s} = \frac{k}{s}\frac{\partial p_p y_p}{\partial k}$ we can rewrite equations (B.30) and (B.31) as

$$\frac{\partial k(s,a,1)}{\partial a} = \phi_a \left(1 - \phi_p \frac{r + \delta + \lambda}{1 + \lambda\phi_p}\right)^{-1} \tag{B.32}$$

$$\frac{\partial k(s,a,1)}{\partial s} = \phi_p \frac{k}{s}\frac{r + \delta + \lambda}{1 + \lambda\phi_p}\left(1 - \phi_p \frac{r + \delta + \lambda}{1 + \lambda\phi_p}\right)^{-1} \tag{B.33}$$

To prove this corollary it is enough to show that the term $(r + \delta + \lambda)/(1 + \lambda\phi_p)$ in equations (B.32) and (B.33) increases with λ, which is proved in Lemma 1. ■

Proposition 8 *When $\phi_g = \phi_p$, the derivative of $\lambda(s,a,1)$ with respect to a is always negative, while the derivative of $\lambda(s,a,1)$ with respect to s is always positive as long as $a > 0$ (and zero otherwise).*

Proof: Note that the FOC for k_p is given by equation (B.1). Because u is invariant in a and s, see Proposition 6, the proof of Proposition 8 for the case $d = 0$ carries over. ■

Corollary 5 *When $\phi_g = \phi_p$, the derivative of $\partial k(s,a,1)/\partial a$ with respect to a is always negative, while the derivative of $\partial k(s,a,1)/\partial a$ with respect to s is positive as long as $a > 0$ (and zero otherwise).*

Proof: By the chain rule we can write

$$\frac{\partial^2 k(s,a,1)}{\partial a^2} = \frac{\partial^2 k(s,a,1)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,1)}{\partial a}$$
$$\frac{\partial^2 k(s,a,1)}{\partial a \partial s} = \frac{\partial^2 k(s,a,1)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,1)}{\partial s}$$

The first derivative in the r.h.s. of these expressions is positive by Corollary 4. Hence, the sign of the derivatives $\frac{\partial^2 k(s,a,1)}{\partial a^2}$ and $\frac{\partial^2 k(s,a,1)}{\partial a \partial s}$ is the same as the sign of the derivatives $\frac{\partial \lambda(s,a,1)}{\partial a}$ and $\frac{\partial \lambda(s,a,1)}{\partial s}$ described in Proposition 8. ■

Proposition 9 *When $\phi_g = \phi_p$, the derivatives of $\pi(s,a,1)$ with respect to a and s are always positive.*

Proof: The derivatives of the profit function with respect to a and s are given by (B.28) and (B.29). These derivatives are positive because $\lambda(s,a,1) > 0$ for constrained agents and $\frac{\partial p_p y_p}{\partial s} > 0$ and $\frac{\partial p_g y_g}{\partial s} > 0$ (see the revenue functions). ■

Corollary 6 *When $\phi_g = \phi_p$, the derivative of $\partial \pi(s,a,1)/\partial a$ with respect to a is always negative, while the derivative of $\partial \pi(s,a,1)/\partial s$ with respect to s is always positive as long as $a > 0$ (and zero otherwise).*

Proof: Using equation (B.28) we can write the second derivatives as,

$$\frac{\partial^2 \pi(s,a,1)}{\partial a^2} = \phi_a \frac{\partial \lambda(s,a,1)}{\partial a}$$
$$\frac{\partial^2 \pi(s,a,1)}{\partial a \partial s} = \phi_a \frac{\partial \lambda(s,a,1)}{\partial s}$$

Then, one only needs to check the signs of the derivatives of λ in Proposition 8. ■

The case $\phi_g > \phi_p$ is more involved because $u(s,a,1)$ changes with a and s. It can be shown that firms with more net worth are less constrained and hence run larger firms and sell a higher fraction of output to the private sector, which offers lower collateral value. More productive firms are able to run larger firms thanks to the earnings-based constraints but are more constrained —because their optimal capital is even larger— and hence sell a lower fraction of output to the private sector. This which means that firms with larger s sell a larger quantity to the pubic sector but they may either sell a larger or smaller quantity to the private sector. This is proved in the following propositions.

Lemma 5 *The sign of the derivative of u with respect to λ is the same as the sign of $(\phi_p - \phi_g)$, that is, more constrained firms shift their output relatively towards the sector whose revenues provide better colateral.*

Proof: Simply note that equation (B.23) implies that $du/d\lambda < 0$ when $\phi_g > \phi_p$ and the opposite when $\phi_g < \phi_p$. ■

Proposition 10 *When $\phi_g > \phi_p$, the derivatives of $u(s,a,1)$, $k(s,a,1)$, and $\lambda(s,a,1)$ with respect to a are positive, positive, and negative respectively,*

Proof: First note that, following Lemma 5, $du/d\lambda < 0$ when $\phi_g > \phi_p$ and that Proposition 1 says that $dk/d\lambda < 0$. That is, more constrained entrepreneurs tilt production towards the sector with higher collateral value and run smaller firms. Next, using the FOC (B.3) and (B.4), the demand equations (4), and the production function we can write,

$$k_p = \left(\frac{\sigma_p - 1}{\sigma_p} B_p \frac{1 + \lambda \phi_p}{r + \delta + \lambda} \right)^{\sigma_p} s^{\sigma_p - 1} \quad \text{and} \quad k_g = \left(\frac{\sigma_p - 1}{\sigma_p} B_g \frac{1 + \lambda \phi_g}{r + \delta + \lambda} \right)^{\sigma_p} s^{\sigma_p - 1}$$

Adding them up, and using the chain rule, let us express $\frac{\partial k}{\partial a}$

$$\frac{\partial k}{\partial a} = \frac{\partial k}{\partial \lambda} \frac{\partial \lambda}{\partial a}$$

Also, using equation (B.23) and the chain rule we can write

$$\frac{\partial u}{\partial a} = \frac{\partial u}{\partial \lambda} \frac{\partial \lambda}{\partial a}$$

These two expressions state that $\frac{\partial k}{\partial a}$ and $\frac{\partial u}{\partial a}$ should have the same sign because both k and u fall with λ. Given this, equation (B.24) implies that $\frac{\partial k}{\partial a} > 0$ and $\frac{\partial u}{\partial a} > 0$. To see why, recall that by Lemma 3 the denominator is positive. In addition, the term $\phi_p \frac{\partial p_p y_p}{\partial u} + \phi_g \frac{\partial p_g y_g}{\partial u}$ is negative whenever $\phi_g > \phi_p$ see Lemma 4. Hence, for $\frac{\partial k}{\partial a} < 0$ we would need $\frac{\partial k}{\partial u} > 0$. That is, given that higher a allows to increase capital through ϕ_a, for higher a to lead to lower capital it must be that entrepreneurs with higher a tilt production towards the sector with lower collateral value. But this would require the signs of $\frac{\partial k}{\partial a}$ and $\frac{\partial u}{\partial a}$ to be different. Instead, $\frac{\partial k}{\partial a} > 0$ can be obtained with $\frac{\partial u}{\partial a} > 0$. It follows that, because $\frac{\partial k}{\partial \lambda} < 0$ and $\frac{\partial k}{\partial a} > 0$, it must be the case that $\frac{\partial \lambda}{\partial a} < 0$. ■

Proposition 11 *When $\phi_g > \phi_p$, the derivatives of $u(s, a, 1)$, $k(s, a, 1)$, and $\lambda(s, a, 1)$ with respect to s are negative, positive, and positive respectively,*

Proof: First note that, following Lemma 5, $du/d\lambda < 0$ when $\phi_g > \phi_p$ and that Proposition 1 says that $dk/d\lambda < 0$. That is, more constrained entrepreneurs tilt production towards the sector with higher collateral value and run smaller firms. Next, by the chain rule (see proof of Proposition 10) we can write

$$\frac{dk}{ds} = \frac{\partial k}{\partial \lambda} \frac{\partial \lambda}{\partial s} + \frac{\partial k}{\partial s} \quad \text{and} \quad \frac{du}{ds} = \frac{\partial u}{\partial \lambda} \frac{\partial \lambda}{\partial s}$$

We learn two things from here. First, $\frac{dk}{ds} \leq 0$ requires $\frac{\partial \lambda}{\partial s} > 0$ (because $\frac{\partial k}{\partial s} > 0$ and $\frac{\partial k}{\partial \lambda} < 0$). Second, $\frac{\partial \lambda}{\partial s} > 0$ requires $\frac{du}{ds} < 0$ (because $du/d\lambda < 0$). But equation (B.25) shows that if $\frac{du}{ds} < 0$ then it must be $\frac{dk}{ds} > 0$ so this enters a contradiction. Therefore, $\frac{dk}{ds} > 0$. Note that from equation (B.25) $\frac{dk}{ds} > 0$ can be achieved with any sign of $\frac{du}{ds}$. Now, regarding the derivatives of $u(s, a, 1)$ and $\lambda(s, a, 1)$ with respect to s, two different things can happen. If $\frac{\partial \lambda}{\partial s} \geq 0$ then $\frac{du}{ds} \leq 0$ (this is an if and only if statement), and then $\frac{dk}{ds} > 0$ according to equation (B.25). Instead, if $\frac{\partial \lambda}{\partial s} < 0$ then $\frac{du}{ds} > 0$ (again an if and only if statement) and we can have both $\frac{dk}{ds} > 0$ or $\frac{dk}{ds} < 0$ according to equation (B.25). ■

Proposition 12 *When $\phi_g > \phi_p$, the derivatives of $\pi(s, a, 1)$ with respect to a and s are always positive.*

Proof: The derivatives of the profit function with respect to a and s are given by (B.28) and (B.29). These derivatives are positive because $\lambda(s,a,1) > 0$ for constrained agents and $\frac{\partial p_p y_p}{\partial s} > 0$ and $\frac{\partial p_g y_g}{\partial s} > 0$ (see the revenue functions). ■

Corollary 7 *When $\phi_g > \phi_p$, the derivative of $\partial\pi(s,a,1)/\partial a$ with respect to a is always negative, while the derivative of $\partial\pi(s,a,1)/\partial s$ with respect to s is always positive as long as $a > 0$ (and zero otherwise).*

Proof: Using equation (B.28) we can write the second derivatives as,

$$\begin{aligned}\frac{\partial^2\pi(s,a,1)}{\partial a^2} &= \phi_a\frac{\partial\lambda(s,a,1)}{\partial a}\\ \frac{\partial^2\pi(s,a,1)}{\partial a\partial s} &= \phi_a\frac{\partial\lambda(s,a,1)}{\partial s}\end{aligned}$$

Then, one only needs to check the signs of the derivatives of λ in Proposition 10 and 11. ■

B.4 A procurement shock

Finally, in this Section we analyze how firm choices change upon arrival of a procurement project for the case $\sigma_p = \sigma_g = \sigma$. To do so, we compare the choices of firms in the $(s,a,1)$ state with firms in the $(s,a,0)$ state.

B.4.1 Unconstrained firms

For unconstrained firms, the increase in total capital is given by,

$$\frac{k^*(s,a,1)}{k^*(s,a,0)} = 1 + \left(\frac{B_g}{B_p}\right)^\sigma = \frac{1}{u^*(s,a,1)}$$

which implies that $u^*(s,a,1)\,k^*(s,a,1) = k^*(s,a,0)$. Hence, the amount of capital used in the private sector for the unconstrained firm with a procurement project equals the capital stock it was using without procurement. This means that unconstrained firms do not change their private sector operations and increase their capital stock to meet the extra demand. The increase in capital $k^*(s,a,1) - k^*(s,a,0)$ is given by $\left(\frac{B_g}{B_p}\right)^\sigma k^*(s,a,0)$. Because $k^*(s,a,0)$ increases with s and is independent from a, so does the capital increase with procurement.

We can also see that the value of a procurement contract increases with firm productivity s and is independent from firm net worth a. This can be seen by use of the expression $\pi = \frac{1}{\sigma-1}(r+\delta)\,k$, which implies that $\pi^*(s,a,1) - \pi^*(s,a,0)$ is proportional to the capital increase $k^*(s,a,1) - k^*(s,a,0)$. This could have also been seen by combining equations (B.9) and (B.22), which allows to express

$$\pi^*(s,a,1) - \pi^*(s,a,0) = \frac{1}{\sigma}\left[\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{r+\delta}\right]^{\sigma-1} B_g^\sigma s^{\sigma-1}$$

B.4.2 Constrained firms

The first thing to note is that a procurement shock worsens the financial situation of firms when $\phi_g \leq \phi_p$. With $\phi_g = \phi_p$ this is because the firm with $d = 1$ has two demands to serve, they are equally pledgeable, and has the same net worth a to finance capital in the two different markets. As a result the firm scales down the operations in the private sector to free up colateral for the production in the public sector, which generates a negative within-firm private sector spillover of the procurement contract, that is, $k_p(s,a,1) \equiv u(s,a,1)\,k(s,a,1) < k(s,a,0)$. When $\phi_g < \phi_p$ the situation is aggravated because the public sector demand can be self-financed to a lesser extent. When $\phi_g > \phi_p$ it could happen otherwise: the public sector demand can be self-financed to a larger extent, which means that for firms with small net worth it could happen that they are less constrained and use the extra financing capacity coming from the public sector to scale up operations in the private sector. This is stated in Proposition 13 below, but we first look at two preliminary results in Lemma 6 and 7.

Lemma 6 *A procurement shock generates a private sector negative spillover if and only if the procurement shock makes the firm more constrained, that is,* $k_p(s,a,1) < k(s,a,0) \Leftrightarrow \lambda(s,a,1) > \lambda(s,a,0)$

Proof: The FOC for the optimal choice of k_p for a firm with $d = 1$ is given by equation (B.1), where recall $\frac{\partial p_p y_p}{\partial k}\frac{1}{u} = \frac{\partial p_p y_p}{\partial k_p}$. The FOC for the optimal choice of k for a firm with $d = 0$ is given by the same equation (B.1) when $u = 1$. The right hand side of equation (B.1) increase with λ (see Lemma 1), so more constrained firms have higher marginal product of capital and a lower level of capital in the private sector. Hence, $k_p(s,a,1) < k(s,a,0) \Leftrightarrow \lambda(s,a,1) > \lambda(s,a,0)$ ■

Lemma 7 *A procurement shock generates a private sector negative spillover for constrained firms if and only if the chosen production for the public sector cannot be self-financed, that is, if and only if* $\phi_g p_g(s,a,1)\,y_g(s,a,1) < k_g(s,a,1)$

Proof: The demand for capital of constrained firms, with or without procurement, is given by equation (9), which allows to write,

$$\begin{aligned} k_p(s,a,0) - \phi_p p_p(s,a,0)\,y_p(s,a,0) &= \phi_a a \\ k_p(s,a,1) - \phi_p p_p(s,a,1)\,y_p(s,a,1) &= \phi_a a - \left[k_g(s,a,1) - \phi_g p_g(s,a,1)\,y_g(s,a,1)\right] \end{aligned}$$

Importantly, the left hand side of these equations increases with k_p. To see how, note that the derivative of the left hand side w.r.t. k_p is equal to $1-\phi_p\frac{\partial p_p y_p}{\partial k_p} = 1-\phi_p\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ according to equation (B.1). Now, $\phi_p\frac{r+\delta+\lambda}{1+\lambda\phi_p} < 1$ according to Lemma 1, so the derivative is positive. Hence, if $k_p(s,a,1) < k_p(s,a,0)$ then $\left[k_p(s,a,1) - \phi_p p_p(s,a,1)\,y_p(s,a,1)\right] < \left[k_p(s,a,0) - \phi_p p_p(s,a,0)\,y_p(s,a,0)\right]$ which requires $\phi_g p_g(s,a,1)\,y_g(s,a,1) < k_g(s,a,1)$.
■

Proposition 13 *When* $\phi_g \le \phi_p$*, a procurement shock for constrained firms generates a private sector negative spillover, that is,* $k_p(s,a,1) < k(s,a,0)$*, makes the firm more constrained, that is,* $\lambda(s,a,1) > \lambda(s,a,0)$*, and production in the government sector cannot be self-financed, that is,* $\phi_g p_g(s,a,1)\,y_g(s,a,1) < k_g(s,a,1)$*. When* $\phi_g > \phi_p$ *the same will happen, with the exception of firms with very small net worth for which the opposite will happen.*

Proof: To prove the first part, let's rewrite the borrowing constraint in (9) for $d = 0$ firms as

$$1 = \phi_a\frac{a}{k(s,a,0)} + \phi_p\frac{p_p(s,a,0)\,y_p(s,a,0)}{k(s,a,0)} \tag{B.34}$$

and for $d = 1$ firms as

$$
\begin{aligned}
1 &= \phi_a \frac{a}{k_p(s,a,1)+k_g(s,a,1)} + \phi_p \frac{p_p(s,a,1)\,y_p(s,a,1)}{k_p(s,a,1)} \\
&+ (1-u(s,a,1))\left[\phi_g \frac{p_g(s,a,1)\,y_g(s,a,1)}{k_g(s,a,1)} - \phi_p \frac{p_p(s,a,1)\,y_p(s,a,1)}{k_p(s,a,1)}\right] \quad \text{(B.35)}
\end{aligned}
$$

If $\phi_g = \phi_p$, firms with $d = 1$ equalize the average product in the public and private sectors, see equations (B.3) and (B.4), so that the third term in equation (B.35) disappears. In this case, if $k_g(s,a,1) = 0$ then equations (B.34) and (B.35) are identical and $k_p(s,a,1) = k(s,a,0)$. However, because the marginal revenue product in the public sector goes to infinity when $k_g(s,a,1) = 0$, it must be that $k_g(s,a,1) > 0$ and hence comparison of equations (B.34) and (B.35) requires $k_p(s,a,1) < k(s,a,0)$. If $\phi_g < \phi_p$, then the third term in equation (B.35) is negative. This can be easily seen by multiplying both sides of equation (B.3) by ϕ_p and both sides of equation (B.4) by ϕ_g. Then whenever $k_g > 0$ and hence $(1-u) > 0$, equation (B.35) requires $k_p(s,a,1) < k(s,a,0)$ to hold. The second and third parts of the Proposition come from Lemma 6 and Lemma 7 respectively. Finally, for the case $\phi_g > \phi_p$ the third term in equation (B.35) is positive. If $a = 0$ this requires $k_p(s,a,1) > k(s,a,0)$ for equation (B.35) to hold as the first term in the right hand side of equation (B.35) disappears. For $a > 0$, the first term in the right hand side of equation (B.35) reappears and offsets this force. More specifically, as a increases, λ falls by Proposition 10, and thus $\frac{p_g(s,a,1)y_g(s,a,1)}{k_g(s,a,1)}$ decreases. In the limit, if a becomes sufficiently large and exceeds the net worth level $a_g^*(s)$ above which the procurement firm is unconstrained, $\frac{p_g(s,a,1)y_g(s,a,1)}{k_g(s,a,1)}$ falls to the unconstrained level of $\frac{\sigma}{\sigma-1}(r+\delta)$, which is strictly smaller than $\frac{1}{\phi_g}$ by Assumption 1. This means that there exists a cutoff level $\bar{a}_g(s)$ such that if $a \in (\bar{a}_g(s), a_g^*(s))$, then $\phi_g p_g(s,a,1) y_g(s,a,1) < k_g(s,a,1)$. And by Lemma 7, this means that the spillover is negative for a in this interval. ■

Proposition 14 *Whenever $\phi_g \geq \phi_p > 0$ having access to procurement always generates an increase in firm size, that is, $k(s,a,1) > k(s,a,0)$ $\forall a, s$. Whenever $\phi_g < \phi_p$ the opposite may happen. In the particular case that $\phi_g = \phi_p = 0$ a procurement shock does not change the size of the firm.*

Proof: We prove the $\phi_g \geq \phi_p > 0$ case by contradiction by showing that if $k(s,a,1) \leq k(s,a,0)$, then the borrowing constraint for the firm with $d = 1$ would not bind, which could not be optimal; so it must be that $k(s,a,1) > k(s,a,0)$. To se why, we start with the case $k(s,a,1) = k(s,a,0)$. In this situation, the firm with $d = 1$ optimally chooses $u(s,a,1) < 1$ because the marginal revenue product of revenues in the public sector tend to infinity as u tends to 1. This generates more revenues and because $\phi_g \geq \phi_p > 0$, Lemma 4 guarantees that this also generates more (unused) borrowing capacity, so it cannot be optimal. If $k(s,a,1) < k(s,a,0)$ and $u(s,a,1) = 1$ this again generates slack in the borrowing constraint because of Lemma 3, and cannot be optimal. But lowering u generates the same or further slack when $\phi_g \geq \phi_p > 0$, see Lemma 4. So $k(s,a,1) < k(s,a,0)$ cannot be optimal either. Note that the argument by contradiction requires that $\phi_g \geq \phi_p > 0$ such that when the firm with $d = 1$ substitutes private revenues with public revenues the borrowing capacity increases. When $\phi_g < \phi_p$, instead, the contrary happens because selling to the government limits the borrowing capacity of the firm, and the proof does not hold. For example, it can be shown that with $0 = \phi_g < \phi_p$ we will have $k(s,a,1) < k(s,a,0)$. Using the financial constraint, the difference in the capital that can be financed with $d = 1$ and $d = 0$ when $\phi_g = 0$ is given by,

$$k(s,a,1) - k(s,a,0) = \phi_p \left[p_p(s,a,1)\, y_p(s,a,1) - p_p(s,a,0)\, y_p(s,a,0) \right]$$

Proposition 13 says that there is a negative private sector spillover, that is $p_p(s,a,1)\, y_p(s,a,1) < p_p(s,a,0)\, y_p(s,a,0)$, whenever $\phi_g < \phi_p$, so we will have $k(s,a,1) < k(s,a,0)$. Finally, note that with $\phi_g = \phi_p = 0$, $k(s,a,1) = k(s,a,0)$ as capital for constrained firms is determined only by a. ■

Proposition 15 *Having access to procurement always generates extra profits, that is,* $\pi(s,a,1) > \pi(s,a,0)\ \forall s,a$. *Whenever* $\phi_g \leq \phi_p$, *the value of procurement is increasing in net worth; whenever* $\phi_g > \phi_p$, *the value of procurement is generally increasing in net worth except for firms with very low net worth when the opposite will happen. The value of procurement is increasing in firm productivity whenever* $\phi_g \geq \phi_p$.

Proof: The first part is trivial. A firm with $d = 1$ has profits equal to

$$\pi(s,a,1) = p_p(s,a,1)\, y_p(s,a,1) + p_g(s,a,1)\, y_g(s,a,1) - (r+\delta)\, k(s,a,1)$$

and can always replicate the profits of a firm with $d = 0$ by choosing $u(s,a,1) = 1$. Because of our functional form assumptions, the marginal revenue product of capital in the public sector, $\partial p_g y_g / \partial k_g$, tends to infinity whenever $u(s,a,1) = 1$, so it means that it is optimal for any firm with $d = 1$ to choose $u(s,a,1) < 1$ and increase profits compared to the case $u(s,a,1) = 1$ and therefore compared to the case of no procurement. For the second part we want to show that $\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial a} > 0$. Equations (B.18) and (B.28) imply

$$\frac{\partial \left[\pi(s,a,1) - \pi(s,a,0)\right]}{\partial a} = \phi_a \Big[\lambda(s,a,1) - \lambda(s,a,0)\Big] > 0$$

and the sign of $\lambda(s,a,1) - \lambda(s,a,0)$ is given by Proposition 13. Finally, for the third part we want to show that $\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} > 0$ whenever $\phi_g \geq \phi_p$. Equations (B.19) and (B.29) imply

$$\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} = (1 + \phi_p \lambda(s,a,1)) \frac{\partial p_p y_p}{\partial s} + (1 + \phi_g \lambda(s,a,1)) \frac{\partial p_g y_g}{\partial s} - (1 + \phi_p \lambda(s,a,0)) \frac{\partial p_p}{\partial s}$$

Note that $\frac{\partial p_p y_p}{\partial s} = \frac{k_p}{s} \frac{\partial p_p y_p}{\partial k_p} = \frac{k_p}{s} \frac{r+\delta+\lambda}{1+\lambda\phi_p}$ and an analogous expression holds for the public good. Substituting these expressions in the above equation gives

$$\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} = \frac{r+\delta+\lambda(s,a,1)}{s} \left[k_p(s,a,1) + k_g(s,a,1)\right] - \frac{r+\delta+\lambda(s,a,0)}{s} k_p(s,a,0)$$

With $\phi_g \geq \phi_p$, Proposition 14 states that $k_p(s,a,1) + k_g(s,a,1) > k_p(s,a,0)$. Therefore, whenever $\lambda(s,a,1) > \lambda(s,a,0)$ we can guarantee that $\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} > 0$. According to Proposition 13 this will generally happen, except for very low a when $\lambda(s,a,1) < \lambda(s,a,0)$. However, in this case we can still show the statement to be true by showing that $\frac{r+\delta+\lambda(s,a,1)}{s} k_p(s,a,1) > \frac{r+\delta+\lambda(s,a,0)}{s} k_p(s,a,0)$. To show this, we take the FOC for k_p in equation (B.1) to obtain an expression for λ as:

$$\lambda = \frac{\frac{\partial p_p y_p}{\partial k_p} - (r+\delta)}{1 - \phi_p \frac{\partial p_p y_p}{\partial k_p}}$$

Then adding $(r+\delta)$ in both sides, rearranging, and multiplying by k_p in both sides we obtain

$$(r+\delta+\lambda)k_p = [1-\phi_p(r+\delta)]\frac{\frac{\partial p_p y_p}{\partial k_p}}{1-\phi_p\frac{\partial p_p y_p}{\partial k_p}}k_p$$

Using our functional form for the revenue function, we can rewrite the last terms as:

$$\frac{\frac{\partial p_p y_p}{\partial k_p}}{1-\phi_p\frac{\partial p_p y_p}{\partial k_p}}k_p = \frac{\sigma-1}{\sigma}\frac{B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{\sigma-1}{\sigma}}}{1-\phi_p\frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}}$$

Taking the derivative of this object w.r.t. k_p, we have:

$$\begin{aligned}
\frac{\partial}{\partial k_p}\left(\frac{\frac{\partial p_p y_p}{\partial k_p}}{1-\phi_p\frac{\partial p_p y_p}{\partial k_p}}k_p\right) &\propto \frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\left[1-\phi_p\frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\right]-B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{\sigma-1}{\sigma}}\frac{\sigma-1}{\sigma}\frac{1}{\sigma}\phi_p B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}-1} \\
&= \frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\left\{\left[1-\phi_p\frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\right]-\phi_p\frac{1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\right\} \\
&= \frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\left\{1-\phi_p B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\right\} \\
&= \frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\left\{1-\phi_p\frac{p_p y_p}{k_p}\right\} > 0
\end{aligned}$$

where the last inequality follows from the fact that by Lemma 3, $\phi_p\frac{p_p y_p}{k_p} < 1$ for constrained firms. This establishes that for constrained firms, the term $(r+\delta+\lambda)k_p$ must be higher whenever k_p is higher. Therefore, if $\lambda(s,a,1) < \lambda(s,a,0)$, the k_p FOC, implies that $k_p(s,a,1) > k_p(s,a,0)$, which in turns implies $[r+\delta+\lambda(s,a,1)]k_p(s,a,1) > [r+\delta+\lambda(s,a,0)]k_p(s,a,0)$. And trivially, this implies $\frac{r+\delta+\lambda(s,a,1)}{s}[k_p(s,a,1)+k_g(s,a,1)] - \frac{r+\delta+\lambda(s,a,0)}{s}k_p(s,a,0)$, proving the statement whenever $\phi_g \geq \phi_p$ and $\lambda(s,a,1) < \lambda(s,a,0)$.
■

C Details on some aggregates

C.1 Equilibrium definition

Let $\mathbf{X} \equiv S \times A \times \{0,1\}$ be the state space of the household problem, $\mathbf{X}_1 \equiv S \times A \times \{1\}$ the subset of the state space for firms with a procurement project, $\mathcal{X}$ a σ-algebra generated by $\mathbf{X}$, and Γ a probability measure over $\mathcal{X}$. Then, given government policy parameters Y_g and m_g and a distribution of entrants Γ_0, the steady state equilibrium requires:

a) Entrepreneurs solve their optimization problem

b) The probability measure Γ is stationary

c) The market for the private good clears:

$$\int_{\mathbf{X}} p_p(s,a,d)\,u(s,a,d)\,y(s,a,d)\,d\Gamma = Y_p = \int_{\mathbf{X}}[b(s,a,d)+c(s,a,d)+\delta k(s,a,d)]\,d\Gamma$$

d) The market for the public good clears:

$$\int_{\mathbf{X}_1} p_g(s,a,1)\,[1-u(s,a,1)]\,y(s,a,1)\,d\Gamma = P_g Y_g$$

e) The probability of obtaining procurement projects is consistent with the measure of goods bought by the public sector,

$$\int_{\mathbf{X}} Pr\left(d' = 1 \mid b\left(s,a,d\right)\right) d\Gamma = \int_{\mathbf{X}_1} d\Gamma = m_g$$

f) The budget constraint of the government holds

$$P_g Y_g = rD + \tau \int_{\mathbf{X}} \pi\left(s,a,d\right) d\Gamma + \left(1-\theta\right)\left[\int_{\mathbf{X}} a'(s,a,d) d\Gamma - \int_{\mathbf{X}} a d\Gamma_0\right]$$

g) By Walras law, the credit market clears.

$$D = \int_{\mathbf{X}} \left[k\left(s,a,d\right) - a\left(s,a,d\right)\right] d\Gamma$$

C.2 Sectorial and aggregate TFP

The TFP for the private and public sectors are given by,

$$\text{TFP}_p \equiv \frac{Y_p}{K_p} = \left[\int_{[0,1]} \left(s_i \frac{\overline{\text{MRPK}}_p}{\text{MRPK}_{ip}}\right)^{\sigma_p - 1} di\right]^{\frac{1}{\sigma_p-1}}, \quad \text{TFP}_g \equiv \frac{Y_g}{K_g} = \left[\int_{I_g} \frac{1}{m_g} \left(s_i \frac{\overline{\text{MRPK}}_g}{\text{MRPK}_{ig}}\right)^{\sigma_g - 1} di\right]^{\frac{1}{\sigma_g-1}} \quad \text{(C.1)}$$

where

$$\frac{1}{\overline{\text{MRPK}}_p} \equiv \int_{[0,1]} \frac{p_{ip} y_{ip}}{P_p Y_p} \frac{1}{\text{MRPK}_{ip}} di, \qquad \frac{1}{\overline{\text{MRPK}}_g} \equiv \int_{I_g} \frac{1}{m_g} \frac{p_{ig} y_{ig}}{P_g Y_g} \frac{1}{\text{MRPK}_{ig}} di \qquad \text{(C.2)}$$

Then aggregate TFP $= \left(Y_p + P_g Y_g\right) / \left(K_p + K_g\right)$ in units of the private sector good is given by the weighted average

$$\text{TFP} = \text{TFP}_p \frac{K_p}{K_p + K_g} + P_g \text{TFP}_g \frac{K_g}{K_p + K_g} \qquad \text{(C.3)}$$

Finally, absent financial frictions there would be no heterogeneity in $\overline{\text{MRPK}}_p$ and $\overline{\text{MRPK}}_g$ and optimal TFP in the private and public sectors (conditional on selection) would be,

$$\text{TFP}_p^* = \left[\int_{[0,1]} s_i^{\sigma_p - 1} di\right]^{\frac{1}{\sigma_p-1}} \quad \text{and} \quad \text{TFP}_g^* = \left[\int_{I_g} \frac{1}{m_g} s_i^{\sigma_g - 1} di\right]^{\frac{1}{\sigma_g-1}} \qquad \text{(C.4)}$$

C.3 Relative price of public sector good

Using the definitions of P_g and P_p in equations (5), the relative price can be written as,

$$\frac{P_g}{P_p} = \frac{\left[\int_{I_g} \frac{1}{m_g} p_{ig}^{1-\sigma_g} di\right]^{\frac{1}{1-\sigma_g}}}{\left[\int_{[0,1]} p_{ip}^{1-\sigma_p} di\right]^{\frac{1}{1-\sigma_p}}} = \frac{\left[\int_{I_g} \frac{1}{m_g} \left(\frac{1}{s_i}\text{MRPK}_{ig}\right)^{1-\sigma_g} di\right]^{\frac{1}{1-\sigma_g}}}{\left[\int_{[0,1]} \left(\frac{1}{s_i}\text{MRPK}_{ip}\right)^{1-\sigma_p} di\right]^{\frac{1}{1-\sigma_p}}}$$

where the last equality follows from using the definition of MRPK_{ip} and the production function as follows,

$$\text{MRPK}_{ip} \equiv \frac{\partial p_{ip} y_{ip}}{\partial k_{ip}} = \frac{\sigma_p - 1}{\sigma_p} \frac{p_{ip} y_{ip}}{k_{ip}} = \frac{\sigma_p - 1}{\sigma_p} p_{ip} s_i \quad \Rightarrow \quad p_{ip} = \frac{\sigma_p}{\sigma_p - 1} \frac{1}{s_i} \text{MRPK}_{ip}$$

and the same applies for MRPK_{ig}. Next multiplying and dividing by $\overline{\text{MRPK}}_g$ in the numerator and by $\overline{\text{MRPK}}_p$ in the denominator we obtain,

$$\frac{P_g}{P_p} = \frac{\overline{\text{MRPK}}_g}{\overline{\text{MRPK}}_p} \frac{\left[\int_{I_g} \frac{1}{m_g} \left(\frac{1}{s_i} \frac{\overline{\text{MRPK}}_g}{\text{MRPK}_{ig}}\right)^{1-\sigma_g} di\right]^{\frac{1}{\sigma_g - 1}}}{\left[\int_{[0,1]} \left(\frac{1}{s_i} \frac{\overline{\text{MRPK}}_p}{\text{MRPK}_{ip}}\right)^{\sigma_p - 1} di\right]^{\frac{1}{1-\sigma_p}}} = \frac{\overline{\text{MRPK}}_p}{\overline{\text{MRPK}}_g} \frac{\text{TFP}_p}{\text{TFP}_g}$$

C.4 Relative sectoral TFP

Given the definition of TFP_p in equation (C.1), we can write

$$\begin{aligned} \text{TFP}_p &= \left[m_g \int_{I_g} \frac{1}{m_g} \left(s_i \frac{\overline{\text{MRPK}}_p}{\text{MRPK}_{ip}}\right)^{\sigma_p - 1} di + (1 - m_g) \int_{I_g^c} \frac{1}{1 - m_g} \left(s_i \frac{\overline{\text{MRPK}}_p}{\text{MRPK}_{ip}}\right)^{\sigma_p - 1} di\right]^{\frac{1}{\sigma_p - 1}} \\ &= \left[m_g \text{TFP}_{p,I_g}^{\sigma_p - 1} + (1 - m_g)\, \text{TFP}_{p,I_g^c}^{\sigma_p - 1}\right]^{\frac{1}{\sigma_p - 1}} \end{aligned}$$

where we have defined TFP_{p,I_g} and TFP_{p,I_g^c} as the average TFP in the private sector within the set of procurement (I_g) and non-procurement (I_g^c) firms respectively. Then, dividing by TFP_g in both sides we get the expression for $\text{TFP}_p/\text{TFP}_g$:

$$\frac{\text{TFP}_p}{\text{TFP}_g} = \left[m_g \left(\frac{\text{TFP}_{p,I_g}}{\text{TFP}_g}\right)^{\sigma_p - 1} + (1 - m_g) \left(\frac{\text{TFP}_{p,I_g^c}}{\text{TFP}_g}\right)^{\sigma_p - 1}\right]^{\frac{1}{\sigma_p - 1}} \tag{C.5}$$

The first term in equation (C.5) reflects the within-firm misallocation. With $\sigma_g = \sigma_p$ this term would be equal to 1 if $\phi_g = \phi_p$ or if there were no financial frictions ($\lambda_i = 0\ \forall i$). Instead, if $\phi_g > \phi_p$ firms switch their output relatively towards the public sector and the dispersion of MRPK_{ig} declines, which makes $\text{TFP}_{p,I_g}/\text{TFP}_g$ fall. The second term in equation (C.5) reflects both between-firm misallocation and selection into procurement. If firms with higher s self-select into procurement, then $\text{TFP}_{p,I_g^c}/\text{TFP}_g$ declines. If there is more dispersion in MRPK_{ip} between non-procurement firms than in MRPK_{ig} between procurement firms, then $\text{TFP}_{p,I_g^c}/\text{TFP}_g$ is lower. In short, absent financial frictions the only reason for $\text{TFP}_p/\text{TFP}_g \neq 1$ would be the selection of firms into procurement. In the first best (no financial frictions and the government selects the firms with highest s) we would have $\text{TFP}_p/\text{TFP}_g < 1$.

C.5 Relative sectoral $\overline{\text{MRPK}}$

Given the definition of $\overline{\text{MRPK}}_p$ in equation (C.2), we can write

$$\begin{aligned} \overline{\text{MRPK}}_p &= \left[\frac{R_{p,I_g}}{P_p Y_p} \int_{I_g} \frac{p_{ip} y_{ip}}{R_{p,I_g}} \text{MRPK}_{ip}^{-1} di + \frac{R_{p,I_g^c}}{P_p Y_p} \int_{I_g^c} \frac{p_{ip} y_{ip}}{R_{p,I_g^c}} \text{MRPK}_{ip}^{-1} di\right]^{-1} \\ &= \left[\frac{R_{p,I_g}}{P_p Y_p} \overline{\text{MRPK}}_{p,I_g}^{-1} + \frac{R_{p,I_g^c}}{P_p Y_p} \overline{\text{MRPK}}_{p,I_g^c}^{-1}\right]^{-1} \end{aligned}$$

where R_{p,I_g} and R_{p,I_g^c} denote total revenues in the private sector by procurement firms and non-procurement firms respectively. Then, dividing by $\overline{\text{MRPK}}_g$ in both sides we obtain the expression for $\overline{\text{MRPK}}_p/\overline{\text{MRPK}}_g$

$$\frac{\overline{\text{MRPK}}_p}{\overline{\text{MRPK}}_g} = \left[\frac{R_{p,I_g}}{P_p Y_p} \left(\frac{\overline{\text{MRPK}}_{p,I_g}}{\overline{\text{MRPK}}_g} \right)^{-1} + \frac{R_{p,I_g^c}}{P_p Y_p} \left(\frac{\overline{\text{MRPK}}_{p,I_g^c}}{\overline{\text{MRPK}}_g} \right)^{-1} \right]^{-1} \tag{C.6}$$

Whenever $\overline{\text{MRPK}}_p \neq \overline{\text{MRPK}}_g$ there is misallocation of capital across sectors. The first term in equation (C.6) reflects the effects of within-firm misallocation on this between-sector misallocation. With $\sigma_g = \sigma_p$ this term would be equal to 1 if $\phi_g = \phi_p$ or if there were no financial frictions ($\lambda_i = 0 \forall i$). Instead, if $\phi_g > \phi_p$ firms switch their output relatively towards the public sector and hence $\overline{\text{MRPK}}_{p,I_g} > \overline{\text{MRPK}}_g$. The second term in equation (C.6) reflects both between-firm misallocation and selection into procurement.

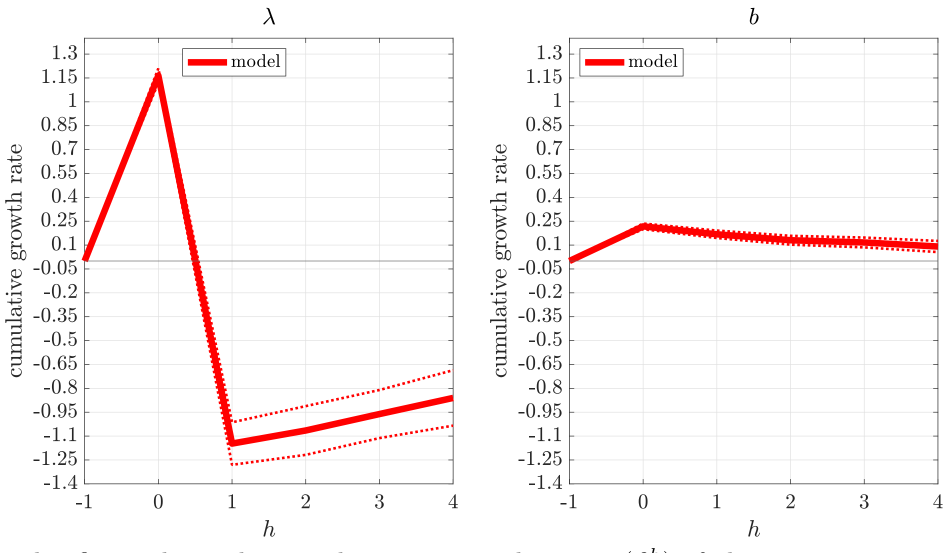
D Details on treatment effects in the benchmark calibration

To analyze dynamic treatment effects of procurement in our model and in the data, we estimate local projection panel regressions (Jordà, 2005). In particular, we regress the cumulative difference of a variable x, $\Delta_h \log(x_{i,t+h}) \equiv \log(x_{i,t+h}) - \log(x_{i,t-1})$ on the regressor PROC_{it}, the firm's lagged credit at $t-1$, firm fixed effects, and sector×year fixed effects:

$$\Delta_h \log(x_{i,t+h}) = \alpha_{ih} + \alpha_{sth} + \beta_1^h \text{PROC}_{it} + \beta_2^h \log\ x_{it-1} + \varepsilon_{ith+h} \tag{D.1}$$

where $x_{i,t+h}$ is either sales to the private sector, λ, or b, measured for firm i at time $t+h$. Therefore, $h = 0, 1, ..., H$ denotes the horizon at which the impact of procurement is estimated. We show the results from running these regressions at annual frequency.

Figure A.III. Treatment effects of procurement in the baseline calibration



Notes: This figure shows the cumulative estimated impact (β_1^h) of obtaining a procurement contract for different time horizons $h = 0, 1, 2, 3, 4$. The left panel shows the effects on λ. The right panel shows the effects on b.

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